Pleiades Work-Precision Diagrams

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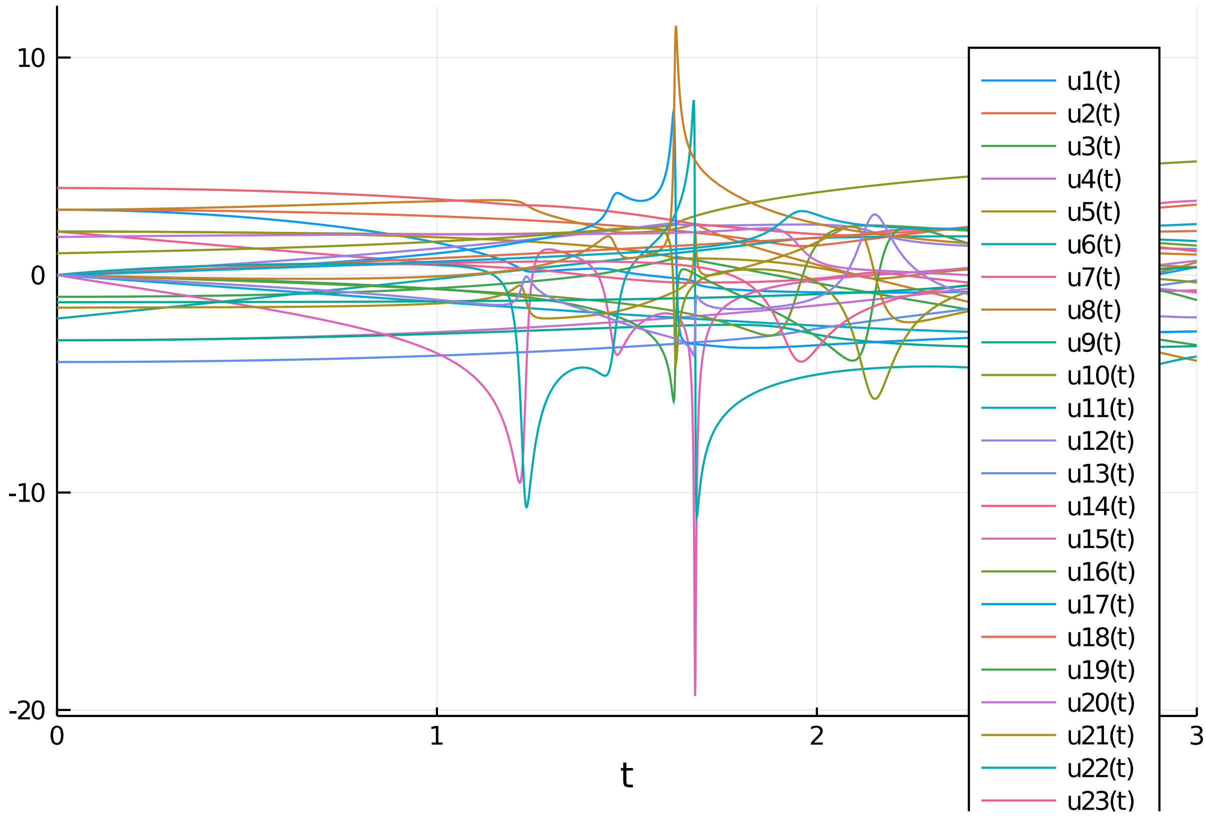
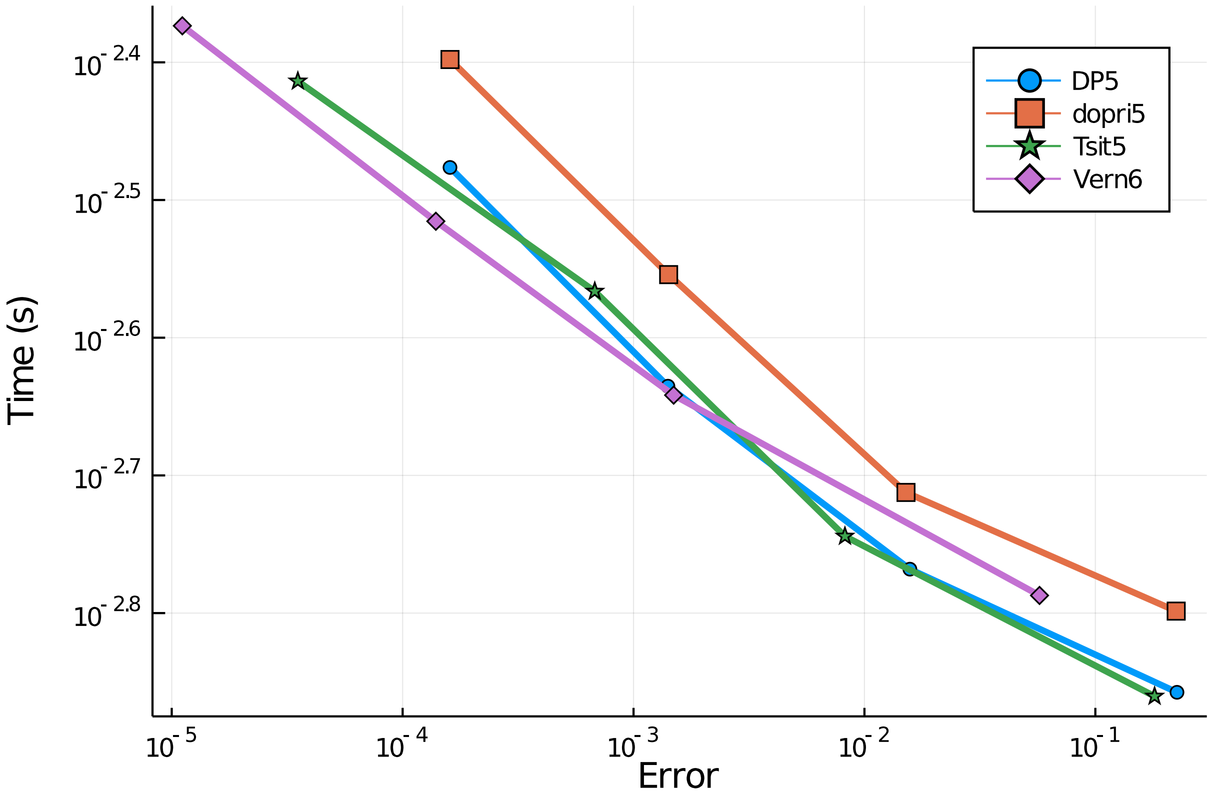
```
using OrdinaryDiffEq, ODE, ODEInterfaceDiffEq, LSODA, Sundials, DiffEqDevTools

f = (du,u,p,t) -> begin
  @inbounds begin
  x = view(u,1:7)   # x
  y = view(u,8:14)  # y
  v = view(u,15:21) # x′
  w = view(u,22:28) # y′
  du[1:7] .= v
  du[8:14].= w
  for i in 15:28
    du[i] = zero(u[1])
  end
  for i=1:7,j=1:7
    if i != j
      r = ((x[i]-x[j])^2 + (y[i] - y[j])^2)^(3/2)
      du[14+i] += j*(x[j] - x[i])/r
      du[21+i] += j*(y[j] - y[i])/r
    end
  end
  end
end

prob =
ODEProblem(f,[3.0,3.0,-1.0,-3.0,2.0,-2.0,2.0,3.0,-3.0,2.0,0,0,-4.0,4.0,0,0,0,0,0,1.75,-1.5,0,0,0,-1.25

abstols = 1.0 ./ 10.0 .^ (6:9)
reltols = 1.0 ./ 10.0 .^ (3:6);

using Plots; gr()

Plots.GRBackend()

sol = solve(prob,Vern8(),abstol=1/10^12,reltol=1/10^10,maxiters=1000000)
test_sol = TestSolution(sol);
plot(sol)
```



0.1 Low Order

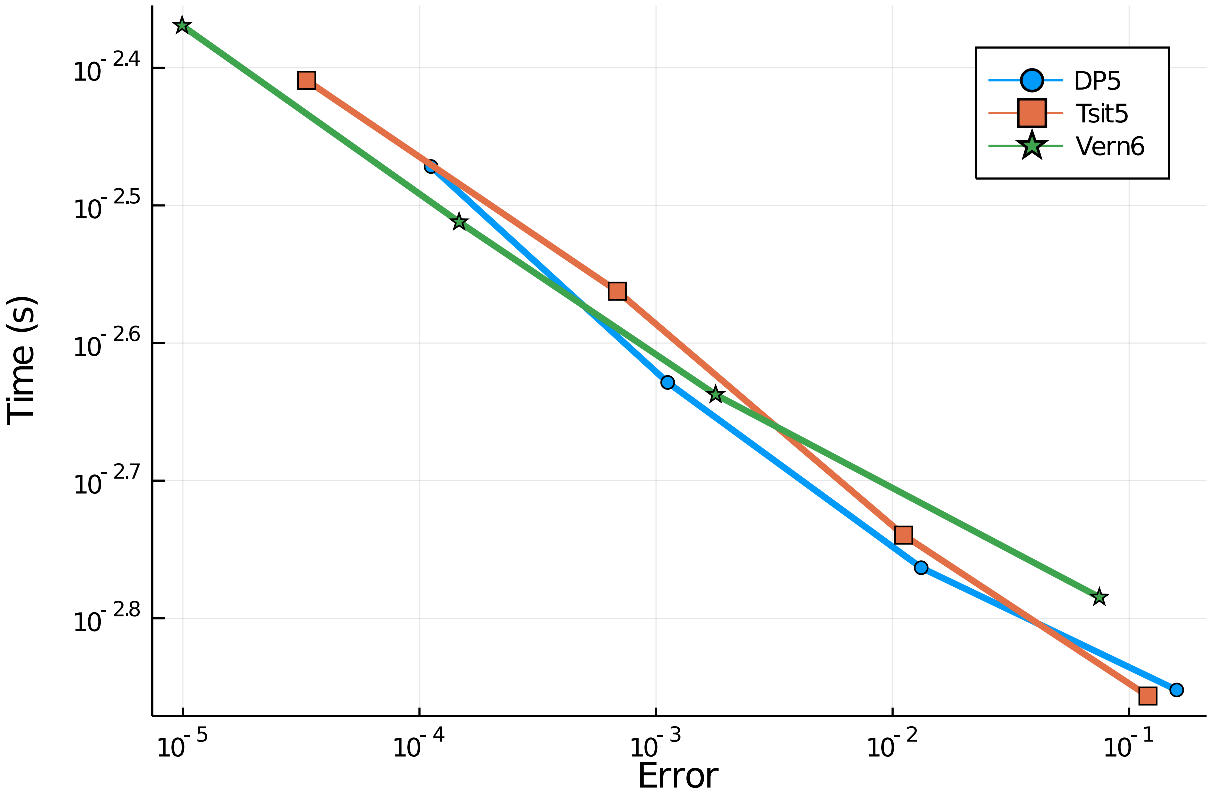
ODE.jl had to be discarded. The error estimate is off since it throws errors and aborts and so that artificially lowers the error the the time is serverly diminished.

```
#setups = [Dict(:alg=>ode45())]
#wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
#plot(wp)

setups = [Dict(:alg=>DP5())
          Dict(:alg=>dopri5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



0.1.1 Interpolation

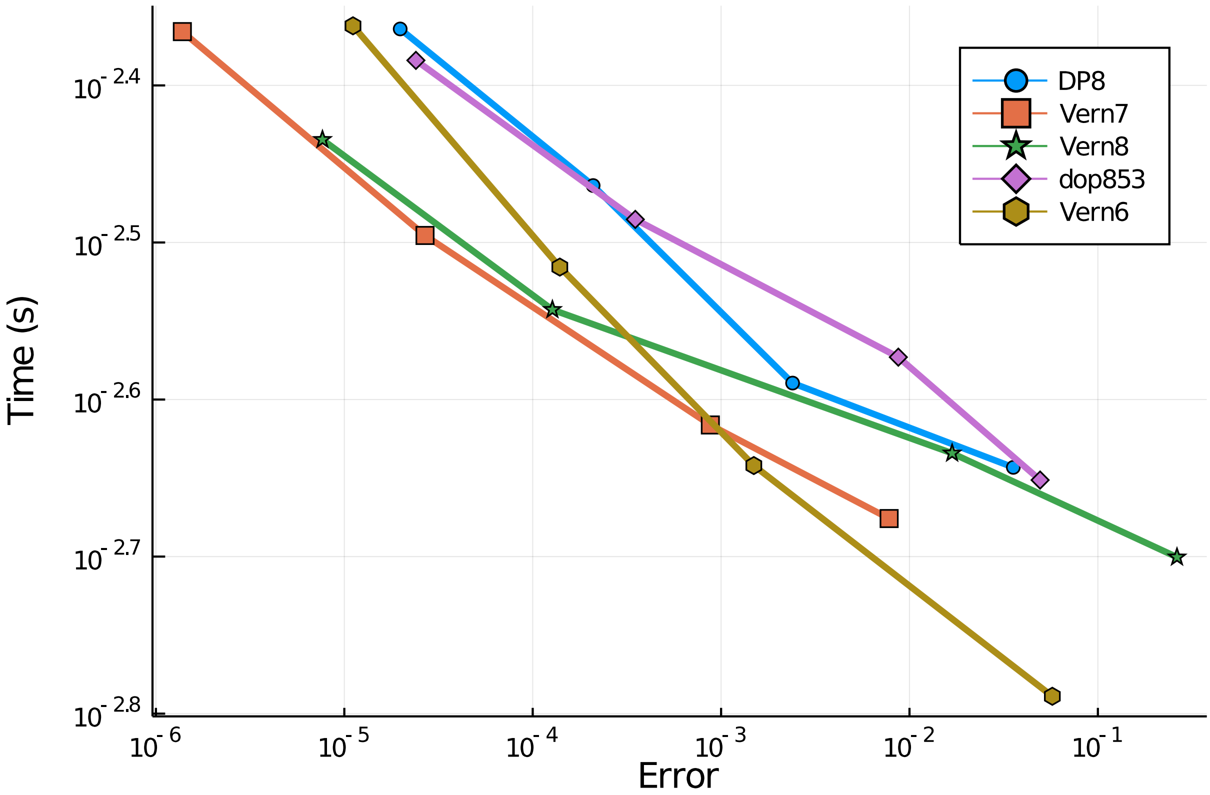
```
setups = [Dict(:alg=>DP5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=10000,error_estimat
plot(wp)
```



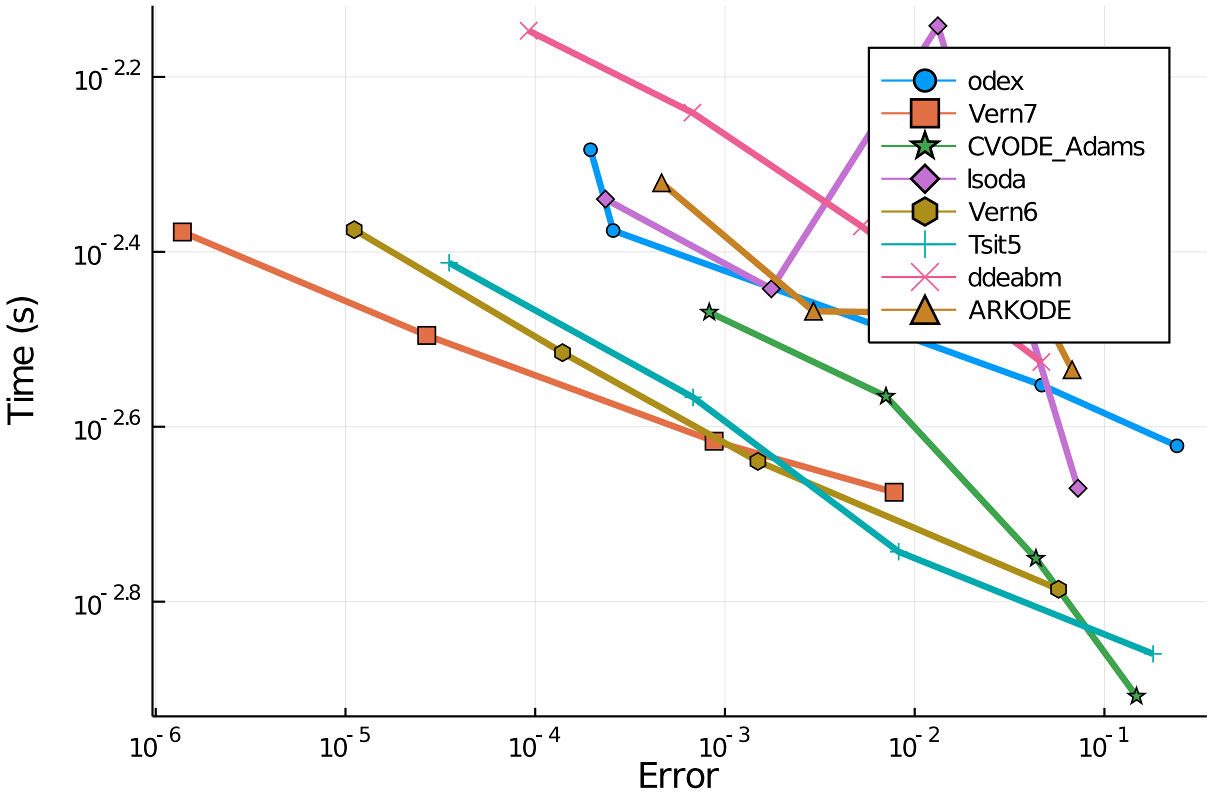
0.2 Higher Order

Once again ODE.jl had to be discarded since it errors.

```
#setups = [Dict(:alg=>ode78())]
#wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxite
#plot(wp)

setups = [Dict(:alg=>DP8())
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>dop853())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxite
plot(wp)
```

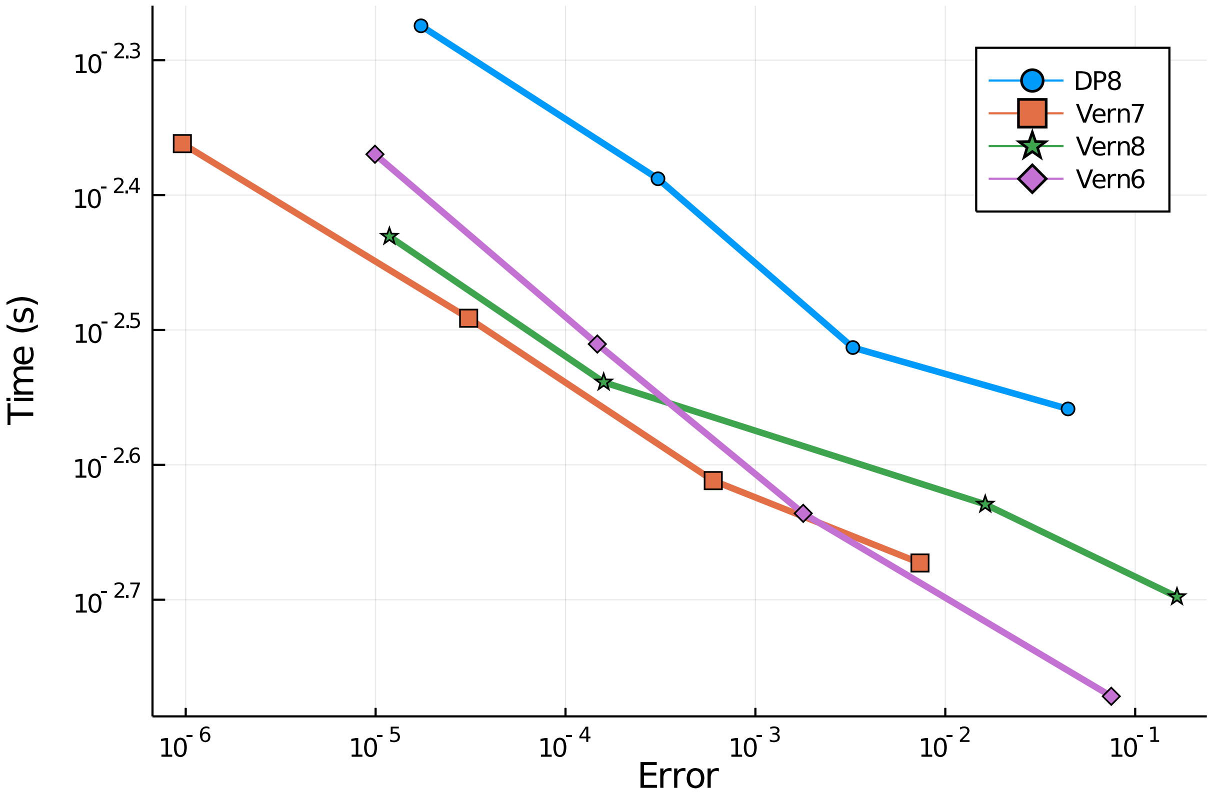



```
setups = [Dict(:alg=>odex())
          Dict(:alg=>Vern7())
          Dict(:alg=>CVODE_Adams())
          Dict(:alg=>lsoda())
          Dict(:alg=>Vern6())
          Dict(:alg=>Tsit5())
          Dict(:alg=>ddeabm())
          Dict(:alg=>ARKODE(Sundials.Explicit(),order=6))
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=20)
plot(wp)
```



0.2.1 Interpolations

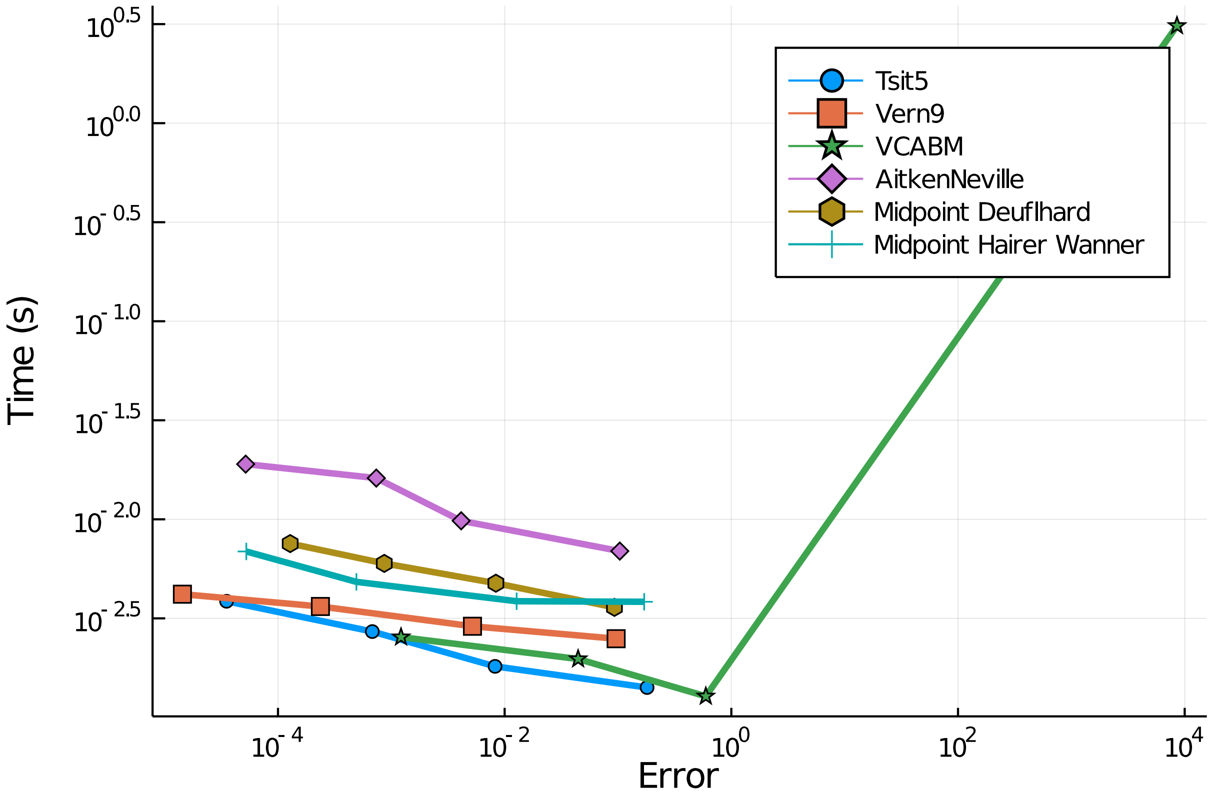
```
setups = [Dict(:alg=>DP8())
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=1000,error_estimate
plot(wp)
```



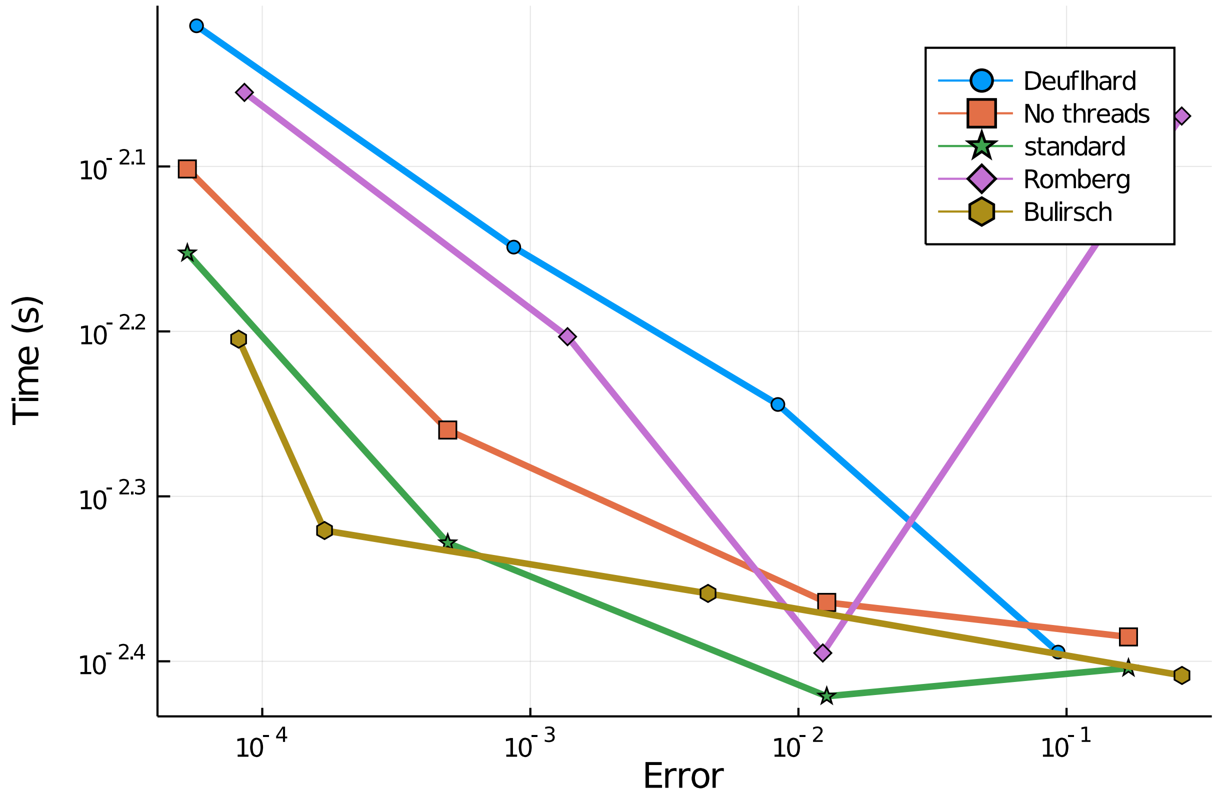
0.3 Comparison with Non-RK methods

Now let's test Tsit5 and Vern9 against parallel extrapolation methods and an Adams-Bashforth-Moulton:

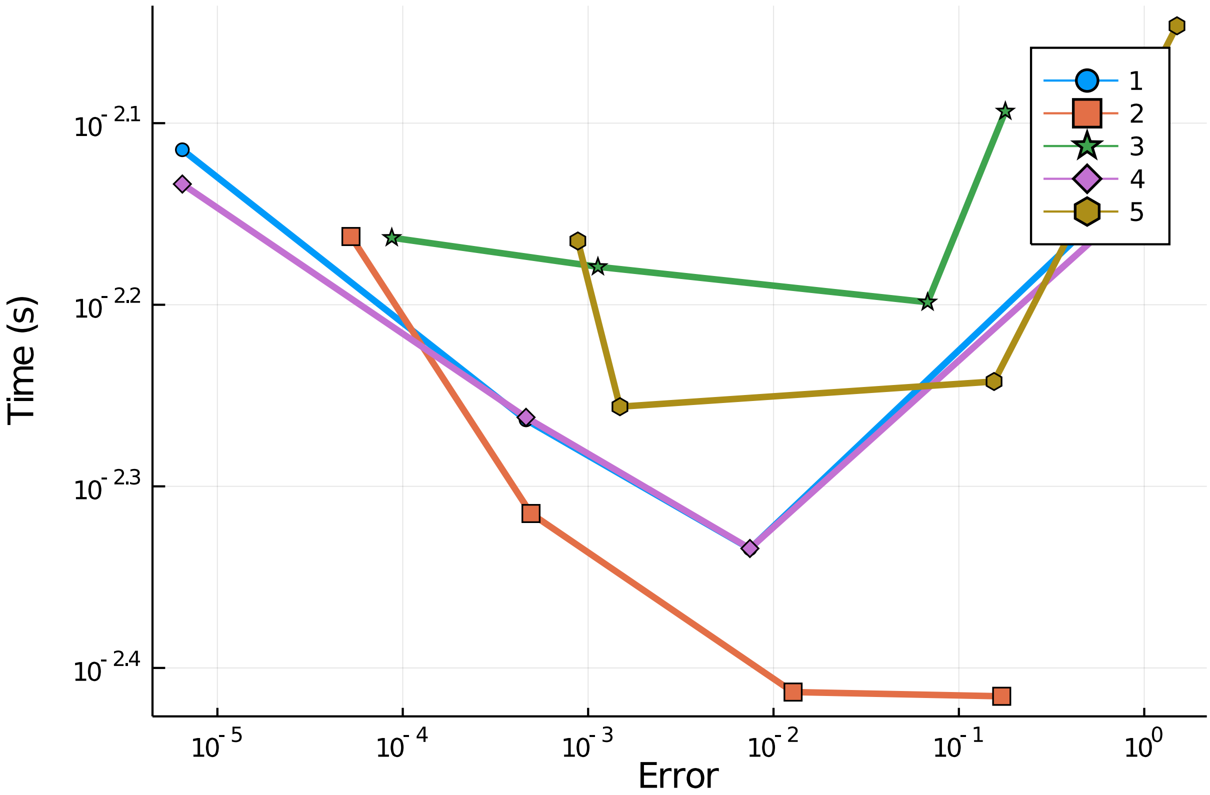
```
setups = [Dict(:alg=>Tsit5())
          Dict(:alg=>Vern9())
          Dict(:alg=>VCABM())
          Dict(:alg=>AitkenNeville(min_order=1, max_order=9, init_order=4,
threading=true))
          Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))]
solnames = ["Tsit5","Vern9","VCABM","AitkenNeville","Midpoint Deuflhard","Midpoint
Hairer Wanner"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```




```
setups = [Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=9, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :romberg, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :bulirsch, threading=true))]
solnames = ["Deuflhard","No threads","standard","Romberg","Bulirsch"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```




```
setups = [Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=15,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=7,
init_order=6, threading=true))]
solnames = ["1","2","3","4","5"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



0.4 Conclusion

One big conclusion is that, once again, the ODE.jl algorithms fail to run on difficult problems. Its minimum timestep is essentially machine epsilon, and so this shows some fatal flaws in its timestepping algorithm. The OrdinaryDiffEq.jl algorithms come out as faster in each case than the ODEInterface algorithms. Overall, the Verner methods have a really good showing once again. The `CVODE_Adams` method does really well here when the tolerances are higher.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

0.5 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("NonStiffODE","Pleiades_wpd.jmd")
```

Computer Information:

```
Julia Version 1.4.2
Commit 44fa15b150* (2020-05-23 18:35 UTC)
Platform Info:
```

```
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_DEPOT_PATH = /builds/JuliaGPU/DiffEqBenchmarks.jl/.julia
  JULIA_CUDA_MEMORY_LIMIT = 2147483648
  JULIA_PROJECT = @.
  JULIA_NUM_THREADS = 4
```

Package Information:

```
Status: `/builds/JuliaGPU/DiffEqBenchmarks.jl/benchmarks/NonStiffODE/Project.toml`
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.22.0
[7f56f5a3-f504-529b-bc02-0b1fe5e64312] LSODA 0.6.1
[c030b06c-0b6d-57c2-b091-7029874bd033] ODE 2.8.0
[54ca160b-1b9f-5127-a996-1867f4bc2a2c] ODEInterface 0.4.6
[09606e27-ecf5-54fc-bb29-004bd9f985bf] ODEInterfaceDiffEq 3.7.0
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.41.0
[65888b18-ceab-5e60-b2b9-181511a3b968] ParameterizedFunctions 5.3.0
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.5.2
[c3572dad-4567-51f8-b174-8c6c989267f4] Sundials 4.2.5
[9a3f8284-a2c9-5f02-9a11-845980a1fd5c] Random
```