King Mongkut's University of Technology | **Name:** ______________________

Machine Learning | **I.D. Number:** ________________

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Take Home Quiz 4 due Sat Feb 25 2023 | **Score:** ______ / 50

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1. The following points (x_i, y_i) are discrete samples from a function $f(x) = ax^3 + bx^2 + cx + d$.

   a. *5 points.* 0.5 hrs. Show the update rule equation used to find the current a, b, c, and d after each iteration. Make sure you show the mathematics on how this is derived.

   b. *15 points.* 2 hrs. Write a program to find the best fit a, b, c, and d using gradient descent. You must write the gradient descent loop yourself and not use any gradient descent libraries. Attach the source code as well. *Hint*: You should get a, b, c, and d close to 0.5, 5.3, -2.7, and 3.5, respectively.

x_i	y_i
-6	103
-5	87
-4	67
-3	46
-2	26
-1	11
0	4
1	7
2	23
3	57
4	110
5	185
6	286

2. *10 points.* 1 hrs. Redo Problem 1b, but use the numerical method to calculate all your partial derivatives, where *h* is a very small number.

$$\frac{\partial}{\partial x_i} f(x_1, \ldots, x_i, \ldots, x_n) = \frac{f(x_1, \ldots, x_i + h, \ldots, x_n) - f(x_1, \ldots, x_i - h, \ldots, x_n)}{2h}$$

3. *10 points.* 1 hrs. Solve Problem 1b using Pseudo-Inverse Linear Regression to find (a, b, c, d). You can use numpy or other tools to invert matrices.

4. *10 points.* 1 hrs. Solve Problem 1b using the Gauss-Newton method to find (a, b, c, d). You can use numpy or other tools to invert matrices in each iteration.

$$X^{t+1} = X^t - \alpha J^{\#1} r(X^t) = X^t - \alpha \left(J^T J\right)^{-1} J^T r(X^t);\ \alpha = 1 \text{ works for linear case.}$$