Discussion 11

1. An optimization problem

Consider the primal optimization problem,

$$p^\star = \min_{\vec{x} \in \mathbb{R}^2} \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 \tag{1}$$

$$\text{s.t.} \quad x_1 \geq 0, \tag{2}$$

$$x_1 + x_2 \geq 2. \tag{3}$$

First we solve the primal problem directly.

(a) Sketch the feasible region and argue that $\vec{x}^\star = (1, 1)$ and $p^\star = 1$.

(b) Next we solve the problem with the help of the dual. First, find the Lagrangian $\mathcal{L}(\vec{x}, \vec{\lambda})$.

(c) Formulate the dual problem.

(d) Solve the dual problem to find $\vec{\lambda}^\star$ and $d^\star$.

(e) Does strong duality hold?

(f) Finally we use the KKT conditions to find $\vec{x}^\star, \vec{\lambda}^\star$. First we write down the KKT conditions and then we find $\tilde{\vec{x}}$ and $\tilde{\vec{\lambda}}$ that satisfy them.

(g) Argue why the optimal primal and dual solutions are given by $\vec{x}^\star = \tilde{\vec{x}}$ and $\vec{\lambda}^\star = \tilde{\vec{\lambda}}$.

2. Complementary Slackness

Consider the problem:

$$p^\star = \min_{x \in \mathbb{R}} \quad x^2 \tag{4}$$

$$\text{s.t.} \quad x \geq 1,\ x \leq 2. \tag{5}$$

(a) Does Slater's condition hold? Is the problem convex? Does strong duality hold?

(b) Find the Lagrangian $\mathcal{L}(x, \lambda_1, \lambda_2)$.

(c) Find the dual function $g(\lambda_1, \lambda_2)$ so that the dual problem is given by,

$$d^\star = \max_{\lambda_1, \lambda_2 \in \mathbb{R}_+} g(\lambda_1, \lambda_2). \tag{6}$$

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(d) Solve the dual problem in (6) for $d^\star$.

(e) Solve for $x^\star, \lambda_1^\star, \lambda_2^\star$ that satisfy the KKT conditions.

(f) Can you spot a connection between the values of $\lambda_1^\star, \lambda_2^\star$ in relation to whether the corresponding inequality constraints are active or not at the optimal $x^\star$?

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