Rigid Body Work-Precision Diagrams

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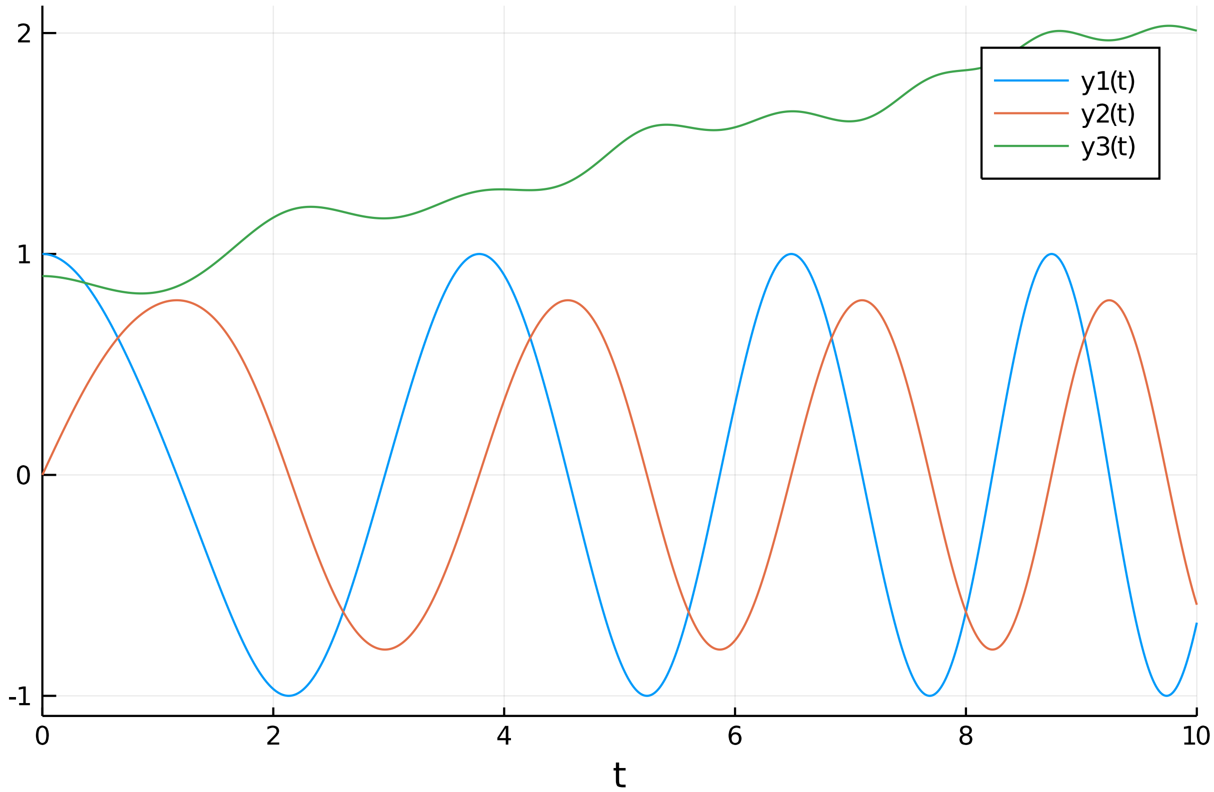
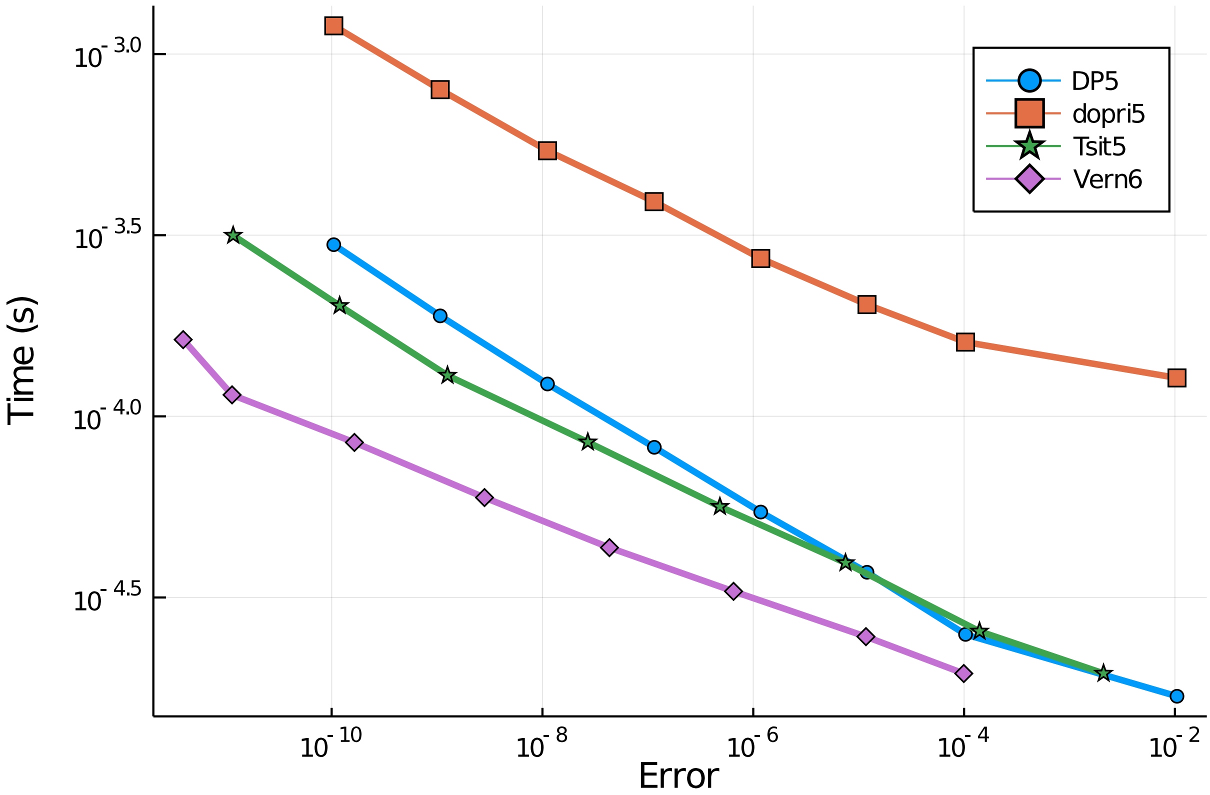
```
using OrdinaryDiffEq, ParameterizedFunctions, ODE, ODEInterfaceDiffEq, LSODA,
      Sundials, DiffEqDevTools

k(t) = 0.25*sin(t)^2

g = @ode_def RigidBody begin
  dy1  = I_1*y2*y3
  dy2  = I_2*y1*y3
  dy3  = I_3*y1*y2 + k(t)
end I_1 I_2 I_3

p = [-2.0,1.25,-0.5]
prob = ODEProblem(g,[1.0;0.0;0.9],(0.0,10.0),p)

abstols = 1.0 ./ 10.0 .^ (6:13)
reltols = 1.0 ./ 10.0 .^ (3:10);
sol = solve(prob,Vern7(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)
using Plots; gr()
```

```
Plots.GRBackend()
```

```
plot(sol)
```



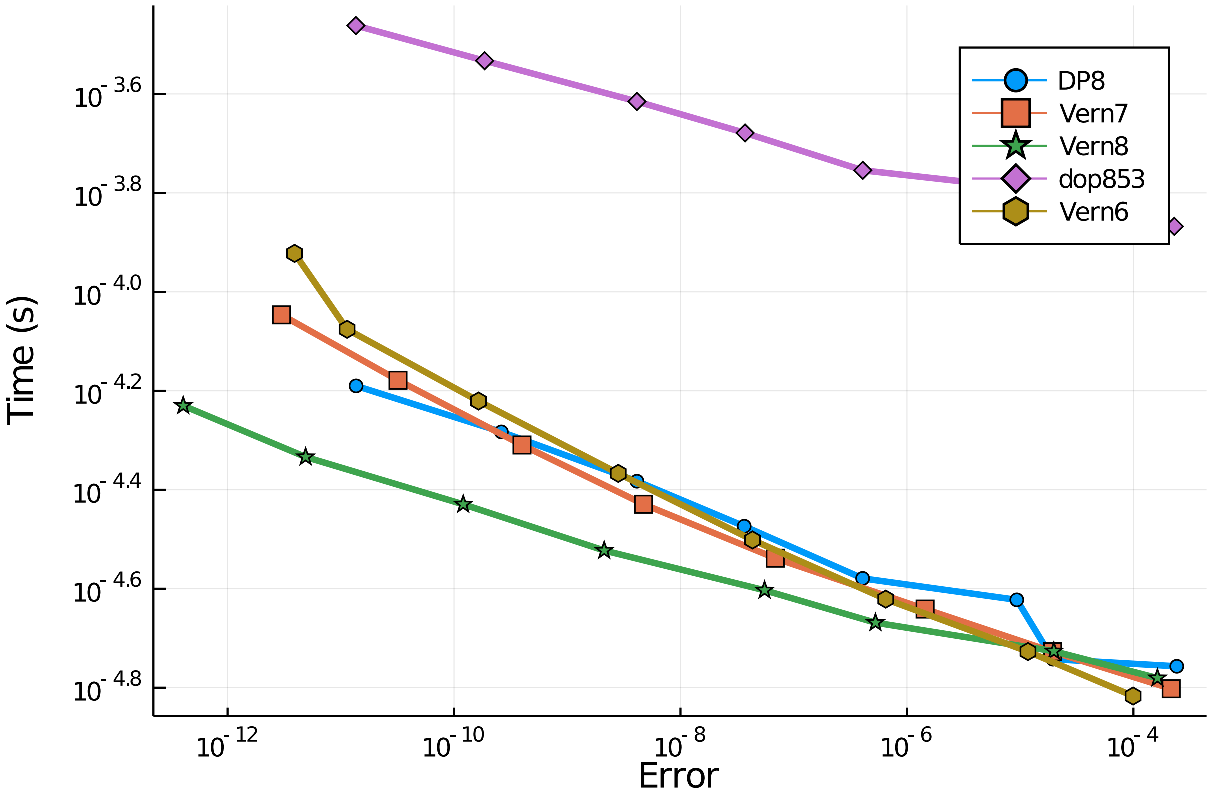

```
setups = [Dict(:alg=>DP5())
          #Dict(:alg=>ode45()) # fails
          Dict(:alg=>dopri5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=true,numruns=100,maxiters
plot(wp)
```



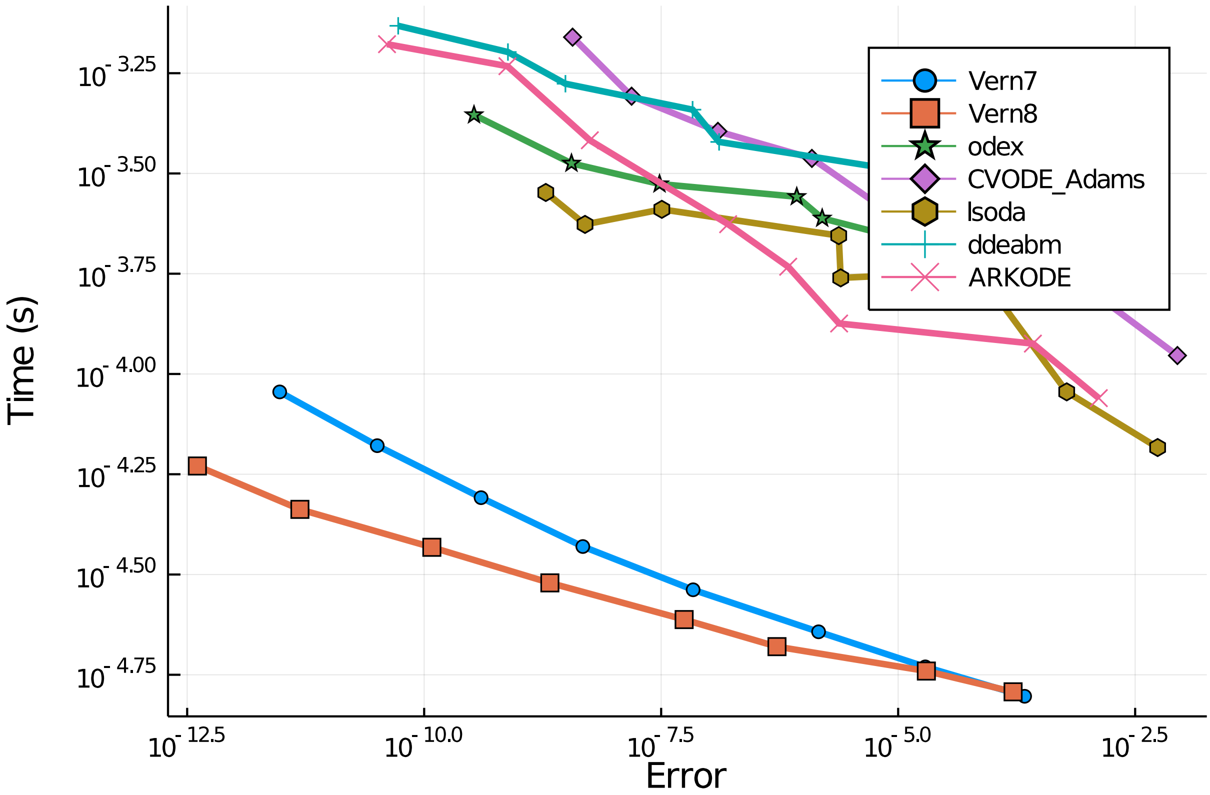
The DifferentialEquations.jl algorithms once again pull ahead. This is the first benchmark we've ran where `ode45` doesn't fail. However, it still doesn't do as well as `Tsit5`. One reason why it does so well is that the maximum norm that ODE.jl uses (as opposed to the L2 norm of Sundials, DifferentialEquations, and ODEInterface) seems to do really well on this problem. `dopri5` does surprisingly bad in this test.

0.1 Higher Order

```
setups = [Dict(:alg=>DP8())
          #Dict(:alg=>ode78()) # fails
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>dop853())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



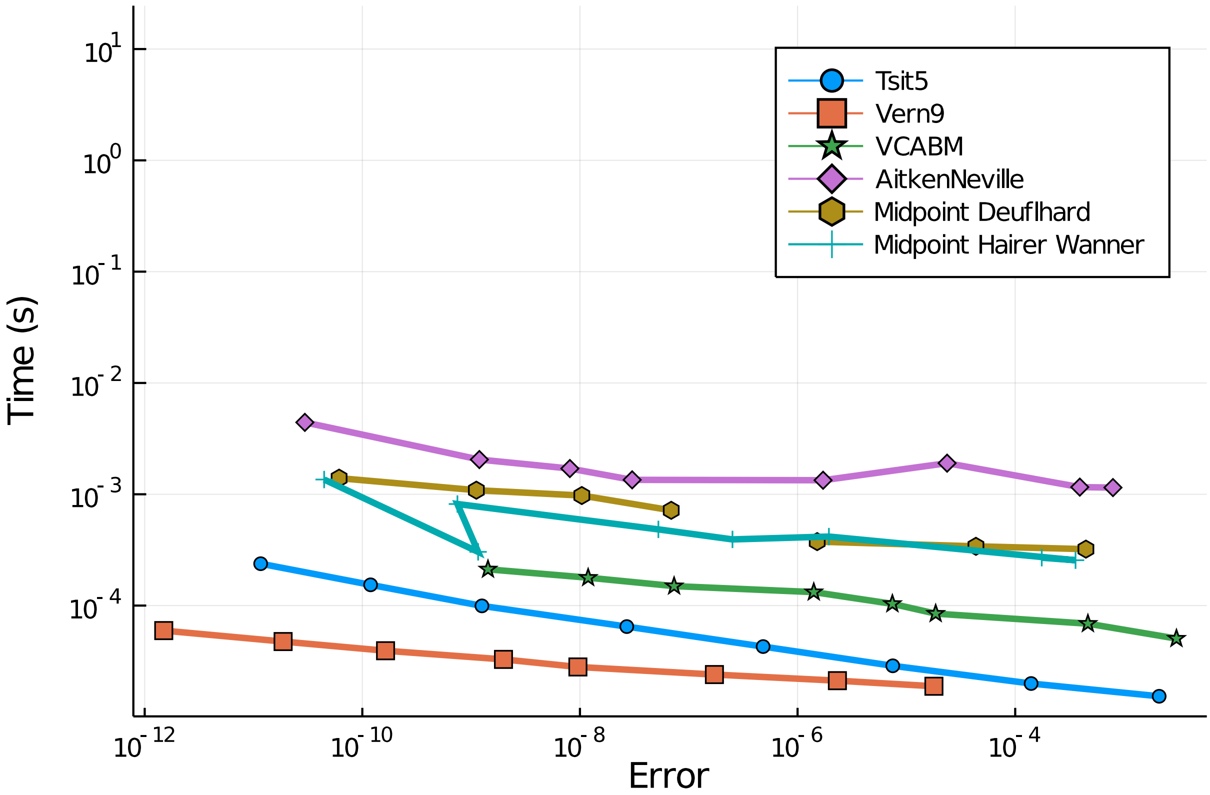

```
setups = [Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>odex())
          Dict(:alg=>CVODE_Adams())
          Dict(:alg=>lsoda())
          Dict(:alg=>ddeabm())
          Dict(:alg=>ARKODE(Sundials.Explicit(),order=6))
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



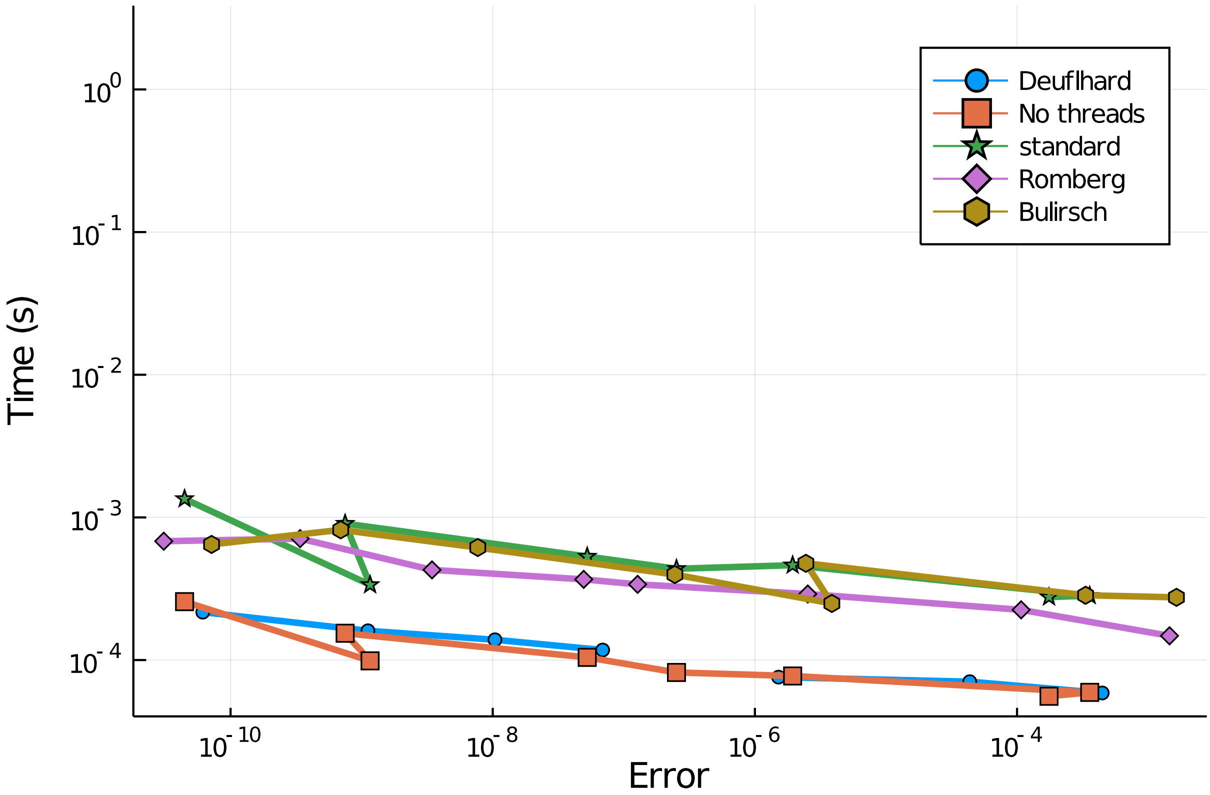
0.2 Comparison with Non-RK methods

Now let's test Tsit5 and Vern9 against parallel extrapolation methods and an Adams-Bashforth-Moulton:

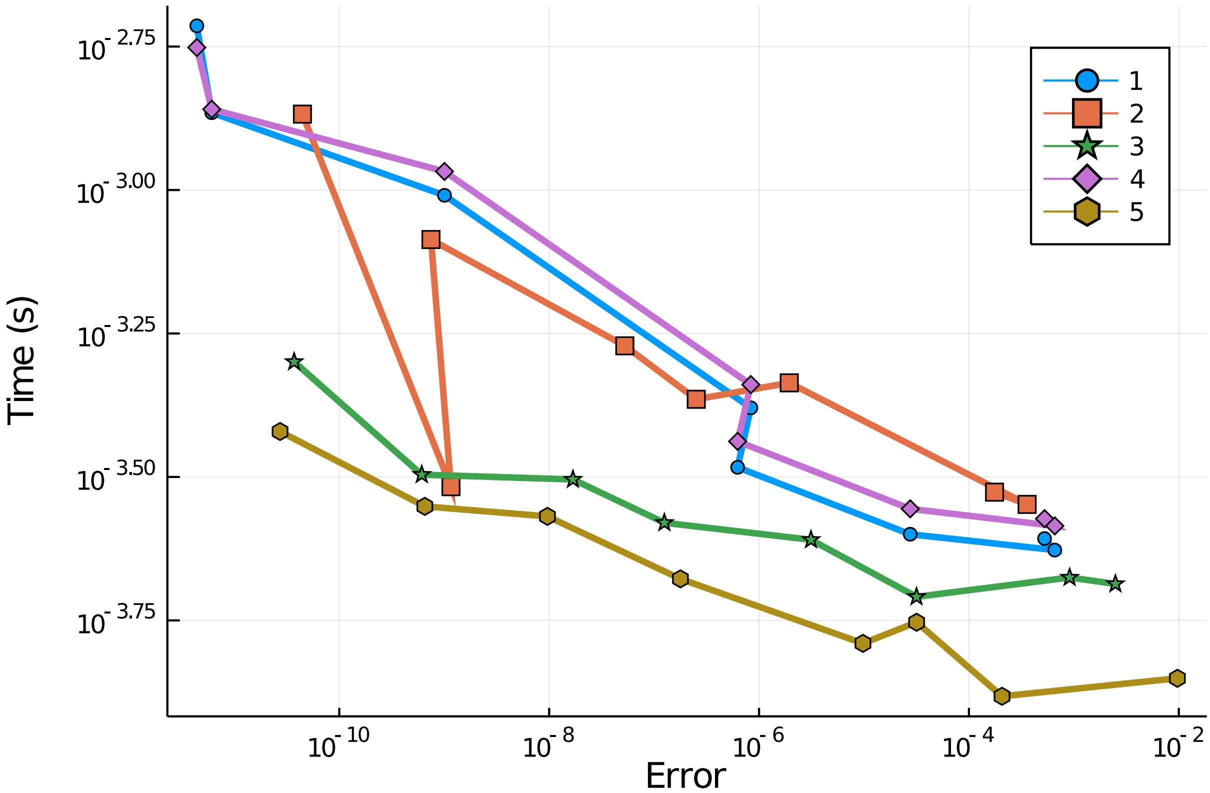
```
setups = [Dict(:alg=>Tsit5())
          Dict(:alg=>Vern9())
          Dict(:alg=>VCABM())
          Dict(:alg=>AitkenNeville(min_order=1, max_order=9, init_order=4,
threading=true))
          Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))]
solnames = ["Tsit5","Vern9","VCABM","AitkenNeville","Midpoint Deuflhard","Midpoint
Hairer Wanner"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```




```
setups = [Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=9, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :romberg, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :bulirsch, threading=true))]
solnames = ["Deuflhard","No threads","standard","Romberg","Bulirsch"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```




```
setups = [Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=15,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=7,
init_order=6, threading=true))]
solnames = ["1","2","3","4","5"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



0.2.1 Conclusion

Once again, the OrdinaryDiffEq.jl pull far ahead in terms of speed and accuracy.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

0.3 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("NonStiffODE","RigidBody_wpd.jmd")
```

Computer Information:

```
Julia Version 1.4.2
Commit 44fa15b150* (2020-05-23 18:35 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
```

```
Environment:
  JULIA_DEPOT_PATH = /builds/JuliaGPU/DiffEqBenchmarks.jl/.julia
  JULIA_CUDA_MEMORY_LIMIT = 2147483648
  JULIA_PROJECT = @.
  JULIA_NUM_THREADS = 4
```

Package Information:

```
Status: `/builds/JuliaGPU/DiffEqBenchmarks.jl/benchmarks/NonStiffODE/Project.toml`
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.22.0
[7f56f5a3-f504-529b-bc02-0b1fe5e64312] LSODA 0.6.1
[c030b06c-0b6d-57c2-b091-7029874bd033] ODE 2.8.0
[54ca160b-1b9f-5127-a996-1867f4bc2a2c] ODEInterface 0.4.6
[09606e27-ecf5-54fc-bb29-004bd9f985bf] ODEInterfaceDiffEq 3.7.0
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.41.0
[65888b18-ceab-5e60-b2b9-181511a3b968] ParameterizedFunctions 5.3.0
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.5.2
[c3572dad-4567-51f8-b174-8c6c989267f4] Sundials 4.2.5
[9a3f8284-a2c9-5f02-9a11-845980a1fd5c] Random
```