



Algorithmen & Datastrukturen 2

WPO – Two-watched Literal Scheme

DPLL (Davis–Putnam–Logemann–Loveland)

Depth-first Tree Search: opbouw van een vervullende interpretatie

1. Kiezen van literals
2. Unit-propagatie
3. Pure Literal Elimination

Unit-propagatie kan versneld worden d.m.v. een speciale datastructuur

Two-watched Literal Scheme (TWLS)

Snel proberen navigeren tussen clauses en literal:

Welke literal zit waar?

Welke clause is unit?



Pointers to the rescue!

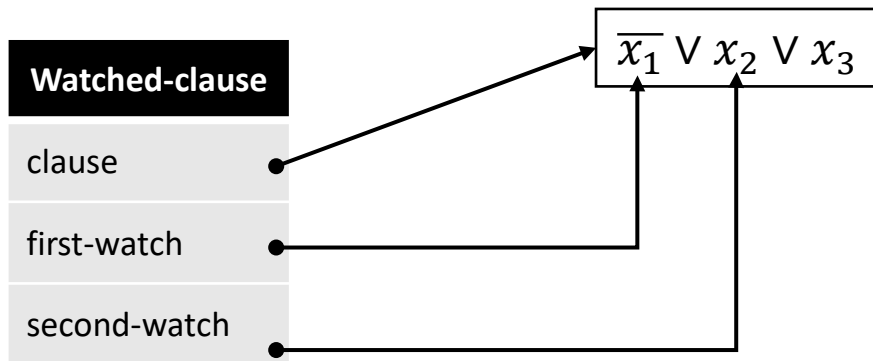
Hoeveel pointers zijn er nodig?

Watched Clause ADT

Wrapper rond Clause ADT waarvan **2 literals** "bekeken" worden

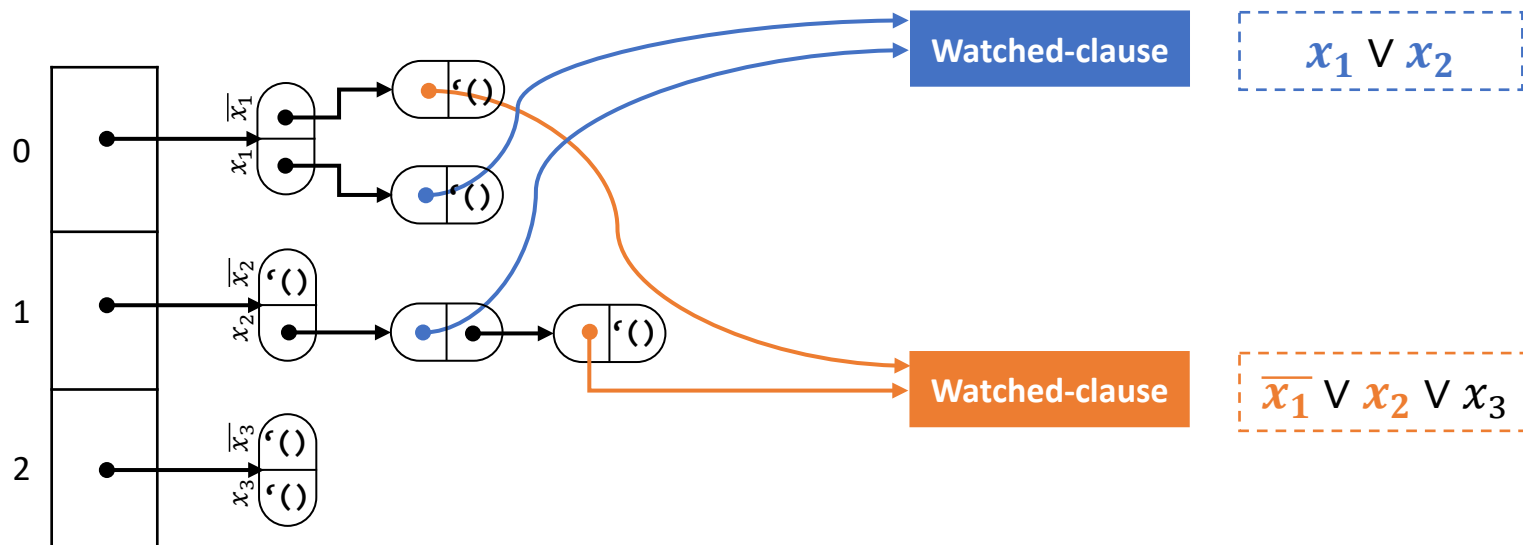
Legt relatie **clause** \rightarrow **literal** voor TWLS vast

Omgekeerde link is ook nodig: **welke literal wordt waar bekeken?**



Variable watchers

Legt relatie **literal** \rightarrow **watched-clause** voor **literals die bekeken worden** vast in een *atom-indexed-vector*



TWLS ADT

Zie `a-d/sat/twls.rkt`

TWLS
var-watchers
watched-clauses
decision-levels
reasons
times

Extra boekhouding voor CDCL (later)
atom-indexed-vectors

Unit-propagatie met TWLS

- Pseudo-code in cursustekst
- Propagatie-queue
 - Literals waarmaken
 - Invloed op meerdere clauses

Algorithm 3: Unit propagation and conflict detection with the two-watched literal scheme

```
1 UNIT-PROP():  
   Output: Either a conflicting clause  $c$  or  $\perp$  if no clause is conflicting  
2 for Clause  $c = \ell$  of length one in  $\mathcal{T}$  do  
3   if  $\ell.value = \mathbf{f}$  then return  $c$ ;  
4   if  $\ell.value = \mathbf{u}$  then MAKETRUE( $\ell, c$ );  
5 end  
6 while propagationQueue is non-empty do  
7    $\ell \leftarrow \text{propagationQueue.DEQUEUE}()$ ;  
8    $\ell.value \leftarrow \mathbf{t}$ ;  
9   for  $c \in \bar{\ell}.watchers$  do  
10    if Some  $\ell' \in c \setminus c.watches$  satisfies  $\ell'.value \neq \mathbf{f}$  then  
11       $\bar{\ell}.watchers \leftarrow \bar{\ell}.watchers \setminus \{c\}$ ;  
12       $\ell'.watchers \leftarrow \ell'.watchers \cup \{c\}$ ;  
13       $c.watches \leftarrow c.watches \setminus \{\bar{\ell}\} \cup \{\ell'\}$   
14    else  
15       $\ell' \leftarrow$  the other watch of  $c$ ;  
      // If  $\ell'.value = \mathbf{t}$  the clause is satisfied and  
      nothing has to happen.  
16      if  $\ell'.value = \mathbf{u}$  then  
17        MAKETRUE( $\ell', c$ );  
18      else if  $\ell'.value = \mathbf{f}$  then  
19        propagationQueue.CLEAR();  
20        return  $c$ ;  
21      end  
22    end  
23  end  
24 end  
25 return  $\perp$ ;
```

Clauses met 1 literal zijn standaard unit



```
(define (propagate-single-lit-clauses! interpret)
  (for-each (lambda (clause)
    (if (= (clause-size clause) 1)
      (let ((lit (car (literals clause))))
        (log-debug "We have a single lit clause:" (clause->string clause))
        (cond ((false? interpret lit)
              (exit clause))
              ((unknown? interpret lit)
               (make-true! lit interpret clause))))))
    (clauses formula))
  interpret)
```

```
2 for Clause  $c = \ell$  of length one in  $\mathcal{T}$  do
3   if  $\ell.value = f$  then return  $c$ ;
4   if  $\ell.value = u$  then MAKETRUE( $\ell, c$ );
5 end
```

maak literal waar + extra boekhouding

Literals propageren vereist boekhouding



```
(define (make-true! lit interpret reason)
```

```
  (log-debug "Making things come TRUE:" (literal->string lit))
```

```
  (true! interpret lit)
```

```
  (twls:reason! twls lit reason)
```

```
  (twls:level! twls lit current-decision-level)
```

```
  (queue:enqueue! prop-q lit))
```

De clause die propagatie toeliet
(nodig voor CDCL)

Enkel in CDCL-code!

```
1 MAKETRUE( $\ell$ , reason):
```

Input : A literal ℓ to be made true and its reason *reason*

```
2 if  $\ell$  is a negative literal  $\ell = \bar{x}$  then
```

```
3    $x.value \leftarrow f$ ;
```

```
4    $x.reason \leftarrow c$ ;
```

```
5    $x.level \leftarrow currentDecisionLevel$ ;
```

```
6 else
```

```
7    $\ell.value \leftarrow t$ ;
```

```
8    $\ell.reason \leftarrow c$ ;
```

```
9    $\ell.level \leftarrow currentDecisionLevel$ ;
```

```
10 end
```

```
11 propagationQueue.ENQUEUE( $\ell$ );
```

```
12 assignmentStack.PUSH( $\ell$ );
```

Unit-propagatie met TWLS

```
6 while propagationQueue is non-empty do
7    $\ell \leftarrow \text{propagationQueue.DEQUEUE}()$ ;
8    $\ell.value \leftarrow \mathbf{t}$ ;
9   for  $c \in \bar{\ell}.watchers$  do
10    if  $\text{Some } \ell' \in c \setminus c.watches \text{ satisfies } \ell'.value \neq \mathbf{f}$  then
11       $\bar{\ell}.watchers \leftarrow \bar{\ell}.watchers \setminus \{c\}$ ;
12       $\ell'.watchers \leftarrow \ell'.watchers \cup \{c\}$ ;
13       $c.watches \leftarrow c.watches \setminus \{\bar{\ell}\} \cup \{\ell'\}$ 
14    else
```

Unit-propagatie met TWLS

```
15      |      |      |  $\ell' \leftarrow$  the other watch of  $c$ ;  
      |      |      | // If  $\ell'.value = t$  the clause is satisfied and  
      |      |      | nothing has to happen.  
16      |      |      | if  $\ell'.value = u$  then  
17      |      |      | | MAKETRUE( $\ell', c$ );  
18      |      |      | else if  $\ell'.value = f$  then  
19      |      |      | | propagationQueue.CLEAR();  
20      |      |      | | return  $c$ ;  
21      |      |      | end  
22      |      | end  
23      | end  
24 end  
25 return  $\perp$ ;
```

Opdracht

Implementeer unit-propagatie in DPLL met TWLS

- Inspecteer `a-d/sat/twls.rkt`
- Vul bestand `iterative.rkt` van de opgave aan
 - Procedure `unit-prop!` afwerken
 - Wanneer propagatie-queue opvullen?
 - Unit clauses propageren telkens opnieuw?
- Plaats dit bestand in `a-d/sat/dpll/optimized`

Test jouw implementatie via `test.rkt`

- Pas `a-d/sat/dpll/config.rkt` aan
- Zie bijgeleverde folder `cnf-samples`

Denkvraagjes (extra oefeningen)

- In DPLL wordt het **current-decision-level** niet aangepast
 - Hoe correct wijzigen?
- Interpretatie wordt telkens gekopieerd en op stack geplaatst?
 - Kunnen we een enkel object gebruiken? (m.a.w. zonder kopieën)