

# Assignment1

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EE24BTECH11007

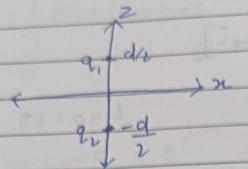
March 7, 2025

## Assignment 1

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1)

2)



$$q_2 = q_1 = q_2 = q$$

$$E_z = E_{q_1} + E_{q_2}$$

$$E_z = \frac{kq}{(z-\frac{d}{2})^2} \hat{i} + \frac{kq}{(z+\frac{d}{2})^2} \hat{i} \quad k = \frac{1}{4\pi\epsilon_0}$$

$$E_z = \begin{cases} \frac{2kqz}{(z^2 - \frac{d^2}{4})^2} \hat{i} & z < \frac{d}{2} \\ \frac{kq(z - \frac{d}{2})}{(z^2 - \frac{d^2}{4})^2} \hat{i} & z > \frac{d}{2} \end{cases}$$

$$b) E_x = E_{q_1} + E_{q_2}$$

$$E_x = \frac{2kqx}{(x^2 + \frac{d^2}{4})^{3/2}} \hat{x} + 0 \hat{i}$$

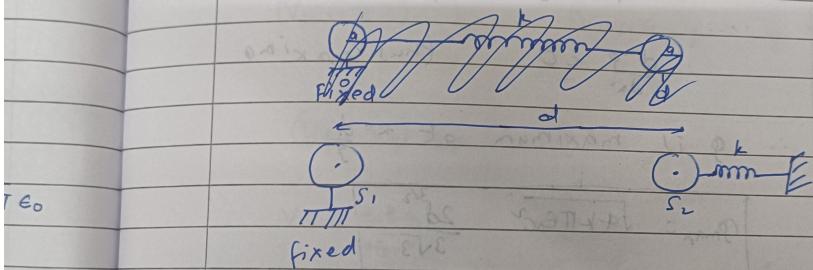
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c)  $E_z = \frac{-kq}{(z^2 + \frac{d^2}{4})^{3/2}} \hat{z}$

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$$E_x = 0 \quad 0 \hat{x} - \frac{kq}{\left(z^2 + \frac{d^2}{4}\right)^{3/2}}$$

2)



as  $q \ll d$  we can assume field due to  $S_1$  at  $S_2$  to be parallel.

$$E_{S_2 S_1} q_{S_2} = -kx$$

$$\frac{1}{4\pi\epsilon_0(d-x)} = -kx$$

$$q^L = k \cdot 4\pi\epsilon_0 C^2 ((d-x)^2)$$

$$Q = \sqrt{k \cdot 4\pi\epsilon_0 C^2 (d-x) \sqrt{x}}$$

$$dq = k \sqrt{(d-x) + x} \cdot \frac{C}{\sqrt{x}}$$

$\frac{d\varphi}{dx} = \frac{k_0(d-3x)}{2\sqrt{x}}$   
 $\frac{d\varphi}{dx} = 0 \text{ at } x = \frac{d}{3}$   
 $\frac{d^2\varphi}{dx^2} = k_0 \left( \frac{-3}{2\sqrt{x}} - \frac{(d-3x)}{4x^{3/2}} \right)$   
 putting  $x = \frac{d}{3}$ ,  
 $\therefore \frac{d^2\varphi}{dx^2} < 0$  thus maxima  
 $\therefore \varphi$  is maximum at  $x = \frac{d}{3}$   
 $Q_{\max} = \sqrt{4k\pi\epsilon_0 c^2} \frac{2d^{3/2}}{3\sqrt{3}}$

Now  
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But why are we getting this  $Q_{\max}$  at  $d=3$  if a higher charge is applied then the charge  $S_2$  will cross the  $d$  mark and the electrostatic force will dominate the spring force and the spheres will neutralise the charges.

Assumption made that the sphere 2 is moved very slowly to its eqm position or else oscillation may happen and the spheres might meet.

Now lets forget this assumption

Charger are dropped instantaneously and the sphere 2 gains kinetic energy in middle and the stops at extremum.  $x = x_0$   
momentarily and oscillates back

Initial potential = Final potential  
Energy

$$(V_{ss1} q_{ss1})_i = (V_{ss1} q_{ss1})_f + \frac{1}{2} k x_0^2$$

$$(V_i - V_f) q_{ss1} = \frac{1}{2} k x_0^2$$

$$V = \frac{q_1 k q_2}{4\pi \epsilon_0 r}$$

$$\therefore \frac{q_2}{C + 4\pi \epsilon_0} \left( \frac{1}{x_0} \right) = \frac{1}{2} k x_0^2$$

will

$$\Phi = \frac{1}{2} \frac{k x_0^3}{C + 4\pi \epsilon_0}$$

meet

$$\Phi = \sqrt{\frac{k C^2 + 4\pi \epsilon_0 C d}{2}} \sqrt{\frac{x_0^3}{d - x_0}}$$

$$\frac{d\Phi}{dx} = \therefore \Phi = k_0 \frac{x_0^2}{\sqrt{d-x}}$$

$$\frac{d\Phi}{dx} = k_0 \left( \frac{3 \cdot x_0}{2 \sqrt{d-x}} + \frac{x_0^2}{(d-x)^{3/2}} \right) = 0$$

$$3 \sqrt{\frac{x_0}{d-x}} = \left( \frac{x_0}{d-x} \right)^{1/2} \Rightarrow \frac{4x}{d-x} = \frac{x_0^2}{(d-x)^2} \Rightarrow 3d - 3x = x_0$$

$$q_{\max} = \frac{1}{2} \sqrt{\frac{r^3}{d^2}}$$

$$\frac{1}{2} \sqrt{\frac{27d^3 \times 3}{64(2d)}}$$

$$q_{\max} = \sqrt{1c^4 \pi \epsilon_0 d} \left( \frac{9d}{16} \right)$$

again if more than this charge is applied spheres meet.

PS4

1)

$$3) F_d L = q_0 (3V - qV)$$

$$F_d = 5 \text{ N}$$

$$r = a \quad 0 < \theta < \frac{\pi}{2} \quad 0 < \phi < 2\pi$$

$$\int F \cdot \hat{n} ds$$

2)

$$3) \quad F_i = 5a_2$$

$$\sqrt{x^2 + y^2 + z^2} = a \quad 0 < \theta < \frac{\pi}{2} \quad 0 < \phi < 2\pi$$

$$\text{Flux} = \iint_S \vec{F}_i \cdot \hat{n} dS$$

$$d\hat{n} \cdot n$$

$$\hat{n} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{a}$$

$$\text{Flux} = \iint_S \vec{F}(5\hat{k}) \cdot \frac{x\hat{i} + y\hat{j} + z\hat{k}}{a} dS$$

$$= \iint_S 5 \frac{z}{a} dS$$

$$\text{Let } z = a \cos \theta$$

$$dS = a \sin \theta d\theta d\phi$$

$$\text{Flux} = 5a^2 \int_{0}^{\pi} \int_{0}^{\pi} \frac{a \cos \theta}{a} \sin \theta \cos \theta d\theta d\phi$$

$$\text{Flux} = \frac{5a^2}{2} \int_{0}^{\pi} d\phi$$

$$\text{Flux} = 5\pi a^2$$

&lt;27

as there is symmetry along z axis  
we can use cylindrical coordinate

Now trying symmetrical

$$x = \rho \sin\theta \cos\phi$$

$$y = \rho \sin\theta \sin\phi$$

$$z = \rho \cos\theta$$

$$\hat{n} dS = \hat{r} a^2 \sin\theta d\theta d\phi$$

$$F_1 = 5 \cos\theta \hat{r} - 5 \sin\theta \hat{\theta}$$

$$\text{Flux} = \iint F_1 \cdot \hat{n} dS$$

$$= \iint (5 \cos\theta \hat{r} - 5 \sin\theta \hat{\theta}) (\hat{r} a^2 \sin\theta d\theta d\phi)$$

$$= \iint 5 \cos\theta \sin\theta a^2 d\theta d\phi$$

$$= 5\pi a^2$$

iii)

$$E_2 = 5z \hat{k}$$

$$\hat{n} = \frac{x \hat{i} + y \hat{j} + z \hat{k}}{\sqrt{a^2}}$$

$$\text{Flux} = \iint \frac{5z^2}{a} dS$$

$$= 5a^3 \iint_{0}^{\pi/2} \cos^2\theta \sin^2\theta d\theta d\phi$$

$$= \frac{70a^3 \pi}{3}$$

$$\text{FLUX} = \iiint \nabla E_n \cdot dV$$

$$= \iiint_{a^2 \times \pi h} 5 d\theta d\phi dr$$

$$= \frac{9}{3} 10 a^3 \pi$$

4)

5) If

$s_r = ar^2$  cylindrical  
let a gaussian surface inside with same axis.  
Using Gauss law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E(2\pi r L) = \frac{1}{\epsilon_0} \int_0^r (2\pi r) ar^2 dr$$

$$E = \frac{1}{2\epsilon_0} \frac{1}{4} \frac{r^3 a}{\epsilon_0}$$

Outside cylinder

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int_0^b 2\pi r l ar dr$$

$$E(\sqrt{\pi}rlb) = \frac{1}{4\epsilon_0} 2\pi b^4$$

$$E = \frac{b^4 q}{4\pi r \epsilon_0}$$

$$\therefore E = \begin{cases} \frac{r^3}{q\epsilon_0} & r \leq b \\ \frac{b^4 q}{4\pi r \epsilon_0} & r > b \end{cases}$$

5)

If  $\mathbf{a}$ 

$$\vec{F} = \vec{a} s + b \hat{\phi} + c \hat{z}$$

$$\text{then } |\vec{F}| = \sqrt{a^2 + c^2}$$

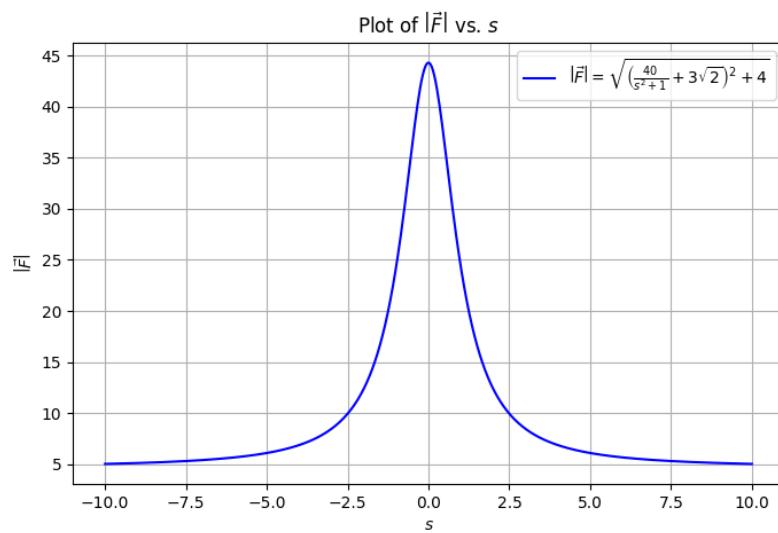
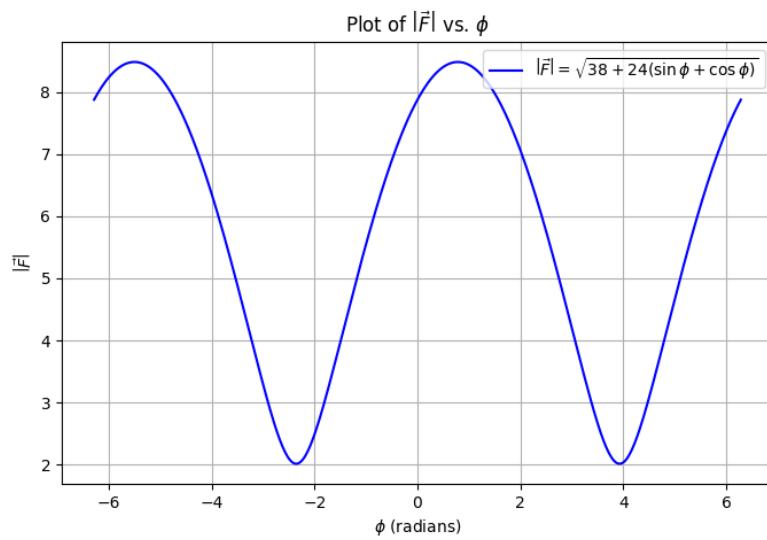
CS(lind) a)  $\vec{F} = \underbrace{(4 + 3\cos\phi + 3\sin\phi)\hat{s}} + 3(\cos\phi - \sin\phi)\hat{\phi} - 2\hat{z}$

$$|\vec{F}| = \sqrt{(4 + 3\cos\phi + 3\sin\phi)^2 + 4}$$

$$= \sqrt{38 + 24(\sin\phi + \cos\phi)}$$

b)  $|\vec{F}| = \left( \frac{40}{s^2+1} + 3\sqrt{1} \right) \hat{s} - 2 \hat{z}$

$$|\vec{F}| = \sqrt{\left( \frac{40}{s^2+1} + 3\sqrt{1} \right)^2 + 4}$$



$$c) \nabla \cdot \vec{F} = \frac{1}{s} \frac{\partial}{\partial s} (s F_s) + \frac{1}{s} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

6)

$$= \frac{1}{s} \frac{\partial}{\partial s} \left( \frac{40}{s^2+1} + 3s(\cos\phi + \sin\phi) \right) + \frac{1}{s} \frac{\partial}{\partial \phi} (3(\cos\phi - \sin\phi))$$

$\alpha = 2$

$$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} + \frac{3\cos\phi + 3\sin\phi}{s} + \frac{-3s\cos\phi - 3s\sin\phi}{s}$$

$$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} - 2$$

$$d) \nabla \times \vec{F} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_s & F_\phi & F_z \end{vmatrix}$$

$$= 0\hat{s} + 0\hat{\phi} + -2(-\sin\phi + \cos\phi)\hat{z}$$

$\therefore$  Not conservative

$$= -2 \frac{(-\sin\phi + \cos\phi)}{s}$$

$\therefore$  Not Conservative

6)

c) Considering all interactions

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• Energy

Let charges be

1 C at  $(0, 1) q_1$ -1 C  $(1, 0) q_2$ +1 C  $(0, -1) q_3$ -1 C  $(-1, 0) q_4$ 

$$\text{Energy} = \sum \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

$$= 4 \left( \frac{-1}{4\pi\epsilon_0 (\sqrt{2})} \right) + 2 \left( \frac{1}{4\pi\epsilon_0 2} \right)$$

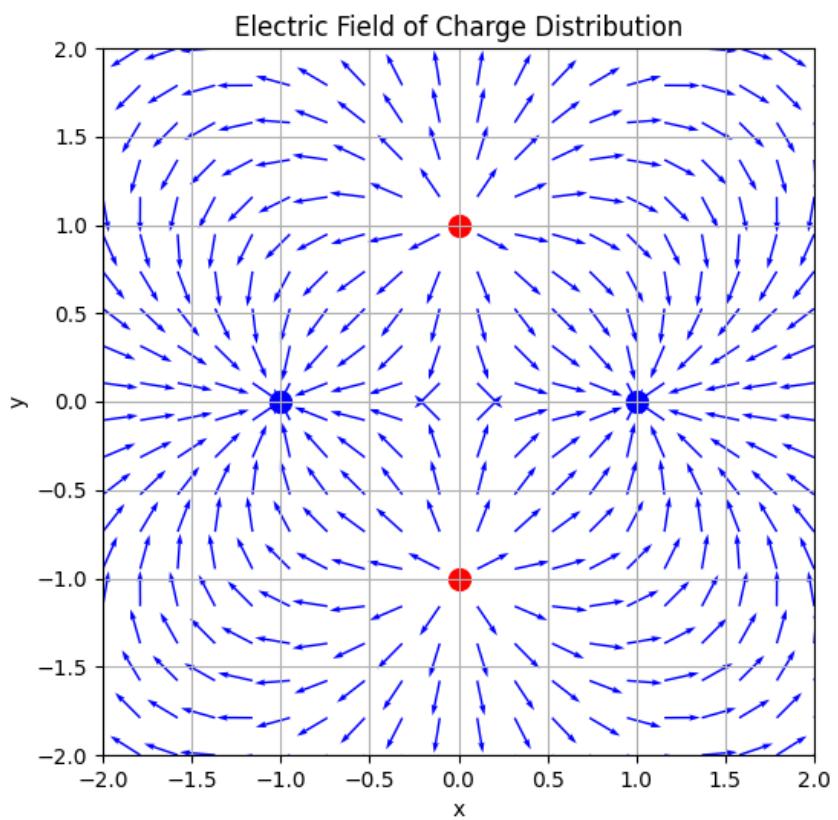
 $(q_i, q_{i+1})$  $(q_i, q_{i+2})$ 

$$= \frac{-1 - 2\sqrt{2}}{4\pi\epsilon_0}$$

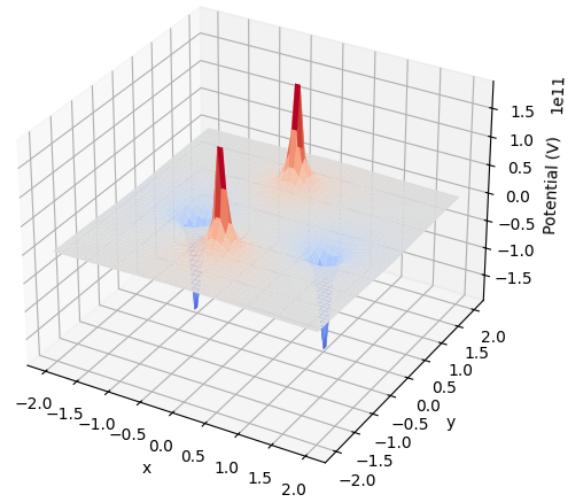
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$$V_b = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2} + \frac{1}{4\pi\epsilon_0} \frac{q_3}{r_3} + \frac{1}{4\pi\epsilon_0} \frac{q_4}{r_4}$$



Electric Potential Distribution



Page No. : 12  
Date: YOUVA

a)

$$\vec{E} = -\nabla V$$

$$\vec{E} = \frac{V_0 e^{-r/a}}{a} \hat{r}$$

$$\nabla \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \frac{V_0 r^2 e^{-r/a}}{a} \right)$$

$$\nabla \cdot \vec{E} = \frac{V_0}{r^2 a} \left( 2r e^{-r/a} - \frac{r^2 e^{-r/a}}{a} \right)$$

$$-\nabla^2 V = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{V_0 \epsilon}{r^2 a} \left( 2r e^{-r/a} - \frac{r^2 e^{-r/a}}{a} \right)$$

put  $r=a$

$\rho = \frac{V_0 \epsilon_0 e^{-1}}{a^2}$

b)  $\vec{E} = \frac{V_0 e^{-r/a}}{a} \hat{r}$

$$\vec{E}' = \frac{V_0 e^{-1}}{a}$$

c)

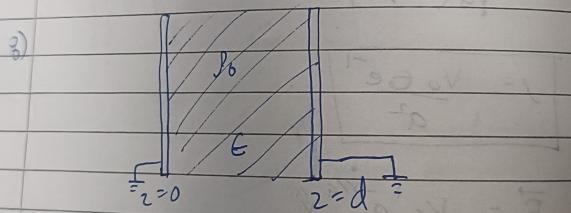
$$\int \int \int \rho dV$$

$$= 2\pi \int_0^a \frac{V_0 \epsilon_0}{r^2} \left( 2r e^{-r/a} - \frac{r^2 e^{-r/a}}{a} \right) dr$$

$$= 2\pi^2 V_0 \epsilon_0 \int_0^\infty e^{-r/a} \left( \frac{2}{r} - \frac{1}{a} \right) dr$$

b)  $\frac{\vec{E}}{E} = -$

c)  $V =$



a)

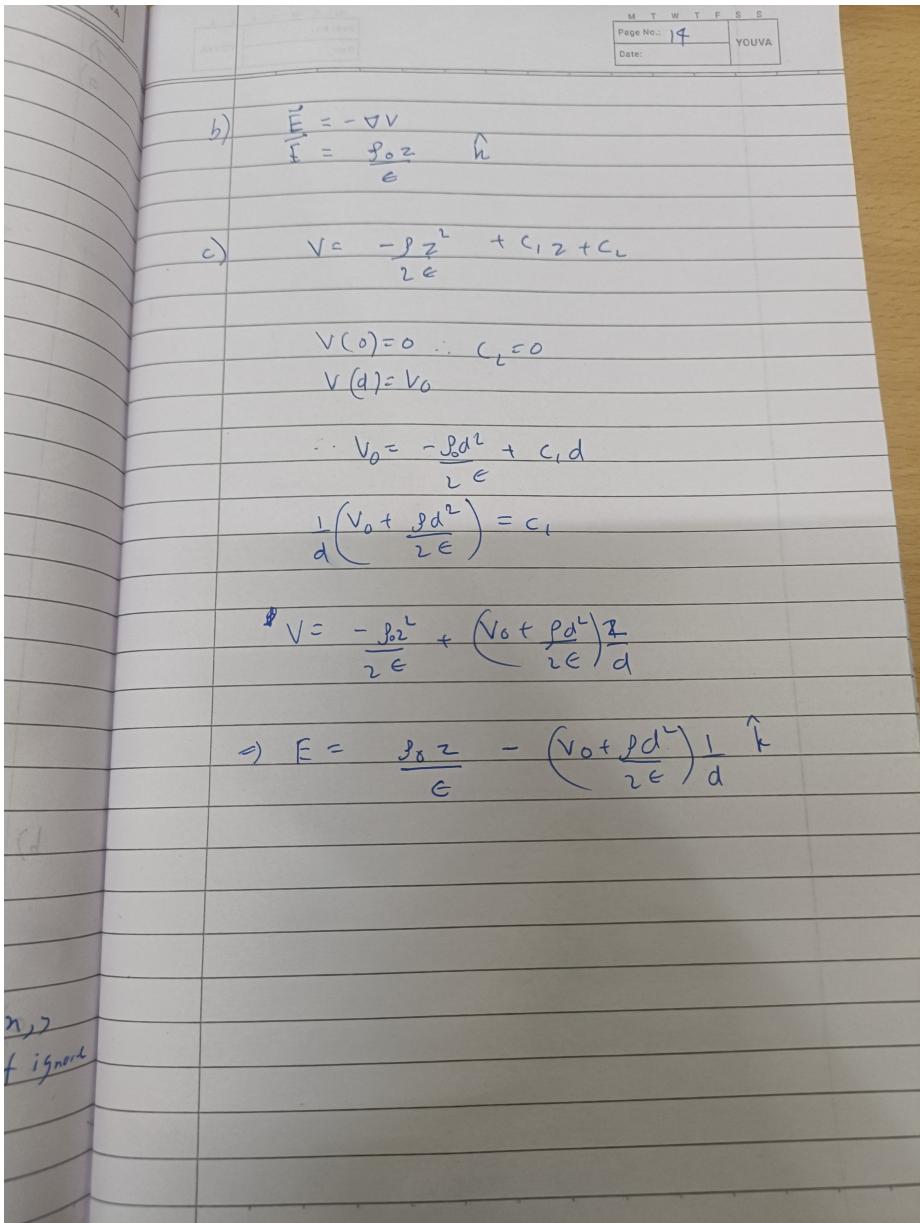
$$-\nabla^2 V = \frac{\rho_0}{\epsilon_0}$$

$$V = -\frac{\rho_0 z}{2\epsilon_0} + c_1 z + c_2$$

$$V(0) = 0 \Rightarrow c_2 = 0$$

$$V(d) = 0 \Rightarrow c_1 = 0$$

$$\therefore V(z) = -\frac{\rho_0 z}{2\epsilon_0}$$



For codes refer to  
<https://github.com/ArnavYadnopavit/Electromagnetics/tree/main/Assignment1>