Fitzhugh-Nagumo Work-Precision Diagrams

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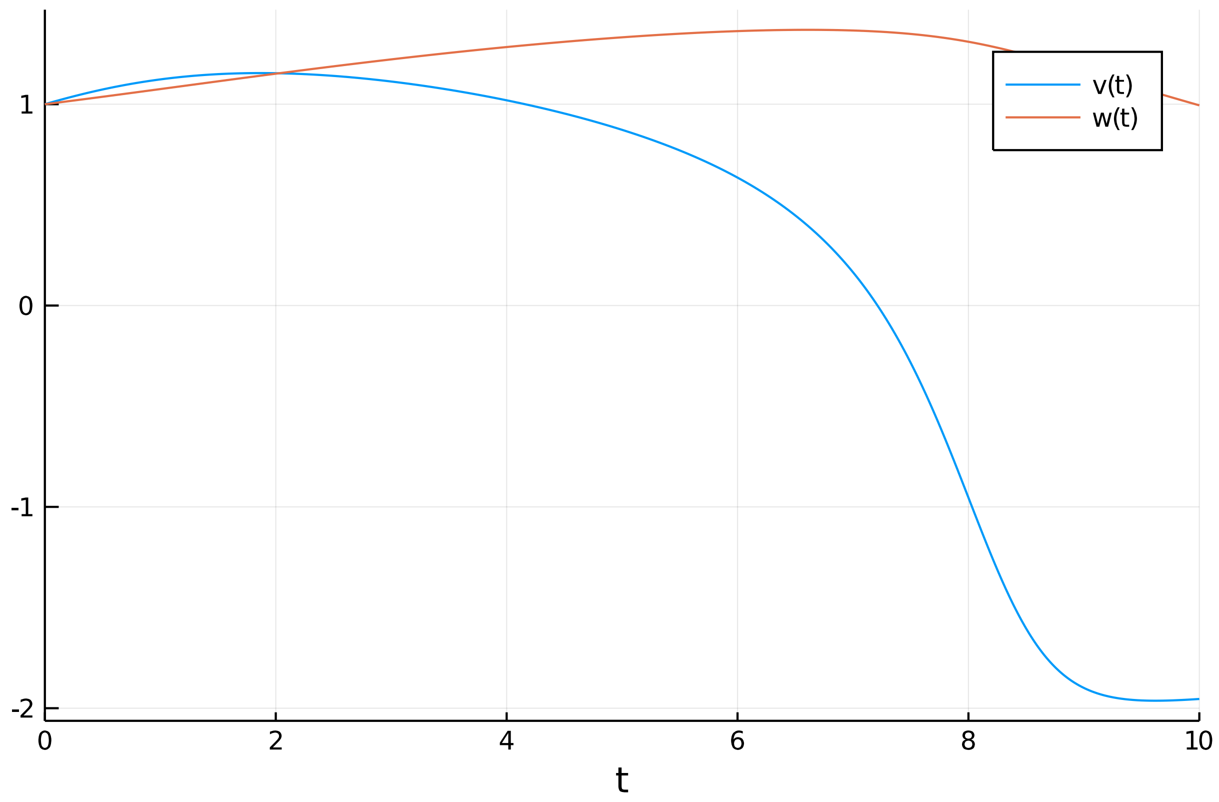
1 Fitzhugh-Nagumo

The purpose of this is to see how the errors scale on a standard nonlinear problem.

```
using OrdinaryDiffEq, ParameterizedFunctions, ODE, ODEInterface,
      ODEInterfaceDiffEq, LSODA, Sundials, DiffEqDevTools

f = @ode_def FitzhughNagumo begin
  dv = v - v^3/3 -w + l
  dw = τinv*(v +  a - b*w)
end a b τinv l

p = [0.7,0.8,1/12.5,0.5]
prob = ODEProblem(f,[1.0;1.0],(0.0,10.0),p)

abstols = 1.0 ./ 10.0 .^ (6:13)
reltols = 1.0 ./ 10.0 .^ (3:10);

sol = solve(prob,Vern7(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)
using Plots; gr()
```

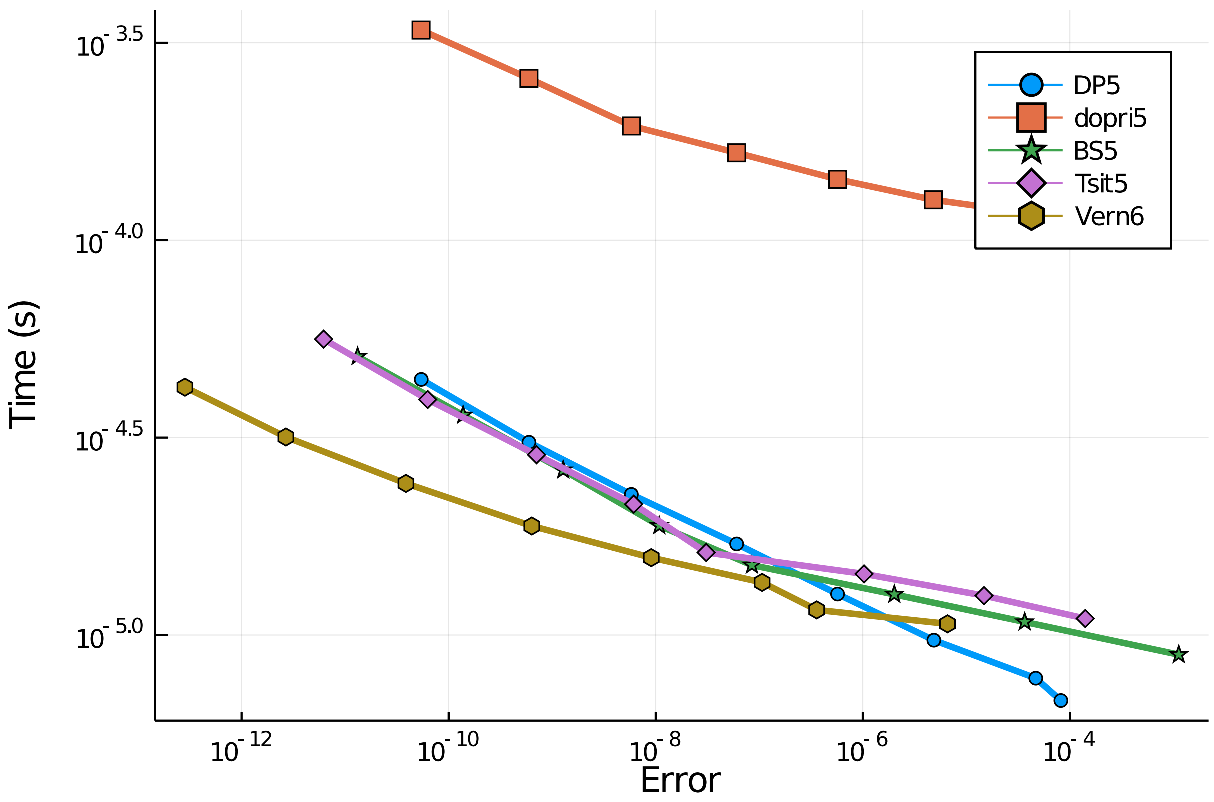
```
Plots.GRBackend()
```

```
plot(sol)
```



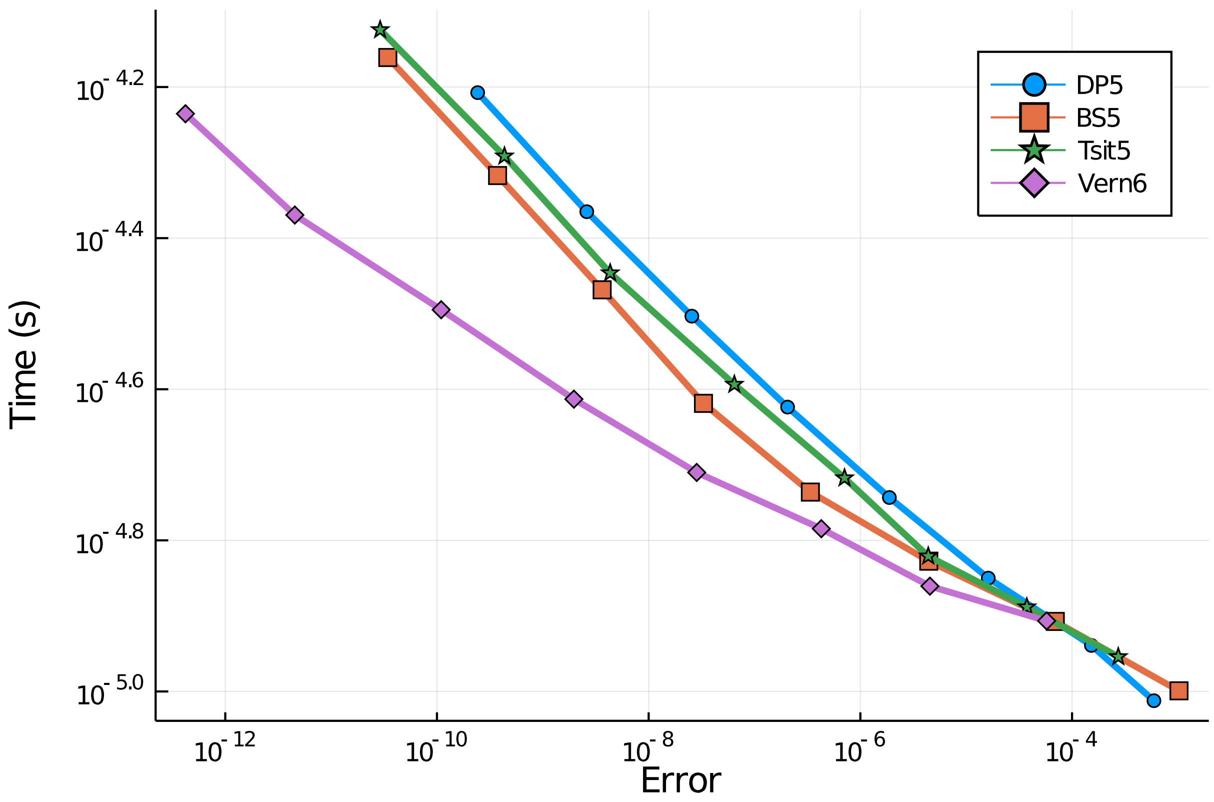
1.1 Low Order

```
setups = [Dict(:alg=>DP5())
          #Dict(:alg=>ode45()) #fails
          Dict(:alg=>dopri5())
          Dict(:alg=>BS5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



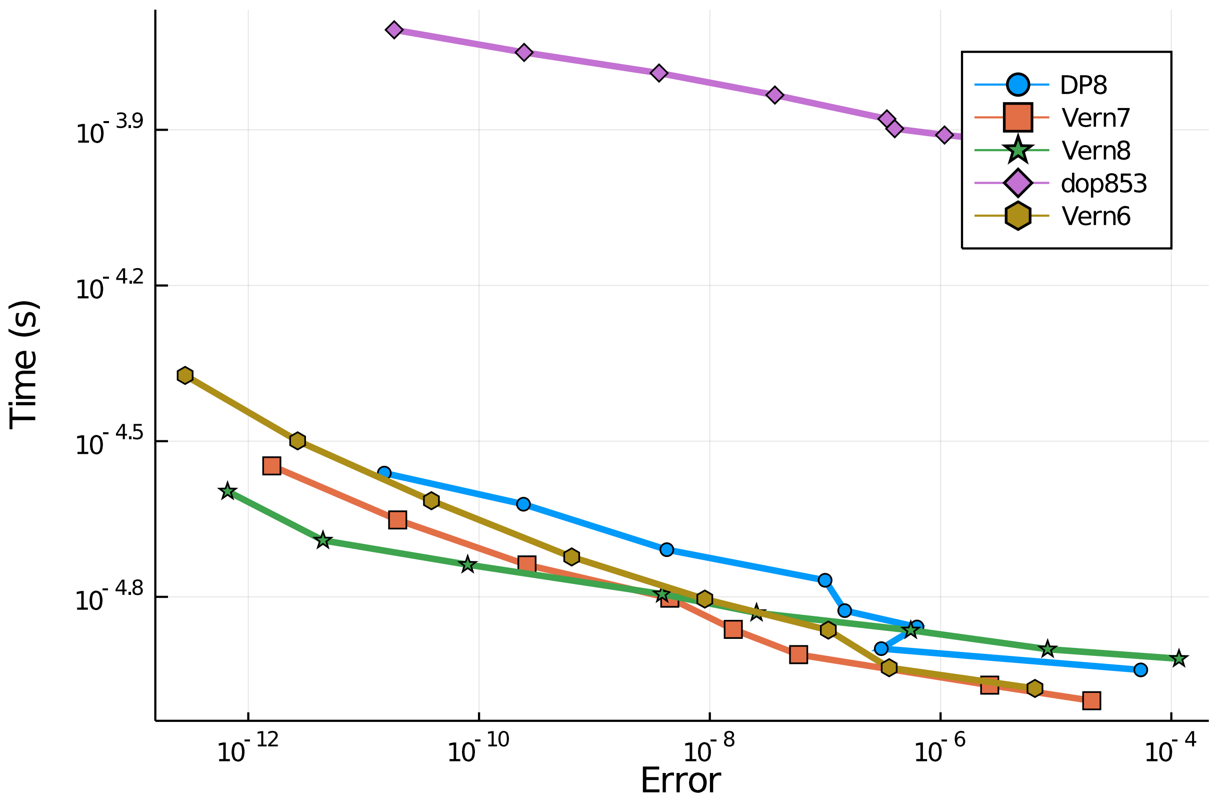
1.1.1 Interpolation

```
setups = [Dict(:alg=>DP5())
          #Dict(:alg=>ode45()) # fails
          Dict(:alg=>BS5())
          Dict(:alg=>Tsit5())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=10000,error_estimat
plot(wp)
```

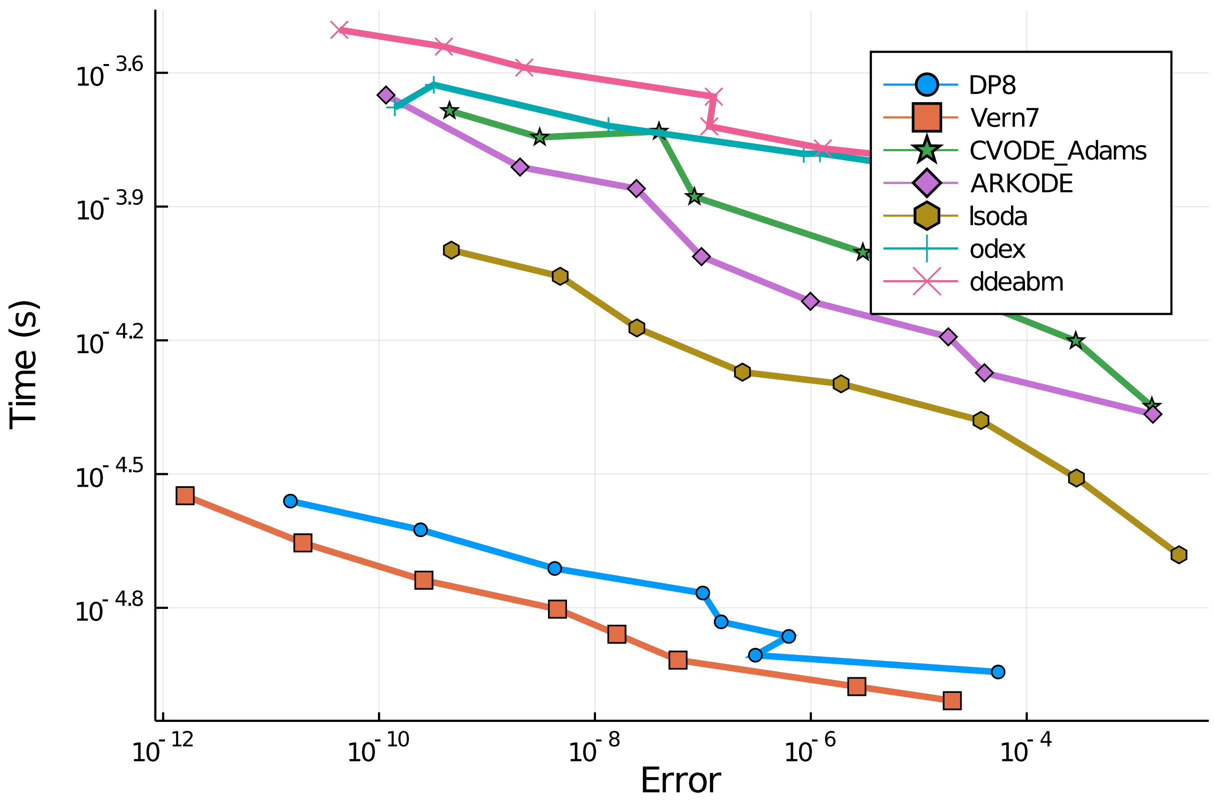


1.2 Higher Order

```
setups = [Dict(:alg=>DP8())
          #Dict(:alg=>ode78()) # fails
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>dop853())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```

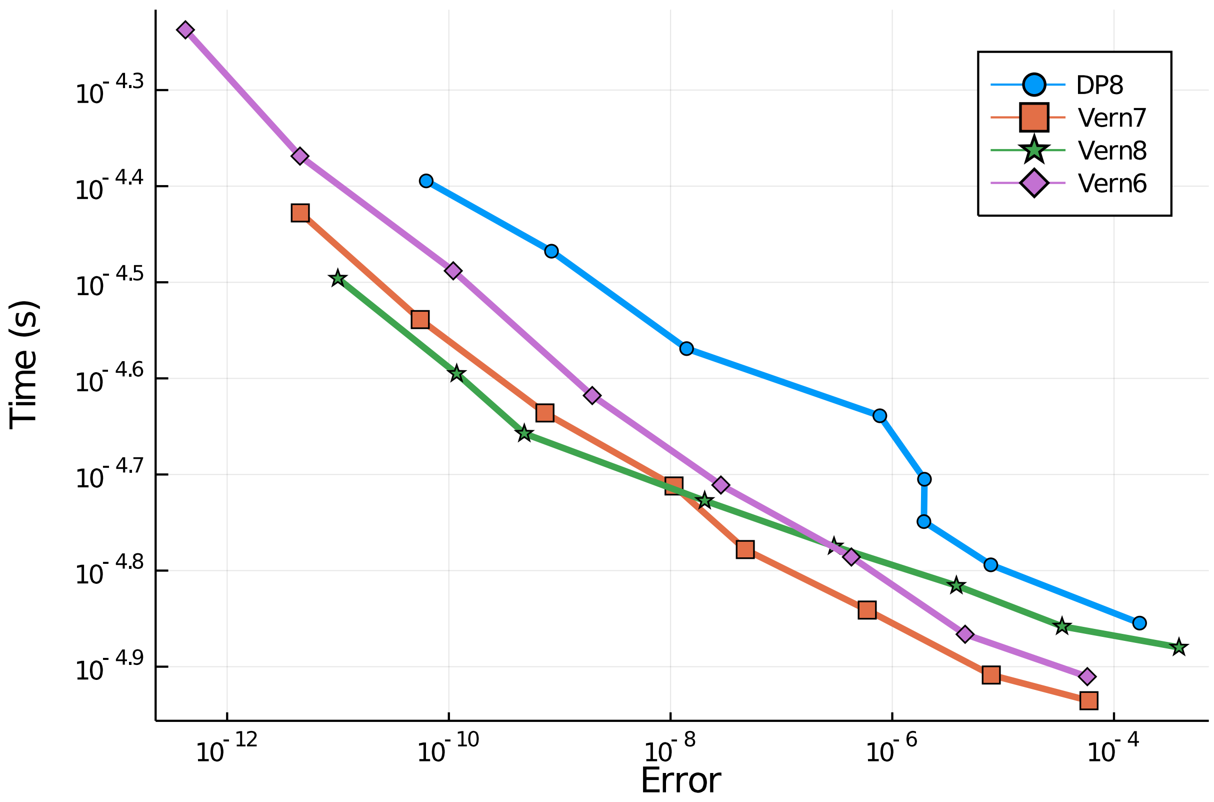


```
setups = [Dict(:alg=>DP8())
          Dict(:alg=>Vern7())
          Dict(:alg=>CVODE_Adams())
          Dict(:alg=>ARKODE(Sundials.Explicit(),order=6))
          Dict(:alg=>lsoda())
          Dict(:alg=>odex())
          Dict(:alg=>ddeabm())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,save_everystep=false,numruns=100,maxiter
plot(wp)
```



1.2.1 Interpolation

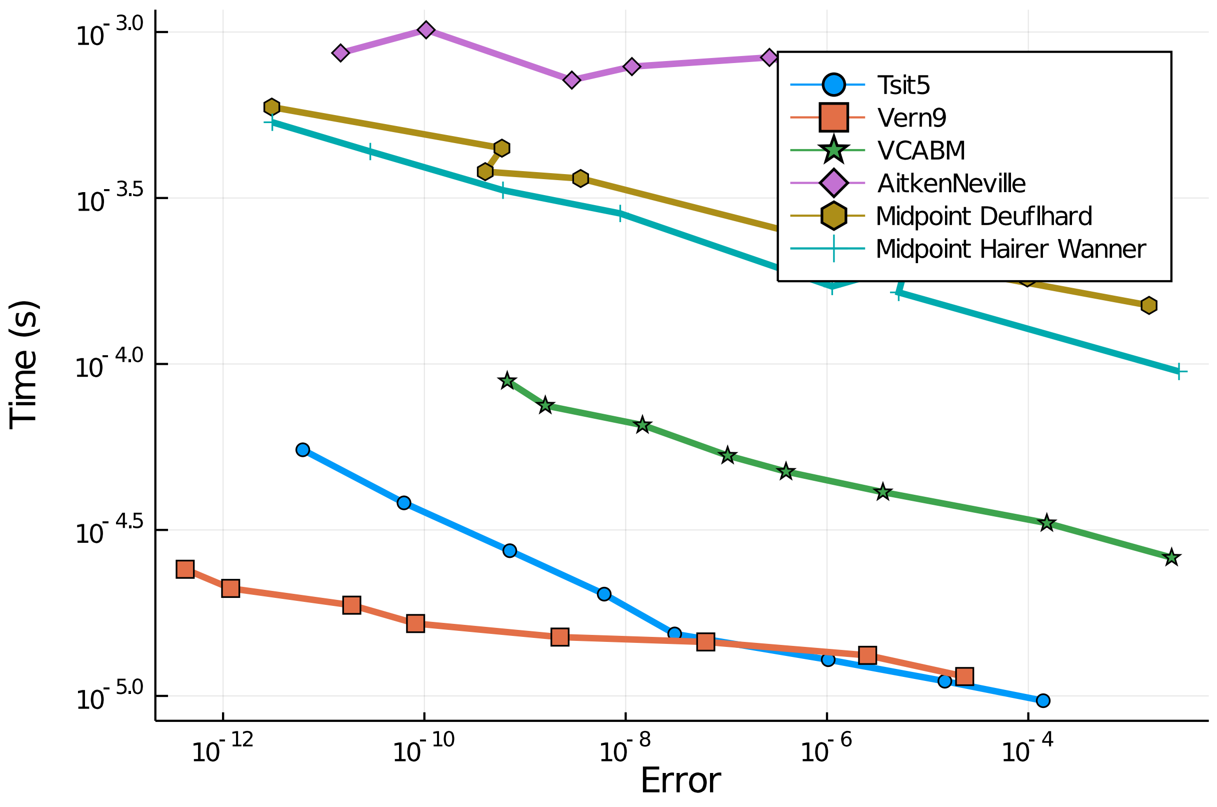
```
setups = [Dict(:alg=>DP8())
          #Dict(:alg=>ode78()) # fails
          Dict(:alg=>Vern7())
          Dict(:alg=>Vern8())
          Dict(:alg=>Vern6())
]
wp =
WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,numruns=100,maxiters=1000,error_estimate
plot(wp)
```



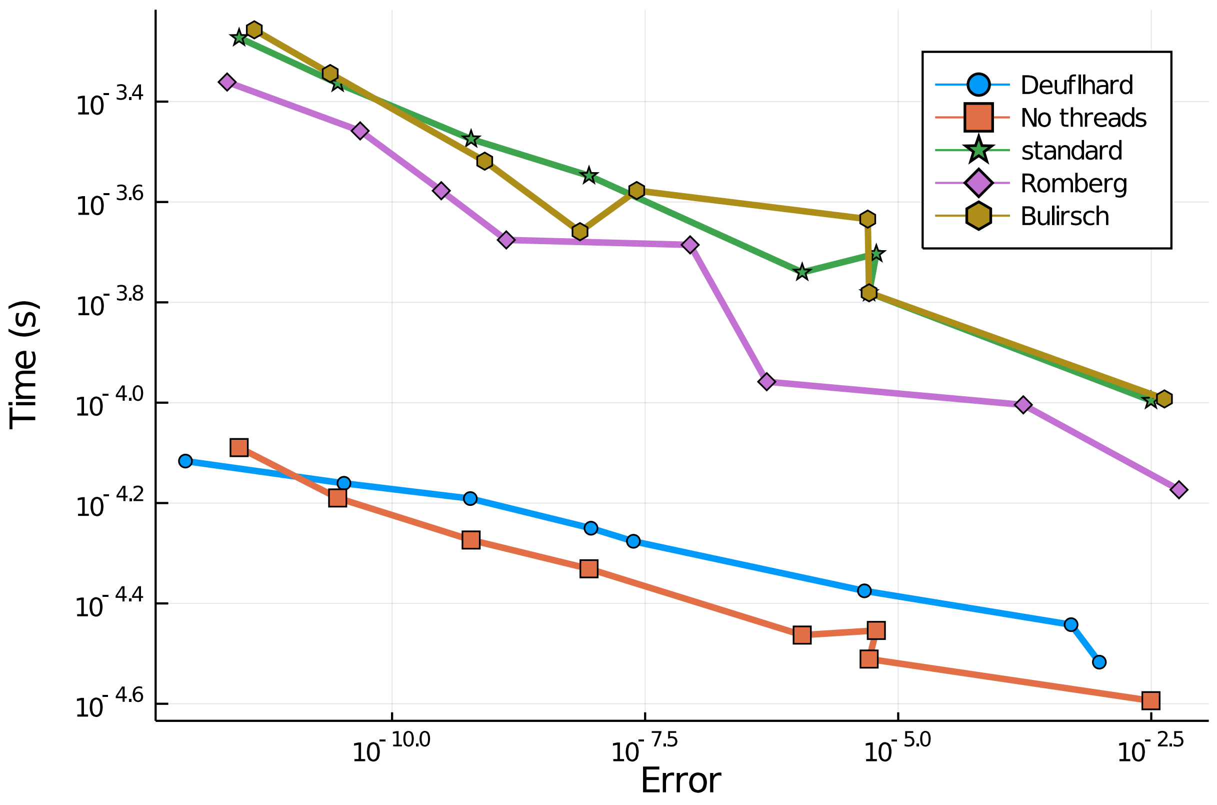
1.3 Comparison with Non-RK methods

Now let's test Tsit5 and Vern9 against parallel extrapolation methods and an Adams-Bashforth-Moulton:

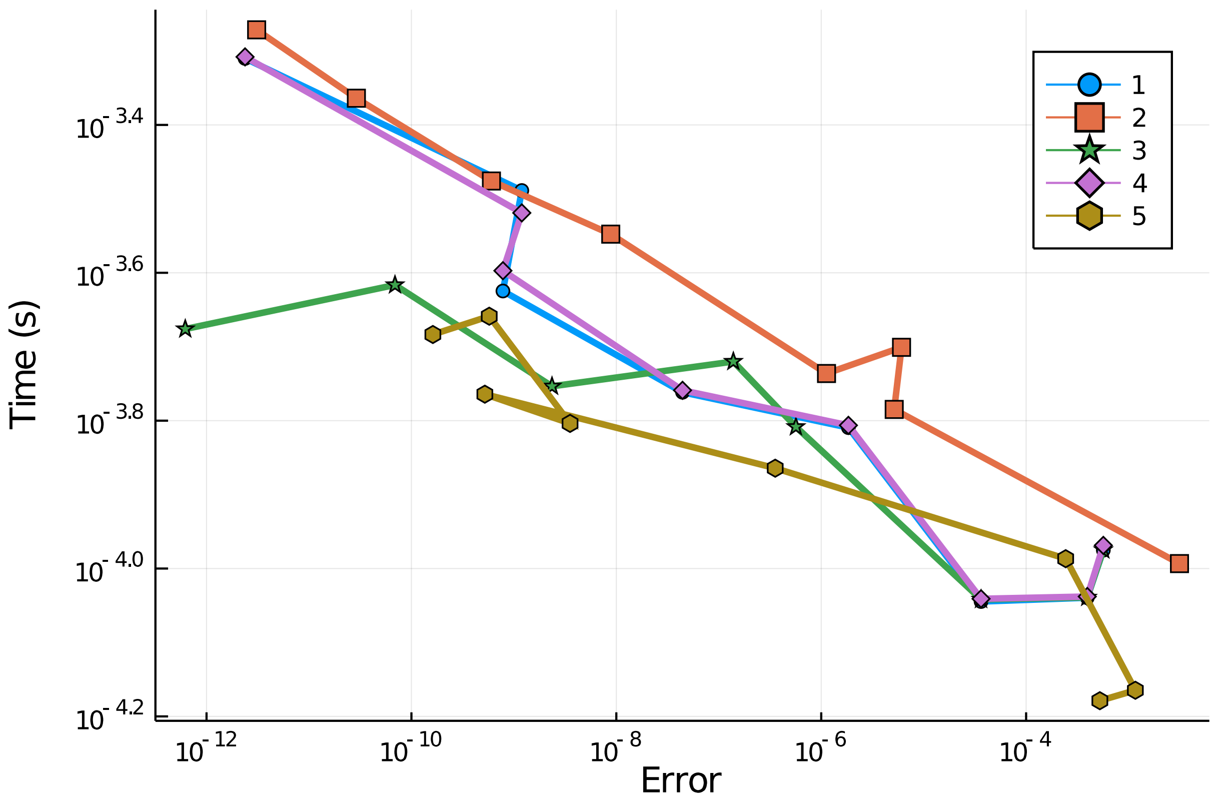
```
setups = [Dict(:alg=>Tsit5())
          Dict(:alg=>Vern9())
          Dict(:alg=>VCABM())
          Dict(:alg=>AitkenNeville(min_order=1, max_order=9, init_order=4,
threading=true))
          Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))]
solnames = ["Tsit5","Vern9","VCABM","AitkenNeville","Midpoint Deuflhard","Midpoint
Hairer Wanner"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



```
setups = [Dict(:alg=>ExtrapolationMidpointDeuflhard(min_order=1, max_order=9,
init_order=9, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=false))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :romberg, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, sequence = :bulirsch, threading=true))]
solnames = ["Deuflhard","No threads","standard","Romberg","Bulirsch"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



```
setups = [Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=11,
init_order=4, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=11,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=2, max_order=15,
init_order=10, threading=true))
          Dict(:alg=>ExtrapolationMidpointHairerWanner(min_order=5, max_order=7,
init_order=6, threading=true))]
solnames = ["1","2","3","4","5"]
wp = WorkPrecisionSet(prob,abstols,reltols,setups;appxsol=test_sol,names=solnames,
                      save_everystep=false,verbose=false,numruns=100)
plot(wp)
```



1.4 Conclusion

As expected, the algorithms are all pretty matched on time for this problem. However, you can clearly see the OrdinaryDiffEq.jl algorithms solving to a much higher accuracy and still faster, especially when the interpolations are involved.

```
using DiffEqBenchmarks
DiffEqBenchmarks.bench_footer(WEAVE_ARGS[:folder],WEAVE_ARGS[:file])
```

1.5 Appendix

These benchmarks are a part of the DiffEqBenchmarks.jl repository, found at: https://github.com/JuliaDi

To locally run this tutorial, do the following commands:

```
using DiffEqBenchmarks
DiffEqBenchmarks.weave_file("NonStiffODE","FitzhughNagumo_wpd.jmd")
```

Computer Information:

```
Julia Version 1.4.2
Commit 44fa15b150* (2020-05-23 18:35 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_DEPOT_PATH = /builds/JuliaGPU/DiffEqBenchmarks.jl/.julia
  JULIA_CUDA_MEMORY_LIMIT = 2147483648
  JULIA_PROJECT = @.
  JULIA_NUM_THREADS = 4
```

Package Information:

```
Status: `/builds/JuliaGPU/DiffEqBenchmarks.jl/benchmarks/NonStiffODE/Project.toml`
[f3b72e0c-5b89-59e1-b016-84e28bfd966d] DiffEqDevTools 2.22.0
[7f56f5a3-f504-529b-bc02-0b1fe5e64312] LSODA 0.6.1
[c030b06c-0b6d-57c2-b091-7029874bd033] ODE 2.8.0
[54ca160b-1b9f-5127-a996-1867f4bc2a2c] ODEInterface 0.4.6
[09606e27-ecf5-54fc-bb29-004bd9f985bf] ODEInterfaceDiffEq 3.7.0
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.41.0
[65888b18-ceab-5e60-b2b9-181511a3b968] ParameterizedFunctions 5.3.0
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.5.2
[c3572dad-4567-51f8-b174-8c6c989267f4] Sundials 4.2.5
[9a3f8284-a2c9-5f02-9a11-845980a1fd5c] Random
```