Fitzhugh-Nagumo Work-Precision Diagrams

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1 Fitzhugh-Nagumo

The purpose of this is to see how the errors scale on a standard nonlinear problem.

```
using OrdinaryDiffEq, ParameterizedFunctions, ODE, ODEInterface,
      ODEInterfaceDiffEq, LSODA, Sundials, DiffEqDevTools

f = @ode_def FitzhughNagumo begin
  dv = v - v^3/3 -w + l
  dw = τinv*(v +  a - b*w)
end a b τinv l

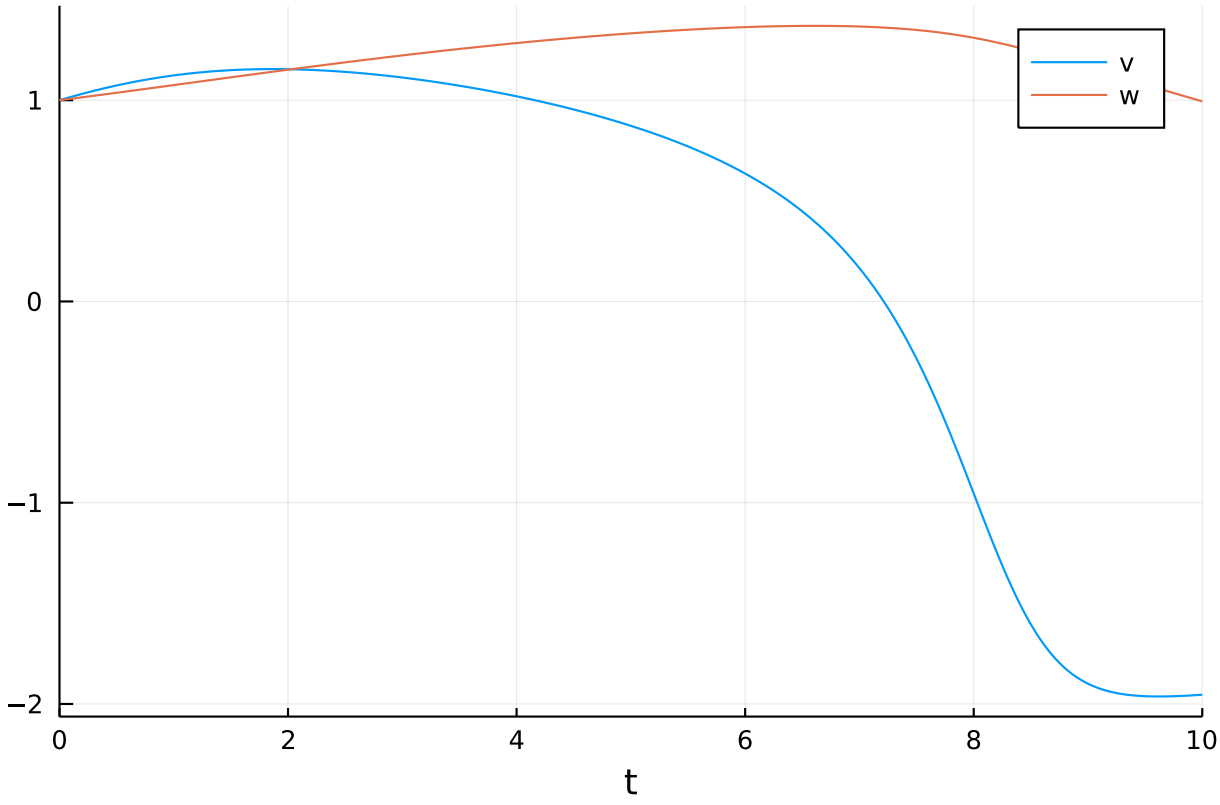
p = [0.7,0.8,1/12.5,0.5]
prob = ODEProblem(f,[1.0;1.0],(0.0,10.0),p)

abstols = 1.0 ./ 10.0 .^ (6:13)
reltols = 1.0 ./ 10.0 .^ (3:10);

sol = solve(prob,Vern7(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)
using Plots; gr()

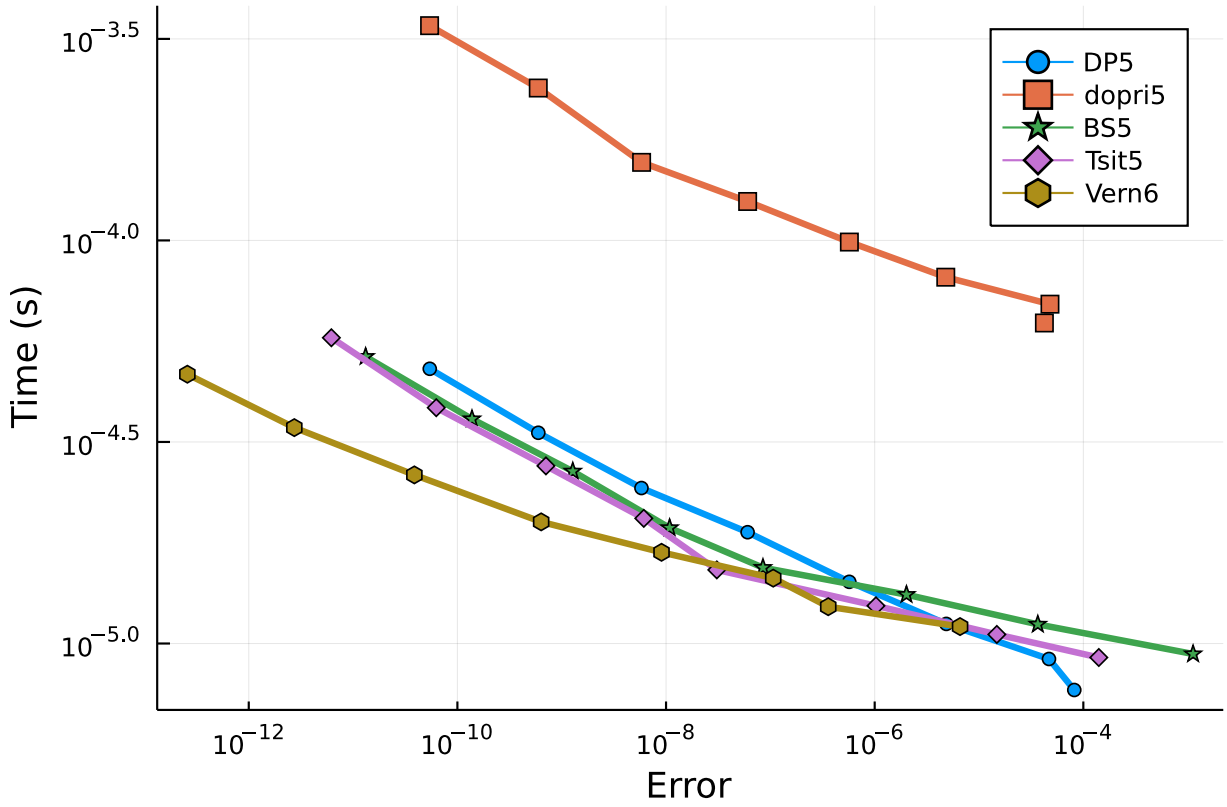
Plots.GRBackend()

plot(sol)
```



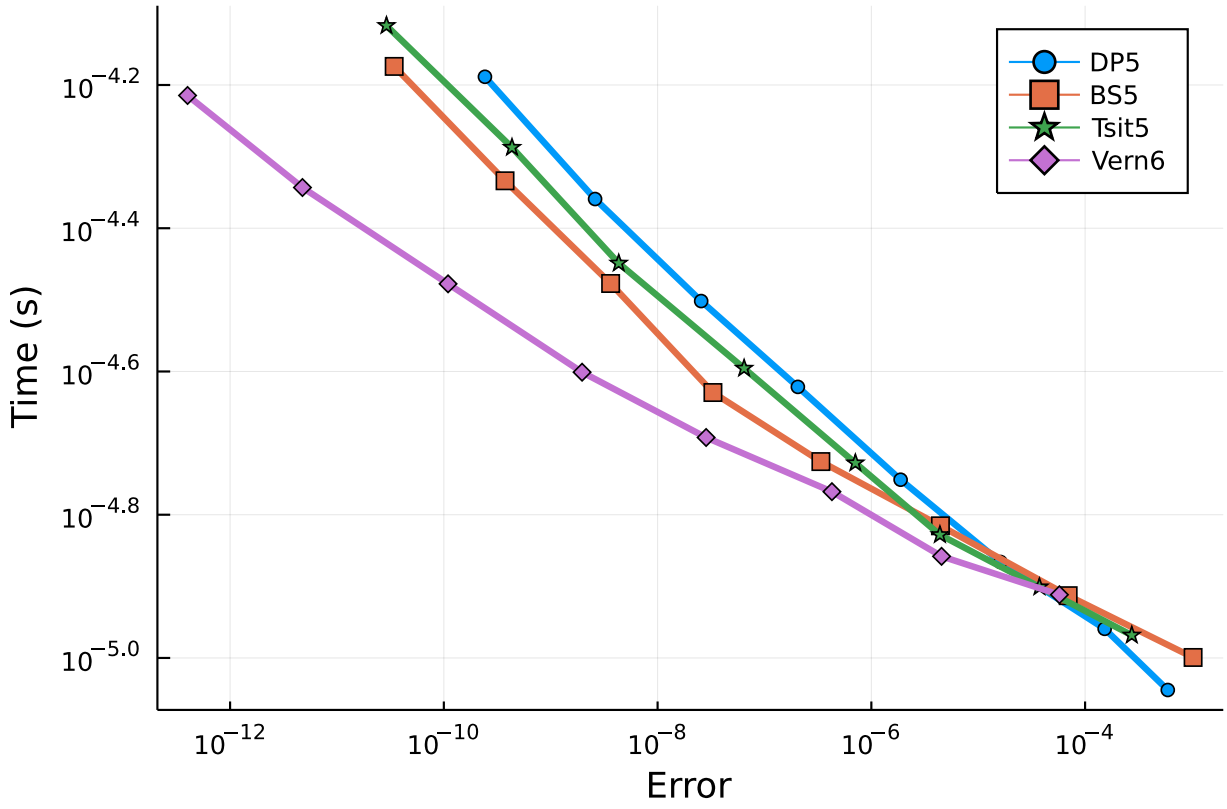
1.1 Low Order

```
setups = [Dict(:alg=>DP5())
          #Dict(:alg=>ode45()) #fails
          Dict(:alg=>dopri5())
          Dict(:alg=>BS5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



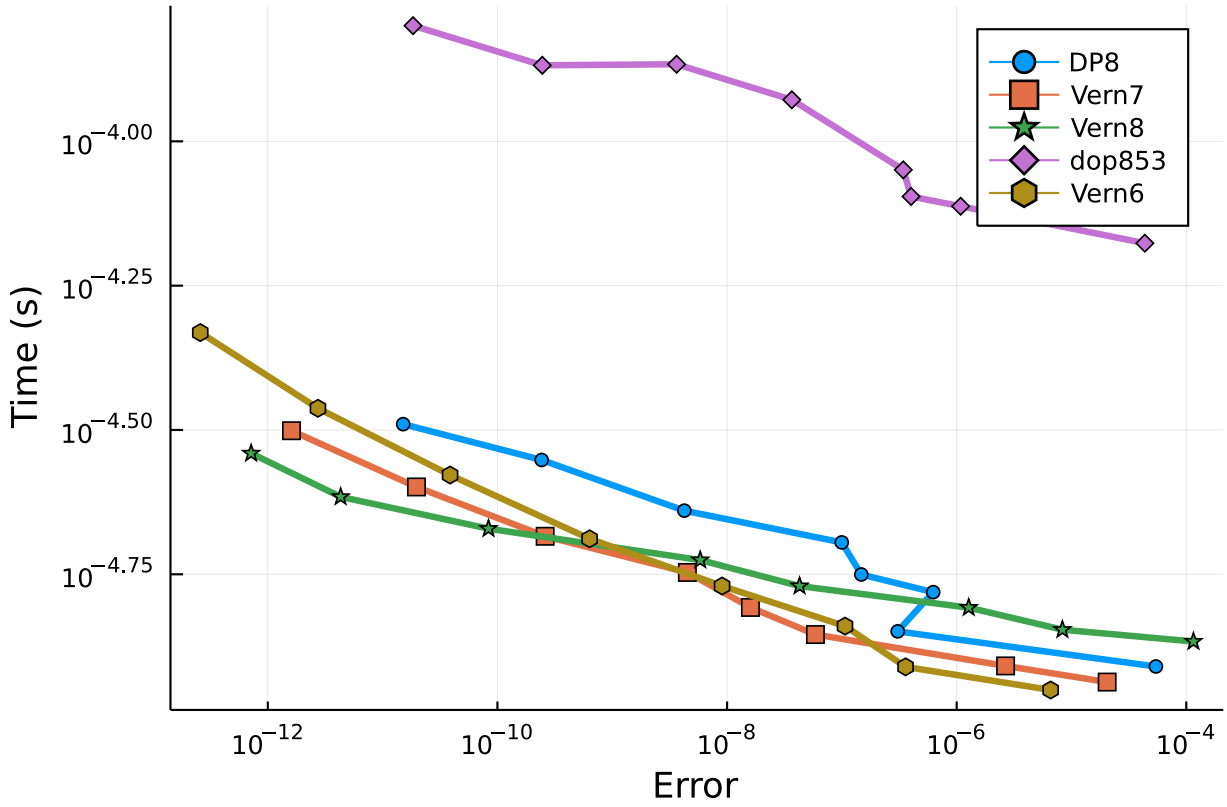
1.1.1 Interpolation

```
setups = [Dict(:alg=>DP5())
          #Dict(:alg=>ode45()) # fails
          Dict(:alg=>BS5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=10000,error_estimat
plot(wp)
```

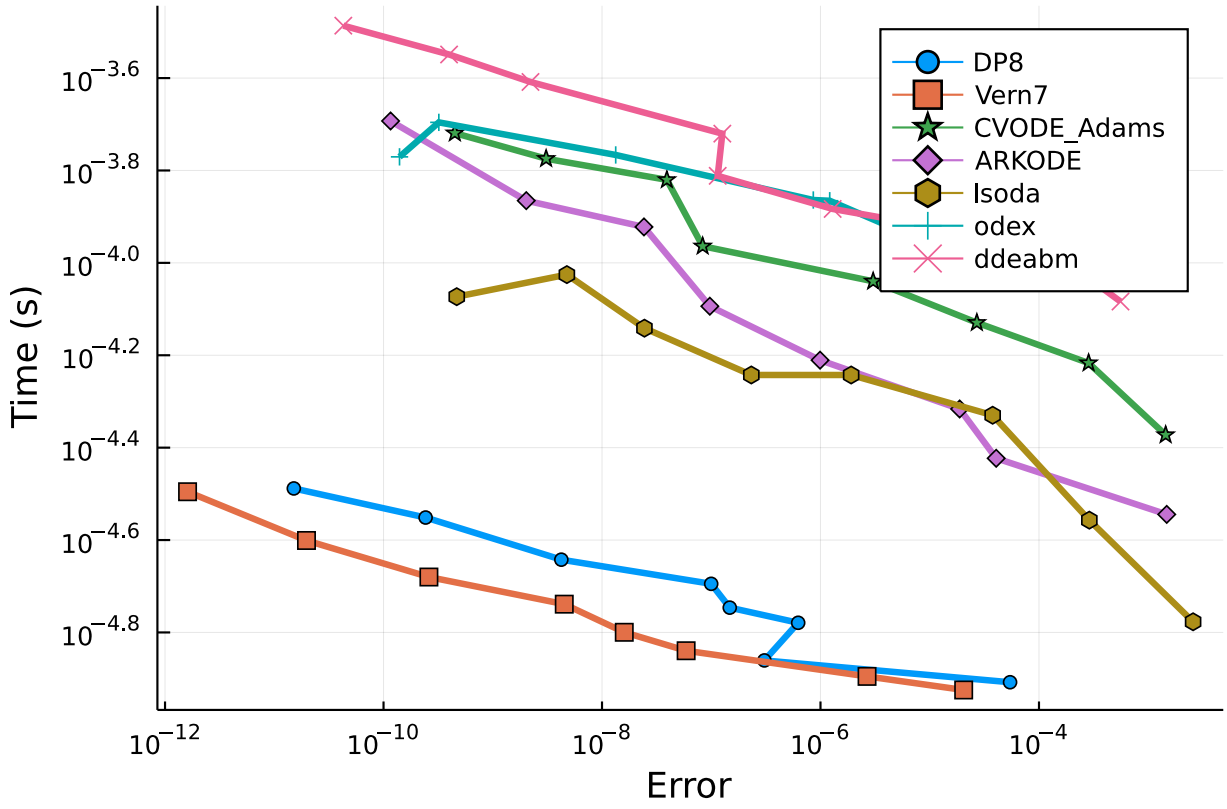


1.2 Higher Order

```
setups = [Dict(:alg=>DP8())
          #Dict(:alg=>ode78()) # fails
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>dop853())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```

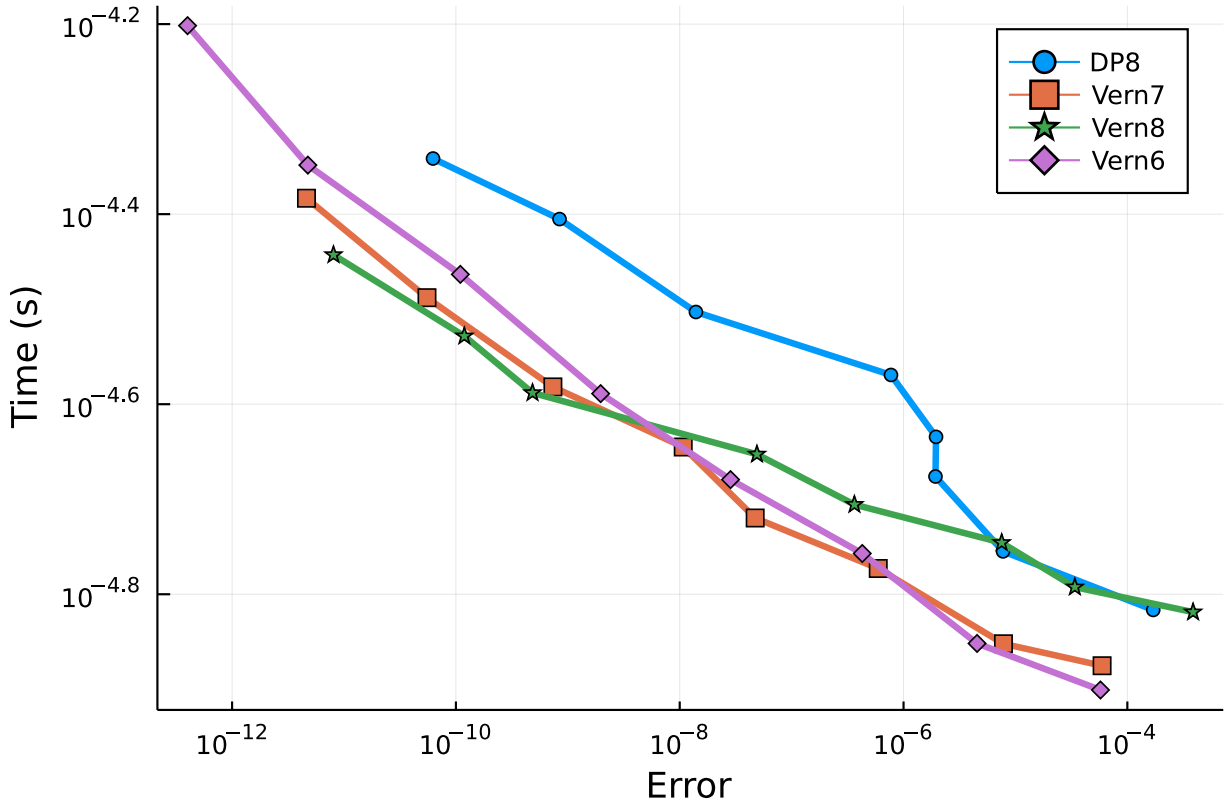



```
setups = [Dict(:alg=>DP8())
          Dict(:alg=>Vern7())
          Dict(:alg=>CVODE_Adams())
          Dict(:alg=>ARKODE(Sundials.Explicit(),order=6))
          Dict(:alg=>lsoda())
          Dict(:alg=>odex())
          Dict(:alg=>ddeabm())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



1.2.1 Interpolation

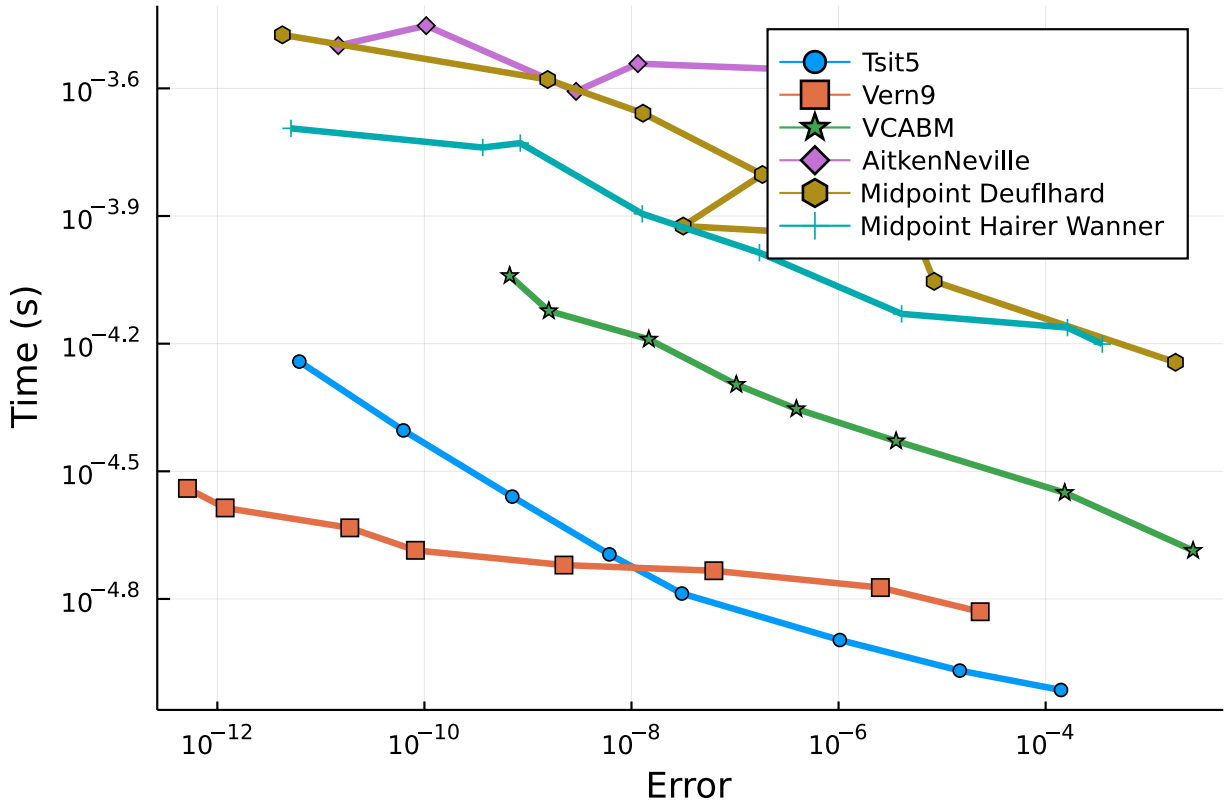
```
setups = [Dict(:alg=>DP8())
          #Dict(:alg=>ode78()) # fails
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=1000,error_estimate
plot(wp)
```



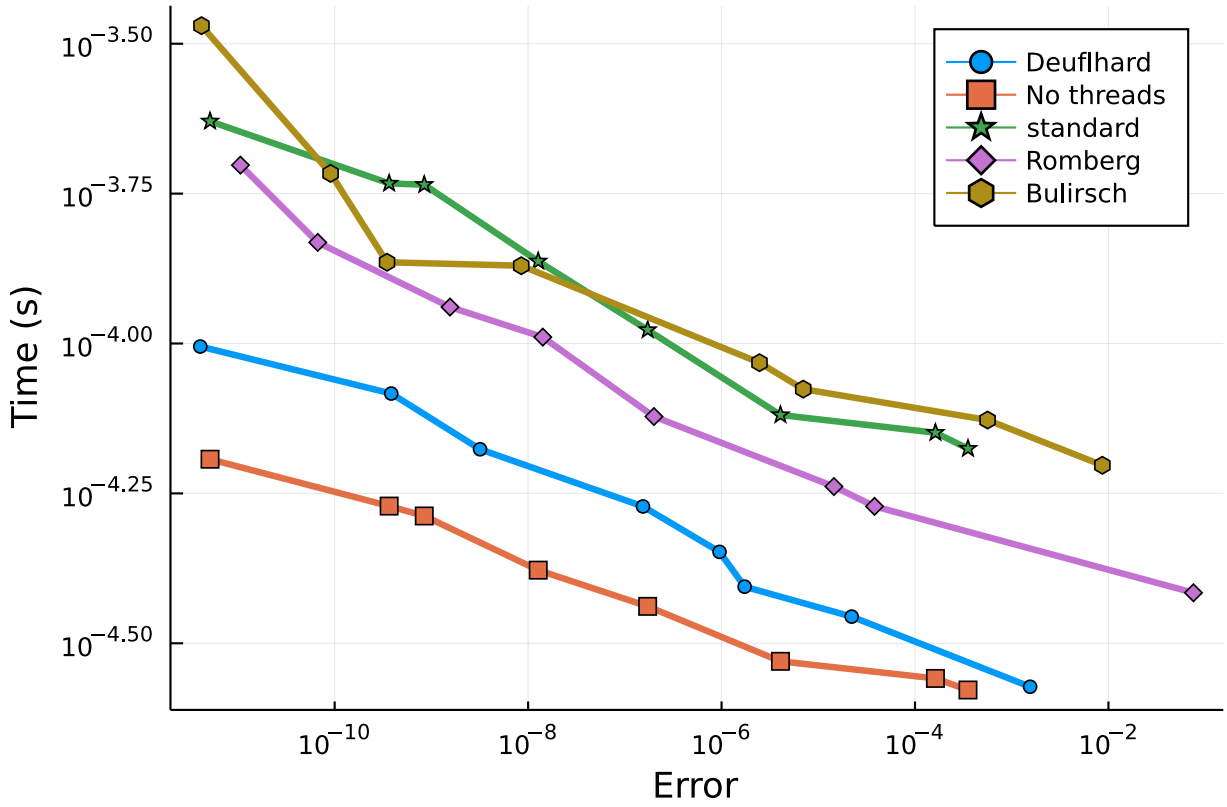
1.3 Comparison with Non-RK methods

Now let's test Tsit5 and Vern9 against parallel extrapolation methods and an Adams-Bashforth-Moulton:

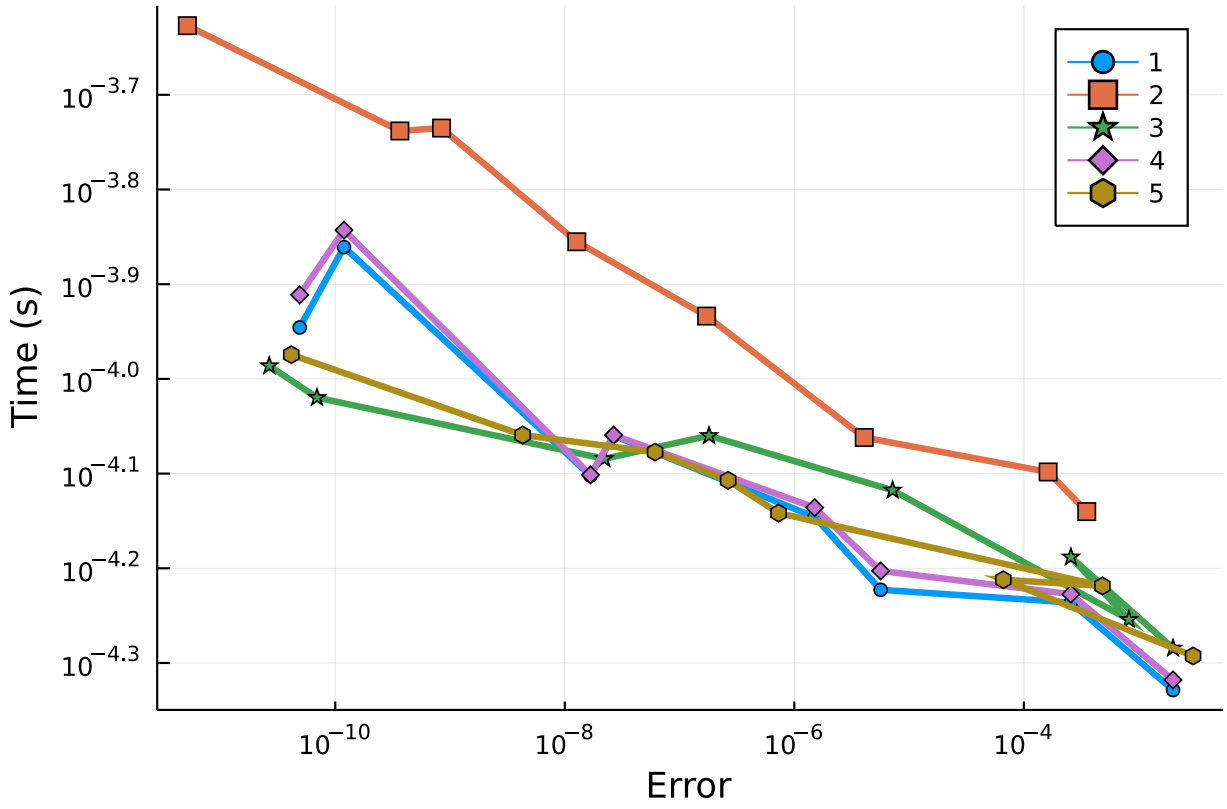
```
setups = [Dict(:alg=>Tsit5())
          Dict(:alg=>Vern9())
          Dict(:alg=>VCABM())
          Dict(:alg=>AitkenNeville(min_order=1, max_order=9, init_order=4,
threading=true))
          Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))]
solnames = ["Tsit5","Vern9","VCABM","AitkenNeville","Midpoint Deuflhard","Midpoint
Hairer Wanner"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```




```
setups = [Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=9, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :romberg, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :bulirsch, threading=true))]
solnames = ["Deuflhard","No threads","standard","Romberg","Bulirsch"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```




```
setups = [Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=15,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=7,
init_order=6, threading=true))]
solnames = ["1","2","3","4","5"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



1.4 Conclusion

As expected, the algorithms are all pretty matched on time for this problem. However, you can clearly see the OrdinaryDiffEq.jl algorithms solving to a much higher accuracy and still faster, especially when the interpolations are involved.

1.5 Appendix

These benchmarks are a part of the SciMLBenchmarks.jl repository, found at: https://github.com/SciML/
For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization https://sciml.ai.

To locally run this benchmark, do the following commands:

```
using SciMLBenchmarks
SciMLBenchmarks.weave_file("benchmarks/NonStiffODE","FitzhughNagumo_wpd.jmd")
```

Computer Information:

```
Julia Version 1.6.1
Commit 6aaedecc44 (2021-04-23 05:59 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
  WORD_SIZE: 64
```

```
  LIBM: libopenlibm
  LLVM: libLLVM-11.0.1 (ORCJIT, skylake)
Environment:
  JULIA_DEPOT_PATH = /root/.cache/julia-buildkite-plugin/depots/5b300254-1738-4989-ae0a
  JULIA_NUM_THREADS = 3
```

Package Information:

```
      Status `/var/lib/buildkite-agent/builds/rtx2070-gpuci1-julia-csail-mit-edu/julial
  [f3b72e0c] DiffEqDevTools v2.27.2
  [7f56f5a3] LSODA v0.7.0
  [c030b06c] ODE v2.13.0
  [54ca160b] ODEInterface v0.5.0
  [09606e27] ODEInterfaceDiffEq v3.10.0
  [1dea7af3] OrdinaryDiffEq v5.53.0
  [65888b18] ParameterizedFunctions v5.10.0
  [91a5bcdd] Plots v1.13.2
  [31c91b34] SciMLBenchmarks v0.1.0 `../..`
  [c3572dad] Sundials v4.4.3
  [9a3f8284] Random
```

And the full manifest:

```
      Status `/var/lib/buildkite-agent/builds/rtx2070-gpuci1-julia-csail-mit-edu/julial
  [c3fe647b] AbstractAlgebra v0.16.0
  [1520ce14] AbstractTrees v0.3.4
  [79e6a3ab] Adapt v3.3.0
  [ec485272] ArnoldiMethod v0.1.0
  [4fba245c] ArrayInterface v3.1.11
  [9e28174c] BinDeps v1.0.2
  [fa961155] CEnum v0.4.1
  [d360d2e6] ChainRulesCore v0.9.41
  [b630d9fa] CheapThreads v0.2.3
  [35d6a980] ColorSchemes v3.12.1
  [3da002f7] ColorTypes v0.11.0
  [5ae59095] Colors v0.12.8
  [861a8166] Combinatorics v1.0.2
  [38540f10] CommonSolve v0.2.0
  [bbf7d656] CommonSubexpressions v0.3.0
  [34da2185] Compat v3.28.0
  [8f4d0f93] Conda v1.5.2
  [187b0558] ConstructionBase v1.2.1
  [d38c429a] Contour v0.5.7
  [9a962f9c] DataAPI v1.6.0
  [864edb3b] DataStructures v0.18.9
  [e2d170a0] DataValueInterfaces v1.0.0
  [2b5f629d] DiffEqBase v6.61.0
```

```
[f3b72e0c] DiffEqDevTools v2.27.2
[c894b116] DiffEqJump v6.14.1
[77a26b50] DiffEqNoiseProcess v5.7.2
[163ba53b] DiffResults v1.0.3
[b552c78f] DiffRules v1.0.2
[b4f34e82] Distances v0.10.3
[31c24e10] Distributions v0.24.18
[ffbed154] DocStringExtensions v0.8.4
[d4d017d3] ExponentialUtilities v1.8.4
[e2ba6199] ExprTools v0.1.3
[8f5d6c58] EzXML v1.1.0
[c87230d0] FFMPEG v0.4.0
[7034ab61] FastBroadcast v0.1.4
[9aa1b823] FastClosures v0.3.2
[1a297f60] FillArrays v0.11.7
[6a86dc24] FiniteDiff v2.8.0
[53c48c17] FixedPointNumbers v0.8.4
[59287772] Formatting v0.4.2
[f6369f11] ForwardDiff v0.10.18
[069b7b12] FunctionWrappers v1.1.2
[28b8d3ca] GR v0.57.4
[5c1252a2] GeometryBasics v0.3.12
[d7ba0133] Git v1.2.1
[42e2da0e] Grisu v1.0.2
[cd3eb016] HTTP v0.9.8
[eafb193a] Highlights v0.4.5
[0e44f5e4] Hwloc v2.0.0
[7073ff75] IJulia v1.23.2
[615f187c] IfElse v0.1.0
[d25df0c9] Inflate v0.1.2
[83e8ac13] IniFile v0.5.0
[d8418881] Intervals v1.5.0
[c8e1da08] IterTools v1.3.0
[42fd0dbc] IterativeSolvers v0.9.0
[82899510] IteratorInterfaceExtensions v1.0.0
[692b3bcd] JLLWrappers v1.3.0
[682c06a0] JSON v0.21.1
[7f56f5a3] LSODA v0.7.0
[b964fa9f] LaTeXStrings v1.2.1
[2ee39098] LabelledArrays v1.6.0
[23fbe1c1] Latexify v0.15.5
[093fc24a] LightGraphs v1.3.5
[d3d80556] LineSearches v7.1.1
[2ab3a3ac] LogExpFunctions v0.2.3
[bdcacae8] LoopVectorization v0.12.18
[1914dd2f] MacroTools v0.5.6
[739be429] MbedTLS v1.0.3
[442fdcdd] Measures v0.3.1
[e1d29d7a] Missings v1.0.0
```

```
[78c3b35d] Mocking v0.7.1
[961ee093] ModelingToolkit v5.16.0
[46d2c3a1] MuladdMacro v0.2.2
[ffc61752] Mustache v1.0.10
[d8a4904e] MutableArithmetics v0.2.19
[d41bc354] NLSolversBase v7.8.0
[2774e3e8] NLsolve v4.5.1
[77ba4419] NaNMath v0.3.5
[8913a72c] NonlinearSolve v0.3.8
[c030b06c] ODE v2.13.0
[54ca160b] ODEInterface v0.5.0
[09606e27] ODEInterfaceDiffEq v3.10.0
[6fe1bfb0] OffsetArrays v1.7.0
[429524aa] Optim v1.3.0
[bac558e1] OrderedCollections v1.4.0
[1dea7af3] OrdinaryDiffEq v5.53.0
[90014a1f] PDMats v0.11.0
[65888b18] ParameterizedFunctions v5.10.0
[d96e819e] Parameters v0.12.2
[69de0a69] Parsers v1.1.0
[ccf2f8ad] PlotThemes v2.0.1
[995b91a9] PlotUtils v1.0.10
[91a5bcdd] Plots v1.13.2
[e409e4f3] PoissonRandom v0.4.0
[f27b6e38] Polynomials v2.0.10
[85a6dd25] PositiveFactorizations v0.2.4
[21216c6a] Preferences v1.2.1
[1fd47b50] QuadGK v2.4.1
[74087812] Random123 v1.3.1
[fb686558] RandomExtensions v0.4.3
[e6cf234a] RandomNumbers v1.4.0
[3cdcf5f2] RecipesBase v1.1.1
[01d81517] RecipesPipeline v0.3.2
[731186ca] RecursiveArrayTools v2.11.3
[f2c3362d] RecursiveFactorization v0.1.12
[189a3867] Reexport v1.0.0
[ae029012] Requires v1.1.3
[ae5879a3] ResettableStacks v1.1.0
[79098fc4] Rmath v0.7.0
[47965b36] RootedTrees v1.0.0
[7e49a35a] RuntimeGeneratedFunctions v0.5.2
[476501e8] SLEEFPirates v0.6.17
[1bc83da4] SafeTestsets v0.0.1
[0bca4576] SciMLBase v1.13.2
[31c91b34] SciMLBenchmarks v0.1.0 `../..`
[6c6a2e73] Scratch v1.0.3
[efcf1570] Setfield v0.7.0
[992d4aef] Showoff v1.0.3
[699a6c99] SimpleTraits v0.9.3
```

```
[b85f4697] SoftGlobalScope v1.1.0
[a2af1166] SortingAlgorithms v1.0.0
[47a9eef4] SparseDiffTools v1.13.2
[276daf66] SpecialFunctions v1.3.0
[aedffcd0] Static v0.2.4
[90137ffa] StaticArrays v1.1.3
[82ae8749] StatsAPI v1.0.0
[2913bbd2] StatsBase v0.33.8
[4c63d2b9] StatsFuns v0.9.8
[7792a7ef] StrideArraysCore v0.1.7
[09ab397b] StructArrays v0.5.1
[c3572dad] Sundials v4.4.3
[d1185830] SymbolicUtils v0.11.2
[0c5d862f] Symbolics v0.1.25
[3783bdb8] TableTraits v1.0.1
[bd369af6] Tables v1.4.2
[8290d209] ThreadingUtilities v0.4.1
[f269a46b] TimeZones v1.5.4
[a759f4b9] TimerOutputs v0.5.8
[a2a6695c] TreeViews v0.3.0
[30578b45] URIParser v0.4.1
[5c2747f8] URIs v1.3.0
[3a884ed6] UnPack v1.0.2
[1986cc42] Unitful v1.7.0
[3d5dd08c] VectorizationBase v0.19.37
[81def892] VersionParsing v1.2.0
[19fa3120] VertexSafeGraphs v0.1.2
[44d3d7a6] Weave v0.10.8
[ddb6d928] YAML v0.4.6
[c2297ded] ZMQ v1.2.1
[700de1a5] ZygoteRules v0.2.1
[6e34b625] Bzip2_jll v1.0.6+5
[83423d85] Cairo_jll v1.16.0+6
[5ae413db] EarCut_jll v2.1.5+1
[2e619515] Expat_jll v2.2.7+6
[b22a6f82] FFMPEG_jll v4.3.1+4
[a3f928ae] Fontconfig_jll v2.13.1+14
[d7e528f0] FreeType2_jll v2.10.1+5
[559328eb] FriBidi_jll v1.0.5+6
[0656b61e] GLFW_jll v3.3.4+0
[d2c73de3] GR_jll v0.57.2+0
[78b55507] Gettext_jll v0.20.1+7
[f8c6e375] Git_jll v2.31.0+0
[7746bdde] Glib_jll v2.59.0+4
[e33a78d0] Hwloc_jll v2.4.1+0
[aacddb02] JpegTurbo_jll v2.0.1+3
[c1c5ebd0] LAME_jll v3.100.0+3
[aae0fff6] LSODA_jll v0.1.1+0
[dd4b983a] LZO_jll v2.10.0+3
```

```
[dd192d2f] LibVPX_jll v1.9.0+1
[e9f186c6] Libffi_jll v3.2.1+4
[d4300ac3] Libgcrypt_jll v1.8.5+4
[7e76a0d4] Libglvnd_jll v1.3.0+3
[7add5ba3] Libgpg_error_jll v1.36.0+3
[94ce4f54] Libiconv_jll v1.16.0+7
[4b2f31a3] Libmount_jll v2.34.0+3
[89763e89] Libtiff_jll v4.1.0+2
[38a345b3] Libuuid_jll v2.34.0+7
[c771fb93] ODEInterface_jll v0.0.1+0
[e7412a2a] Ogg_jll v1.3.4+2
[458c3c95] OpenSSL_jll v1.1.1+6
[efe28fd5] OpenSpecFun_jll v0.5.4+0
[91d4177d] Opus_jll v1.3.1+3
[2f80f16e] PCRE_jll v8.42.0+4
[30392449] Pixman_jll v0.40.0+0
[ea2cea3b] Qt5Base_jll v5.15.2+0
[f50d1b31] Rmath_jll v0.3.0+0
[fb77eaff] Sundials_jll v5.2.0+1
[a2964d1f] Wayland_jll v1.17.0+4
[2381bf8a] Wayland_protocols_jll v1.18.0+4
[02c8fc9c] XML2_jll v2.9.11+0
[aed1982a] XSLT_jll v1.1.33+4
[4f6342f7] Xorg_libX11_jll v1.6.9+4
[0c0b7dd1] Xorg_libXau_jll v1.0.9+4
[935fb764] Xorg_libXcursor_jll v1.2.0+4
[a3789734] Xorg_libXdmcp_jll v1.1.3+4
[1082639a] Xorg_libXext_jll v1.3.4+4
[d091e8ba] Xorg_libXfixes_jll v5.0.3+4
[a51aa0fd] Xorg_libXi_jll v1.7.10+4
[d1454406] Xorg_libXinerama_jll v1.1.4+4
[ec84b674] Xorg_libXrandr_jll v1.5.2+4
[ea2f1a96] Xorg_libXrender_jll v0.9.10+4
[14d82f49] Xorg_libpthread_stubs_jll v0.1.0+3
[c7cfdc94] Xorg_libxcb_jll v1.13.0+3
[cc61e674] Xorg_libxkbfile_jll v1.1.0+4
[12413925] Xorg_xcb_util_image_jll v0.4.0+1
[2def613f] Xorg_xcb_util_jll v0.4.0+1
[975044d2] Xorg_xcb_util_keysyms_jll v0.4.0+1
[0d47668e] Xorg_xcb_util_renderutil_jll v0.3.9+1
[c22f9ab0] Xorg_xcb_util_wm_jll v0.4.1+1
[35661453] Xorg_xkbcomp_jll v1.4.2+4
[33bec58e] Xorg_xkeyboard_config_jll v2.27.0+4
[c5fb5394] Xorg_xtrans_jll v1.4.0+3
[8f1865be] ZeroMQ_jll v4.3.2+6
[3161d3a3] Zstd_jll v1.4.8+0
[0ac62f75] libass_jll v0.14.0+4
[f638f0a6] libfdk_aac_jll v0.1.6+4
[b53b4c65] libpng_jll v1.6.37+6
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[a9144af2] libsodium_jll v1.0.20+0
[f27f6e37] libvorbis_jll v1.3.6+6
[1270edf5] x264_jll v2020.7.14+2
[dfaa095f] x265_jll v3.0.0+3
[d8fb68d0] xkbcommon_jll v0.9.1+5
[0dad84c5] ArgTools
[56f22d72] Artifacts
[2a0f44e3] Base64
[ade2ca70] Dates
[8bb1440f] DelimitedFiles
[8ba89e20] Distributed
[f43a241f] Downloads
[7b1f6079] FileWatching
[9fa8497b] Future
[b77e0a4c] InteractiveUtils
[b27032c2] LibCURL
[76f85450] LibGit2
[8f399da3] Libdl
[37e2e46d] LinearAlgebra
[56ddb016] Logging
[d6f4376e] Markdown
[a63ad114] Mmap
[ca575930] NetworkOptions
[44cfe95a] Pkg
[de0858da] Printf
[3fa0cd96] REPL
[9a3f8284] Random
[ea8e919c] SHA
[9e88b42a] Serialization
[1a1011a3] SharedArrays
[6462fe0b] Sockets
[2f01184e] SparseArrays
[10745b16] Statistics
[4607b0f0] SuiteSparse
[fa267f1f] TOML
[a4e569a6] Tar
[8dfed614] Test
[cf7118a7] UUIDs
[4ec0a83e] Unicode
[e66e0078] CompilerSupportLibraries_jll
[deac9b47] LibCURL_jll
[29816b5a] LibSSH2_jll
[c8ffd9c3] MbedTLS_jll
[14a3606d] MozillaCACerts_jll
[4536629a] OpenBLAS_jll
[efcefdf7] PCRE2_jll
[bea87d4a] SuiteSparse_jll
[83775a58] Zlib_jll
[8e850ede] nghttp2_jll
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[3f19e933] p7zip_jll
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