

# 100 Independent Linear Work-Precision Diagrams

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For these tests we will solve a diagonal 100 independent linear differential equations. This will demonstrate the efficiency of the implementation of the methods for handling large systems, since the system is both large enough that array handling matters, but `f` is cheap enough that it is not simply a game of calculating `f` as few times as possible. We will be mostly looking at the efficiency of the work-horse Dormand-Prince Order 4/5 Pairs: one from `DifferentialEquations.jl` (DP5), one from `ODE.jl` `rk45`, one from `ODEInterface` (Hairer's famous `dopri5`, and one from SUNDIALS' ARKODE suite.

Also included is `Tsit5`. While all other ODE programs have gone with the traditional choice of using the Dormand-Prince 4/5 pair as the default, `DifferentialEquations.jl` uses `Tsit5` as one of the default algorithms. It's a very new (2011) and not widely known, but the theory and the implementation shows it's more efficient than DP5. Thus we include it just to show off how re-designing a library from the ground up in a language for rapid code and rapid development has its advantages.

## 0.1 Setup

```
using OrdinaryDiffEq, Sundials, DiffEqDevTools, Plots, ODEInterfaceDiffEq, ODE, LSODA
using Random
Random.seed!(123)
gr()
# 2D Linear ODE
function f(du,u,p,t)
    @inbounds for i in eachindex(u)
        du[i] = 1.01*u[i]
    end
end
function f_analytic(u_0,p,t)
    u_0*exp(1.01*t)
end
tspan = (0.0,10.0)
prob = ODEProblem(ODEFunction(f,analytic=f_analytic),rand(100,100),tspan)

abstols = 1.0 ./ 10.0 .^ (3:13)
reltols = 1.0 ./ 10.0 .^ (0:10);
```

### 0.1.1 Speed Baseline

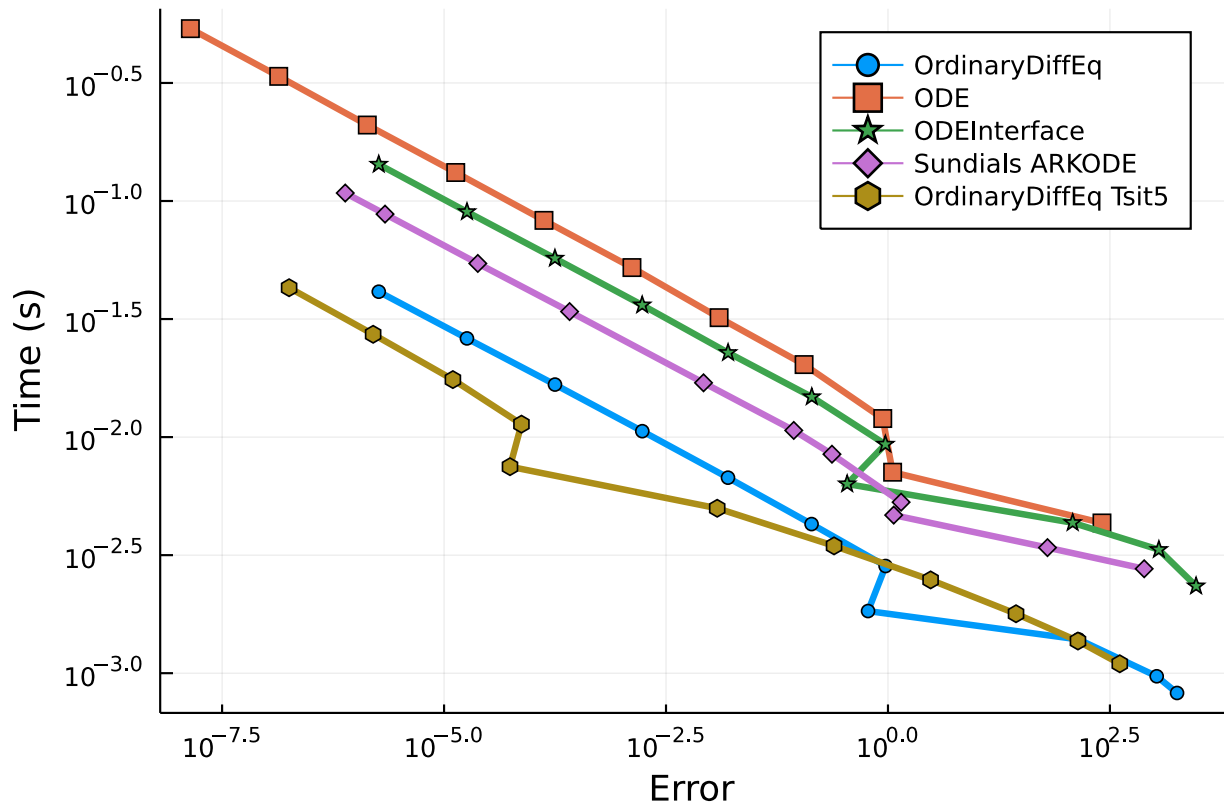
First a baseline. These are all testing the same Dormand-Prince order 5/4 algorithm of each package. While all the same Runge-Kutta tableau, they exhibit different behavior due to

different choices of adaptive timestepping algorithms and tuning. First we will test with all extra saving features are turned off to put DifferentialEquations.jl in "speed mode".

```

setups = [Dict(:alg=>DP5())
          Dict(:alg=>ode45())
          Dict(:alg=>dopri5())
          Dict(:alg=>ARKODE(Sundials.Explicit(), etable=Sundials.DORMAND_PRINCE_7_4_5))
          Dict(:alg=>Tsit5())]
solnames = ["OrdinaryDiffEq"; "ODE"; "ODEInterface"; "Sundials ARKODE"; "OrdinaryDiffEq
Tsit5"]
wp =
WorkPrecisionSet(prob, abstols, reltols, setups; names=solnames, save_everystep=false, numruns=100)
plot(wp)

```



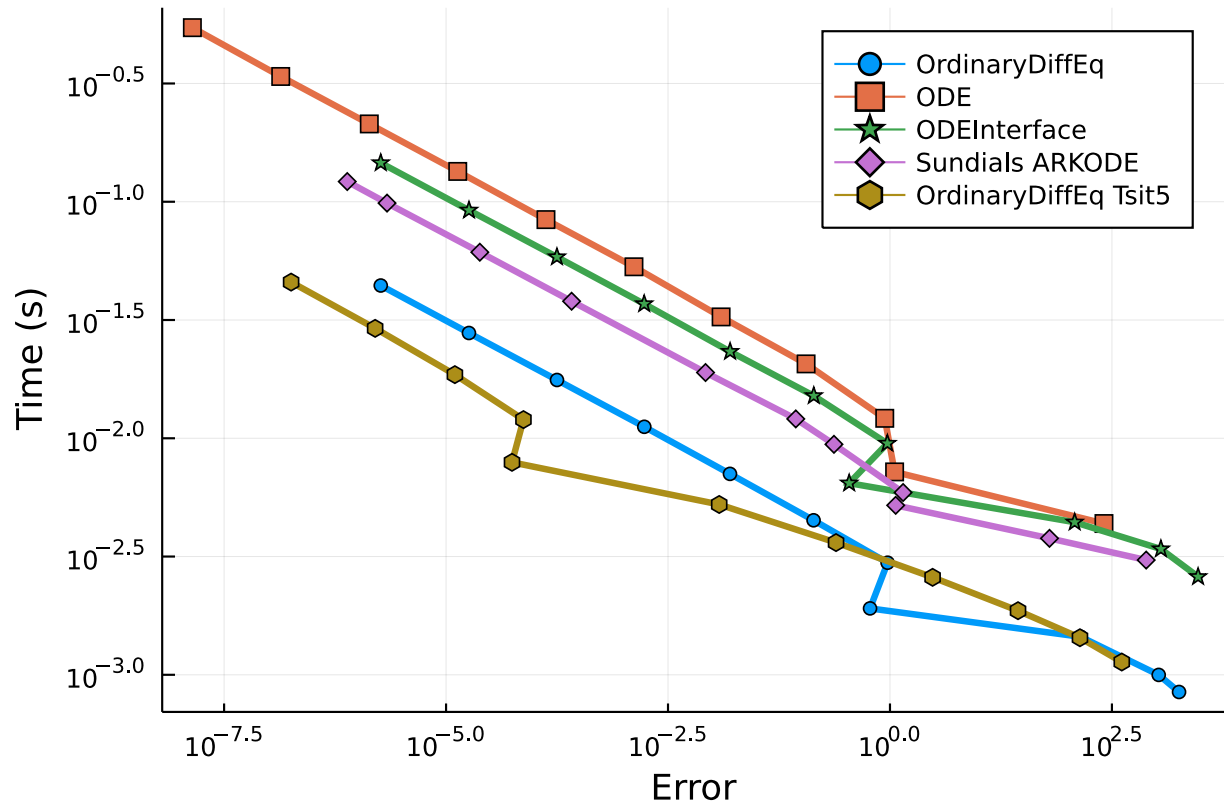
### 0.1.2 Full Saving

```

setups = [Dict(:alg=>DP5(), :dense=>false)
          Dict(:alg=>ode45(), :dense=>false)
          Dict(:alg=>dopri5()) # dense=false by default: no nonlinear interpolation

          Dict(:alg=>ARKODE(Sundials.Explicit(), etable=Sundials.DORMAND_PRINCE_7_4_5), :dense=>false)
          Dict(:alg=>Tsit5(), :dense=>false)]
solnames = ["OrdinaryDiffEq"; "ODE"; "ODEInterface"; "Sundials ARKODE"; "OrdinaryDiffEq
Tsit5"]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; names=solnames, numruns=100)
plot(wp)

```



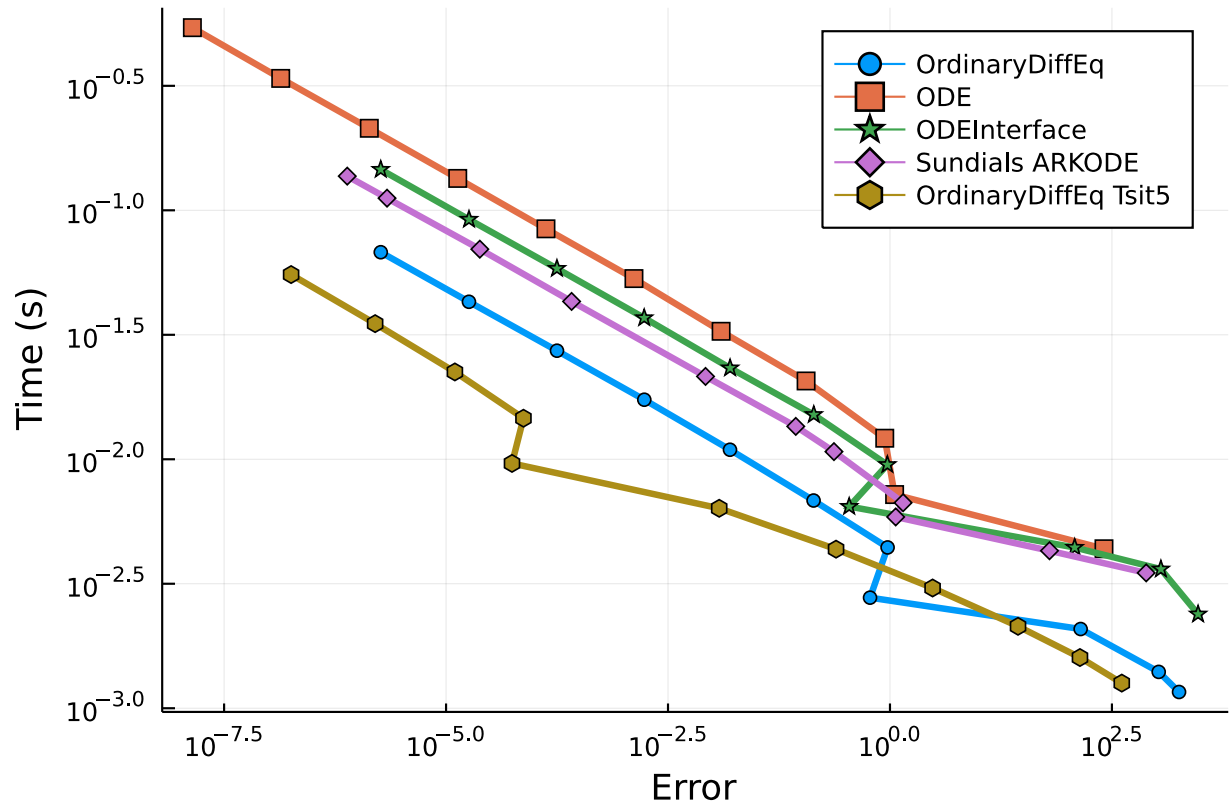
### 0.1.3 Continuous Output

Now we include continuous output. This has a large overhead because at every timepoint the matrix of rates  $k$  has to be deep copied.

```

setups = [Dict(:alg=>DP5())
           Dict(:alg=>ode45())
           Dict(:alg=>dopri5())
           Dict(:alg=>ARKODE(Sundials.Explicit(), etable=Sundials.DORMAND_PRINCE_7_4_5))
           Dict(:alg=>Tsit5())]
solnames = ["OrdinaryDiffEq"; "ODE"; "ODEInterface"; "Sundials ARKODE"; "OrdinaryDiffEq Tsit5"]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; names=solnames, numruns=100)
plot(wp)

```



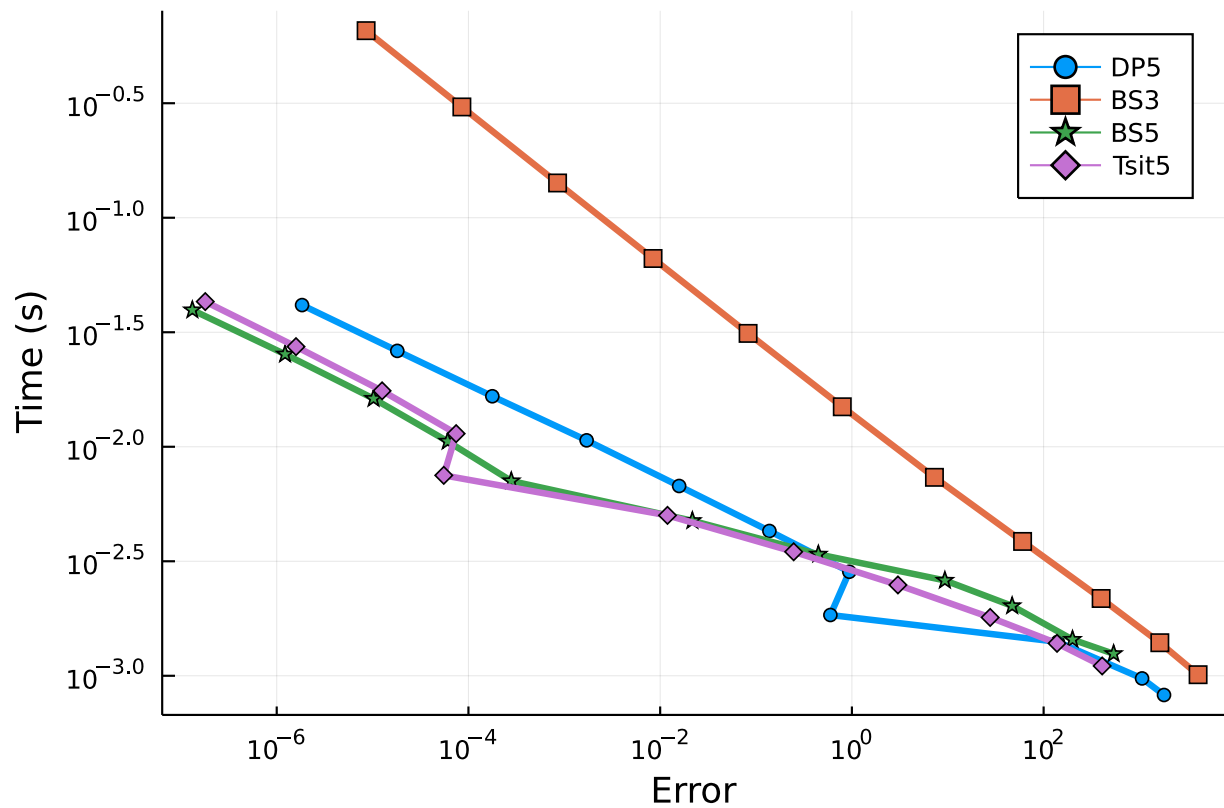
#### 0.1.4 Other Runge-Kutta Algorithms

Now let's test it against a smattering of other Runge-Kutta algorithms. First we will test it with all overheads off. Let's do the Order 5 (and the 2/3 pair) algorithms:

```

setups = [Dict(:alg=>DP5())
           Dict(:alg=>BS3())
           Dict(:alg=>BS5())
           Dict(:alg=>Tsit5())]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; save_everystep=false, numruns=100)
plot(wp)

```



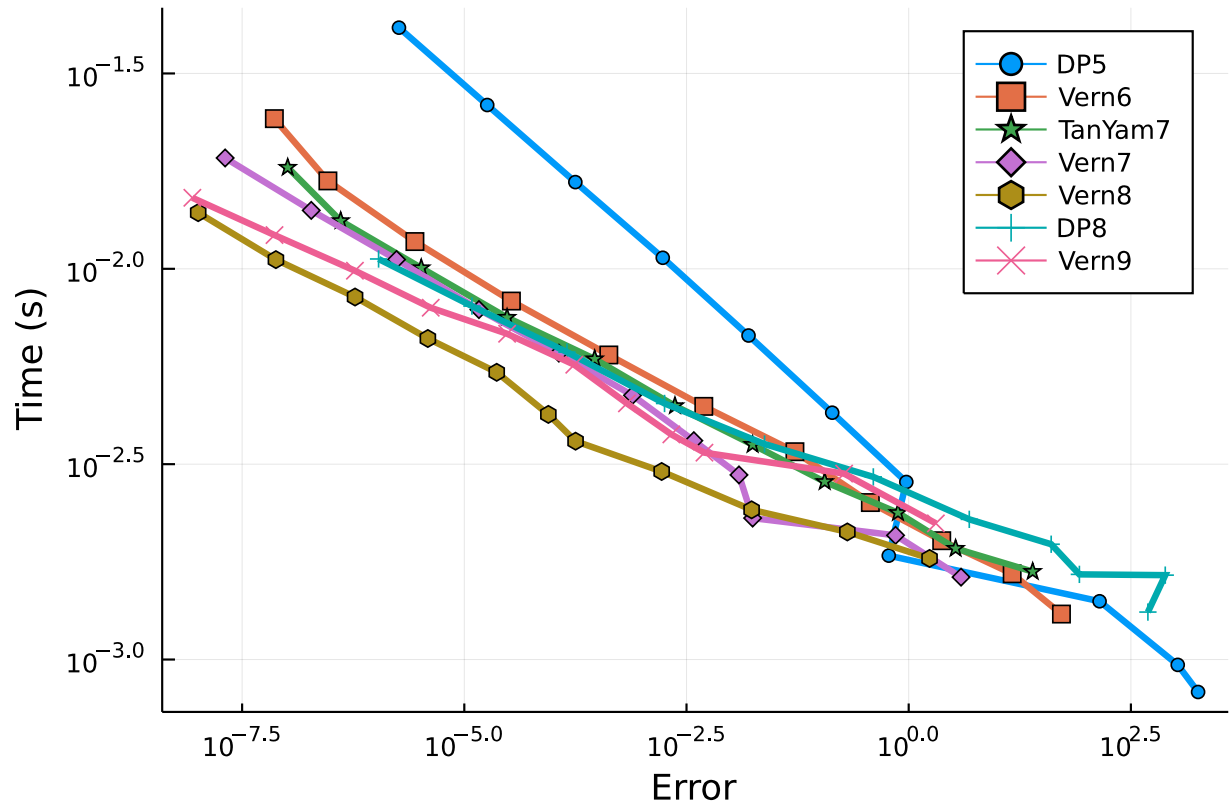
## 0.2 Higher Order

Now let's see how OrdinaryDiffEq.jl fairs with some higher order algorithms:

```

setups = [Dict{:alg=>DP5()}
          Dict{:alg=>Vern6()}
          Dict{:alg=>TanYam7()}
          Dict{:alg=>Vern7()}
          Dict{:alg=>Vern8()}
          Dict{:alg=>DP8()}
          Dict{:alg=>Vern9()}]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; save_everystep=false, numruns=100)
plot(wp)

```



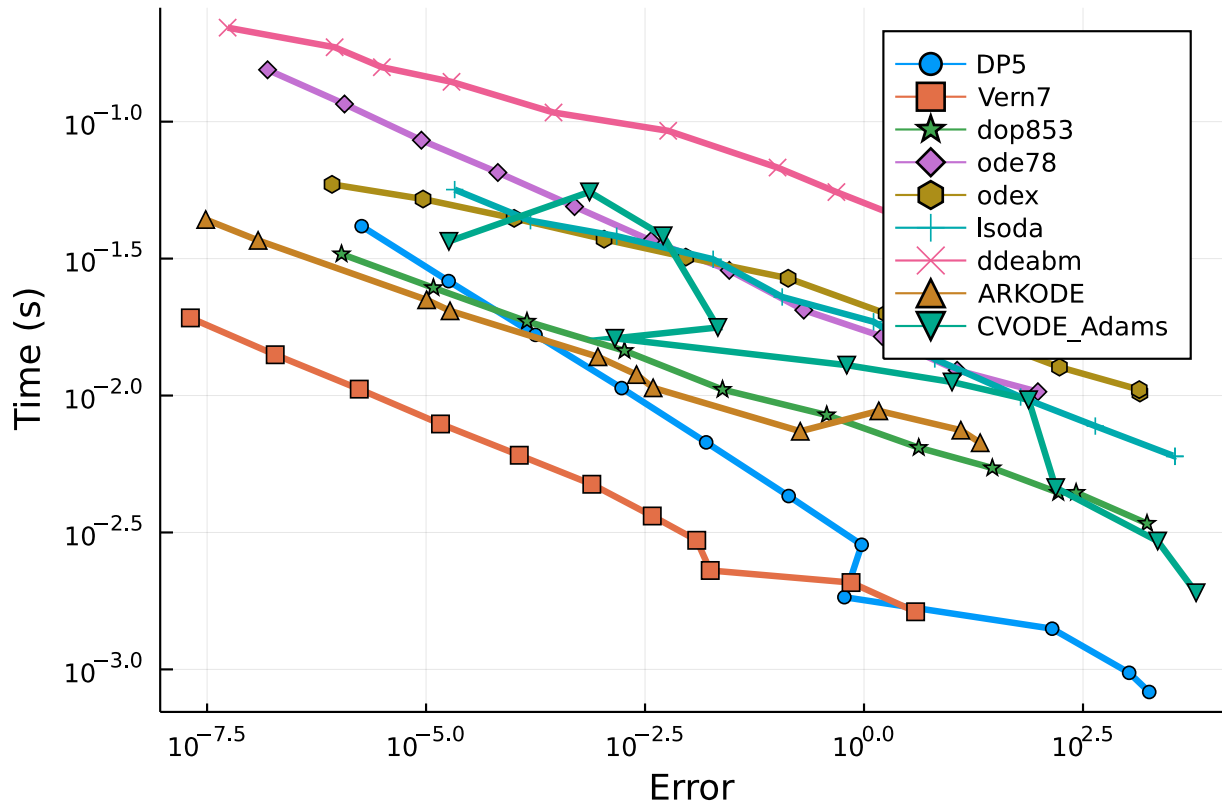
### 0.3 Higher Order With Many Packages

Now we test OrdinaryDiffEq against the high order methods of the other packages:

```

setups = [Dict(:alg=>DP5())
          Dict(:alg=>Vern7())
          Dict(:alg=>dop853())
          Dict(:alg=>ode78())
          Dict(:alg=>odex())
          Dict(:alg=>lsoda())
          Dict(:alg=>ddeabm())
          Dict(:alg=>ARKODE(Sundials.Explicit(),order=8))
          Dict(:alg=>CVODE_Adams())]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; save_everystep=false, numruns=100)
plot(wp)

```



## 0.4 Interpolation Error

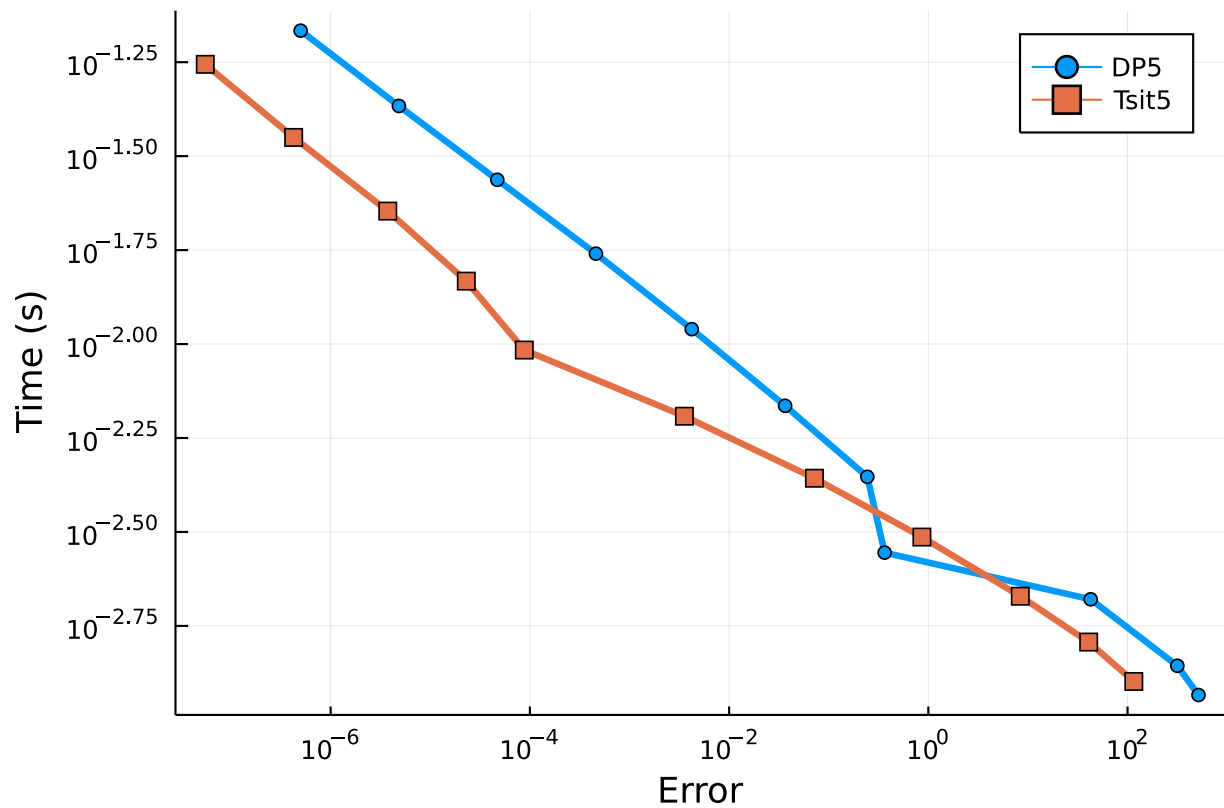
Now we will look at the error using an interpolation measurement instead of at the timestep-ping points. Since the DifferentialEquations.jl algorithms have higher order interpolants than the ODE.jl algorithms, one would expect this would magnify the difference. First the order 4/5 comparison:

```

setups = [Dict(:alg=>DP5())
           #Dict(:alg=>ode45())
           Dict(:alg=>Tsit5())]

wp =
WorkPrecisionSet(prob, abstols, reltols, setups; error_estimate=:L2, dense_errors=true, numruns=100)
plot(wp)

```



Note that all of ODE.jl uses a 3rd order Hermite interpolation, while the DifferentialEquations algorithms interpolations which are specialized to the algorithm. For example, DP5 and Tsit5 both use "free" order 4 interpolations, which are both as fast as the Hermite interpolation while achieving far less error. At higher order:

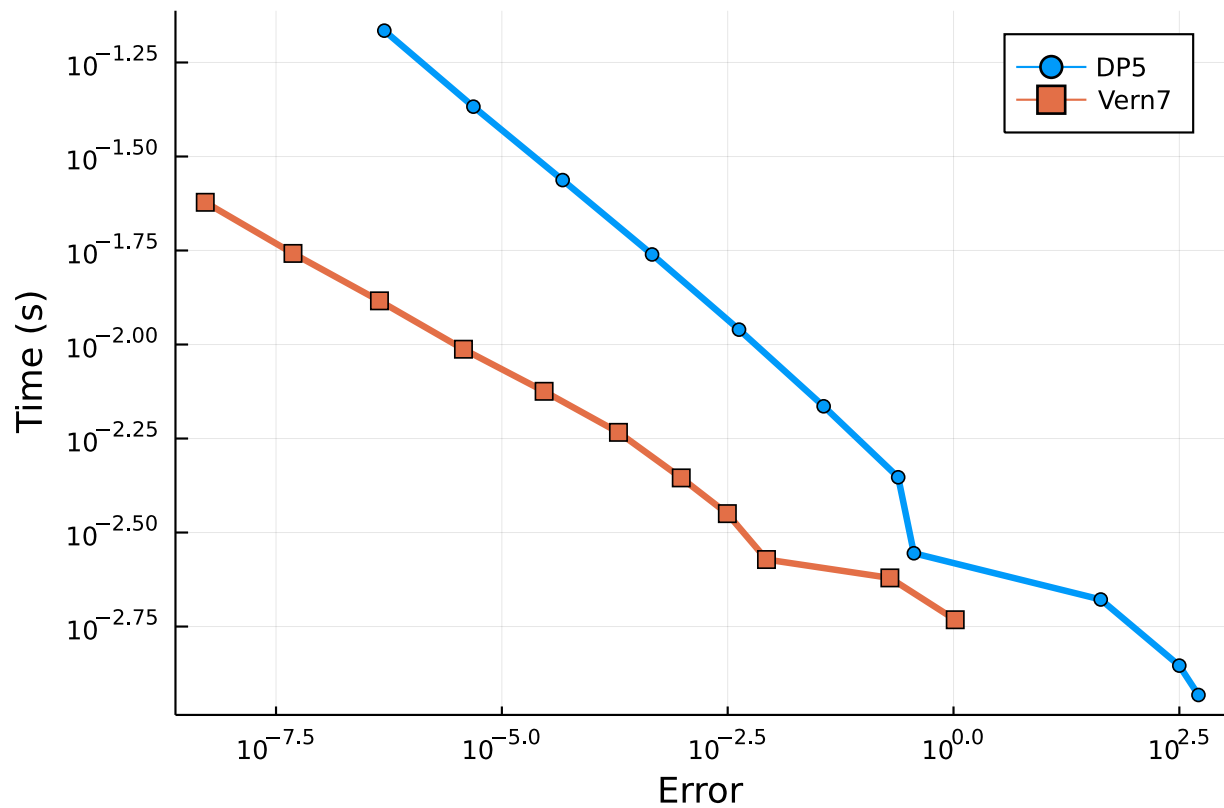
```

setups = [Dict{:alg=>DP5()}
           Dict{:alg=>Vern7()}
           #Dict{:alg=>ode78()}
          ]

wp =
WorkPrecisionSet(prob, abstols, reltols, setups; error_estimate=:L2, dense_errors=true, numruns=100)
plot(wp)

```

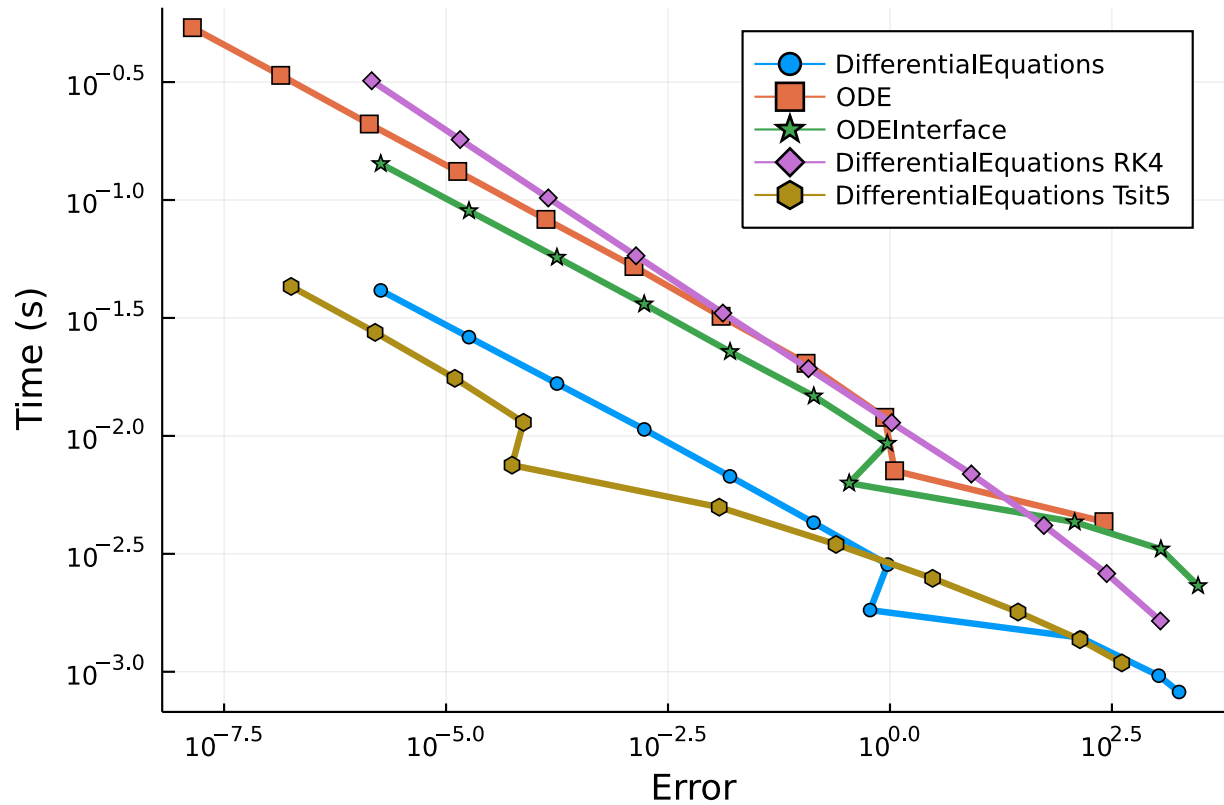




## 0.5 Comparison with Fixed Timestep RK4

Let's run the first benchmark but add some fixed timestep RK4 methods to see the difference:

```
abstols = 1.0 ./ 10.0 .^ (3:13)
reltols = 1.0 ./ 10.0 .^ (0:10);
dts = [1,1/2,1/4,1/10,1/20,1/40,1/60,1/80,1/100,1/140,1/240]
setups = [Dict(:alg=>DP5())
          Dict(:alg=>ode45())
          Dict(:alg=>dopri5())
          Dict(:alg=>RK4(),:dts=>dts)
          Dict(:alg=>Tsit5())]
solnames = ["DifferentialEquations";"ODE";"ODEInterface";"DifferentialEquations
RK4";"DifferentialEquations Tsit5"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



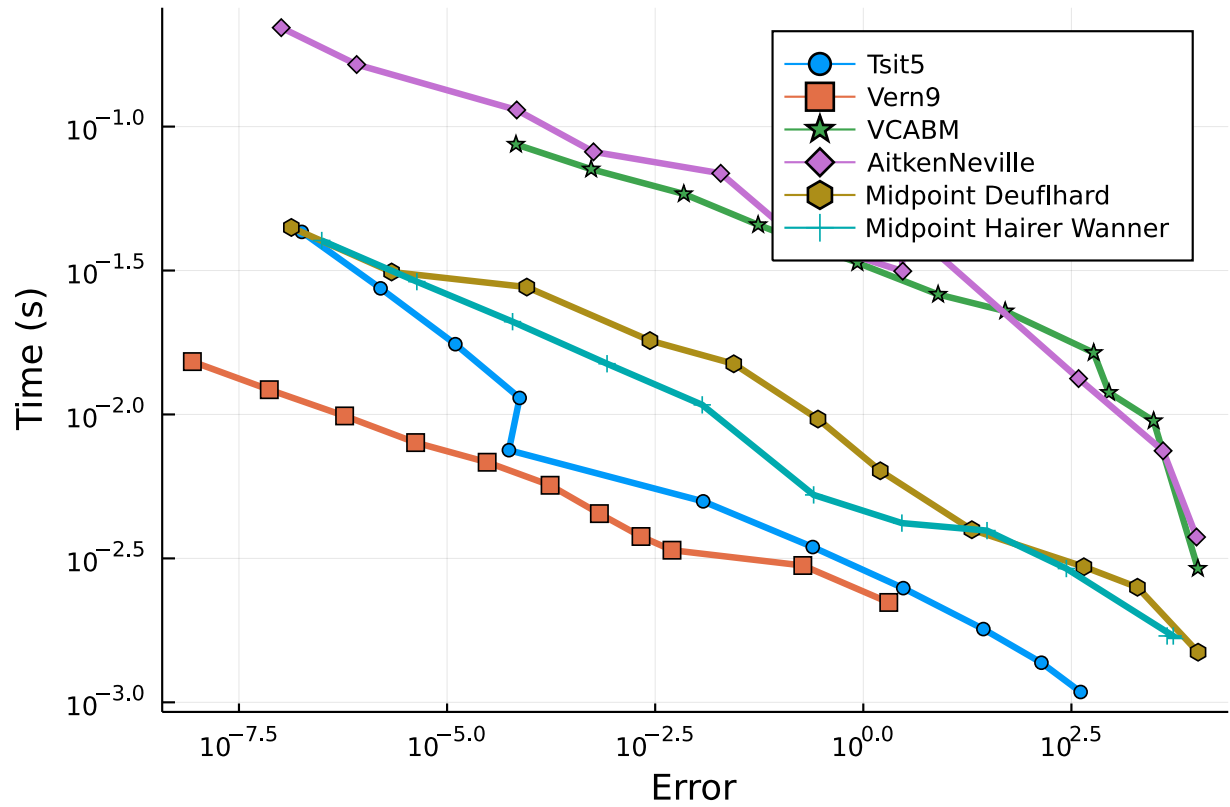
## 0.6 Comparison with Non-RK methods

Now let's test Tsit5 and Vern9 against parallel extrapolation methods and an Adams-Bashforth-Moulton:

```

setups = [Dict(:alg=>Tsit5())
          Dict(:alg=>Vern9())
          Dict(:alg=>VCABM())
          Dict(:alg=>AitkenNeville(min_order=1, max_order=9, init_order=4,
threading=true))
          Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))]
solnames = ["Tsit5", "Vern9", "VCABM", "AitkenNeville", "Midpoint Deuflhard", "Midpoint
Hairer Wanner"]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; names=solnames,
                      save_everystep=false, verbose=false, numruns=100)
plot(wp)

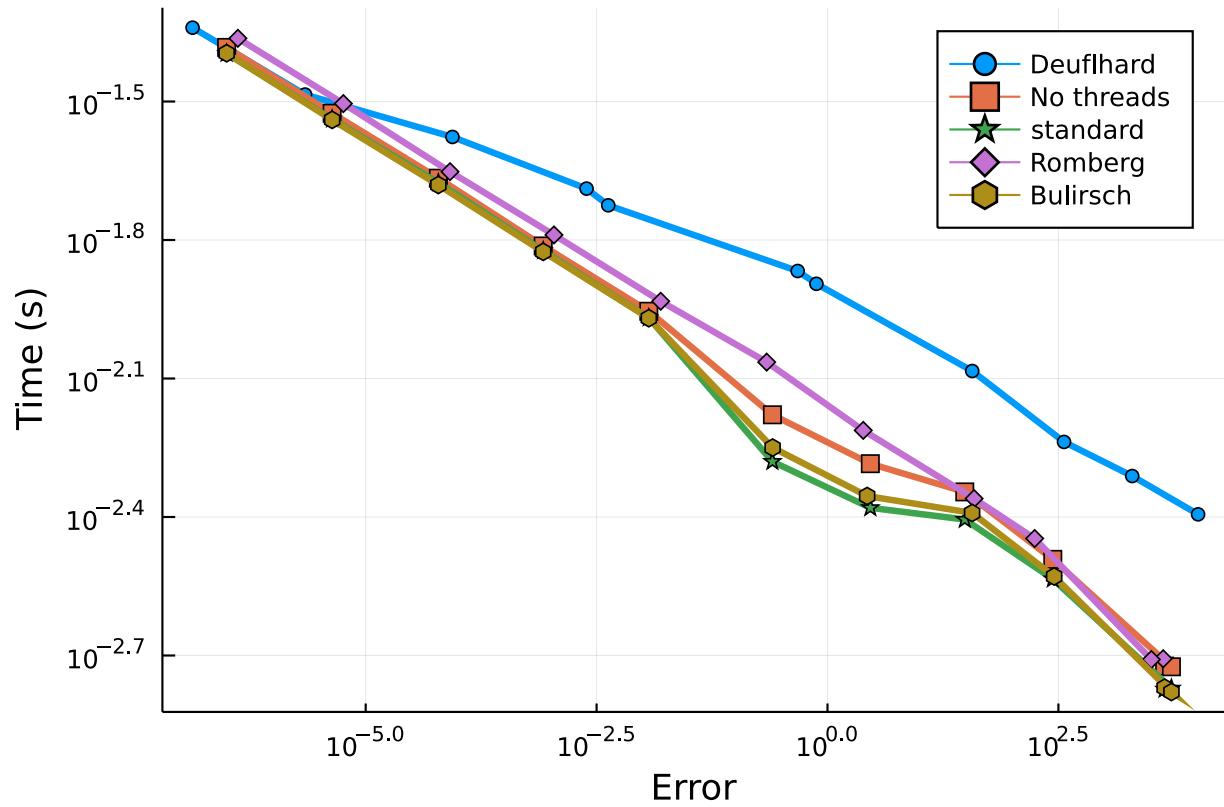
```



```

setups = [Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=9, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :romberg, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :bulirsch, threading=true))]
solnames = ["Deuflhard", "No threads", "standard", "Romberg", "Bulirsch"]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; names=solnames,
                      save_everystep=false, verbose=false, numruns=100)
plot(wp)

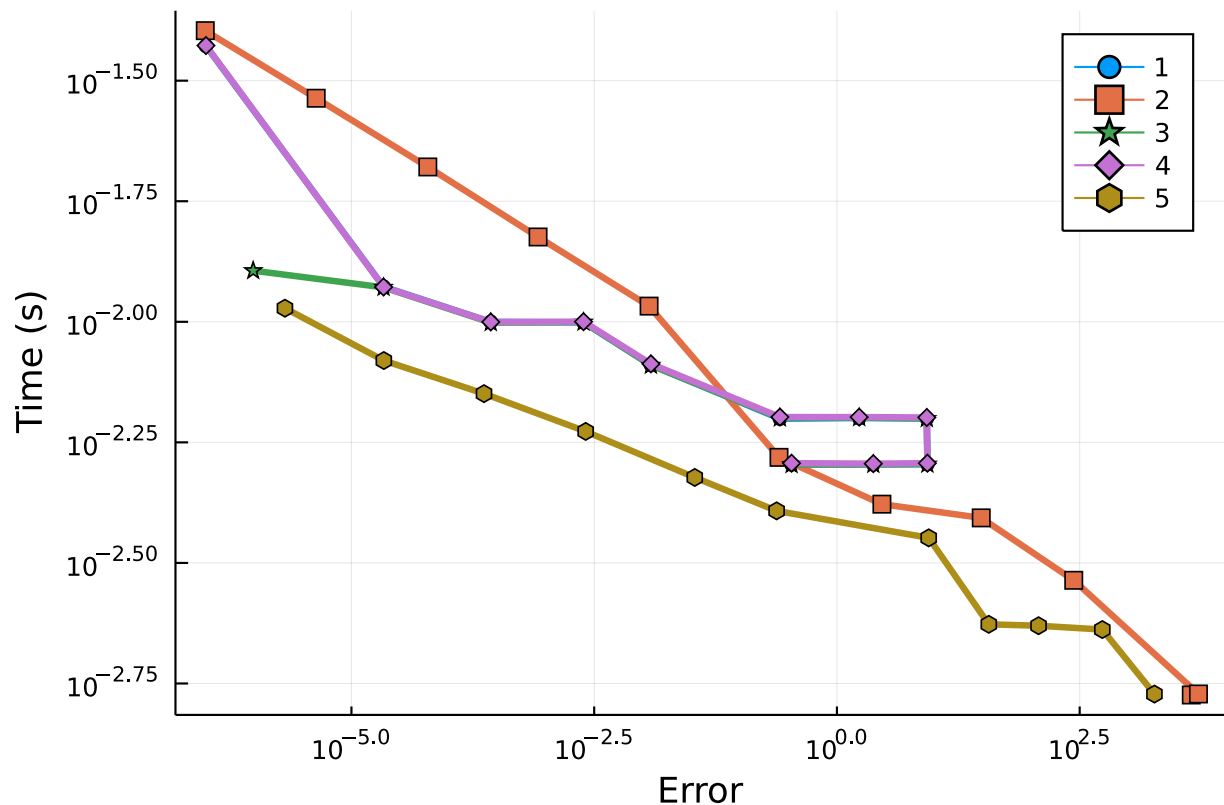
```



```

setups = [Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=15,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=7,
init_order=6, threading=true))]
solnames = ["1", "2", "3", "4", "5"]
wp = WorkPrecisionSet(prob, abstols, reltols, setups; names=solnames,
                      save_everystep=false, verbose=false, numruns=100)
plot(wp)

```



## 0.7 Conclusion

DifferentialEquations's default choice of `Tsit5` does well for quick and easy solving at normal tolerances. However, at low tolerances the higher order algorithms are faster. In every case, the DifferentialEquations algorithms are far in the lead, many times an order of magnitude faster than the competitors. `Vern7` with its included 7th order interpolation looks to be a good workhorse for scientific computing in floating point range. These along with many other benchmarks are why these algorithms were chosen as part of the defaults.

## 0.8 Appendix

These benchmarks are a part of the `SciMLBenchmarks.jl` repository, found at: <https://github.com/SciML/SciMLBenchmarks.jl>. For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization <https://sciml.ai>.

To locally run this benchmark, do the following commands:

```
using SciMLBenchmarks
SciMLBenchmarks.weave_file("benchmarks/NonStiffODE", "linear_wpd.jmd")
```

Computer Information:

```
Julia Version 1.6.1
Commit 6aaedec44 (2021-04-23 05:59 UTC)
Platform Info:
```

OS: Linux (x86\_64-pc-linux-gnu)  
CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz  
WORD\_SIZE: 64  
LIBM: libopenlibm  
LLVM: libLLVM-11.0.1 (ORCJIT, skylake)

Environment:

JULIA\_DEPOT\_PATH = /root/.cache/julia-buildkite-plugin/depots/5b300254-1738-4989-ae0  
JULIA\_NUM\_THREADS = 3

Package Information:

```
Status `~/var/lib/buildkite-agent/builds/rtx2070-gpuci1-julia-csail-mit-edu/julia
[f3b72e0c] DiffEqDevTools v2.27.2
[7f56f5a3] LSODA v0.7.0
[c030b06c] ODE v2.13.0
[54ca160b] ODEInterface v0.5.0
[09606e27] ODEInterfaceDiffEq v3.10.0
[1dea7af3] OrdinaryDiffEq v5.53.0
[65888b18] ParameterizedFunctions v5.10.0
[91a5bcdd] Plots v1.13.2
[31c91b34] SciMLBenchmarks v0.1.0 `../..`
[c3572dad] Sundials v4.4.3
[9a3f8284] Random
```

And the full manifest:

```
Status `~/var/lib/buildkite-agent/builds/rtx2070-gpuci1-julia-csail-mit-edu/julia
[c3fe647b] AbstractAlgebra v0.16.0
[1520ce14] AbstractTrees v0.3.4
[79e6a3ab] Adapt v3.3.0
[ec485272] ArnoldiMethod v0.1.0
[4fba245c] ArrayInterface v3.1.10
[9e28174c] BinDeps v1.0.2
[fa961155] CEnum v0.4.1
[d360d2e6] ChainRulesCore v0.9.41
[b630d9fa] CheapThreads v0.2.3
[35d6a980] ColorSchemes v3.12.1
[3da002f7] ColorTypes v0.11.0
[5ae59095] Colors v0.12.8
[861a8166] Combinatorics v1.0.2
[38540f10] CommonSolve v0.2.0
[bbf7d656] CommonSubexpressions v0.3.0
[34da2185] Compat v3.27.0
[8f4d0f93] Conda v1.5.2
[187b0558] ConstructionBase v1.1.0
[d38c429a] Contour v0.5.7
[9a962f9c] DataAPI v1.6.0
```

[864edb3b] DataStructures v0.18.9  
[e2d170a0] DataValueInterfaces v1.0.0  
[2b5f629d] DiffEqBase v6.60.1  
[f3b72e0c] DiffEqDevTools v2.27.2  
[c894b116] DiffEqJump v6.14.1  
[77a26b50] DiffEqNoiseProcess v5.7.1  
[163ba53b] DiffResults v1.0.3  
[b552c78f] DiffRules v1.0.2  
[b4f34e82] Distances v0.10.3  
[31c24e10] Distributions v0.24.18  
[ffbed154] DocStringExtensions v0.8.4  
[d4d017d3] ExponentialUtilities v1.8.4  
[e2ba6199] ExprTools v0.1.3  
[8f5d6c58] EzXML v1.1.0  
[c87230d0] FFMPEG v0.4.0  
[9aa1b823] FastClosures v0.3.2  
[1a297f60] FillArrays v0.11.7  
[6a86dc24] FiniteDiff v2.8.0  
[53c48c17] FixedPointNumbers v0.8.4  
[59287772] Formatting v0.4.2  
[f6369f11] ForwardDiff v0.10.18  
[069b7b12] FunctionWrappers v1.1.2  
[28b8d3ca] GR v0.57.4  
[5c1252a2] GeometryBasics v0.3.12  
[d7ba0133] Git v1.2.1  
[42e2da0e] Grisu v1.0.2  
[cd3eb016] HTTP v0.9.8  
[eafb193a] Highlights v0.4.5  
[0e44f5e4] Hwloc v1.3.0  
[7073ff75] IJulia v1.23.2  
[615f187c] IfElse v0.1.0  
[d25df0c9] Inflate v0.1.2  
[83e8ac13] IniFile v0.5.0  
[d8418881] Intervals v1.5.0  
[c8e1da08] IterTools v1.3.0  
[42fd0dbc] IterativeSolvers v0.9.0  
[82899510] IteratorInterfaceExtensions v1.0.0  
[692b3bcd] JLLWrappers v1.3.0  
[682c06a0] JSON v0.21.1  
[7f56f5a3] LSODA v0.7.0  
[b964fa9f] LaTeXStrings v1.2.1  
[2ee39098] LabelledArrays v1.6.0  
[23fbe1c1] Latexify v0.15.5  
[093fc24a] LightGraphs v1.3.5  
[d3d80556] LineSearches v7.1.1  
[2ab3a3ac] LogExpFunctions v0.2.3  
[bdcacae8] LoopVectorization v0.12.15  
[1914dd2f] MacroTools v0.5.6  
[739be429] MbedTLS v1.0.3

[442fdcdd] Measures v0.3.1  
 [e1d29d7a] Missings v1.0.0  
 [78c3b35d] Mocking v0.7.1  
 [961ee093] ModelingToolkit v5.16.0  
 [46d2c3a1] MuladdMacro v0.2.2  
 [ffc61752] Mustache v1.0.10  
 [d8a4904e] MutableArithmetics v0.2.18  
 [d41bc354] NLSolversBase v7.8.0  
 [2774e3e8] NLSolve v4.5.1  
 [77ba4419] NaNMath v0.3.5  
 [8913a72c] NonlinearSolve v0.3.8  
 [c030b06c] ODE v2.13.0  
 [54ca160b] ODEInterface v0.5.0  
 [09606e27] ODEInterfaceDiffEq v3.10.0  
 [6fe1bfb0] OffsetArrays v1.7.0  
 [429524aa] Optim v1.3.0  
 [bac558e1] OrderedCollections v1.4.0  
 [1dea7af3] OrdinaryDiffEq v5.53.0  
 [90014a1f] PDMats v0.11.0  
 [65888b18] ParameterizedFunctions v5.10.0  
 [d96e819e] Parameters v0.12.2  
 [69de0a69] Parsers v1.1.0  
 [ccf2f8ad] PlotThemes v2.0.1  
 [995b91a9] PlotUtils v1.0.10  
 [91a5bcdd] Plots v1.13.2  
 [e409e4f3] PoissonRandom v0.4.0  
 [f27b6e38] Polynomials v2.0.10  
 [85a6dd25] PositiveFactorizations v0.2.4  
 [21216c6a] Preferences v1.2.1  
 [1fd47b50] QuadGK v2.4.1  
 [74087812] Random123 v1.3.1  
 [fb686558] RandomExtensions v0.4.3  
 [e6cf234a] RandomNumbers v1.4.0  
 [3cdcf5f2] RecipesBase v1.1.1  
 [01d81517] RecipesPipeline v0.3.2  
 [731186ca] RecursiveArrayTools v2.11.3  
 [f2c3362d] RecursiveFactorization v0.1.12  
 [189a3867] Reexport v1.0.0  
 [ae029012] Requires v1.1.3  
 [ae5879a3] ResettableStacks v1.1.0  
 [79098fc4] Rmath v0.7.0  
 [47965b36] RootedTrees v1.0.0  
 [7e49a35a] RuntimeGeneratedFunctions v0.5.2  
 [476501e8] SLEEFPirates v0.6.15  
 [1bc83da4] SafeTestsets v0.0.1  
 [0bca4576] SciMLBase v1.13.0  
 [31c91b34] SciMLBenchmarks v0.1.0 `.../...`  
 [6c6a2e73] Scratch v1.0.3  
 [efcf1570] Setfield v0.7.0



[992d4aef] Showoff v1.0.3  
 [699a6c99] SimpleTraits v0.9.3  
 [b85f4697] SoftGlobalScope v1.1.0  
 [a2af1166] SortingAlgorithms v1.0.0  
 [47a9eef4] SparseDiffTools v1.13.2  
 [276daf66] SpecialFunctions v1.3.0  
 [aedffcd0] Static v0.2.4  
 [90137ffa] StaticArrays v1.1.2  
 [82ae8749] StatsAPI v1.0.0  
 [2913bbd2] StatsBase v0.33.8  
 [4c63d2b9] StatsFuns v0.9.8  
 [7792a7ef] StrideArraysCore v0.1.6  
 [09ab397b] StructArrays v0.5.1  
 [c3572dad] Sundials v4.4.3  
 [d1185830] SymbolicUtils v0.11.2  
 [0c5d862f] Symbolics v0.1.25  
 [3783bdb8] TableTraits v1.0.1  
 [bd369af6] Tables v1.4.2  
 [8290d209] ThreadingUtilities v0.4.1  
 [f269a46b] TimeZones v1.5.4  
 [a759f4b9] TimerOutputs v0.5.8  
 [a2a6695c] TreeViews v0.3.0  
 [30578b45] URIParser v0.4.1  
 [5c2747f8] URIs v1.3.0  
 [3a884ed6] UnPack v1.0.2  
 [1986cc42] Unitful v1.7.0  
 [3d5dd08c] VectorizationBase v0.19.34  
 [81def892] VersionParsing v1.2.0  
 [19fa3120] VertexSafeGraphs v0.1.2  
 [44d3d7a6] Weave v0.10.8  
 [ddb6d928] YAML v0.4.6  
 [c2297ded] ZMQ v1.2.1  
 [700de1a5] ZygoteRules v0.2.1  
 [6e34b625] Bzip2\_jll v1.0.6+5  
 [83423d85] Cairo\_jll v1.16.0+6  
 [5ae413db] EarCut\_jll v2.1.5+1  
 [2e619515] Expat\_jll v2.2.7+6  
 [b22a6f82] FFMPEG\_jll v4.3.1+4  
 [a3f928ae] Fontconfig\_jll v2.13.1+14  
 [d7e528f0] FreeType2\_jll v2.10.1+5  
 [559328eb] FriBidi\_jll v1.0.5+6  
 [0656b61e] GLFW\_jll v3.3.4+0  
 [d2c73de3] GR\_jll v0.57.2+0  
 [78b55507] Gettext\_jll v0.20.1+7  
 [f8c6e375] Git\_jll v2.31.0+0  
 [7746bdde] Glib\_jll v2.59.0+4  
 [e33a78d0] Hwloc\_jll v2.4.1+0  
 [aacddb02] JpegTurbo\_jll v2.0.1+3  
 [c1c5ebd0] LAME\_jll v3.100.0+3

[aae0fff6] LSODA\_jll v0.1.1+0  
 [dd4b983a] LZ0\_jll v2.10.0+3  
 [dd192d2f] LibVPX\_jll v1.9.0+1  
 [e9f186c6] Libffi\_jll v3.2.1+4  
 [d4300ac3] Libgcrypt\_jll v1.8.5+4  
 [7e76a0d4] Libglvnd\_jll v1.3.0+3  
 [7add5ba3] Libgpg\_error\_jll v1.36.0+3  
 [94ce4f54] Libiconv\_jll v1.16.0+7  
 [4b2f31a3] Libmount\_jll v2.34.0+3  
 [89763e89] Libtiff\_jll v4.1.0+2  
 [38a345b3] Libuuid\_jll v2.34.0+7  
 [c771fb93] ODEInterface\_jll v0.0.1+0  
 [e7412a2a] Ogg\_jll v1.3.4+2  
 [458c3c95] OpenSSL\_jll v1.1.1+6  
 [efe28fd5] OpenSpecFun\_jll v0.5.4+0  
 [91d4177d] Opus\_jll v1.3.1+3  
 [2f80f16e] PCRE\_jll v8.42.0+4  
 [30392449] Pixman\_jll v0.40.0+0  
 [ea2cea3b] Qt5Base\_jll v5.15.2+0  
 [f50d1b31] Rmath\_jll v0.3.0+0  
 [fb77eaff] Sundials\_jll v5.2.0+1  
 [a2964d1f] Wayland\_jll v1.17.0+4  
 [2381bf8a] Wayland\_protocols\_jll v1.18.0+4  
 [02c8fc9c] XML2\_jll v2.9.11+0  
 [aed1982a] XSLT\_jll v1.1.33+4  
 [4f6342f7] Xorg\_libX11\_jll v1.6.9+4  
 [0c0b7dd1] Xorg\_libXau\_jll v1.0.9+4  
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 [d091e8ba] Xorg\_libXfixes\_jll v5.0.3+4  
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 [d1454406] Xorg\_libXinerama\_jll v1.1.4+4  
 [ec84b674] Xorg\_libXrandr\_jll v1.5.2+4  
 [ea2f1a96] Xorg\_libXrender\_jll v0.9.10+4  
 [14d82f49] Xorg\_libpthread\_stubs\_jll v0.1.0+3  
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 [cc61e674] Xorg\_libxkbfile\_jll v1.1.0+4  
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 [33bec58e] Xorg\_xkeyboard\_config\_jll v2.27.0+4  
 [c5fb5394] Xorg\_xtrans\_jll v1.4.0+3  
 [8f1865be] ZeroMQ\_jll v4.3.2+6  
 [3161d3a3] Zstd\_jll v1.4.8+0  
 [0ac62f75] libass\_jll v0.14.0+4

[f638f0a6] libfdk\_aac\_jll v0.1.6+4  
[b53b4c65] libpng\_jll v1.6.37+6  
[a9144af2] libsodium\_jll v1.0.20+0  
[f27f6e37] libvorbis\_jll v1.3.6+6  
[1270edf5] x264\_jll v2020.7.14+2  
[dfaa095f] x265\_jll v3.0.0+3  
[d8fb68d0] xkbcommon\_jll v0.9.1+5  
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[76f85450] LibGit2  
[8f399da3] Libdl  
[37e2e46d] LinearAlgebra  
[56ddb016] Logging  
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[de0858da] Printf  
[3fa0cd96] REPL  
[9a3f8284] Random  
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[9e88b42a] Serialization  
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[6462fe0b] Sockets  
[2f01184e] SparseArrays  
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[4607b0f0] SuiteSparse  
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[a4e569a6] Tar  
[8dfed614] Test  
[cf7118a7] UUIDs  
[4ec0a83e] Unicode  
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[deac9b47] LibCURL\_jll  
[29816b5a] LibSSH2\_jll  
[c8ffd9c3] MbedTLS\_jll  
[14a3606d] MozillaCACerts\_jll  
[4536629a] OpenBLAS\_jll  
[efcefd7f] PCRE2\_jll  
[bea87d4a] SuiteSparse\_jll

[83775a58] Zlib\_jll  
[8e850ede] nghttp2\_jll  
[3f19e933] p7zip\_jll