Invariant Operations on Discrete Neural Functions over Galois Field

Fedir E. Geche[1], Anatolii Ye. Batyuk[2], V.Yu. Buchok[1]
[1]Uzhhorod National University, 88000, Ukraine, Uzhhorod, Narodna Square, 3, fgeche@hotmail.com
[2]Lviv Polytechnic National University, 79013, Ukraine, Lviv, S. Bandera, 12, abatyuk@gmail.com

***Abstract*—The paper introduces concepts of a discrete neural function and a neural element over a Galois field. A criterion of the possibility of discrete function spectral analysis over a finite field is established. Discrete neural function invariant operations over a Galois field are described.**

Keywords—neural element; structure vector; discrete function; group character; finite field; spectrum of discrete function; invariant operation.

I. Introduction

Finite fields and groups are widely used in the theory of logic functions and automata [1-3]. Particularly important role belongs to finite fields in coding theory [1]. In [4], it is shown that any finite automaton has an isomorphic image in the form of a linear automaton over some finite field. Analysis and synthesis of linear automata over an arbitrary finite field is carried out by traditional methods of spectral analysis.

Basic methods of spectral analysis [5,6] can be successfully used to check the possibility of discrete functions implementation by a neural element [13, 14] over a finite Galois field for finding invariant operations on them.

II. Spectral analysis of discrete functions over a Galois field

Let $F = GF\left(p^m\right)$ be a Galois field, which contains the cyclic groups

$$H_{k_1} = \langle a_1 \mid a_1^{k_1} = 1\rangle, H_{k_2} = \langle a_2 \mid a_2^{k_2} = 1\rangle, \ldots,$$
$$H_{k_n} = \langle a_n \mid a_n^{k_n} = 1\rangle, G_n = H_{k_1} \otimes H_{k_2} \otimes \ldots \otimes H_{k_n}$$

is a direct product of cyclic groups H_{k_i}.

As a discrete function of n variables over the field F we define the unambiguous mapping of the form $f : G_n \to F$.

Note that the spectral analysis of discrete functions $f : G_n \to C$ over the field C is always possible because the field C contains a primitive k-th root of unity for any $k = HCK\left(k_1, k_2, \ldots, k_n\right)$, – the least common multiple of numbers $k_1, k_2, \ldots, k_n$.

If to choose F to be the Galois field $F = GF\left(p^m\right)$, the spectral analysis of a discrete functions $f : G_n \to F$ is not always possible [15, 16]. The spectral analysis of discrete functions over $F = GF\left(p^m\right)$ will be possible only when the dimension of the vector space $V_F^n = \left\{ f \mid f : G_n \to GF\left(p^m\right)\right\}$ and the order of the characters group $X(G_n)$ over the field F are the same.

Consider the problem: whether it is possible to expand a function $f : G_n \to F$ in terms of characters of the group G_n over the field F If $k_1 = k_2 = \ldots = k_n = 2$ and to choose F to be the field of real numbers R, the group of characters $X\left(G_n\right)$ of the group G_n over the field R coincides with the system of Walsh-Hadamard basic functions and the spectral analysis of discrete functions is possible.

If $k_1 = k_2 = \ldots = k_n = k\left(k > 2\right)$, and as a field F to be choosen the field of complex numbers C, the group of characters $X\left(G_n\right)$ over the field C coincides with the system of Vilenkyna-Krestenson basic functions and the spectral analysis of discrete functions is also possible.

Let $k = HCK\left(k_1, k_2, \ldots, k_n\right)$. Let us show that in case the field $F = GF(p^m)$ contains a primitive k-th root of unity, k evenly divides $u = p^m - 1$. Assume, that a field with the primitive element ε contains the primitive k-th root of unity σ. Then the cyclic group $H_k = \left\langle \sigma \mid \sigma^k = 1\right\rangle$ is a subgroup of the cyclic group of the field F and according to the Lagrange theorem [7], we obtain that k is a divisor of u. Let k evenly divides u. Consider the element $\sigma = \varepsilon^{u/k}$. Show that it is a primitive root of unity. Taking into account the properties of primitive element ε of the field F [6,8.9], we can state that for any $i, j, r \in \{1, 2, \ldots, k-1\}$ $\sigma^i \neq \sigma^j$, if $i \neq j$ and $\sigma^r \neq 1$. Thus, k is the least natural number so that $\sigma^k = 1$. Hence, we conclude that the spectral analysis of the

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discrete functions $f: G_n \to F$ is possible if and only if k evenly divides u.

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Let $k = HCK(k_1, k_2, \ldots, k_n)$. Show that in case the field $F = GF(p^m)$ contains a primitive k-th root of unity, k evenly divides $u = p^m - 1$. Assume, that the field with the primitive element ε contains the primitive k-th root of unity σ. Then the cyclic group $H_k = \langle \sigma \mid \sigma^k = 1 \rangle$ is a subgroup of the cyclic group of the field F and according to the Lagrange theorem [7, 11], we obtain that k is a divisor of u. Let k evenly divides u. Consider the element $\sigma = \varepsilon^{u/k}$. Show that it is a primitive k-th root of unity. Taking into account the properties of the primitive element ε of the field F [8,9], we can state that for any $i, j, r \in \{1, 2, \ldots, k-1\}$ $\sigma^i \neq \sigma^j$, if $i \neq j$ and $\sigma^r \neq 1$. Thus, k is the least natural number so that $\sigma^k = 1$. Hence, we conclude that the spectral analysis of discrete functions $f: G_n \to F$ is possible if and only if k evenly divides u.

Let us find an analytical form of characters of the group G_n over the field $F = GF(p^m)$. Let $k = HCK(k_1, k_2, \ldots, k_n)$, ε be a primitive element of the field $F = GF(p^m)$, $H_k = \langle a \mid a^k = 1 \rangle$ be a cyclic group of the k-th order and k be a divisor of u. Then for an arbitrary element $h_i \in H_{k_i}$ there exists such $j_{k_i} \in \{0, 1, \ldots, k_i - 1\}$ that $h_i = a_i^{j_{k_i}}$, where $a_i = a^{k/k_i}$ is a generator of the cyclic group H_{k_i} $(i = 1, 2, \ldots, n)$. The characters χ_{r_i} of the group H_{k_i} over the field $F = GF(p^m)$ can be written as follows:

$$\chi_{r_i}(h_i) = \sigma_i^{r_i j_{k_i}}, \tag{1}$$

where $\sigma_i = \varepsilon^{u/k_i}$, $r_i \in \{0, 1, \ldots, k_i - 1\}$.

Since the group G_n is a direct product of the cyclic groups $H_{k_1}, \ldots, H_{k_n}$, for an arbitrary $\mathbf{g} \in G_n$ there exists such $j_i \in \{0, 1, \ldots, k_i - 1\}, i = 1, 2, \ldots, n$ that $\mathbf{g} = \left(a_1^{j_1}, \ldots, a_n^{j_n}\right) = \left(a^{kj_1/k_1}, \ldots, a^{kj_n/k_n}\right)$.

From the multiplicative properties of characters [5,6] and from (1) we have, that all characters of the group are functions

$$\chi_{(r_1, \ldots, r_n)}(\mathbf{g}) = \sigma^{t_1 r_1 j_1 + \ldots + t_n r_n j_n}, \tag{2}$$

where:

$\sigma = \varepsilon^{u/k}$, $t_i = \dfrac{k}{k_i}$, $r_i \in \{0, 1, \ldots, k_i - 1\}$, $i = 1, 2, \ldots, n$

If on the set of all characters of the group G_n to define the product of two characters $\chi_{(r_1, \ldots, r_n)}$, $\chi_{(q_1, \ldots, q_n)}$ as follows:

$$\forall \mathbf{g} \in G_n \quad \chi_{(r_1, \ldots, r_n)}(\mathbf{g}) \cdot \chi_{(q_1, \ldots, q_n)}(\mathbf{g}) = \chi_{(r_1 \oplus_1 q_1, \ldots, r_n \oplus_n q_n)}(\mathbf{g}),$$

where $\oplus_i$ is the addition modulo k_i, they constitute a multiplicative group of characters $X(G_n)$. On the basis of (2) it can be stated that the number of different characters of the group G_n over the field F equals the order of the group G_n. Then the fact that the characters are orthogonal [5,6] and $|X(G_n)| = \dim_F V_F^n = k_1 k_2 \ldots k_n$ yields that $X(G_n)$ forms an orthogonal basis of the space V_F^n. Thus, an arbitrary element $f \in V_F^n$ can be written unambiguously:

$$f(\mathbf{g}) = \sum_{r_1=0}^{k_1-1} \cdots \sum_{r_n=0}^{k_n-1} s_{(r_1, \ldots, r_n)} \chi_{(r_1, \ldots, r_n)}(\mathbf{g}), \tag{3}$$

where the addition and the multiplication are performed in the field F.

The expansion (3) is called the spectral expansion of the discrete function $f: G_n \to F$ in terms of characters of the group G_n over the field F.

Multiplying both terms of (3) by $\chi^{-1}_{(q_1, \ldots, q_n)}$ and adding up the left and right sides of the resulting equality for all elements of the group G_n, we obtain:

$$\sum_{\mathbf{g} \in G_n} f(\mathbf{g}) \chi^{-1}_{(q_1, \ldots, q_n)}(\mathbf{g}) =$$

$$= \sum_{\mathbf{g} \in G_n} \left(\sum_{r_1=0}^{k_1-1} \cdots \sum_{r_n=0}^{k_n-1} s_{(r_1, \ldots, r_n)} \chi_{(r_1, \ldots, r_n)}(\mathbf{g}) \right) \chi^{-1}_{(q_1, \ldots, q_n)}(\mathbf{g}).$$

Considering the orthogonality of the characters, the right side of the last equality can be written as:

$$\sum_{\mathbf{g}\in G_n}\left(\sum_{r_1=0}^{k_1-1}\cdots\sum_{r_n=0}^{k_n-1}s_{(r_1,\ldots,r_n)}\chi_{(r_1,\ldots,r_n)}(\mathbf{g})\right)\chi^{-1}_{(q_1,\ldots,q_n)}(\mathbf{g})=$$
$$\sum_{r_1=0}^{k_1-1}\cdots\sum_{r_n=0}^{k_n-1}s_{(r_1,\ldots,r_n)}\left(\sum_{\mathbf{g}\in G_n}\chi_{(r_1,\ldots,r_n)}(\mathbf{g})\chi^{-1}_{(q_1,\ldots,q_n)}(\mathbf{g})\right)=$$
$$= s_{(q_1,\ldots,q_n)}\left|G_n\right|.$$

Thus, the spectral coefficients of the function can be calculated according to the following formula:

$$s_{(q_1,\ldots,q_n)}=\left|G_n\right|^{-1}\sum_{\mathbf{g}\in G_n}f(\mathbf{g})\chi^{-1}_{(q_1,\ldots,q_n)}(\mathbf{g}), \tag{4}$$

where $q_i\in\{0,1,\ldots,k_i-1\}$ $(i\in\{1,2,\ldots,n\})$.

The aforesaid results of spectral analysis of discrete functions, functions of characters we shall illustrate by the following examples.

Example. Let $n=2, k=k_1=k_2=2$ and $F=GF(3^2)$ is a Galois field with the modular polynomial x^2+x+2. Spectral analysis of the functions $f\in V_F^2=\{f\mid f:G_2\to F\}$ $(G_2=H_2\otimes H_2)$ is possible, because k divides $u=3^2-1$. Denote ε to be a generator of the cyclic group of the field $GF(3^2)$, i.e. $GF(3^2)\setminus\{0\}=\{\varepsilon^j\mid j=0,1,\ldots,7\}$. Then $\sigma=\varepsilon^4=2$. The characters of the group G_2 we calculate according to the formula (2)

TABLE 1. GROUP OF CHARACTERS $X(G_2)$ OVER THE FIELD $GF(3^2)$

$G_2\setminus\chi(G_2)$	$\chi_{(0,0)}$	$\chi_{(0,1)}$	$\chi_{(1,0)}$	$\chi_{(1,1)}$
$(1,1)$	1	1	1	1
$(1,a)$	1	σ	1	σ
$(a,1)$	1	1	σ	σ
(a,a)	1	σ	σ	1

Let $f(1,1)=f(1,a)=f(a,1)=\sigma$ and $f(a,a)=0$. By the formula (4) we find spectral coefficients $s_{(0,0)},s_{(0,1)},s_{(1,0)},s_{(1,1)}$ of the function f

$$s_{(0,0)}=\left|G_2\right|^{-1}(\sigma\cdot1+\sigma\cdot1+\sigma\cdot1+0\cdot1)=1\cdot0=0,$$
$$s_{(0,1)}=\left|G_2\right|^{-1}(\sigma\cdot1+\sigma\cdot\sigma+\sigma\cdot1+0\cdot\sigma)=1\cdot(2\sigma+1)=2,$$
$$s_{(1,0)}=\left|G_2\right|^{-1}(\sigma\cdot1+\sigma\cdot1+\sigma\cdot\sigma+0\cdot\sigma)=1\cdot(2\sigma+1)=2,$$
$$s_{(1,1)}=\left|G_2\right|^{-1}(\sigma\cdot1+\sigma\cdot\sigma+\sigma\cdot\sigma+0\cdot\sigma)=1\cdot(\sigma+2)=1.$$

Thus,

$\forall\mathbf{g}\in G_2$ $\quad f(\mathbf{g})=2\chi_{(0,1)}(\mathbf{g})+2\chi_{(1,0)}(\mathbf{g})+\chi_{(1,1)}(\mathbf{g})$.

III. INVARIANT OPERATIONS ON DISCRETE NEURAL FUNCTIONS

Let $k_1,k_2,\ldots,k_n,q$ be natural numbers $(k_i\geq2, i=1,\ldots\ldots,n, q\geq2)$ and $k=HCK(k_1,k_2,\ldots,k_n,q)$. Further we consider only those fields $F=GF(p^m)$, which satisfy the condition: p^m-1 is evenly divisible by k. This means that the field $F=GF(p^m)$ contains cyclic groups H_{k_i},H_q with corresponding generators

$$\sigma_i=\varepsilon^{u/k_i}\ (i=1,2,\ldots,n),\ \sigma=\varepsilon^{u/q},$$

where ε is a primitive element of the field F, $u=cardF-1$.

Define on the set $F\setminus\{0\}$ a function $\mathrm{Fsign}\xi$ as follows:

$\forall\xi\in F\setminus\{0\}$ $\mathrm{Fsign}\xi=\sigma^j$, if $\dfrac{ju}{q}\leq\deg\xi<\dfrac{(j+1)u}{q}$,

where $\deg\xi$ is the degree of $\xi\ (\xi=\varepsilon^{\deg\xi})$, . $j\in\{0,1,\ldots,q-1\}$

A neural elements over the field $F=GF(p^m)$ is called a logic unit with $n+1$ inputs $x_1,\ldots,x_n;x_0$ $(n\geq1)$, which respectively take values of sets H_{k_i} $(i=1,\ldots,n)$ and $H_0=\{1\}$, and the output, which takes the value of the set H_q. Each input is assigned to a corresponding element ω_i of the field F and the values of the output signal are found as follows: the values of input signals are multiplied by the corresponding elements ω_i, then the obtaines results are added up and at the output we have values $\mathrm{Fsign}\xi$ by obtained sum.

Let $G_n=H_{k_1}\otimes H_{k_2}\otimes\ldots\otimes H_{k_n}$ be a direct product of cyclic groups H_{k_i}. A discrete function $f:G_n\to H_q$ is realized by one neural element over the field $F=GF(p^m)$, if there is such $n+1$-dimensional vector $\mathbf{w}=(\omega_1,\ldots,\omega_n;\omega_0)$ $(\omega_1\in F)$ that for all

$\mathbf{g}=(\gamma_1,\ldots,\gamma_n)\in G_n$ $f(\mathbf{g})=$Fsign w$(\mathbf{g})$, where $\mathrm{w}(\mathbf{g})=\omega_1\gamma_1+\ldots+\omega_n\gamma_n+\omega_0$, and the addition and the multiplication are performed in the field F [12]. A vector $\mathbf{w}=(\omega_1,\ldots,\omega_n;\omega_0)$ is called a structure vector of neural element (NE) on the field F.

A discrete function $f:G_n\to H_q$, which is realised by one NE over the field F is called a neural function over F.

In the view of compact representation of classes of neural functions over F, it is important to know those transformations on discrete neural functions, that maintain the feature of their implementation by one NE. In the following theorems, there is obtained the summarizing the results of [10], which mentions the operations, concerning which the classes of many-valued neural functions over F are closed.

Theorem 1. *If the function $f:G_n\to H_q$ is realized by one neural element over the field $F=GF(p^m)$ with the structure vector $\mathbf{w}=(\omega_1,\ldots,\omega_i,\ldots,\omega_n;\omega_0)$, then the function $f_1(x_1,\ldots,x_i,\ldots,x_n)=f(x_1,\ldots,\xi_i x_i,\ldots,x_n)$, where $\xi_i\in H_{k_i}$, is also realised by one NE over the field F with the structure vector $\mathbf{w}_1=(\omega_1,\ldots,\xi_i\omega_i,\ldots,\omega_n;\omega_0)$.*

Theorem 2. *If the discrete function $f:G_n\to H_q$ is realised by one NE over the field $F=GF(p^m)$ with structure vector $\mathbf{w}=(\omega_1,\ldots,\omega_i,\ldots,\omega_j\ldots,\omega_n;\omega_0)$ and $k_i=k_j$, then the function $f_2(x_1,\ldots,x_i,\ldots,x_j\ldots,x_n)=$ $=f(x_1,\ldots,x_j,\ldots,x_i\ldots,x_n)$ is also realised by one NE with the structure vector $\mathbf{w}_2=(\omega_1,\ldots,\omega_j,\ldots\ldots,\omega_i\ldots,\omega_n;\omega_0)$.*

Theorem 3. *If the discrete function $f:G_n\to H_q$ is realized by one neural element over the field $F=GF(p^m)$ with the structure vector $\mathbf{w}=(\omega_1,\ldots,\omega_n;\omega_0)$, then the function $f_3(x_1,\ldots,x_n)==\,=\xi f(\xi_1x_1,\ldots,\xi_nx_n)$, where $\xi\in H_q$, $\xi_i\in H_{k_i}(i=1,2,\ldots,n)$, is also realised by one NE over the field F with the structure vector $\mathbf{w}_3=(\xi\cdot\xi_1\omega_1,\ldots,\xi\cdot\xi_n\omega_n;\xi\cdot\omega_0)$.*

The feasibility and the practical value of the development of methods of synthesis of neural elements and the study of invariant operations on boolean and multiple-valued logic functions over the Galois field are confirmed by the results of software experiments. The programs which simulate the action of multi-valued neural elements and implement the spectral method for the synthesis of these elements over the Galois field indicate that the power of discrete classes neuro functions increases with increasing power of the Galois field. To confirm this fact, we give the following simple examples, e.g. $\mathrm{n}=2,\mathrm{k}_1=k_2=q=2$ i $F=GF(3)$. Number u is divided on $k=HCK(k_1,k_2,q)$, therefore, the spectral analysis of Boolean functions over F is possible. $\varepsilon=2$ is the simple element of the field *F*. We will show all the Boolean functions of two variables in the following table shows that are implemented by one neuron element (NE) over the field F:

Table2. Boolean neuron functions over the field GF(3)

x_1	x_2	g_0	g_1	g_2	g_3	g_4	g_5
1	1	1	1	1	2	2	2
1	2	1	1	2	2	2	1
2	1	1	2	1	2	1	2
2	2	1	2	2	2	1	1

Vector patterns of NE that implement these functions are the following:

$$\mathbf{w}_{g_0}=(0,0;1),\mathbf{w}_{g_1}=(1,0;0),\mathbf{w}_{g_2}=(0,1;0),\ ,$$
$$\mathbf{w}_{g_3}=(0,0;2)\ \mathbf{w}_{g_4}=(2,0;0),\ \mathbf{w}_{g_5}=(0,2;0).$$

So neuro classes of Boolean functions of two variables over $GF(3)$ consists of 6 functions g_0,g_1,g_2,g_3,g_4,g_5.

Let's consider $F=GF(5)$ field. $\varepsilon=2$ will be taken as a primitive element of F field . Than $\sigma=\varepsilon^{\frac{5-1}{2}}=4$. Boolean neuron functions from 2 variables over $GF(5)$ will be illustrated in the following table:

Table 3. Boolean neuron functions over the field $GF(5)$

x_1	x_2	f_0	f_1	f_2	f_3	f_4	f_5	f_6
1	1	1	1	1	1	1	1	1
1	4	1	1	1	1	4	4	4
4	1	1	1	4	4	1	1	4
4	4	1	4	1	4	1	4	1

f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
1	4	4	4	4	4	4	4	4
4	1	1	1	1	4	4	4	4
4	1	1	4	4	1	1	4	4
4	1	4	1	4	1	4	1	4

Vector patterns of NE will be the following:

$$\mathbf{w}_{f_0} = (0,0;1), \mathbf{w}_{f_1} = (2,2;2), \mathbf{w}_{f_2} = (2,3;2),$$
$$\mathbf{w}_{f_3} = (1,0;0), \mathbf{w}_{f_4} = (3,2;2), \mathbf{w}_{f_5} = (0,1;0),$$
$$\mathbf{w}_{f_6} = (1,1;4), \mathbf{w}_{f_7} = (2,2;3), \mathbf{w}_{f_8} = (3,3;2),$$
$$\mathbf{w}_{f_9} = (4,4;1), \mathbf{w}_{f_{10}} = (0,4;0), \mathbf{w}_{f_{11}} = (2,3;3),$$
$$\mathbf{w}_{f_{12}} = (4,0;0), \mathbf{w}_{f_{13}} = (3,2;3), \mathbf{w}_{f_{14}} = (3,3;3),$$
$$\mathbf{w}_{f_{15}} = (0,0;4).$$

IV. CONCLUSIONS

Based on Table 3 we can claim that all Boolean functions of two variables are neuron functions over $GF(5)$ field. The field $GF(5)$ is a minimal Galois field (field with the minimum number of elements), where all the boolean functions of two variables are implemented by a NE. It should be noted that over the fields of real and complex numbers, not all functions of algebra of logic from two variables are implemented by one neural element.

Let $k = HCK(k_1,\ldots,k_n,q)\,(k_i \geq 2, q \geq 2)$ and $G_n = H_{k_1} \otimes \ldots \otimes H_{k_n}$. Program experiments shows that there is used the following hypothesis :

Hypothesis. For the improvised k and improvised n the minimum $F_{\min}$ Galois field can be specified where all discrete functions $f : G_n \to H_q$ are implemented by one NE.

This result is important for coding, compression and transmission of information.

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