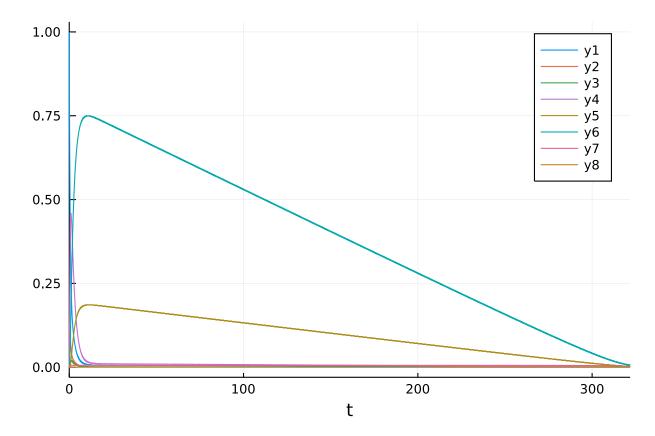
# HIRES Work-Precision Diagrams

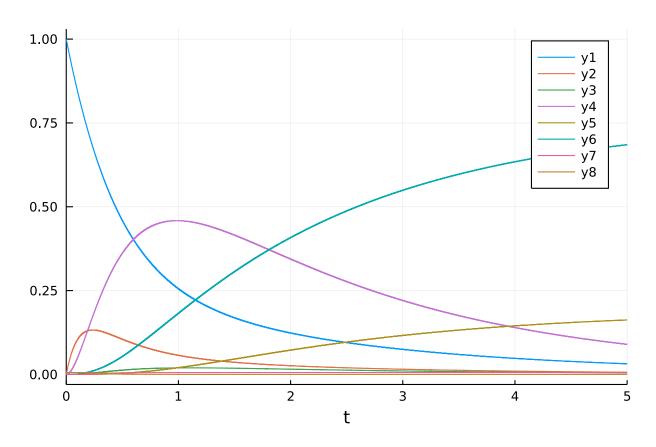
## Chris Rackauckas

May 10, 2021

```
using OrdinaryDiffEq, ParameterizedFunctions, Plots, ODE, ODEInterfaceDiffEq, LSODA,
DiffEqDevTools, Sundials
using LinearAlgebra
gr() #gr(fmt=:png)
f = @ode_def Hires begin
  dy1 = -1.71*y1 + 0.43*y2 + 8.32*y3 + 0.0007
  dy2 = 1.71*y1 - 8.75*y2
  dy3 = -10.03*y3 + 0.43*y4 + 0.035*y5
  dy4 = 8.32*y2 + 1.71*y3 - 1.12*y4
  dy5 = -1.745*y5 + 0.43*y6 + 0.43*y7
  dy6 = -280.0*y6*y8 + 0.69*y4 + 1.71*y5 -
           0.43*y6 + 0.69*y7
 dy7 = 280.0*y6*y8 - 1.81*y7
  dy8 = -280.0*y6*y8 + 1.81*y7
u0 = zeros(8)
u0[1] = 1
u0[8] = 0.0057
prob = ODEProblem(f,u0,(0.0,321.8122))
sol = solve(prob,Rodas5(),abstol=1/10^14,reltol=1/10^14)
test_sol = TestSolution(sol)
abstols = 1.0 ./ 10.0 .^{(4:11)}
reltols = 1.0 ./ 10.0 .^{(1:8)};
plot(sol)
```



plot(sol,tspan=(0.0,5.0))



#### 0.1 Omissions

The following were omitted from the tests due to convergence failures. ODE.jl's adaptivity is not able to stabilize its algorithms, while GeometricIntegratorsDiffEq has not upgraded to Julia 1.0. GeometricIntegrators.jl's methods used to be either fail to converge at comparable dts (or on some computers errors due to type conversions).

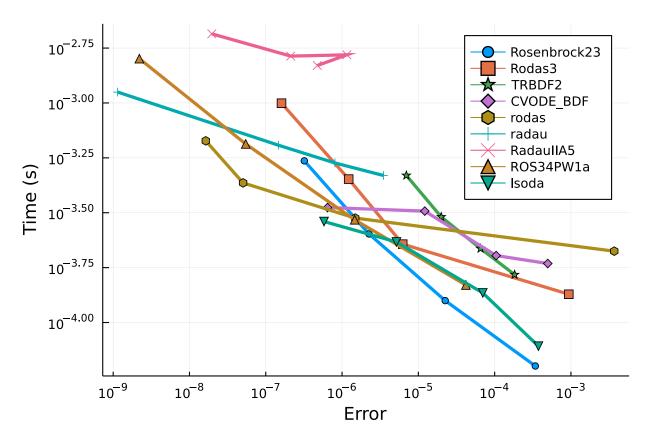
```
#sol = solve(prob,ode23s()); println("Total ODE.jl steps: $(length(sol))")
#using GeometricIntegratorsDiffEq
#try
# sol = solve(prob,GIRadIIA3(),dt=1/10)
#catch e
# println(e)
#end
```

The stabilized explicit methods are not stable enough to handle this problem well. While they don't diverge, they are really slow.

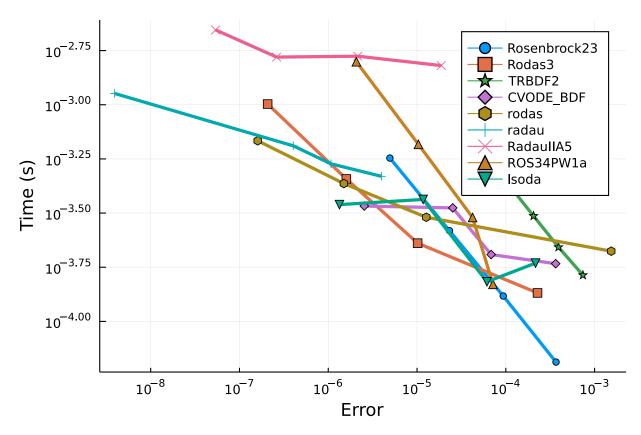
```
setups = [
#Dict(:alg=>ROCK2()),
#Dict(:alg=>ROCK4())
#Dict(:alg=>ESERK5())
]
Any[]
```

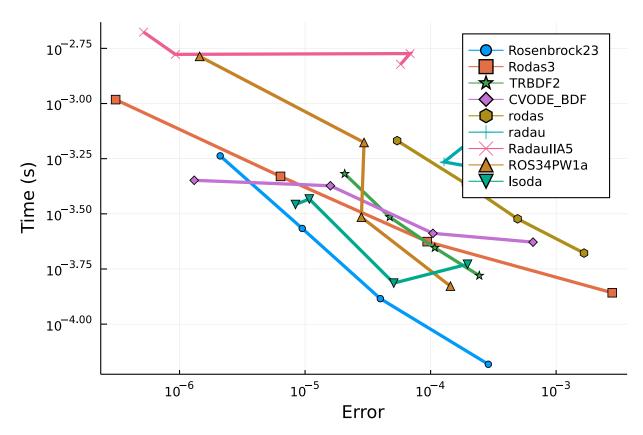
## 0.2 High Tolerances

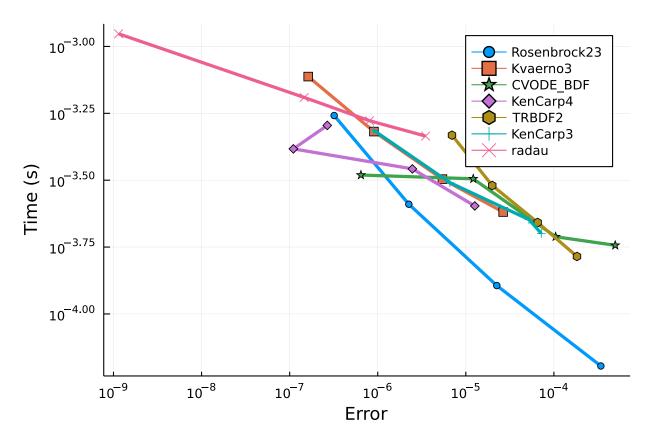
This is the speed when you just want the answer.

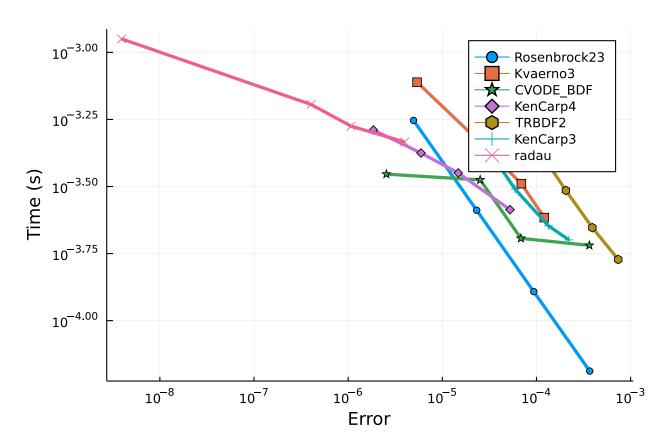


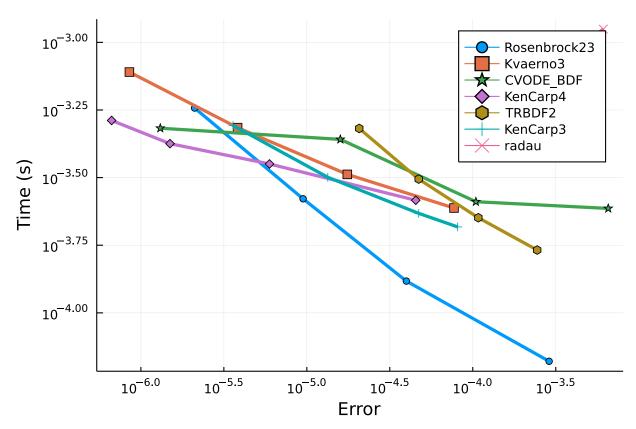
 $\label{eq:wp_recisionSet} $$ wp = WorkPrecisionSet(prob,abstols,reltols,setups;dense = false,verbose=false, appxsol=test_sol,maxiters=Int(1e5),error_estimate=:12) $$ plot(wp) $$$ 

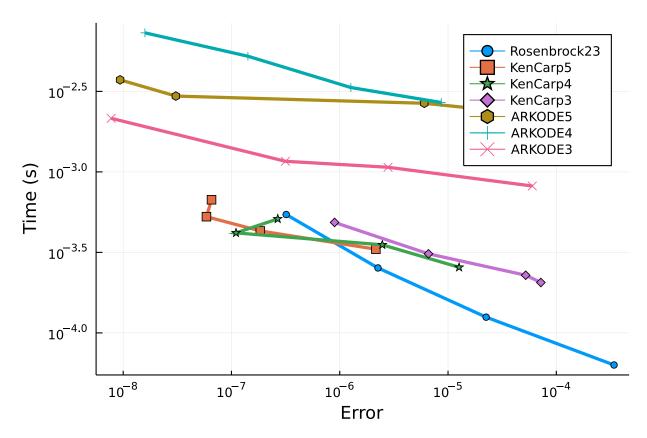


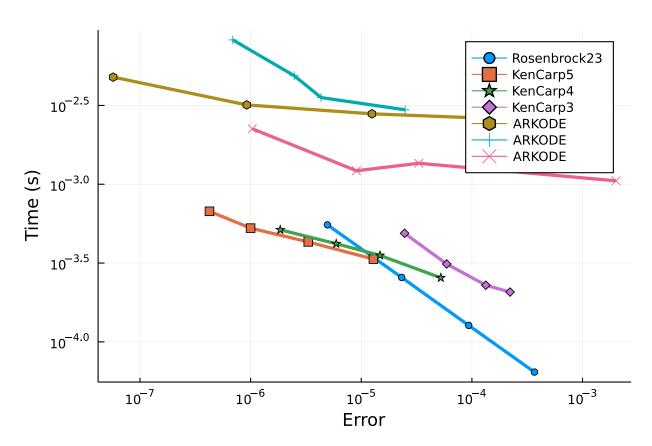










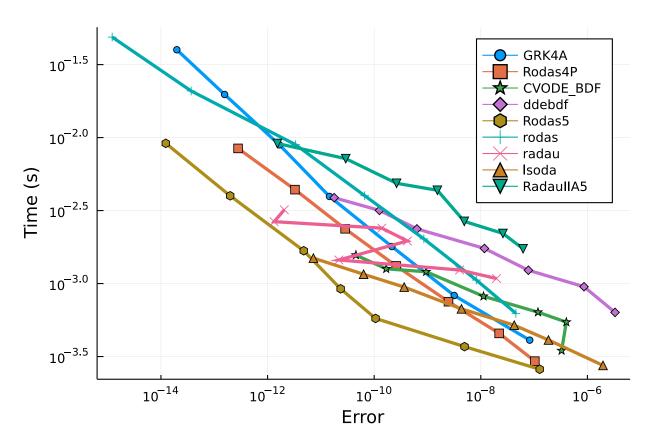


```
Dict(:alg=>ImplicitEulerExtrapolation()),
           #Dict(:alg=>ImplicitDeuflhardExtrapolation()), # Diverges
           #Dict(:alg=>ImplicitHairerWannerExtrapolation()), # Diverges
           Dict(:alg=>ABDF2()),
           Dict(:alg=>QNDF()),
           Dict(:alg=>Exprb43()),
           Dict(:alg=>Exprb32()),
wp = WorkPrecisionSet(prob,abstols,reltols,setups;
                        save_everystep=false,appxsol=test_sol,maxiters=Int(1e5))
plot(wp)
    10^{-1}
                                                                  Rosenbrock23
                                                                  TRBDF2
                                                                  ImplicitEulerExtrapolation
                                                                  ABDF2
                                                                  ONDF
    10<sup>-2</sup>
                                                                  Exprb43
                                                                  Exprb32
Time (s)
    10<sup>-3</sup>
    10
                   10<sup>-8</sup>
                                                                                 10-2
                                       10^{-6}
                                                            10^{-4}
```

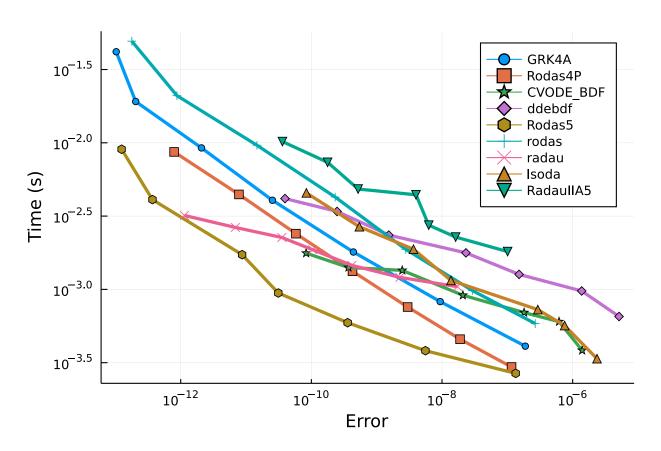
#### 0.2.1 Low Tolerances

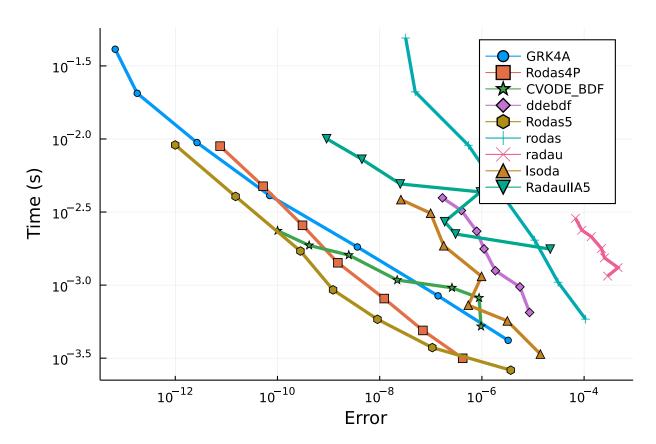
This is the speed at lower tolerances, measuring what's good when accuracy is needed.

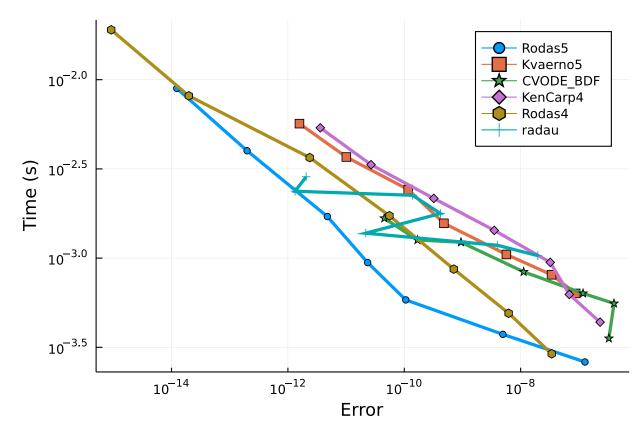
Error



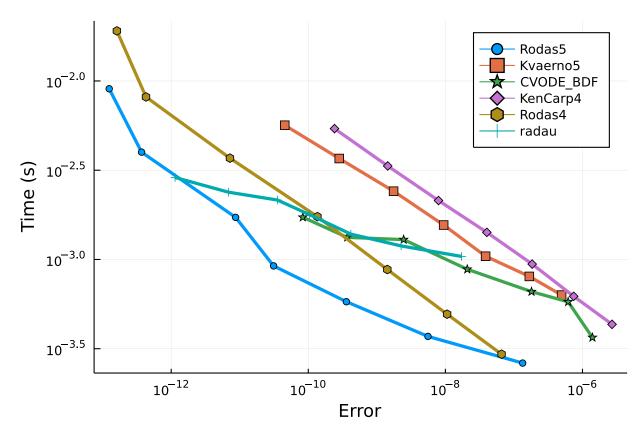
 $\label{eq:wp_recisionSet} $$ wp = WorkPrecisionSet(prob,abstols,reltols,setups;verbose=$false$, $$ dense=$false$,appxsol=test_sol,maxiters=Int(1e5),error_estimate=:12) $$ plot(wp) $$$ 

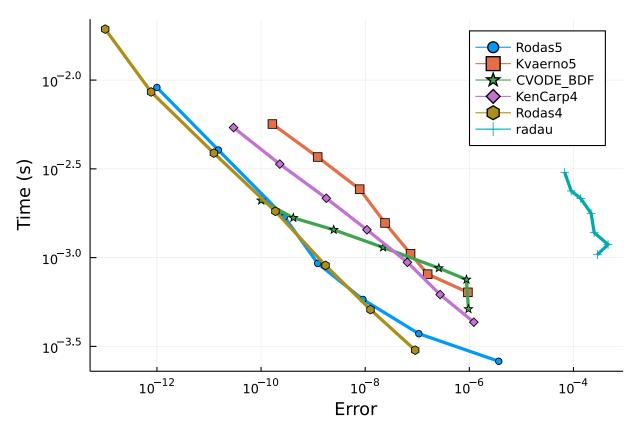


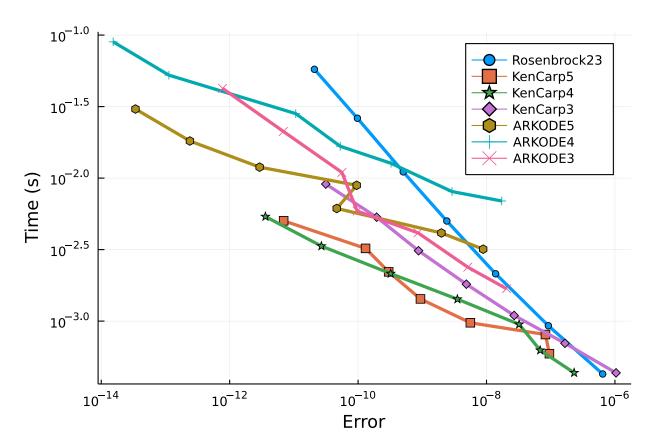




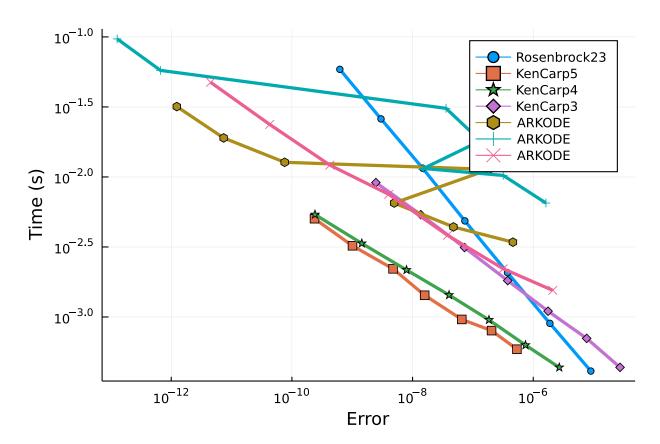
 $\label{eq:wp_recisionSet} $$ wp = WorkPrecisionSet(prob,abstols,reltols,setups;verbose=$false$, $$ dense=$false$,appxsol=test_sol,maxiters=Int(1e5),error_estimate=:12) $$ plot(wp) $$$ 







 $\label{eq:wp_recisionSet} $$ wp = WorkPrecisionSet(prob,abstols,reltols,setups;verbose=$false$, $$ dense=$false$,appxsol=test_sol,maxiters=Int(1e5),error_estimate=:12) $$ plot(wp) $$$ 



The following algorithms were removed since they failed.

#### 0.2.2 Conclusion

At high tolerances, Rosenbrock23 and lsoda hits the the error estimates and are fast. At lower tolerances and normal user tolerances, Rodas5 is extremely fast. When you get down to reltol=1e-10 radau begins to become as efficient as Rodas4, and it continues to do well below that.

# 0.3 Appendix

These benchmarks are a part of the SciMLBenchmarks.jl repository, found at: https://github.com/SciML, For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization https://sciml.ai.

To locally run this benchmark, do the following commands:

```
using SciMLBenchmarks
SciMLBenchmarks.weave_file("benchmarks/StiffODE","Hires.jmd")

Computer Information:

Julia Version 1.6.1
Commit 6aaedecc44 (2021-04-23 05:59 UTC)
Platform Info:
    OS: Linux (x86_64-pc-linux-gnu)
    CPU: Intel(R) Core(TM) i7-9700K CPU @ 3.60GHz
    WORD_SIZE: 64
    LIBM: libopenlibm
    LLVM: libLLVM-11.0.1 (ORCJIT, skylake)
Environment:
    JULIA_DEPOT_PATH = /root/.cache/julia-buildkite-plugin/depots/5b300254-1738-4989-ae06
    JULIA_NUM_THREADS = 3
```

Package Information:

Status `/var/lib/buildkite-agent/builds/rtx2070-gpuci1-julia-csail-mit-edu/julia: [f3b72e0c] DiffEqDevTools v2.27.2

```
[5a33fad7] GeometricIntegratorsDiffEq v0.2.0 [7f56f5a3] LSODA v0.7.0 [c030b06c] ODE v2.13.0 [09606e27] ODEInterfaceDiffEq v3.10.0 [1dea7af3] OrdinaryDiffEq v5.53.1 [65888b18] ParameterizedFunctions v5.10.0 [91a5bcdd] Plots v1.13.2 [31c91b34] SciMLBenchmarks v0.1.0 `../..#` [c3572dad] Sundials v4.4.3 [a759f4b9] TimerOutputs v0.5.8 [37e2e46d] LinearAlgebra
```

#### And the full manifest:

```
Status \( \tau \)/var/lib/buildkite-agent/builds/rtx2070-gpuci1-julia-csail-mit-edu/julia
[c3fe647b] AbstractAlgebra v0.16.0
[621f4979] AbstractFFTs v1.0.1
[1520ce14] AbstractTrees v0.3.4
[79e6a3ab] Adapt v3.3.0
[4c88cf16] Aqua v0.5.0
[ec485272] ArnoldiMethod v0.1.0
[4fba245c] ArrayInterface v3.1.11
[4c555306] ArrayLayouts v0.4.12
[9e28174c] BinDeps v1.0.2
[b99e7846] BinaryProvider v0.5.10
[a74b3585] Blosc v0.7.0
[fa961155] CEnum v0.4.1
[d360d2e6] ChainRulesCore v0.9.41
[b630d9fa] CheapThreads v0.2.3
[35d6a980] ColorSchemes v3.12.1
[3da002f7] ColorTypes v0.11.0
[5ae59095] Colors v0.12.8
[861a8166] Combinatorics v1.0.2
[38540f10] CommonSolve v0.2.0
[bbf7d656] CommonSubexpressions v0.3.0
[34da2185] Compat v3.28.0
[8f4d0f93] Conda v1.5.2
[187b0558] ConstructionBase v1.2.1
[d38c429a] Contour v0.5.7
[717857b8] DSP v0.6.10
[9a962f9c] DataAPI v1.6.0
[864edb3b] DataStructures v0.18.9
[e2d170a0] DataValueInterfaces v1.0.0
[55939f99] DecFP v1.1.0
[2b5f629d] DiffEqBase v6.61.0
[f3b72e0c] DiffEqDevTools v2.27.2
[c894b116] DiffEqJump v6.14.1
[77a26b50] DiffEqNoiseProcess v5.7.2
```

[163ba53b] DiffResults v1.0.3

```
[b552c78f] DiffRules v1.0.2
```

[b4f34e82] Distances v0.10.3

[31c24e10] Distributions v0.24.18

[ffbed154] DocStringExtensions v0.8.4

[e30172f5] Documenter v0.26.3

[d4d017d3] ExponentialUtilities v1.8.4

[e2ba6199] ExprTools v0.1.3

[8f5d6c58] EzXML v1.1.0

[c87230d0] FFMPEG v0.4.0

[7a1cc6ca] FFTW v1.4.1

[7034ab61] FastBroadcast v0.1.5

[9aa1b823] FastClosures v0.3.2

[442a2c76] FastGaussQuadrature v0.4.7

[057dd010] FastTransforms v0.11.3

[1a297f60] FillArrays v0.10.2

[6a86dc24] FiniteDiff v2.8.0

[53c48c17] FixedPointNumbers v0.8.4

[59287772] Formatting v0.4.2

[f6369f11] ForwardDiff v0.10.18

[069b7b12] FunctionWrappers v1.1.2

[28b8d3ca] GR v0.57.4

[14197337] GenericLinearAlgebra v0.2.5

[dcce2d33] GeometricIntegrators v0.6.2

[5a33fad7] GeometricIntegratorsDiffEq v0.2.0

[5c1252a2] GeometryBasics v0.3.12

[d7ba0133] Git v1.2.1

[42e2da0e] Grisu v1.0.2

[f67ccb44] HDF5 v0.14.3

[cd3eb016] HTTP v0.9.8

[eafb193a] Highlights v0.4.5

[0e44f5e4] Hwloc v2.0.0

[7073ff75] IJulia v1.23.2

[b5f81e59] IOCapture v0.1.1

[615f187c] IfElse v0.1.0

[d25df0c9] Inflate v0.1.2

[83e8ac13] IniFile v0.5.0

[d8418881] Intervals v1.5.0

[c8e1da08] IterTools v1.3.0

[42fd0dbc] IterativeSolvers v0.9.0

[82899510] IteratorInterfaceExtensions v1.0.0

[692b3bcd] JLLWrappers v1.3.0

[682c06a0] JSON v0.21.1

[7f56f5a3] LSODA v0.7.0

[b964fa9f] LaTeXStrings v1.2.1

[2ee39098] LabelledArrays v1.6.0

[23fbe1c1] Latexify v0.15.5

[093fc24a] LightGraphs v1.3.5

[d3d80556] LineSearches v7.1.1

[2ab3a3ac] LogExpFunctions v0.2.3

```
[bdcacae8] LoopVectorization v0.12.18
```

[1914dd2f] MacroTools v0.5.6

[739be429] MbedTLS v1.0.3

[442fdcdd] Measures v0.3.1

[c03570c3] Memoize v0.4.4

[e1d29d7a] Missings v1.0.0

[78c3b35d] Mocking v0.7.1

[961ee093] ModelingToolkit v5.16.0

[46d2c3a1] MuladdMacro v0.2.2

[ffc61752] Mustache v1.0.10

[d41bc354] NLSolversBase v7.8.0

[2774e3e8] NLsolve v4.5.1

[77ba4419] NaNMath v0.3.5

[8913a72c] NonlinearSolve v0.3.8

[c030b06c] ODE v2.13.0

[54ca160b] ODEInterface v0.5.0

[09606e27] ODEInterfaceDiffEq v3.10.0

[6fe1bfb0] OffsetArrays v1.8.0

[429524aa] Optim v1.3.0

[bac558e1] OrderedCollections v1.4.0

[1dea7af3] OrdinaryDiffEq v5.53.1

[90014a1f] PDMats v0.11.0

[65888b18] ParameterizedFunctions v5.10.0

[d96e819e] Parameters v0.12.2

[69de0a69] Parsers v1.1.0

[ccf2f8ad] PlotThemes v2.0.1

[995b91a9] PlotUtils v1.0.10

[91a5bcdd] Plots v1.13.2

[e409e4f3] PoissonRandom v0.4.0

[f27b6e38] Polynomials v1.2.1

[85a6dd25] PositiveFactorizations v0.2.4

[21216c6a] Preferences v1.2.2

[92933f4c] ProgressMeter v1.6.2

[1fd47b50] QuadGK v2.4.1

[74087812] Random123 v1.3.1

[fb686558] RandomExtensions v0.4.3

[e6cf234a] RandomNumbers v1.4.0

[3cdcf5f2] RecipesBase v1.1.1

[01d81517] RecipesPipeline v0.3.2

[731186ca] RecursiveArrayTools v2.11.3

[f2c3362d] RecursiveFactorization v0.1.12

[189a3867] Reexport v0.2.0

[ae029012] Requires v1.1.3

[ae5879a3] ResettableStacks v1.1.0

[79098fc4] Rmath v0.7.0

[47965b36] RootedTrees v1.0.0

[7e49a35a] RuntimeGeneratedFunctions v0.5.2

[476501e8] SLEEFPirates v0.6.17

[1bc83da4] SafeTestsets v0.0.1

```
[Obca4576] SciMLBase v1.13.2
```

[31c91b34] SciMLBenchmarks v0.1.0 `../..#`

[6c6a2e73] Scratch v1.0.3

[efcf1570] Setfield v0.7.0

[992d4aef] Showoff v1.0.3

[699a6c99] SimpleTraits v0.9.3

[b85f4697] SoftGlobalScope v1.1.0

[a2af1166] SortingAlgorithms v1.0.0

[47a9eef4] SparseDiffTools v1.13.2

[276daf66] SpecialFunctions v0.10.3

[a25cea48] SpecialPolynomials v0.1.0

[aedffcd0] Static v0.2.4

[90137ffa] StaticArrays v1.1.3

[82ae8749] StatsAPI v1.0.0

[2913bbd2] StatsBase v0.33.8

[4c63d2b9] StatsFuns v0.9.8

[7792a7ef] StrideArraysCore v0.1.7

[09ab397b] StructArrays v0.5.1

[c3572dad] Sundials v4.4.3

[d1185830] SymbolicUtils v0.11.2

[0c5d862f] Symbolics v0.1.25

[3783bdb8] TableTraits v1.0.1

[bd369af6] Tables v1.4.2

[8290d209] ThreadingUtilities v0.4.1

[f269a46b] TimeZones v1.5.4

[a759f4b9] TimerOutputs v0.5.8

[c751599d] ToeplitzMatrices v0.6.3

[a2a6695c] TreeViews v0.3.0

[30578b45] URIParser v0.4.1

[5c2747f8] URIs v1.3.0

[3a884ed6] UnPack v1.0.2

[1986cc42] Unitful v1.7.0

[3d5dd08c] VectorizationBase v0.19.37

[81def892] VersionParsing v1.2.0

[19fa3120] VertexSafeGraphs v0.1.2

[44d3d7a6] Weave v0.10.8

[ddb6d928] YAML v0.4.6

[c2297ded] ZMQ v1.2.1

[700de1a5] ZygoteRules v0.2.1

[0b7ba130] Blosc\_jll v1.14.3+1

[6e34b625] Bzip2 jll v1.0.6+5

[83423d85] Cairo\_jll v1.16.0+6

[47200ebd] DecFP\_jll v2.0.2+0

[5ae413db] EarCut\_jll v2.1.5+1

[2e619515] Expat jll v2.2.7+6

[b22a6f82] FFMPEG jll v4.3.1+4

[f5851436] FFTW\_jll v3.3.9+7

[34b6f7d7] FastTransforms jll v0.4.1+0

[a3f928ae] Fontconfig jll v2.13.1+14

```
[d7e528f0] FreeType2 jll v2.10.1+5
[559328eb] FriBidi jll v1.0.5+6
[0656b61e] GLFW_jll v3.3.4+0
[d2c73de3] GR jll v0.57.2+0
[78b55507] Gettext_jll v0.20.1+7
[f8c6e375] Git_jll v2.31.0+0
[7746bdde] Glib jll v2.59.0+4
[0234f1f7] HDF5_jll v1.12.0+1
[e33a78d0] Hwloc jll v2.4.1+0
[1d5cc7b8] IntelOpenMP_jll v2018.0.3+2
[aacddb02] JpegTurbo_jll v2.0.1+3
[c1c5ebd0] LAME_jll v3.100.0+3
[aaeOfff6] LSODA_jll v0.1.1+0
[dd4b983a] LZO jll v2.10.0+3
[dd192d2f] LibVPX jll v1.9.0+1
[e9f186c6] Libffi jll v3.2.1+4
[d4300ac3] Libgcrypt_jll v1.8.5+4
[7e76a0d4] Libglvnd_jll v1.3.0+3
[7add5ba3] Libgpg_error_jll v1.36.0+3
[94ce4f54] Libiconv jll v1.16.0+7
[4b2f31a3] Libmount jll v2.34.0+3
[89763e89] Libtiff_jll v4.1.0+2
[38a345b3] Libuuid jll v2.34.0+7
[5ced341a] Lz4_jll v1.9.2+2
[856f044c] MKL_jll v2021.1.1+1
[c771fb93] ODEInterface jll v0.0.1+0
[e7412a2a] Ogg jll v1.3.4+2
[458c3c95] OpenSSL jll v1.1.1+6
[efe28fd5] OpenSpecFun_jll v0.5.4+0
[91d4177d] Opus jll v1.3.1+3
[2f80f16e] PCRE_jll v8.42.0+4
[30392449] Pixman jll v0.40.0+0
[ea2cea3b] Qt5Base_jll v5.15.2+0
[f50d1b31] Rmath jll v0.3.0+0
[fb77eaff] Sundials jll v5.2.0+1
[a2964d1f] Wayland_jll v1.17.0+4
[2381bf8a] Wayland_protocols_jll v1.18.0+4
[02c8fc9c] XML2 jll v2.9.11+0
[aed1982a] XSLT jll v1.1.33+4
[4f6342f7] Xorg_libX11_jll v1.6.9+4
[OcOb7dd1] Xorg libXau jll v1.0.9+4
[935fb764] Xorg libXcursor jll v1.2.0+4
[a3789734] Xorg_libXdmcp_jll v1.1.3+4
[1082639a] Xorg_libXext_jll v1.3.4+4
[d091e8ba] Xorg libXfixes jll v5.0.3+4
```

[a51aa0fd] Xorg\_libXi\_jll v1.7.10+4

[d1454406] Xorg\_libXinerama\_jll v1.1.4+4
[ec84b674] Xorg\_libXrandr\_jll v1.5.2+4
[ea2f1a96] Xorg libXrender jll v0.9.10+4

```
[14d82f49] Xorg_libpthread_stubs_jll v0.1.0+3
```

[c7cfdc94] Xorg libxcb jll v1.13.0+3

[cc61e674] Xorg\_libxkbfile\_jll v1.1.0+4

[12413925] Xorg\_xcb\_util\_image\_jll v0.4.0+1

[2def613f] Xorg\_xcb\_util\_jll v0.4.0+1

[975044d2] Xorg\_xcb\_util\_keysyms\_jll v0.4.0+1

[Od47668e] Xorg\_xcb\_util\_renderutil\_jll v0.3.9+1

[c22f9ab0] Xorg\_xcb\_util\_wm\_jll v0.4.1+1

[35661453] Xorg xkbcomp jll v1.4.2+4

[33bec58e] Xorg\_xkeyboard\_config\_jll v2.27.0+4

[c5fb5394] Xorg\_xtrans\_jll v1.4.0+3

[8f1865be] ZeroMQ jll v4.3.2+6

[3161d3a3] Zstd\_jll v1.4.8+0

[0ac62f75] libass jll v0.14.0+4

[f638f0a6] libfdk\_aac\_jll v0.1.6+4

[b53b4c65] libpng jll v1.6.37+6

[a9144af2] libsodium jll v1.0.20+0

[f27f6e37] libvorbis\_jll v1.3.6+6

[1270edf5] x264\_jll v2020.7.14+2

[dfaa095f] x265 jll v3.0.0+3

[d8fb68d0] xkbcommon\_jll v0.9.1+5

[Odad84c5] ArgTools

[56f22d72] Artifacts

[2a0f44e3] Base64

[ade2ca70] Dates

[8bb1440f] DelimitedFiles

[8ba89e20] Distributed

[f43a241f] Downloads

[7b1f6079] FileWatching

[9fa8497b] Future

[b77e0a4c] InteractiveUtils

[4af54fe1] LazyArtifacts

[b27032c2] LibCURL

[76f85450] LibGit2

[8f399da3] Libdl

[37e2e46d] LinearAlgebra

[56ddb016] Logging

[d6f4376e] Markdown

[a63ad114] Mmap

[ca575930] NetworkOptions

[44cfe95a] Pkg

[de0858da] Printf

[3fa0cd96] REPL

[9a3f8284] Random

[ea8e919c] SHA

[9e88b42a] Serialization

[1a1011a3] SharedArrays

[6462fe0b] Sockets

[2f01184e] SparseArrays

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[10745b16] Statistics
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[4607b0f0] SuiteSparse

[fa267f1f] TOML

[a4e569a6] Tar

[8dfed614] Test

[cf7118a7] UUIDs

[4ec0a83e] Unicode

[e66e0078] CompilerSupportLibraries\_jll

[781609d7] GMP\_jll

[deac9b47] LibCURL\_jll

[29816b5a] LibSSH2\_jll

[3a97d323] MPFR\_jll

[c8ffd9c3] MbedTLS\_jll

[14a3606d] MozillaCACerts\_jll

[4536629a] OpenBLAS\_jll

[efcefdf7] PCRE2\_j11

[bea87d4a] SuiteSparse\_jll

[83775a58] Zlib\_jll

[8e850ede] nghttp2\_jll

[3f19e933] p7zip\_jll