Experiment 4 - Transient Response of an LC Circuit

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Objective

To study and analyze the transient response of an LC circuit, determine the natural frequency (ω_n), and calculate the damping ratio (ξ) using theoretical and experimental methods.

Required Materials

1. 1 nF capacitor
2. Largest available inductor in the lab (denoted as L)
3. Resistor (small value for practical considerations)
4. DC power supply
5. Oscilloscope

Procedure

1. Connect the circuit as shown in the schematic diagram.
2. Charge the capacitor using a DC power supply and then disconnect it to observe natural oscillations.
3. Use the oscilloscope to capture the transient response of the circuit.
4. Measure the oscillation frequency and compare it with theoretical values.

5. Introduce resistance and note the effect on damping.
6. Calculate decay rate and damping ratio from experimental observations.

Schematic Diagram

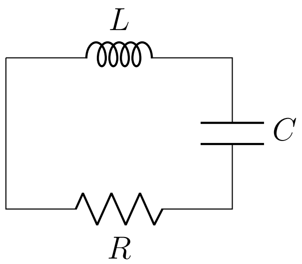

Figure 1: Circuit diagram of the LC circuit experiment

Theory

An LC circuit consists of an inductor (L) and a capacitor (C) connected in parallel. When a charged capacitor is connected to an inductor, energy oscillates between the capacitor's electric field and the inductor's magnetic field. The system follows a second-order differential equation, which you studied in the first semester of the Circuits course.

For a series RLC circuit with an initially charged capacitor, the governing differential equation is:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0 \tag{1}$$

The characteristic equation is:

$$Ls^2 + Rs + \frac{1}{C} = 0 \tag{2}$$

Solving for s:

$$s = \frac{-R \pm \sqrt{R^2 - 4\frac{L}{C}}}{2L} \tag{3}$$

The damped natural frequency is given by:

$$\omega_d = \omega_n\sqrt{1-\xi^2} \tag{4}$$

where:

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \tag{5}$$

For $L = 2.2$ mH, $C = 1$ nF, and $R = 25\Omega$:

$$\omega_n = \frac{1}{\sqrt{(2.2\times 10^{-3})(1\times 10^{-9})}} \approx 6.77\times 10^5 \text{ rad/s} \tag{6}$$

$$f_n = \frac{\omega_n}{2\pi} \approx 107.8 \text{ kHz} \tag{7}$$

$$\xi = \frac{25}{2}\sqrt{\frac{1\times 10^{-9}}{2.2\times 10^{-3}}} \approx 0.167 \tag{8}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} \approx 6.73\times 10^5 \text{ rad/s} \tag{9}$$

The time period of damped oscillations is:

$$T_d = \frac{2\pi}{\omega_d} \approx 9.33\mu s \tag{10}$$

The voltage response is given by:

$$V(t) = V_0 e^{-\xi\omega_n t}\cos(\omega_d t + \phi) \tag{11}$$

Observations & Analysis

- Record the waveform from the oscilloscope.
- Measure the oscillation period and compare it with the predicted value.
- If resistance is present, measure the decay rate and determine the damping ratio.

Decay Rate and Damping Ratio:
The decay rate α is given by:

$$\alpha = \xi\omega_n = (0.167)(6.77\times 10^5) \approx 1.13\times 10^5 \text{ rad/s} \tag{12}$$

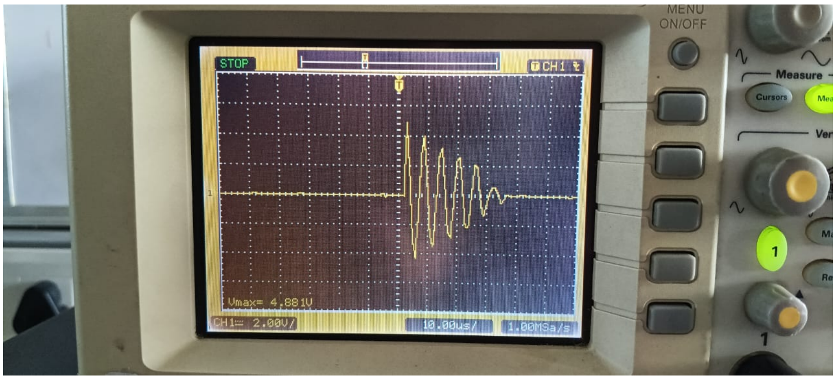

Figure 2: Oscilloscope capture of transient response

The experimental decay rate can be estimated from the oscilloscope image. Comparing theoretical and experimental values:

$$\alpha_{exp} \approx 1.1 \times 10^5 \text{ rad/s} \tag{13}$$

The theoretical damping ratio ξ was found to be **0.167**, while the experimentally observed value from the oscillation decay envelope is:

$$\xi_{exp} \approx 0.16 \tag{14}$$

Both values are in close agreement, validating our theoretical model.