Feagin's Order 10, 12, and 14 Methods

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DifferentialEquations.jl includes Feagin's explicit Runge-Kutta methods of orders 10/8, 12/10, and 14/12. These methods have such high order that it's pretty much required that one uses numbers with more precision than Float64. As a prerequisite reference on how to use arbitrary number systems (including higher precision) in the numerical solvers, please see the Solving Equations in With Chosen Number Types notebook.

0.1 Investigation of the Method's Error

We can use Feagin's order 16 method as follows. Let's use a two-dimensional linear ODE. Like in the Solving Equations in With Chosen Number Types notebook, we change the initial condition to BigFloats to tell the solver to use BigFloat types.

```
using DifferentialEquations
const linear_bigα = big(1.01)
f(u,p,t) = (linear_bigα*u)

# Add analytical solution so that errors are checked
f_analytic(u0,p,t) = u0*exp(linear_bigα*t)
ff = ODEFunction(f,analytic=f_analytic)
prob = ODEProblem(ff,big(0.5),(0.0,1.0))
sol = solve(prob,Feagin14(),dt=1//16,adaptive=false);

println(sol.errors)
```

```
Dict{Symbol, BigFloat}(:l∞ => 2.19751040342660991781470263264956056068365936
7683780324635801610297349872909655e-23, :final => 2.197510403426609917814
70263264956056068365936768378032463580161029734987290965 5e-23, :l2 => 1.0615
01597814768635894514677590712762248364686527596359902826841740549975688161e
-23)
```

Compare that to machine ϵ for Float64:

```
eps(Float64)
```

```
2.220446049250313e-16
```

The error for Feagin's method when the stepsize is 1/16 is 8 orders of magnitude below machine ϵ! However, that is dependent on the stepsize. If we instead use adaptive timestepping with the default tolerances, we get

```
sol =solve(prob,Feagin14());
println(sol.errors); print("The length was $(length(sol))")
```

```
Dict{Symbol, BigFloat}(:l∞ => 1.5457388839431409625465375986097592198164147
90728029220638828884206395861982752e-09, :final => 1.5457388839431409625465
37598609759219816414790728029220638828884206395861982752e-09, :l2 => 8.9250
66870202330409924421192162193462506388332261074725109949218067763405137993e
-10)
The length was 3
```

Notice that when the stepsize is much higher, the error goes up quickly as well. These super high order methods are best when used to gain really accurate approximations (using still modest timesteps). Some examples of where such precision is necessary is astrodynamics where the many-body problem is highly chaotic and thus sensitive to small errors.

0.2 Convergence Test

The Order 14 method is awesome, but we need to make sure it's really that awesome. The following convergence test is used in the package tests in order to make sure the implementation is correct. Note that all methods have such tests in place.

```
using DiffEqDevTools
dts = 1.0 ./ 2.0 .^(10:-1:4)
sim = test_convergence(dts,prob,Feagin14())
```

```
DiffEqDevTools.ConvergenceSimulation{SciMLBase.ODESolution{BigFloat, 1, Vec
tor{BigFloat}, Vector{BigFloat}, Dict{Symbol, BigFloat}, Vector{Float64}, V
ector{Vector{BigFloat}}, SciMLBase.ODEProblem{BigFloat, Tuple{Float64, Floa
t64}, false, SciMLBase.NullParameters, SciMLBase.ODEFunction{false, typeof(
Main.##WeaveSandBox#2259.f), LinearAlgebra.UniformScaling{Bool}, typeof(Mai
n.##WeaveSandBox#2259.f_analytic), Nothing, Nothing, Nothing, Nothing, Noth
ing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, typeof(SciMLBase
.DEFAULT_OBSERVED), Nothing}, Base.Iterators.Pairs{Union{}, Union{}, Tuple{
}, NamedTuple{(), Tuple{}}}, SciMLBase.StandardODEProblem}, OrdinaryDiffEq.
Feagin14, OrdinaryDiffEq.InterpolationData{SciMLBase.ODEFunction{false, typ
eof(Main.##WeaveSandBox#2259.f), LinearAlgebra.UniformScaling{Bool}, typeof
(Main.##WeaveSandBox#2259.f_analytic), Nothing, Nothing, Nothing, Nothing,
Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, Nothing, typeof(SciML
Base.DEFAULT_OBSERVED), Nothing}, Vector{BigFloat}, Vector{Float64}, Vector
{Vector{BigFloat}}, OrdinaryDiffEq.Feagin14ConstantCache{BigFloat, Float64}
}, DiffEqBase.DEStats}}(SciMLBase.ODESolution{BigFloat, 1, Vector{BigFloat}
, Vector{BigFloat}, Dict{Symbol, BigFloat}, Vector{Float64}, Vector{Vector{
BigFloat}}, SciMLBase.ODEProblem{BigFloat, Tuple{Float64, Float64}, false,
SciMLBase.NullParameters, SciMLBase.ODEFunction{false, typeof(Main.##WeaveS
andBox#2259.f), LinearAlgebra.UniformScaling{Bool}, typeof(Main.##WeaveSand
Box#2259.f_analytic), Nothing, Nothing, Nothing, Nothing, Nothing, Nothing,
 Nothing, Nothing, Nothing, Nothing, Nothing, typeof(SciMLBase.DEFAULT_OBSE
RVED), Nothing}, Base.Iterators.Pairs{Union{}, Union{}, Tuple{}, NamedTuple
{(), Tuple{}}}, SciMLBase.StandardODEProblem}, OrdinaryDiffEq.Feagin14, Ord
inaryDiffEq.InterpolationData{SciMLBase.ODEFunction{false, typeof(Main.##We
aveSandBox#2259.f), LinearAlgebra.UniformScaling{Bool}, typeof(Main.##Weave
SandBox#2259.f_analytic), Nothing, Nothing, Nothing, Nothing, Nothing, Noth
ing, Nothing, Nothing, Nothing, Nothing, Nothing, typeof(SciMLBase.DEFAULT_
OBSERVED), Nothing}, Vector{BigFloat}, Vector{Float64}, Vector{Vector{BigFl
oat}}, OrdinaryDiffEq.Feagin14ConstantCache{BigFloat, Float64}}, DiffEqBase
.DEStats}[t: [0.0, 0.0009765625, 0.001953125, 0.0029296875, 0.00390625, 0.0
048828125, 0.005859375, 0.0068359375, 0.0078125, 0.0087890625  ...  0.9912109
375, 0.9921875, 0.9931640625, 0.994140625, 0.9951171875, 0.99609375, 0.9970
703125, 0.998046875, 0.9990234375, 1.0]
u: BigFloat[0.50, 0.5004934073532741442240167407783486492180603021615841294
```

```
52202794115660211219599, 0.500987301608180818440355281223188950584910597683
```

```
274979609386868635781, 1.33014813993405462867526880344471252204348922373995
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319016635913214729849926370533428936466432, 1.34598522869683246090918314024
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046875, 0.0546875, 0.0625, 0.0703125  ...  0.9296875, 0.9375, 0.9453125, 0.95
3125, 0.9609375, 0.96875, 0.9765625, 0.984375, 0.9921875, 1.0]
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749511286039814682961, 1.28882096451229946257972559447843216250288715170826
8911920429452719237031672889, 1.2990307954353467102700618873848859068511184
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 0.109375, 0.125, 0.140625  ...  0.859375, 0.875, 0.890625, 0.90625, 0.921875
, 0.9375, 0.953125, 0.96875, 0.984375, 1.0]
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14211007], t: [0.0, 0.03125, 0.0625, 0.09375, 0.125, 0.15625, 0.1875, 0.218
```

```
75, 0.25, 0.28125  ...  0.71875, 0.75, 0.78125, 0.8125, 0.84375, 0.875, 0.906
25, 0.9375, 0.96875, 1.0]
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76152641046  ...  1.0333338465044267120825803549601079350439706137564931634 51
129462609138481821 4, 1.0664686032469082465446852271794933182809166964255378
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310202099855349439700971640128267134708182647319688 4, 1.372800507508458259 1
877840364500830922627073433062829812545919269654934113726 91]], Dict{Any, An
y}(:l∞ => BigFloat[3.354354545962993017750167938278129130201240818733894747
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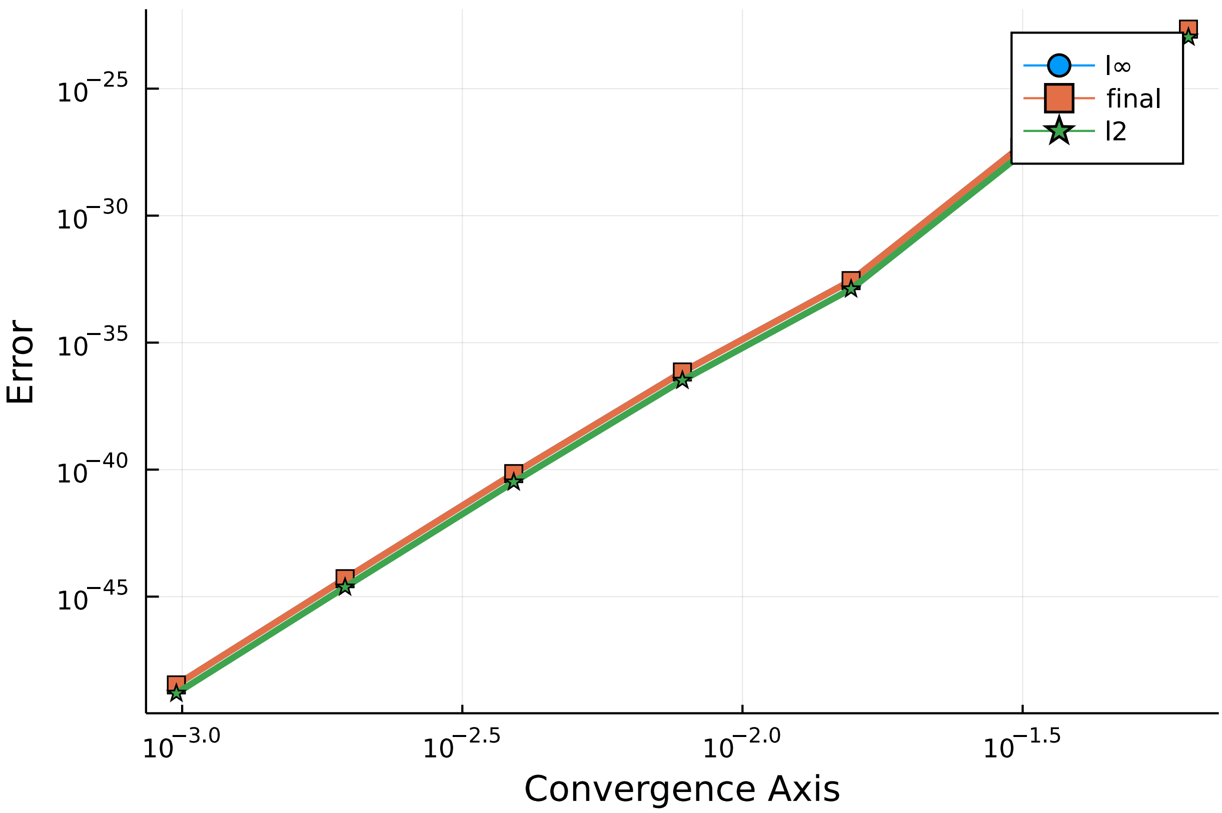
65844653981666663129348690225051975282413106146086946663583979066 8e-41, 6.9
985599579590960027470311487785578310778846432424801704948013801707657539745
79e-37, 2.761604674257899176583497342905526511107065867988065457455490380 82
5745507598647e-33, 4.9650607496729548374201289866032526435726391756486604 23
260171211890675951960709e-28, 2.19751040342660991781470263264956056068365 93
```

```
67683780324635801610297349872909655e-23], :l2 => BigFloat[1.557658061895966
3258462073477008215661222502349518679823852498452784936626769 44e-49, 2.3604
116571975473334985472232128807659899539105231983760849929611214557876 43313e
-45, 3.240607605160746766371785542718280708237168066098329304267778983 90162
3487961701e-41, 3.264565979149024498598621687244084221464920048688554495210
3681724856862288 22379e-37, 1.294777666747386485263611419731111056071389 8649
84176915402703871046417929063523e-33, 2.3514850301910030614259494469823 3564
880118152433224493354561444378609124576 2492e-28, 1.061501597814768635894514
6775907127622483646865275963599028268417405499756 88161e-23]), 7, Dict(:dts
=> [0.0009765625, 0.001953125, 0.00390625, 0.0078125, 0.015625, 0.03125, 0.
0625]), Dict{Any, Any}(:l∞ => 14.293327546103852435000893132848160405565048
162543747153761505341874614116 04701, :final => 14.293327546103852435000 8931
328481604055650481625437471537615053418746141160 4701, :l2 => 14.30280974051
8404232320190576343152425943132331198112128897631829609780 82577142), [0.000
9765625, 0.001953125, 0.00390625, 0.0078125, 0.015625, 0.03125, 0.0625])
```

For a view of what's going on, let's plot the simulation results.

```
using Plots
gr()
plot(sim)
```



This is a clear trend indicating that the convergence is truly Order 14, which is the estimated slope.

0.3 Appendix

These tutorials are a part of the SciMLTutorials.jl repository, found at: https://github.com/SciML/SciML For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization https://sciml.ai.

To locally run this tutorial, do the following commands:

```
using SciMLTutorials
SciMLTutorials.weave_file("tutorials/ode_extras","02-feagin.jmd")
```

Computer Information:

```
Julia Version 1.6.2
Commit 1b93d53fc4 (2021-07-14 15:36 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: AMD EPYC 7502 32-Core Processor
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-11.0.1 (ORCJIT, znver2)
Environment:
  JULIA_DEPOT_PATH = /root/.cache/julia-buildkite-plugin/depots/a6029d3a-f78b-41ea-bc97
  JULIA_NUM_THREADS = 16
```

Package Information:

```
      Status `/var/lib/buildkite-agent/builds/5-amdci4-julia-csail-mit-edu/julialang/sc
  [f3b72e0c] DiffEqDevTools v2.27.2
  [0c46a032] DifferentialEquations v6.17.1
  [961ee093] ModelingToolkit v5.17.3
  [76087f3c] NLopt v0.6.2
  [2774e3e8] NLsolve v4.5.1
  [429524aa] Optim v1.3.0
  [1dea7af3] OrdinaryDiffEq v5.56.0
  [91a5bcdd] Plots v1.15.2
  [30cb0354] SciMLTutorials v0.9.0
  [37e2e46d] LinearAlgebra
  [2f01184e] SparseArrays
```

And the full manifest:

```
      Status `/var/lib/buildkite-agent/builds/5-amdci4-julia-csail-mit-edu/julialang/sc
  [c3fe647b] AbstractAlgebra v0.16.0
  [1520ce14] AbstractTrees v0.3.4
  [79e6a3ab] Adapt v3.3.0
  [ec485272] ArnoldiMethod v0.1.0
  [4fba245c] ArrayInterface v3.1.15
  [4c555306] ArrayLayouts v0.7.0
  [aae01518] BandedMatrices v0.16.9
  [6e4b80f9] BenchmarkTools v1.0.0
  [764a87c0] BoundaryValueDiffEq v2.7.1
  [fa961155] CEnum v0.4.1
```

```
[00ebfdb7] CSTParser v2.5.0
[d360d2e6] ChainRulesCore v0.9.44
[b630d9fa] CheapThreads v0.2.5
[523fee87] CodecBzip2 v0.7.2
[944b1d66] CodecZlib v0.7.0
[35d6a980] ColorSchemes v3.12.1
[3da002f7] ColorTypes v0.11.0
[5ae59095] Colors v0.12.8
[861a8166] Combinatorics v1.0.2
[a80b9123] CommonMark v0.8.1
[38540f10] CommonSolve v0.2.0
[bbf7d656] CommonSubexpressions v0.3.0
[34da2185] Compat v3.30.0
[8f4d0f93] Conda v1.5.2
[187b0558] ConstructionBase v1.2.1
[d38c429a] Contour v0.5.7
[a8cc5b0e] Crayons v4.0.4
[9a962f9c] DataAPI v1.6.0
[864edb3b] DataStructures v0.18.9
[e2d170a0] DataValueInterfaces v1.0.0
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