

1. 采用自底向上的方式

其中类型上下文表示: $f: A \rightarrow A \quad x: A$

$$\begin{array}{c}
 \hline \Gamma \vdash f: A \rightarrow A \quad \Gamma \vdash x: A \\
 \hline \Gamma \vdash f: A \rightarrow A \quad \Gamma \vdash fx: A \\
 \hline \Gamma \vdash f: A \rightarrow A \quad \Gamma \vdash f(fx): A \\
 \hline \Gamma \vdash x: A \quad \Gamma \vdash f(f(fx)): A \\
 \hline \Gamma \vdash f: A \rightarrow A \quad \Gamma \vdash \lambda x^A. f(f(fx)): A \rightarrow A \\
 \hline \Gamma \vdash \lambda f^{A \rightarrow A} x^A. f(f(fx)): (A \rightarrow A) \rightarrow A \rightarrow A
 \end{array}$$

2.

Inductive fruit: Type:=

lapple
lpear.

Inductive food: Type:=

lrice
lsteak
ldessert (f: fruit).

Definition vegetarian (f: food): bool:=

match f with
lsteak => false
lrice => true
ldessert f => true
end.

Example test1: (vegetarian rice) = true.

Proof. simpl. reflexivity. **Qed.**

Example test2: (vegetarian steak) = false.

Proof. simpl. reflexivity. **Qed.**

Example test3: (vegetarian (dessert apple)) = true.

Proof. simpl. reflexivity. **Qed.**

Example test4: (vegetarian (dessert pear)) = true.

Proof. simpl. reflexivity. **Qed.**