

1. 定义函数gtb，使得gtb nm 返回布尔值 true当且仅当 $n > m$ 。至少用两种方法定义这个函数。

方法1

```
Fixpoint gtb (n m: nat) : bool :=  
  match n, m with  
  | O, _   => false  
  | _, O   => true  
  | S n', S m' => gtb n' m'  
  end.
```

```
Example test_gtb1:          gtb 2 2 = false.
```

```
Proof. simpl. reflexivity. Qed.
```

```
Example test_gtb2:          gtb 2 4 = false.
```

```
Proof. simpl. reflexivity. Qed.
```

```
Example test_gtb3:          gtb 4 2 = true.
```

```
Proof. simpl. reflexivity. Qed.
```

方法2

```
Fixpoint eqb (n m : nat) : bool :=  
  match n with  
  | O => match m with  
    | O => true  
    | S m' => false  
  end  
  | S n' => match m with  
    | O => false  
    | S m' => eqb n' m'  
  end  
end
```

end.

```
Fixpoint geb (n m : nat) : bool :=  
  match m with  
  | O => true  
  | S m' =>  
    match n with  
    | O => false  
    | S n' => geb n' m'  
    end  
  end.
```

```
Definition negb (b:bool) : bool :=  
  match b with  
  | true => false  
  | false => true  
  end.
```

```
Definition andb (b1:bool) (b2:bool) : bool :=  
  match b1 with  
  | true => b2  
  | false => false  
  end.
```

```
Definition gtb' (n m : nat) : bool :=  
  andb (negb (eqb n m)) (geb n m).
```

Example test_gtb1': $\text{gtb}' 2 2 = \text{false}.$

Proof. simpl. reflexivity. **Qed.**

Example test_gtb2': $\text{gtb}' 2 4 = \text{false}.$

Proof. simpl. reflexivity. **Qed.**

Example test_gtb3': gtb' 4 2 = true.
Proof. simpl. reflexivity. **Qed.**

2. 证明如下性质。Theorem plus_1_1': forall n m o : nat, n=m->m=o->1+n =So.

Theorem plus_1_1' : forall n m o : nat, n = m -> m = o -> 1 + n = S o.

Proof.

intros n m o Hn Hm.

rewrite -> Hn.

rewrite <- Hm.

reflexivity.

Qed.

1 subgoal

forall n m o : nat, n = m -> m = o -> 1 + n = S o (1/1)

1 subgoal

n, m, o : nat

Hn : n = m

Hm : m = o

(1/1)

1 + n = S o |

1 subgoal

n, m, o : nat

Hn : n = m

Hm : m = o

(1/1)

1 + m = S o |

1 subgoal

$n, m, o : \text{nat}$

$Hn : n = m$

$Hm : m = o$

(1/1)

$1 + m = S\ m$

No more subgoals.