# Lorenz Bayesian Parameter Estimation Benchmarks

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## 0.1 Parameter estimation of Lorenz Equation using DiffEqBayes.jl

```
using DiffEqBayes
using Distributions
using OrdinaryDiffEq, RecursiveArrayTools, ParameterizedFunctions
using Plots

gr(fmt=:png)

Plots.GRBackend()

g1 = @ode_def LorenzExample begin
  dx = σ*(y-x)
  dy = x*(ρ-z) - y
  dz = x*y - β*z
end σ ρ β

(::Main.WeaveSandBox1.LorenzExample{getfield(Main.WeaveSandBox1, Symbol("##
1#5")),getfield(Main.WeaveSandBox1, Symbol("##2#6")),getfield(Main.WeaveSan
dBox1, Symbol("##3#7")),Nothing,Nothing,getfield(Main.WeaveSandBox1, Symbol
("###4#8")),Expr,Expr}) (generic function with 2 methods)

r0 = [1.0; 0.0; 0.0]
tspan = (0.0, 30.0)
p = [10.0,28.0,2.66]

3-element Array{Float64,1}:
 10.0
 28.0
  2.66

prob = ODEProblem(g1,r0,tspan,p)
sol = solve(prob,Tsit5())

retcode: Success
Interpolation: specialized 4th order "free" interpolation
t: 357-element Array{Float64,1}:
  0.0
  3.5678604836301404e-5
  0.0003924646531993154
  0.0032623362978895253
  0.00905769384828584
  0.016955582524407417
  0.02768842697456643
  0.04185394003674388
```

```
  0.060236921553619587
  0.08368074642983417
  ⋮
 29.43765805702477
 29.510631239565896
 29.578880567314734
 29.659017275004008
 29.727496214049776
 29.810471259396742
 29.90700141130956
 29.993678718729086
 30.0
u: 357-element Array{Array{Float64,1},1}:
 [1.0, 0.0, 0.0]
 [0.999643, 0.000998805, 1.78143e-8]
 [0.996105, 0.0109654, 2.14696e-6]
 [0.96936, 0.0897687, 0.000143797]
 [0.924207, 0.242279, 0.0010461]
 [0.880049, 0.438715, 0.00342406]
 [0.848333, 0.691528, 0.0084873]
 [0.8495, 1.01449, 0.0182119]
 [0.913889, 1.44248, 0.0366944]
 [1.08882, 2.05219, 0.074029]
 ⋮
 [-7.11525, -1.1001, 32.1527]
 [-3.6978, -0.426863, 26.6792]
 [-2.22809, -1.03217, 22.3688]
 [-1.89224, -2.12005, 18.2938]
 [-2.36411, -3.51716, 15.6141]
 [-3.88697, -6.52388, 13.6665]
 [-7.69711, -13.0948, 15.3928]
 [-12.9863, -18.6942, 25.7278]
 [-13.3371, -18.7185, 26.8431]
```

```
t = collect(linspace(1,30,30))
```

```
Error: UndefVarError: linspace not defined
```

```
sig = 0.49
data = convert(Array, VectorOfArray([(sol(t[i]) + sig*randn(3)) for i in 1:length(t)]))
```

```
Error: UndefVarError: t not defined
```

```
Plots.scatter(t, data[1,:],markersize=4,color=:purple)
```

```
Error: UndefVarError: data not defined
```

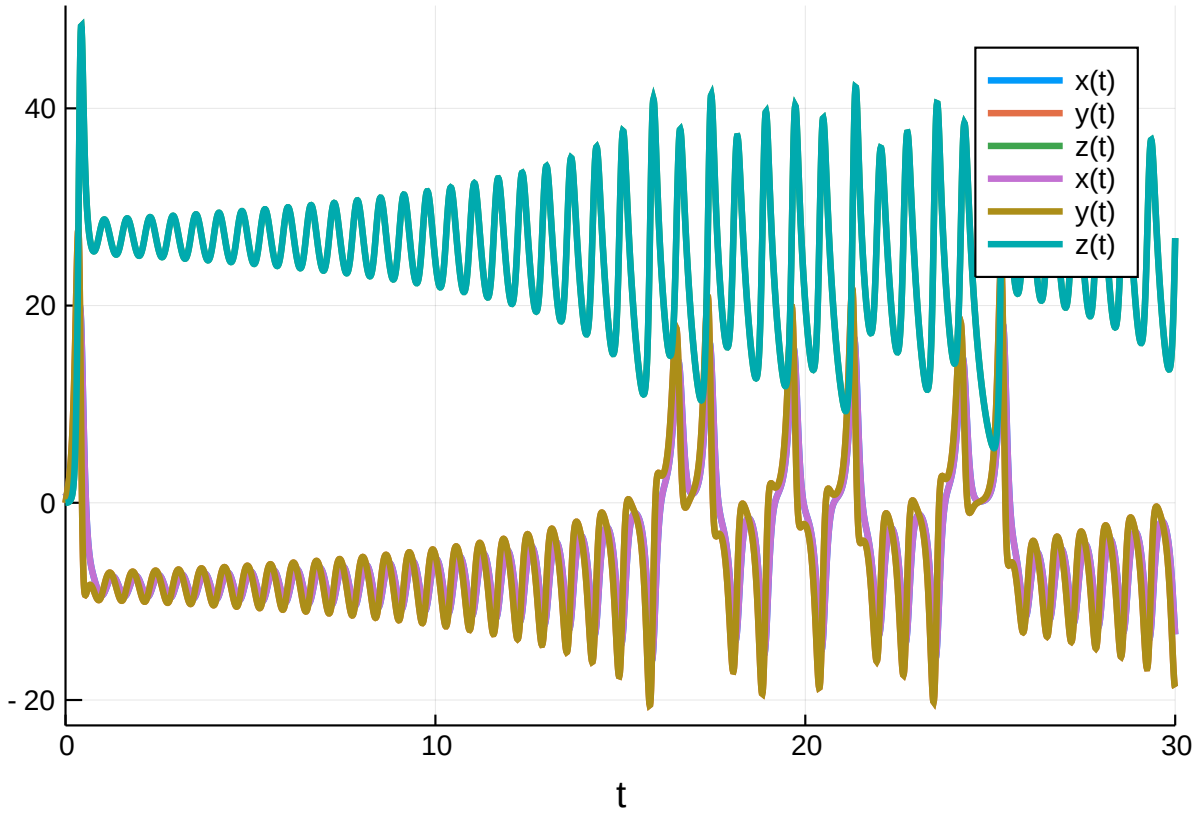
```
Plots.scatter!(t, data[2,:],markersize=4,color=:yellow)
```

```
Error: UndefVarError: data not defined
```

```
Plots.scatter!(t, data[3,:],markersize=4,color=:black)
```

```
Error: UndefVarError: data not defined
```

```
plot!(sol)
```




```
cb = AdaptiveProbIntsUncertainty(5)
```

```
Error: UndefVarError: AdaptiveProbIntsUncertainty not defined
```

```
monte_prob = MonteCarloProblem(prob)
sim = solve(monte_prob,Tsit5(),num_monte=100,callback=cb,reltol=1e-5,abstol=1e-5)
```

```
Error: UndefVarError: cb not defined
```

```
plot(sim,vars=(0,1),linealpha=0.4)
```

```
Error: UndefVarError: sim not defined
```

```
cb = AdaptiveProbIntsUncertainty(5)
```

```
Error: UndefVarError: AdaptiveProbIntsUncertainty not defined
```

```
monte_prob = MonteCarloProblem(prob)
sim = solve(monte_prob,Tsit5(),num_monte=100,callback=cb,reltol=1e-6,abstol=1e-6)
```

```
Error: UndefVarError: cb not defined
```

```
plot(sim,vars=(0,1),linealpha=0.4)
```

```
Error: UndefVarError: sim not defined
```

```
cb = AdaptiveProbIntsUncertainty(5)
```

```
Error: UndefVarError: AdaptiveProbIntsUncertainty not defined
```

```
monte_prob = MonteCarloProblem(prob)
sim = solve(monte_prob,Tsit5(),num_monte=100,callback=cb,reltol=1e-8,abstol=1e-8)
```

```
Error: UndefVarError: cb not defined
```

```
plot(sim,vars=(0,1),linealpha=0.4)
```

```
Error: UndefVarError: sim not defined
```

```
priors =
    [Truncated(Normal(10,2),1,15),Truncated(Normal(30,5),1,45),Truncated(Normal(2.5,0.5),1,4)]
```

```
3-element Array{Distributions.Truncated{Distributions.Normal{Float64},Distr
ibutions.Continuous},1}:
 Truncated(Distributions.Normal{Float64}(μ=10.0, σ=2.0), range=(1.0, 15.0))
 Truncated(Distributions.Normal{Float64}(μ=30.0, σ=5.0), range=(1.0, 45.0))
 Truncated(Distributions.Normal{Float64}(μ=2.5, σ=0.5), range=(1.0, 4.0))
```

## 0.2 Parameter estimation using Stan.jl backend.

Lorenz equation is a chaotic system hence requires very low tolerance to be estimated in a reasonable way, we use 1e-8 obtained from the uncertainity plots. Use of Truncated priors is necessary to prevent Stan from stepping into negative and other improbable areas.

```
@time bayesian_result =
    stan_inference(prob,t,data,priors;reltol=1e-8,abstol=1e-8,vars=(StanODEData(),InverseGamma(3,2)))
```

```
Error: UndefVarError: t not defined
```

```
plot_chain(bayesian_result)
```

```
Error: UndefVarError: bayesian_result not defined
```

### 0.2.1 Parameter estimation using Turing.jl backend

```
@time bayesian_result_turing = turing_inference(prob,Tsit5(),t,data,priors)
```

```
Error: UndefVarError: t not defined
```

```
plot_chain(bayesian_result_turing)
```

```
Error: UndefVarError: bayesian_result_turing not defined
```

## 0.3 Conclusion

Due to the chaotic nature of Lorenz Equation, it is a very hard problem to estimate as it has the property of exponentially increasing errors. Its uncertainity plot points to its chaotic

behaviour and goes awry for different values of tolerance, we use 1e-8 as the tolerance as it makes its uncertainity small enough to be trusted in `(0,30)` time span.

The behaviour is estimation using Stan.jl backend is as expected and it gives more accurate results as we decrease the tolerance, for 1e-8 we obtain quite accurate results as compared to higher tolerance values but lowering the tolerance leads to longer sampling time, incase of 1e-8 it took 11 hours. We also pass 500 warmup samples for proper convergence, as the plots provide evidence of non-convergence without it which observed over multiple runs.