Assignment1

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EE24BTECH11007

March 7, 2025

Assignment 1

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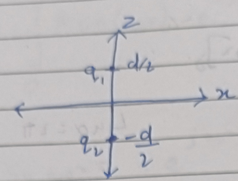
c)

1)

a)



2)

a) $q_1 = q_2 = q$

$E_z = E_{q_1} + E_{q_2}$

$$E_z = \frac{kq}{\left(z-\frac{d}{2}\right)^2}\hat{k} + \frac{kq}{\left(z+\frac{d}{2}\right)^2}\hat{k} \qquad k = \frac{1}{4\pi\epsilon_0}$$

$$E_z = \begin{cases} \dfrac{2kq\,z}{\left(z^2-\frac{d^2}{4}\right)^2}\hat{k} & z < \frac{d}{2} \\[2ex] \dfrac{kq\left(z^2+\frac{d^2}{4}\right)}{\left(z^2-\frac{d^2}{4}\right)^2} & z > \frac{d}{2} \end{cases}$$

b) $E_x = E_{q_1} + E_{q_2}$

$$E_x = \frac{2kqx}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}\hat{i} + 0\hat{k}$$

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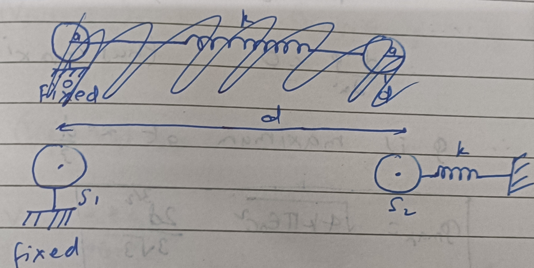
c) $E_z = \frac{-2kqd}{\left(z^2 - \frac{d^2}{4}\right)^2} \hat{k}$

$E_x = 0\hat{i} - \frac{kqd}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$

2)



as $a << d$ we can assume field due to S_1 at S_2 to be parallel.

$\therefore \quad E_{S_2S_1} q_{S_2} = -kx$

$\frac{1 \cdot q q}{4\pi\epsilon_0 (d-x)^2} = kx$

$q^2 = k\, 4\pi\epsilon_0 C^2 \left((d-x)^2 x\right)$

$q = \sqrt{k\, 4\pi\epsilon_0 C^2}\ (d-x)\sqrt{x}$

$dq = k\left((d-x) + -\sqrt{x}\right) \therefore$ monotonically

$$\frac{d\phi}{dx} = \frac{k_0 (d - 3x)}{2\sqrt{x}}$$

$$\frac{d\phi}{dx} = 0 \text{ at } x = \frac{d}{3}$$

$$\frac{d^2\phi}{dx^2} = k_0\left(\frac{-3}{2\sqrt{x}} - \frac{(d-3x)}{4x^{3/2}}\right)$$

putting $x = \frac{d}{3}$

$\therefore \frac{d^2\phi}{dx^2} < 0$ thus maxima

$\therefore$ ϕ is maximum at $x = \frac{d}{3}$

$$\boxed{\phi_{max} = \sqrt{4k\pi\epsilon_0 c^2} \quad \frac{2d^{3/2}}{3\sqrt{3}}}$$

But why are we getting this ϕ_{max} at $d=3$ if a higher charge is applied then ~~the charge~~ S_2 will cross the $\frac{d}{3}$ mark and the electrostatic force will dominate the spring force and the spheres will meet neutralising the charges.

Assumption made that the sphere 2 is moved very slowly to its eqm position or else oscillation may happen and again the spheres might meet.

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Now lets forget this assumption

Charges are dropped instantly and the sphere 2 gains kinetic energy in middle and the stops at extremum $x = x_0$ momentarily and oscillates back

Initial potential energy = Final potential Energy

$$(V_{s_2 s_1} q_{s_2})_i = (V_{s_2 s_1} q_{s_2})_f + \frac{1}{2} k x_0^2$$

$$(V_i - V_f) q_{s_2} = \frac{1}{2} k x^2$$

$$V = \frac{q_1}{4\pi\epsilon_0 r}$$

$$\therefore \frac{q^2}{C^2 4\pi\epsilon_0} \left(\frac{1}{x_0} - \frac{1}{d}\right) = \frac{1}{2} k x^2$$

$$q^2 (d-x) = \frac{1}{2} \frac{k x^3 d C^2 4\pi\epsilon_0}{(d-x)}$$

$$q = \sqrt{\frac{k C^2 4\pi\epsilon_0 d}{2}} \sqrt{\frac{x_0^3}{d-x_0}}$$

$$\therefore q = k_0 \frac{x^{3/2}}{\sqrt{d-x}}$$

$$\frac{dq}{dx} = k_0 \left(\frac{3}{2} \frac{x_0^{1/2}}{\sqrt{d-x_0}} + \frac{1}{2} \frac{x_0^{3/2}}{(d-x_0)^{3/2}} \right) = 0$$

$$3\sqrt{\frac{x_0}{d-x_0}} = \left(\frac{x_0}{d-x_0}\right)^{3/2} \Rightarrow \frac{3x}{d-x} = \frac{x_0^2}{(d-x)_0^2} \Rightarrow 3d - 3x = x_0$$

$$x_0 = \frac{3d}{4}$$

$$q_{max} = k_0 \sqrt{\frac{x^3}{d-x}}$$

$$k_0 \sqrt{\frac{27d^3 \times 3}{64(2d)}}$$

$$q_{max} = \sqrt{kc^2 4\pi\epsilon_0 d}\left(\frac{9d}{16}\right)$$

again if more than this charge is applied spheres meet.

3)

$$F_1 = 5\hat{k}$$

$$r = a \qquad 0 < \theta < \frac{\pi}{2} \qquad 0 < \phi < 2\pi$$

$$\int F \cdot \hat{n}\, ds$$

PS 4

1)

2)

3) $F_1 = 5 a_z$

$x^2+y^2+z^2 = a^2 \quad 0 < \theta < \frac{\pi}{2} \quad 0 < \phi < 2\pi$

$$\text{Flux} = \iint_S \vec{F_1} \cdot \hat{n}\, ds$$

$ds\hat{n} \quad n$

$$\hat{n} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{a}$$

$$\text{Flux} = \iint (5\hat{k}) \cdot \frac{x\hat{i}+y\hat{j}+z\hat{k}}{a}\, ds$$

$$= \iint 5\frac{z}{a}\, ds$$

let $z = a\cos\theta$

$ds = a^2 \sin\theta\, d\theta\, d\phi$

$$\text{Flux} = 5a^2 \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta\, d\phi$$

$$\text{Flux} = \frac{5a^2}{2}\int_0^{2\pi} d\phi$$

$$\text{Flux} = 5\pi a^2$$

~~as there is symmetry along z axis we can use cylinderical coordinal~~

Now trying symmetrical

$x = \rho \sin\theta \cos\phi$

$y = \rho \sin\theta \sin\phi$

$z = \rho \cos\theta$

$$\hat{n}\, dS = \hat{\rho}\, a^2 \sin\theta \, d\theta \, d\phi$$

$$F_1 = 5 \cos\theta \, \hat{r} - 5 \sin\theta \, \hat{\theta}$$

$$\text{Flux} = \iint F_1 \cdot \hat{n} \, dS$$

$$= \iint (5\cos\theta \, \hat{r} - 5 \sin\theta \, \hat{\theta})(\hat{r})\, a^2 \sin\theta \, d\theta \, d\phi$$

$$= \iint 5\cos\theta \sin\theta \, a^2 \, d\theta \, d\phi$$

$$= 5\pi a^2$$

ii) $F_2 = 5z\, \hat{k}$

$$\hat{n} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{a}$$

$$\text{Flux} = \iint \frac{5z^2}{a} \, dS$$

$$= 5a^3 \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta \, d\phi$$

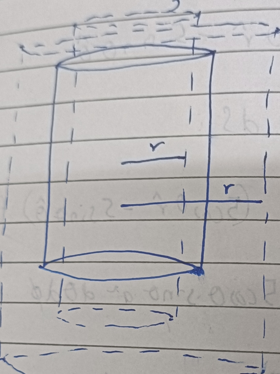
$$\frac{10 a^3 \pi}{3}$$

$$\text{flux} = \iiint \nabla \cdot F \, dV$$

$$= \int_0^{a}\int_0^{2\pi}\int_0^{\pi/2} 5 \; d\theta \, d\phi \, d\rho$$

$$= \frac{10a^3\pi}{3}$$

4)



$\rho_v = a r^2$

Let a gaussian cylinderical surface inside cyli… with same axis.

Using Gauss law

$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$E(2\pi r L) = \frac{1}{\epsilon_0}\int_0^r (2\pi r)(a r^2)\,dr$$

$$E = \frac{1}{4}\frac{r^3 a}{\epsilon_0}$$

Outside

$\oint E$

$\therefore E$

5)

If

a) $\vec{F}$

$|\vec{F}$

b) $\oint \vec{F}$

$|$

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Outside cylinder

$$\oint E \cdot dA = \frac{1}{\epsilon_0}\int_0^b 2\pi r L \, a r^2 dr$$

$$E(2\pi r L) = \frac{1}{4\epsilon_0} 2\pi b^4 L a$$

$$E = \frac{b^4 a}{4 r \epsilon_0}$$

$$\therefore E = \begin{cases} \frac{r^3}{4\epsilon_0} & r \le b \\ \frac{b^4 a}{4 r \epsilon_0} & r > b \end{cases}$$

5)

If

$$\vec{F} = a\hat{s} + b\hat{\phi} + c\hat{z}$$

then $|\vec{F}| = \sqrt{a^2 + c^2}$

cylinder

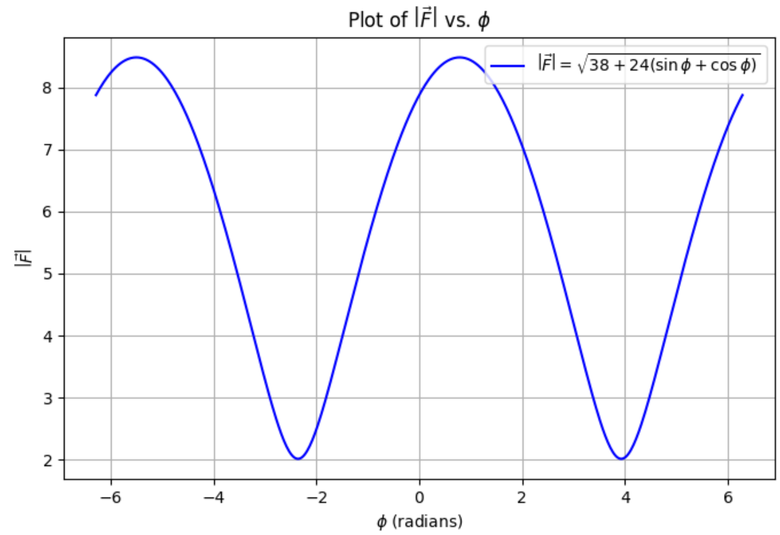
a) $\vec{F} = (4 + 3\cos\phi + 3\sin\phi)\hat{s} + 3(\cos\phi - \sin\phi)\hat{\phi} - 2\hat{z}$

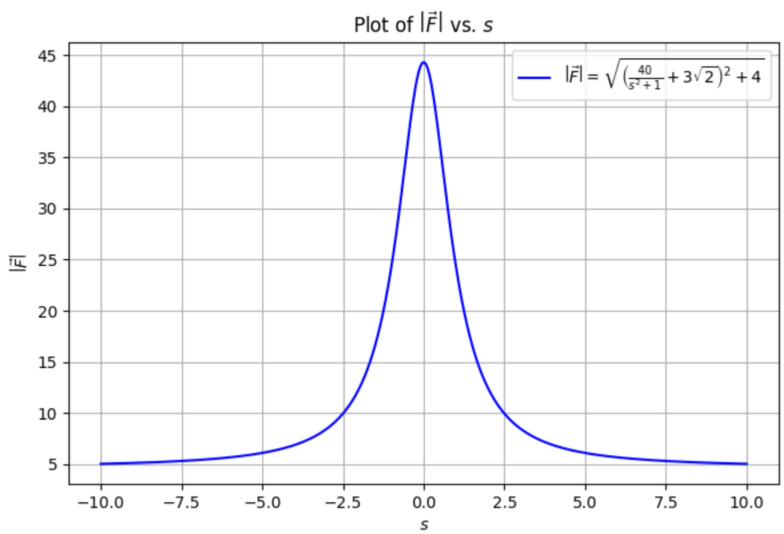
$$|\vec{F}| = \sqrt{(4 + 3\cos\phi + 3\sin\phi)^2 + 4}$$

$$= \sqrt{38 + 24(\sin\phi + \cos\phi)}$$

b) $\vec{F} = \left(\frac{40}{s^2+1} + 3\sqrt{2}\right)\hat{s} - 2\hat{z}$

$$|\vec{F}| = \sqrt{\left(\frac{40}{s^2+1} + 3\sqrt{2}\right)^2 + 4}$$


Plot of $|\vec{F}|$ vs. ϕ
$|\vec{F}| = \sqrt{38 + 24(\sin\phi + \cos\phi)}$
$|\vec{F}|$
ϕ (radians)
8
7
6
5
4
3
2
−6
−4
−2
0
2
4
6


Plot of $|\vec{F}|$ vs. s
$|\vec{F}| = \sqrt{\left(\frac{40}{s^2+1} + 3\sqrt{2}\right)^2 + 4}$
$|\vec{F}|$
s
45
40
35
30
25
20
15
10
5
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

c) $\nabla \cdot \vec{F} = \frac{1}{s}\frac{\partial}{\partial s}(s F_s) + \frac{1}{s}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$

$= \frac{1}{s}\frac{\partial}{\partial s}\left(\frac{40s}{s^2+1} + 3s(\cos\phi + \sin\phi)\right) + \frac{1}{s}\frac{\partial}{\partial \phi}\left(3(\cos\phi - \sin\phi)\right)$

$- 2$

$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} + \cancel{\frac{3\cos\phi + 3\sin\phi}{s}} + \cancel{\frac{-3\sin\phi}{s}} - \cancel{\frac{3\cos\phi}{s}}$

$= \frac{40}{(s^2+1)s} - \frac{80s}{(s^2+1)^2} - 2$

d) $\nabla \times \vec{F} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{s\,\partial \phi} & \frac{\partial}{\partial z} \\ F_s & F_\phi & F_z \end{vmatrix}$

$= 0\,\hat{s} + 0\,\hat{\phi} - \cancel{\frac{3(-\sin\phi + \cos\phi)}{s}}\,\hat{z}$

∴ ~~Not conservative~~

$= 0 \quad - \frac{3(-\sin\phi + \cos\phi)}{s}$

∴ Not Conservative

6)

c) Considering all interactions

• ~~Energy =~~

let charges be

1C at $(0,1)$ q_1

-1C $(1,0)$ q_2

1C $(0,-1)$ q_3

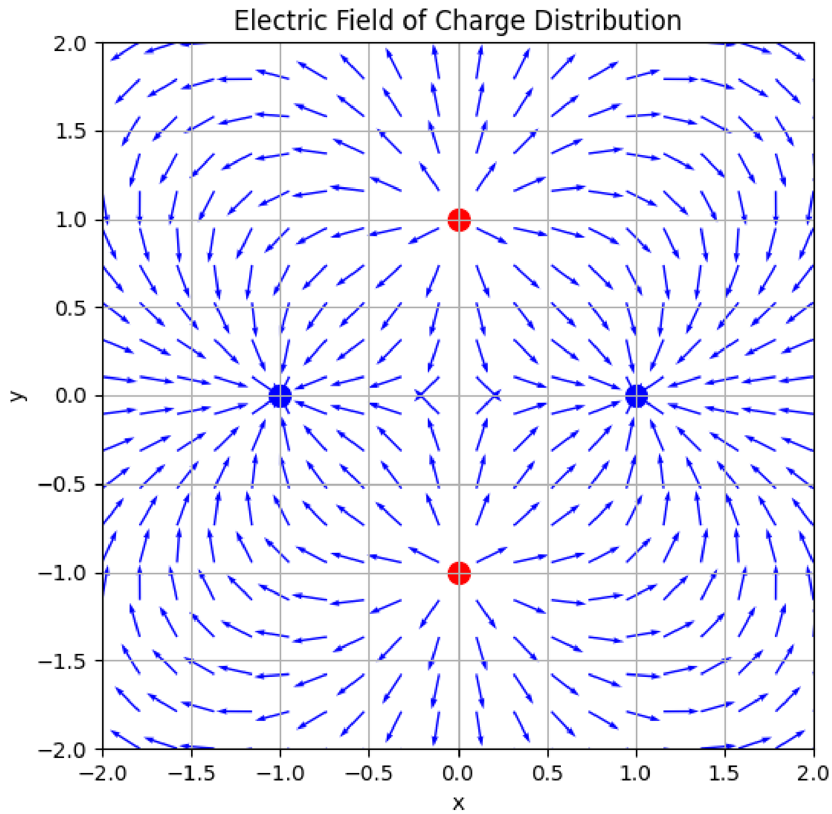
-1C $(-1,0)$ q_4

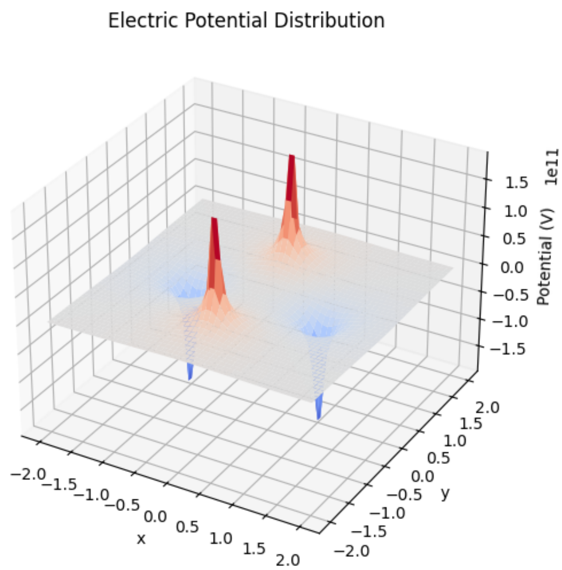
$$\text{Energy} = \sum \frac{qq}{4\pi\epsilon_0 r}$$

$$= 4\left(\frac{-1}{4\pi\epsilon_0(\sqrt{2})}\right) + 2\left(\frac{1}{4\pi\epsilon_0 2}\right)$$

$(q_i q_{i+1})$ (q_i, q_{i+2})

$$= \frac{1-2\sqrt{2}}{4\pi\epsilon_0} \text{ J}$$


Electric Field of Charge Distribution
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x


Electric Potential Distribution
1e11
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Potential (V)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y

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7)
a)

$$E = -\nabla V$$

$$\vec{E} = \frac{V_0 e^{-r/a}}{a}\hat{r}$$

$$\nabla\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{V_0 r^2 e^{-r/a}}{a}\right)$$

$$\nabla\cdot\vec{E} = \frac{V_0}{r^2 a}\left(2re^{-r/a} - \frac{r^2 e^{-r/a}}{a}\right)$$

$$-\nabla^2 V = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{V_0 \epsilon}{r^2 a}\left(2re^{-r/a} - \frac{r^2 e^{-r/a}}{a}\right)$$

put $r = a$

$$\boxed{\rho = \frac{V_0 \epsilon_0 e^{-1}}{a^2}}$$

b) $$\vec{E} = \frac{V_0 e^{-r/a}}{a}\hat{r}$$

$$\vec{E} = \frac{V_0 e^{-1}}{a}$$

c) $$\iiint \rho\, dV$$

$$= 2\pi^2 \int_0^a \frac{V_0 \epsilon_0}{r^2}\left(2re^{-r/a} - \frac{r^2 e^{-r/a}}{a}\right)$$

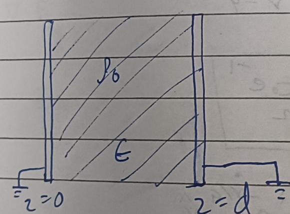
$$= 2\pi^2 V_0 \epsilon_0 \int_0^\infty e^{-r/a}\left(\frac{2}{r} - \frac{1}{a}\right) dr$$

b) $\vec{E} = -$

$\vec{E} =$

c) $V =$

3)



a)

$$-\nabla^2 V = \frac{\rho_0}{\epsilon_0}$$

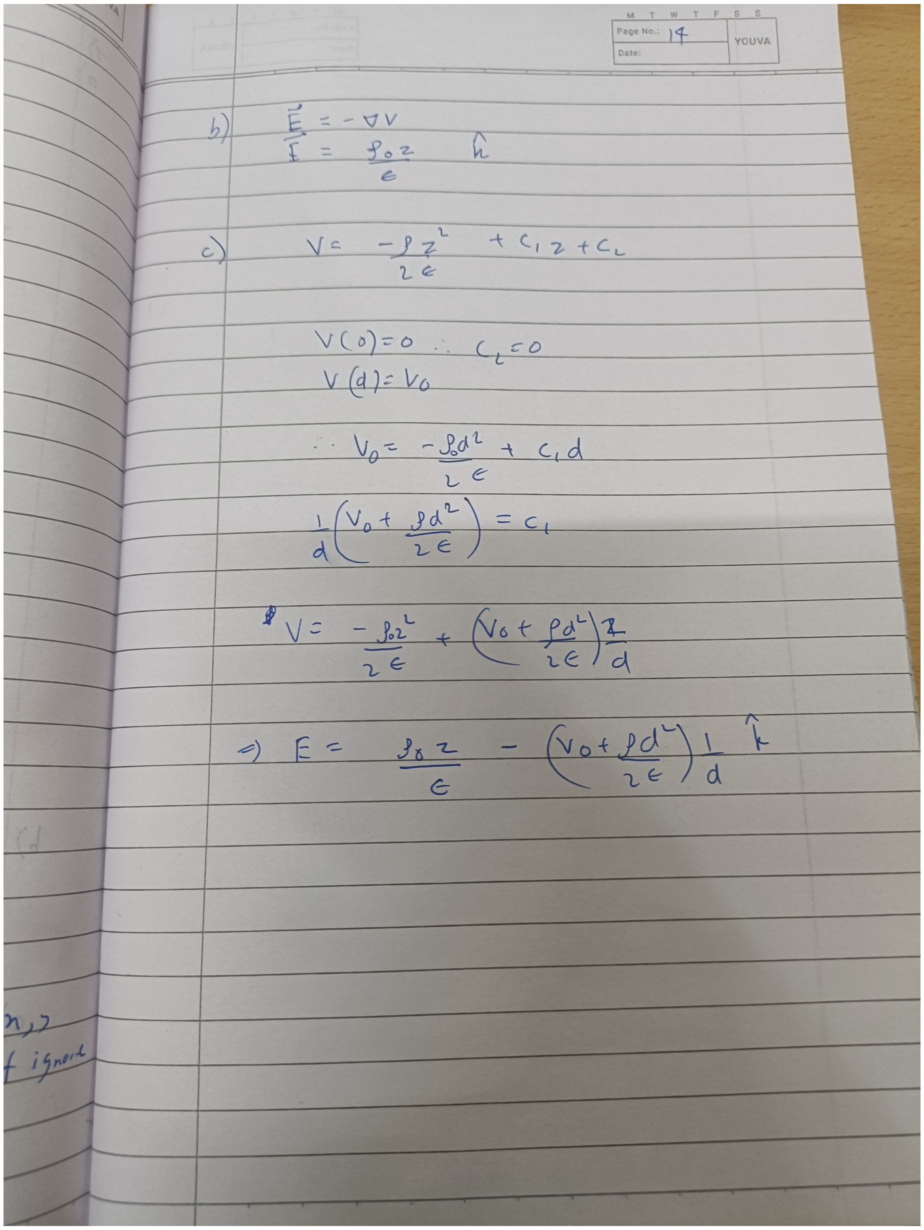
$$V = -\frac{\rho_0 z^2}{2\epsilon_0} + c_1 z + c_2$$

as x, y inf igno

$V(0) = 0 \Rightarrow c_2 = 0$

$V(d) = 0 \Rightarrow c_1 = 0$

$\therefore V(z) = -\frac{\rho_0 z^2}{}$



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b) $\vec{E} = -\nabla V$

$$\vec{E} = \frac{\rho_0 z}{\epsilon} \hat{k}$$

c) $$V = -\frac{\rho z^2}{2\epsilon} + C_1 z + C_2$$

$V(0) = 0 \therefore C_2 = 0$

$V(d) = V_0$

$$\therefore V_0 = -\frac{\rho_0 d^2}{2\epsilon} + C_1 d$$

$$\frac{1}{d}\left(V_0 + \frac{\rho d^2}{2\epsilon}\right) = C_1$$

$$V = -\frac{\rho_0 z^2}{2\epsilon} + \left(V_0 + \frac{\rho d^2}{2\epsilon}\right)\frac{z}{d}$$

$$\Rightarrow E = \frac{\rho_0 z}{\epsilon} - \left(V_0 + \frac{\rho d^2}{2\epsilon}\right)\frac{1}{d} \hat{k}$$

For codes refer to
https://github.com/ArnavYadnopavit/Electromagnetics/tree/main/Assignment1