Kolmogorov Backward Equations

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```julia
using Flux, StochasticDiffEq
using NeuralPDE
using Plots
using CUDA
```

0.1 Introduction on Backward Kolmogorov Equations

The backward Kolmogorov Equation deals with a terminal condtion. The one dimensional backward kolmogorov equation that we are going to deal with is of the form :

$$\frac{\partial p}{\partial t} = -\mu(x)\frac{\partial p}{\partial x} - \frac{1}{2}\sigma^2(x)\frac{\partial^2 p}{\partial x^2}, \quad p(T,x) = \varphi(x)$$

for all \$ t \in{ [0 , T] } \$ and for all \$ x \in R^d \$

The Black Scholes Model The Black-Scholes Model governs the price evolution of the European put or call option. In the below equation V is the price of some derivative , S is the Stock Price , r is the risk free interest rate and σ the volatility of the stock returns. The payoff at a time T is known to us. And this makes it a terminal PDE. In case of an European put option the PDE is:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} - rV = 0, \quad V(T,S) = max\{\mathcal{K} - S, 0\}$$

for all \$ t \in{ [0 , T] } \$ and for all \$ S \in R^d \$

In order to make the above equation in the form of the Backward - Kolmogorov PDE we should substitute

$$V(S,t) = e^{r(t-T)}p(S,t)$$

and thus we get

$$e^{r(t-T)}\frac{\partial p}{\partial t} + re^{r(t-T)}p(S,t) = -\mu(x)\frac{\partial p}{\partial x}e^{r(t-T)} - \frac{1}{2}\sigma^2(x)\frac{\partial^2 p}{\partial x^2}e^{r(t-T)} + re^{r(t-T)}p(S,t)$$

And the terminal condition

$$p(S,T) = max\{\mathcal{K} - x, 0\}$$

We will train our model and the model itself will be the solution of the equation

0.2 Defining the problem and the solver

We should start defining the terminal condition for our equation:

```
function phi(xi)
    y = Float64[]
    K = 100
    for x in eachcol(xi)
        val = max(K - maximum(x) , 0.00)
        y = push!(y , val)
    end
    y = reshape(y , 1 , size(y)[1] )
    return y
end
```

```
phi (generic function with 1 method)
```

Now we shall define the problem : We will define the σ and μ by comparing it to the orignal equation. The xspan is the span of initial stock prices.

```
d = 1
r = 0.04
sigma = 0.2
xspan = (80.00 , 115.0)
tspan = (0.0 , 1.0)
σ(du , u , p , t) = du .= sigma.*u
μ(du , u , p , t) = du .= r.*u
prob = KolmogorovPDEProblem(μ , σ , phi , xspan , tspan, d)
```

```
KolmogorovPDEProblem with uType Int64 and tType Float64. In-place: nothing
timespan: (0.0, 1.0)
u0: 0
```

Now once we have defined our problem it is necessary to define the parameters for the solver.

```
sdealg = EM()
ensemblealg = EnsembleThreads()
dt = 0.01
dx = 0.01
trajectories = 100000
```

```
100000
```

Now lets define our model m and the optimiser

```
m = Chain(Dense(d, 64, elu),Dense(64, 128, elu),Dense(128 , 16 , elu) , Dense(16 , 1))
use_gpu = false
if CUDAnative.functional() == true
  m = fmap(CuArrays.cu , m)
  use_gpu = true
end
opt = Flux.ADAM(0.0005)
```

```
Error: UndefVarError: CUDAnative not defined
```

And then finally call the solver

```
@time sol = solve(prob, NeuralNetDiffEq.NNKolmogorov(m, opt, sdealg, ensemblealg),
verbose = true, dt = dt,
            dx = dx , trajectories = trajectories , abstol=1e-6, maxiters = 1000 ,
use_gpu = use_gpu)
```

```
Error: UndefVarError: NeuralNetDiffEq not defined
```

0.3 Analyzing the solution

Now let us find a Monte-Carlo Solution and plot the both:

```
monte_carlo_sol = []
x_out = collect(85:2.00:110.00)
for x in x_out
  u_0= [x]
  g_val(du , u , p , t) = du .= 0.2.*u
  f_val(du , u , p , t) = du .= 0.04.*u
  dt = 0.01
  tspan = (0.0,1.0)
  prob = SDEProblem(f_val,g_val,u_0,tspan)
  output_func(sol,i) = (sol[end],false)
  ensembleprob_val = EnsembleProblem(prob , output_func = output_func )
  sim_val = solve(ensembleprob_val, EM(), EnsembleThreads() , dt=0.01,
trajectories=100000,adaptive=false)
  s = reduce(hcat , sim_val.u)
  mean_phi = sum(phi(s))/length(phi(s))
  global monte_carlo_sol = push!(monte_carlo_sol , mean_phi)
end
```

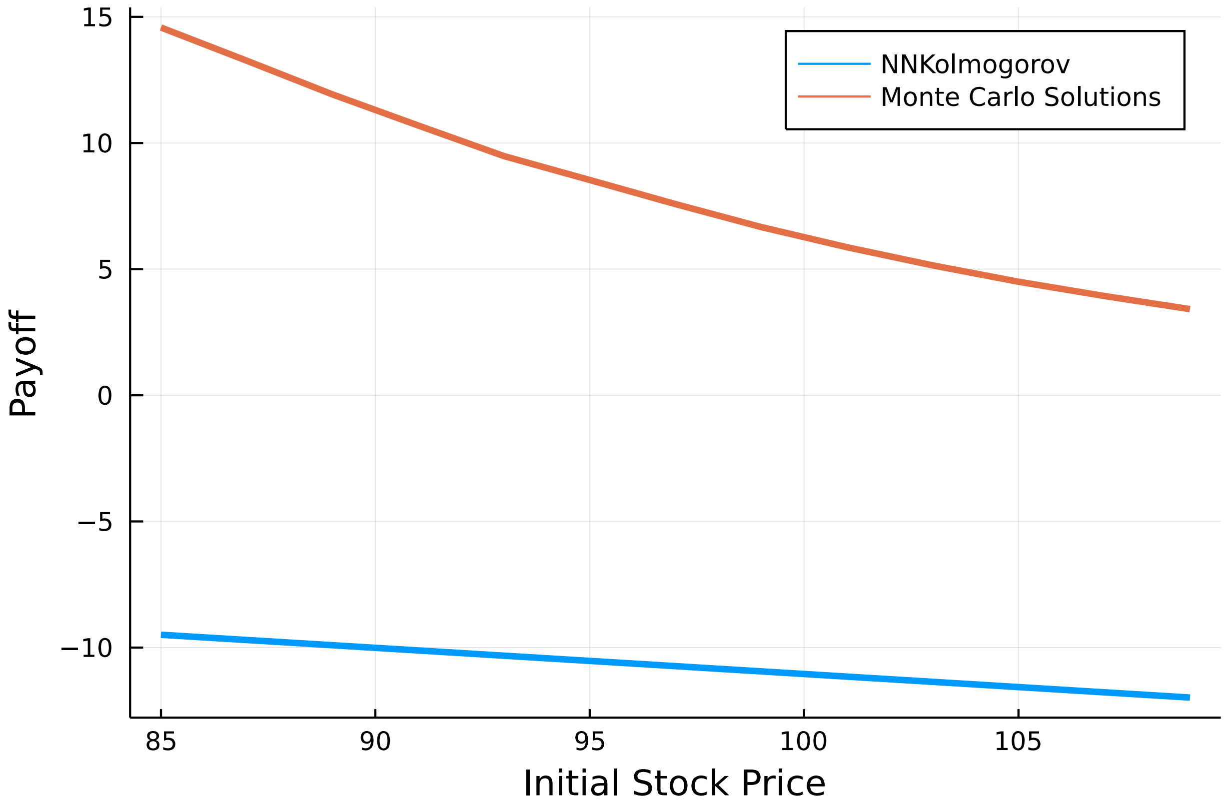
##Plotting the Solutions We should reshape the inputs and outputs to make it compatible with our model. This is the most important part. The algorithm gives a distributed function over all initial prices in the xspan.

```
x_model = reshape(x_out, 1 , size(x_out)[1])
if use_gpu == true
  m = fmap(cpu , m)
end
y_out = m(x_model)
y_out = reshape(y_out , 13 , 1)
```

```
13×1 Matrix{Float64}:
  -9.4921953307678
  -9.699637464487834
  -9.907049911503343
 -10.114438667338526
 -10.321808895412124
 -10.529165026702223
 -10.736510848784336
 -10.943858898018714
 -11.151239188338304
 -11.358655069875732
 -11.566107689385086
 -11.773597883372243
 -11.981126220514565
```

And now finally we can plot the solutions

```
plot(x_out , y_out , lw = 3 ,  xaxis="Initial Stock Price", yaxis="Payoff" , label =
"NNKolmogorov")
plot!(x_out , monte_carlo_sol , lw = 3 ,  xaxis="Initial Stock Price", yaxis="Payoff"
,label = "Monte Carlo Solutions")
```



0.4 Appendix

These tutorials are a part of the SciMLTutorials.jl repository, found at: https://github.com/SciML/SciML For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization https://sciml.ai.

To locally run this tutorial, do the following commands:

```
using SciMLTutorials
SciMLTutorials.weave_file("tutorials/advanced","03-kolmogorov_equations.jmd")
```

Computer Information:

```
Julia Version 1.6.1
Commit 6aaedecc44 (2021-04-23 05:59 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: AMD EPYC 7502 32-Core Processor
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-11.0.1 (ORCJIT, znver2)
```

```
Environment:
  JULIA_DEPOT_PATH = /root/.cache/julia-buildkite-plugin/depots/a6029d3a-f78b-41ea-bc9
  JULIA_NUM_THREADS = 16
```

Package Information:

```
      Status `/var/lib/buildkite-agent/builds/4-amdci4-julia-csail-mit-edu/julialang/s
  [2169fc97] AlgebraicMultigrid v0.4.0
  [6e4b80f9] BenchmarkTools v1.0.0
  [052768ef] CUDA v2.6.3
  [2b5f629d] DiffEqBase v6.62.2
  [9fdde737] DiffEqOperators v4.26.0
  [0c46a032] DifferentialEquations v6.17.1
  [587475ba] Flux v0.12.1
  [961ee093] ModelingToolkit v5.17.3
  [2774e3e8] NLsolve v4.5.1
  [315f7962] NeuralPDE v3.10.1
  [1dea7af3] OrdinaryDiffEq v5.56.0
  [91a5bcdd] Plots v1.15.2
  [0bca4576] SciMLBase v1.13.4
  [30cb0354] SciMLTutorials v0.9.0
  [47a9eef4] SparseDiffTools v1.13.2
  [684fba80] SparsityDetection v0.3.4
  [789caeaf] StochasticDiffEq v6.34.1
  [c3572dad] Sundials v4.4.3
  [37e2e46d] LinearAlgebra
  [2f01184e] SparseArrays
```

And the full manifest:

```
      Status `/var/lib/buildkite-agent/builds/4-amdci4-julia-csail-mit-edu/julialang/s
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  [1520ce14] AbstractTrees v0.3.4
  [79e6a3ab] Adapt v3.3.0
  [2169fc97] AlgebraicMultigrid v0.4.0
  [ec485272] ArnoldiMethod v0.1.0
  [4fba245c] ArrayInterface v3.1.15
  [4c555306] ArrayLayouts v0.7.0
  [13072b0f] AxisAlgorithms v1.0.0
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  [ffab5731] BlockBandedMatrices v0.10.6
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  [fa961155] CEnum v0.4.1
```

```
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[35d6a980] ColorSchemes v3.12.1
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[78b55507] Gettext_jll v0.21.0+0
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[7e76a0d4] Libglvnd_jll v1.3.0+3
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[4b2f31a3] Libmount_jll v2.35.0+0
[89763e89] Libtiff_jll v4.1.0+2
[38a345b3] Libuuid_jll v2.36.0+0
[856f044c] MKL_jll v2021.1.1+1
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```

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[d1454406] Xorg_libXinerama_jll v1.1.4+4
[ec84b674] Xorg_libXrandr_jll v1.5.2+4
[ea2f1a96] Xorg_libXrender_jll v0.9.10+4
[14d82f49] Xorg_libpthread_stubs_jll v0.1.0+3
[c7cfdc94] Xorg_libxcb_jll v1.13.0+3
[cc61e674] Xorg_libxkbfile_jll v1.1.0+4
[12413925] Xorg_xcb_util_image_jll v0.4.0+1
[2def613f] Xorg_xcb_util_jll v0.4.0+1
[975044d2] Xorg_xcb_util_keysyms_jll v0.4.0+1
[0d47668e] Xorg_xcb_util_renderutil_jll v0.3.9+1
[c22f9ab0] Xorg_xcb_util_wm_jll v0.4.1+1
[35661453] Xorg_xkbcomp_jll v1.4.2+4
[33bec58e] Xorg_xkeyboard_config_jll v2.27.0+4
[c5fb5394] Xorg_xtrans_jll v1.4.0+3
[8f1865be] ZeroMQ_jll v4.3.2+6
[3161d3a3] Zstd_jll v1.5.0+0
[0ac62f75] libass_jll v0.14.0+4
[f638f0a6] libfdk_aac_jll v0.1.6+4
[b53b4c65] libpng_jll v1.6.38+0
[a9144af2] libsodium_jll v1.0.20+0
[f27f6e37] libvorbis_jll v1.3.6+6
[1270edf5] x264_jll v2020.7.14+2
[dfaa095f] x265_jll v3.0.0+3
[d8fb68d0] xkbcommon_jll v0.9.1+5
[0dad84c5] ArgTools
[56f22d72] Artifacts
[2a0f44e3] Base64
[ade2ca70] Dates
[8bb1440f] DelimitedFiles
[8ba89e20] Distributed
[f43a241f] Downloads
[7b1f6079] FileWatching
[9fa8497b] Future
[b77e0a4c] InteractiveUtils
[4af54fe1] LazyArtifacts
[b27032c2] LibCURL
[76f85450] LibGit2
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[8f399da3] Libdl
[37e2e46d] LinearAlgebra
[56ddb016] Logging
[d6f4376e] Markdown
[a63ad114] Mmap
[ca575930] NetworkOptions
[44cfe95a] Pkg
[de0858da] Printf
[9abbd945] Profile
[3fa0cd96] REPL
[9a3f8284] Random
[ea8e919c] SHA
[9e88b42a] Serialization
[1a1011a3] SharedArrays
[6462fe0b] Sockets
[2f01184e] SparseArrays
[10745b16] Statistics
[4607b0f0] SuiteSparse
[fa267f1f] TOML
[a4e569a6] Tar
[8dfed614] Test
[cf7118a7] UUIDs
[4ec0a83e] Unicode
[e66e0078] CompilerSupportLibraries_jll
[deac9b47] LibCURL_jll
[29816b5a] LibSSH2_jll
[c8ffd9c3] MbedTLS_jll
[14a3606d] MozillaCACerts_jll
[4536629a] OpenBLAS_jll
[bea87d4a] SuiteSparse_jll
[83775a58] Zlib_jll
[8e850ede] nghttp2_jll
[3f19e933] p7zip_jll
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