Stochastic Heat Equation Benchmarks

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1 Stochastic Heat Equation Benchmarks

In this notebook we will benchmark against the stochastic heat equation with Dirichlet BCs and scalar noise. The function for generating the problem is as follows:

Stochastic Heat Equation with scalar multiplicative noise

S-ROCK: CHEBYSHEV METHODS FOR STIFF STOCHASTIC DIFFERENTIAL EQUATIONS

ASSYR ABDULLE AND STEPHANE CIRILLI

Raising D or k increases stiffness

```julia
using StochasticDiffEq, DiffEqNoiseProcess, LinearAlgebra, Statistics

function generate_stiff_stoch_heat(D=1,k=1;N = 100, t_end = 3.0, adaptivealg = :RSwM3)
    A = Array(Tridiagonal([1.0 for i in 1:N-1],[-2.0 for i in 1:N],[1.0 for i in 1:N-1]))
    dx = 1/N
    A = D/(dx^2) * A
    function f(du,u,p,t)
        mul!(du,A,u)
    end
    #=
    function f(::Type{Val{:analytic}},u0,p,t,W)
        exp((A-k/2)*t+W*I)*u0 # no -k/2 for Strat
    end
    =#
    function g(du,u,p,t)
        @. du = k*u
    end

SDEProblem(f,g,ones(N),(0.0,t_end),noise=WienerProcess(0.0,0.0,0.0,rswm=RSWM(adaptivealg=adaptivealg)))
end

N = 100
D = 1; k = 1
    A = Array(Tridiagonal([1.0 for i in 1:N-1],[-2.0 for i in 1:N],[1.0 for i in 1:N-1]))
    dx = 1/N
    A = D/(dx^2) * A;
```

```
100×100 Array{Float64,2}:
 -20000.0   10000.0       0.0       0.0  …       0.0       0.0       0.0
  10000.0  -20000.0   10000.0       0.0          0.0       0.0       0.0
      0.0   10000.0  -20000.0   10000.0          0.0       0.0       0.0
```

```
    0.0         0.0    10000.0   -20000.0           0.0         0.0         0.0
    0.0         0.0        0.0    10000.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0   ...     0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    ⋮                                     ⋱
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0           0.0         0.0         0.0
    0.0         0.0        0.0        0.0   ...     0.0         0.0         0.0
    0.0         0.0        0.0        0.0       10000.0         0.0         0.0
    0.0         0.0        0.0        0.0      -20000.0    10000.0         0.0
    0.0         0.0        0.0        0.0       10000.0   -20000.0    10000.0
    0.0         0.0        0.0        0.0           0.0    10000.0   -20000.0
```

Now lets solve it with high accuracy.

```
prob = generate_stiff_stoch_heat(1.0,1.0)
@time sol = solve(prob,SRIW1(),progress=true,abstol=1e-6,reltol=1e-6);

18.388128 seconds (29.85 M allocations: 7.832 GiB, 5.78% gc time)
retcode: Success
Interpolation: 1st order linear
t: 700049-element Array{Float64,1}:
 0.0
 4.04628794533697e-9
 4.855545534404364e-9
 5.765960322105182e-9
 6.7901769582686025e-9
 7.94242067395245e-9
 9.23869485409678e-9
 1.067192267109061e-8
 1.2232270092873639e-8
 1.390920860298187e-8
 ⋮
 2.9998101160563486
 2.9998272995550637
 2.9998466309911183
 2.9998683788566796
 2.9998928452054363
 2.9999203698477874
 2.9999513350704325
 2.9999861709459084
 3.0
u: 700049-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.9999039868231989, 0.9999444449989893, 0.9999444458176114, 0.99994444581
76114, 0.9999444458176114, 0.9999444458176114, 0.9999444458176114, 0.999944
4458176114, 0.9999444458176114, 0.9999444458176114  ...  0.9999444458176114,
0.9999444458176114, 0.9999444458176114, 0.9999444458176114, 0.9999444458176
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49989893, 0.9999039868231989]
 [0.9998881441097403, 0.9999366929550392, 0.9999366941337865, 0.99993669413
37945, 0.9999366941337945, 0.9999366941337945, 0.9999366941337945, 0.999936
6941337945, 0.9999366941337945, 0.9999366941337945  ...  0.9999366941337945,
```

0.9999366941337945, 0.9999366941337945, 0.9999366941337945, 0.9999366941337
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3

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90108e-10, 1.5572426735615742e-10, -1.1692446762785197e-10, 7.8013042330754
81e-11, -3.902514944991724e-11]

1.1 Highest dt

Let's try to find the highest possible dt:

```
@time sol = solve(generate_stiff_stoch_heat(1.0,1.0),SRIW1());
```

1.304003 seconds (877.85 k allocations: 124.273 MiB)

```
retcode: Success
Interpolation: 1st order linear
t: 93333-element Array{Float64,1}:
 0.0
 1.535027855349674e-5
 1.842033426419609e-5
 2.1874146938732856e-5
 2.575968619758672e-5
 3.0130917863797315e-5
 3.504855348828424e-5
 4.0580893565832026e-5
 4.6804776153073284e-5
 5.38066440637197e-5
  :
 2.9996337509507427
 2.999664399144817
 2.99969887836315
 2.9997376674837755
 2.9997813052444786
 2.99983039772527
 2.9998856267661598
 2.9999477594371613
 3.0
u: 93333-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.8723163991148746, 0.990883388806052, 1.002664941389549, 1.0026649413895
49, 1.002664941389549, 1.002664941389549, 1.002664941389549, 1.002664941389
549, 1.002664941389549, 1.002664941389549  ...  1.002664941389549, 1.00266494
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41389549, 1.002664941389549, 1.002664941389549, 0.990883388806052, 0.872316
3991148746]
 [0.8489227831991796, 0.9862828883577451, 1.0011862117782817, 1.00158119110
83257, 1.0015867433075767, 1.0015867433075767, 1.0015867433075767, 1.001586
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83577451, 0.8489227831991796]
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74749, 0.9993944859989928, 0.9993948996777072, 0.9993949029892671, 0.999394
9029892671, 0.9993949029892671, 0.9993949029892671  ...  0.9993949029892671,
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11748504, 0.8233185223557571]
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```

5

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38, 0.9941903451431988, 0.993338927368097, 0.9854167951638835, 0.931699600
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@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),SRIW1(),progress=true,adaptive=false,dt=0.00005);

0.667166 seconds (524.94 k allocations: 82.245 MiB)
retcode: Success
Interpolation: 1st order linear
t: 60002-element Array{Float64,1}:
 0.0
 5.0e-5
 0.0001
 0.00015000000000000001
 0.0002
 0.00025
 0.0003000000000000003
 0.00035000000000000005
 0.0004000000000000001
 0.00045000000000000001
 ⋮
 2.9996499999996784
 2.9996999999996783
 2.999749999999678
 2.999799999999678
 2.999849999999678
 2.999899999999678
 2.9999499999996777
 2.9999999999996776
 3.0
u: 60002-element Array{Array{Float64,1},1}:
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```

```julia
@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),EM(),progress=true,adaptive=false,dt=0.00005);
```

```
11.182711 seconds (23.58 M allocations: 7.040 GiB, 9.11% gc time)
retcode: Success
Interpolation: 1st order linear
t: 60002-element Array{Float64,1}:
 0.0
 5.0e-5
 0.0001
 0.00015000000000000001
 0.0002
 0.00025
 0.0003000000000000003
 0.0003500000000000005
 0.0004000000000000001
 0.0004500000000000001
 ⋮
 2.9996499999996784
 2.9996999999996783
 2.999749999999678
 2.999799999999678
 2.999849999999678
 2.999899999999678
 2.9999499999996777
 2.9999999999996776
 3.0
u: 60002-element Array{Array{Float64,1},1}:
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solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.1);
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11.130003 seconds (29.51 M allocations: 7.272 GiB, 7.02% gc time)
retcode: Success
Interpolation: 1st order linear
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05845e-10]

@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.01);
```

```
  0.018362 seconds (6.23 k allocations: 801.344 KiB, 68.69% gc time)
retcode: Success
Interpolation: 1st order linear
t: 61-element Array{Float64,1}:
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 0.0013562500000000003
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 ⋮
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 1.6420566998957509
 1.8477137873827199
 2.07907801080556
 2.339362762156255
 2.632183107425787
 2.7452217062980093
 2.8723901300292596
 3.0
u: 61-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  …  1.0, 1.0, 1.0, 1.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
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15

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@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitRKMil(),progress=true,dt=0.001);

0.005871 seconds (6.40 k allocations: 818.406 KiB)
retcode: Success
Interpolation: 1st order linear
t: 66-element Array{Float64,1}:
 0.0
 0.0002
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 0.000643352969110728
 0.0009074087815957939
 0.0012044715706414932
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 ⋮
 1.2203922663871176
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 1.738251475571066
 1.9557165467087958
 2.2003647517387415
 2.4755939823974304
 2.785226866888456
 3.0
u: 66-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  ...  1.0, 1.0, 1.0, 1.0,
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746688484379e-11, -4.6466210063834787e-11]
```

```julia
@time sol =
solve(generate_stiff_stoch_heat(1.0,1.0),ImplicitEM(),progress=true,dt=0.001);
```

```
10.622823 seconds (27.93 M allocations: 7.198 GiB, 6.30% gc time)
retcode: Success
```

```
Interpolation: 1st order linear
t: 66-element Array{Float64,1}:
 0.0
 0.001
 0.0012000000000000001
 0.001425
 0.0016781250000000001
 0.0019628906250000002
 0.0022832519531250004
 0.0026436584472656257
 0.003049115753173829
 0.003505255222320558
 ⋮
 1.1706813810781274
 1.3170915537128933
 1.481802997927005
 1.6671033726678808
 1.875566294251366
 2.1100870810327867
 2.3739229661618855
 2.6707383369321214
 3.0
u: 66-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  …  1.0, 1.0, 1.0, 1.0,
 1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.27203686396417925, 0.47058127927035076, 0.6154876874491318, 0.721246729
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 477940104674981, 0.9258928519983329, 0.8958848436745754, 0.8547691846638499
 , 0.798434309065084, 0.7212467293184006, 0.6154876874491318, 0.470581279270
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 [0.21465136962716724, 0.4003080388085639, 0.550305755758086, 0.66702933281
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20

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4.70814370860706e-12]
```

1.2 Simple Error Analysis

Now let's check the error at an arbitrary timepoint in there. Our analytical solution only exists in the Stratanovich sense, so we are limited in the methods we can calculate errors for.

```
function simple_error(alg;kwargs...)
    sol = solve(generate_stiff_stoch_heat(1.0,1.0,t_end=0.25),alg;kwargs...);
    sum(abs2,sol[end] - exp(A*sol.t[end]+sol.W[end]*I)*prob.u0)
end
```

```
simple_error (generic function with 1 method)
```

```
mean(simple_error(EulerHeun(),dt=0.00005) for i in 1:400)
```

```
3.3081114973762924e-9
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.1) for i in 1:400)
```

```
0.00276854897317656
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.01) for i in 1:400)
```

```
0.0024058199447384797
```

```
mean(simple_error(ImplicitRKMil(interpretation=:Stratanovich),dt=0.001) for i in 1:400)
```

```
0.0023173476300486374
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.001) for i in 1:400)
```

```
0.0007568092191341506
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.01) for i in 1:400)
```

```
0.0010298495611328299
```

```
mean(simple_error(ImplicitEulerHeun(),dt=0.1) for i in 1:400)
```

```
0.0011922697128923844
```

1.3 Interesting Property

Note that RSwM1 and RSwM2 are not stable on this problem.

```
sol = solve(generate_stiff_stoch_heat(1.0,1.0,adaptivealg=:RSwM1),SRIW1());
```

```
retcode: Success
Interpolation: 1st order linear
t: 92046-element Array{Float64,1}:
 0.0
 1.535027855349674e-5
 1.842033426419609e-5
 2.1874146938732856e-5
 2.575968619758672e-5
 3.0130917863797315e-5
 3.504855348828424e-5
 4.0580893565832026e-5
 4.6804776153073284e-5
 5.38066440637197e-5
 ⋮
 2.9996947951090287
 2.999720730090053
 2.999749906943705
 2.9997827309040637
 2.9998196578594674
 2.9998612006842964
 2.999907936362229
 2.9999605139999033
 3.0
u: 92046-element Array{Array{Float64,1},1}:
 [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0  …  1.0, 1.0, 1.0, 1.0,
 1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
 [0.8711879134057668, 0.9895511769581284, 1.0013327295416257, 1.00133272954
16257, 1.0013327295416257, 1.0013327295416257, 1.0013327295416257, 1.001332
7295416257, 1.0013327295416257, 1.0013327295416257  …  1.0013327295416257,
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1.4 Conclusion

In this problem, the implicit methods do not have a stepsize limit. This is because the
stiffness almost entirely deteriministic due to diffusion. In that case, if we do not care about
the error too much, the implicit methods dominate. Of course, as the tolerance gets lower
there is a tradeoff point where the higher order methods will become more efficient. The
explicit methods are clearly stability-bound and thus unless we want an error of like 10^-10
we are better off using an implicit method here.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

1.5 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi
To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("StiffSDE","StochasticHeat.jmd")
```

Computer Information:

```
Julia Version 1.4.2
Commit 44fa15b150* (2020-05-23 18:35 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_DEPOT_PATH = /builds/JuliaGPU/DiffEqBenchmarks.jl/.julia
  JULIA_CUDA_MEMORY_LIMIT = 2147483648
  JULIA_PROJECT = @.
  JULIA_NUM_THREADS = 8
```

Package Information:

```
Status: `/builds/JuliaGPU/DiffEqBenchmarks.jl/benchmarks/StiffSDE/Project.toml`
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.22.0
[77a26b50-5914-5dd7-bc55-306e6241c503] DiffEqNoiseProcess 5.0.2
[a077e3f3-b75c-5d7f-a0c6-6bc4c8ec64a9] DiffEqProblemLibrary 4.8.0
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.5.3
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.24.0
[37e2e46d-f89d-539d-b4ee-838fcccc9c8e] LinearAlgebra
[9a3f8284-a2c9-5f02-9a11-845980a1fd5c] Random
[10745b16-79ce-11e8-11f9-7d13ad32a3b2] Statistics
```