

# The Outer Solar System

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## 0.1 Data

The chosen units are: masses relative to the sun, so that the sun has mass 1. We have taken  $m_0 = 1.00000597682$  to take account of the inner planets. Distances are in astronomical units, times in earth days, and the gravitational constant is thus  $G = 2.95912208286 \cdot 10^{-4}$ .

planet	mass	
Jupiter	$m_1 = 0.000954786104043$	<ul><li>-3.5023653</li><li>-3.8169847</li><li>-1.5
Saturn	$m_2 = 0.000285583733151$	<ul><li>9.0755314</li><li>-3.0458353</li><li>-1.6
Uranus	$m_3 = 0.0000437273164546$	<ul><li>8.3101420</li><li>-16.2901086</li><li>-7.2
Neptune	$m_4 = 0.0000517759138449$	<ul><li>11.4707666</li><li>-25.7294829</li><li>-10.8
Pluto	$m_5 = 1/(1.3 \cdot 10^8)$	<ul><li>-15.5387357</li><li>-25.2225594</li><li>-3.1

The data is taken from the book "Geometric Numerical Integration" by E. Hairer, C. Lubich and G. Wanner.

```
using Plots, OrdinaryDiffEq, DiffEqPhysics, RecursiveArrayTools
gr()
```

```
G = 2.95912208286e-4
M = [1.00000597682, 0.000954786104043, 0.000285583733151, 0.0000437273164546,
0.0000517759138449, 1/1.3e8]
planets = ["Sun", "Jupiter", "Saturn", "Uranus", "Neptune", "Pluto"]
```

```
pos_x = [0.0, -3.5023653, 9.0755314, 8.3101420, 11.4707666, -15.5387357]
pos_y = [0.0, -3.8169847, -3.0458353, -16.2901086, -25.7294829, -25.2225594]
pos_z = [0.0, -1.5507963, -1.6483708, -7.2521278, -10.8169456, -3.1902382]
pos = ArrayPartition(pos_x, pos_y, pos_z)
```

```
vel_x = [0.0, 0.00565429, 0.00168318, 0.00354178, 0.00288930, 0.00276725]
vel_y = [0.0, -0.00412490, 0.00483525, 0.00137102, 0.00114527, -0.00170702]
vel_z = [0.0, -0.00190589, 0.00192462, 0.00055029, 0.00039677, -0.00136504]
vel = ArrayPartition(vel_x, vel_y, vel_z)
```

```
tspan = (0., 200_000)
```

```
(0.0, 200000)
```

The N-body problem's Hamiltonian is

$$H(p, q) = \frac{1}{2} \sum_{i=0}^N \frac{p_i^T p_i}{m_i} - G \sum_{i=1}^N \sum_{j=0}^{i-1} \frac{m_i m_j}{\|q_i - q_j\|}$$

Here, we want to solve for the motion of the five outer planets relative to the sun, namely, Jupiter, Saturn, Uranus, Neptune and Pluto.

```
const sum = sum
const N = 6
potential(p, t, x, y, z, M) = -G*sum(i->sum(j->(M[i]*M[j])/sqrt((x[i]-x[j])^2 +
(y[i]-y[j])^2 + (z[i]-z[j])^2), 1:i-1), 2:N)

potential (generic function with 1 method)
```

## 0.2 Hamiltonian System

NBodyProblem constructs a second order ODE problem under the hood. We know that a Hamiltonian system has the form of

$$\dot{p} = -H_q(p, q) \quad \dot{q} = H_p(p, q)$$

For an N-body system, we can symplify this as:

$$\dot{p} = -\nabla V(q) \quad \dot{q} = M^{-1}p.$$

Thus  $\dot{q}$  is defined by the masses. We only need to define  $\dot{p}$ , and this is done internally by taking the gradient of  $V$ . Therefore, we only need to pass the potential function and the rest is taken care of.

```
nprob = NBodyProblem(potential, M, pos, vel, tspan)
sol = solve(nprob, Yoshida6(), dt=100);
```

```
Error: UndefVarError: NBodyProblem not defined
```

```
orbitplot(sol, body_names=planets)
```

```
Error: UndefVarError: sol not defined
```

## 0.3 Appendix

These tutorials are a part of the SciMLTutorials.jl repository, found at: <https://github.com/SciML/SciMLTutorials.jl>. For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization <https://sciml.ai>.

To locally run this tutorial, do the following commands:

```
using SciMLTutorials
SciMLTutorials.weave_file("tutorials/models", "07-outer_solar_system.jmd")
```

Computer Information:

```
Julia Version 1.6.2
Commit 1b93d53fc4 (2021-07-14 15:36 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
```

CPU: AMD EPYC 7502 32-Core Processor  
WORD\_SIZE: 64  
LIBM: libopenlibm  
LLVM: libLLVM-11.0.1 (ORCJIT, znver2)

Environment:

JULIA\_DEPOT\_PATH = /root/.cache/julia-buildkite-plugin/depots/a6029d3a-f78b-41ea-bc9  
JULIA\_NUM\_THREADS = 16

Package Information:

```
Status `~/var/lib/buildkite-agent/builds/1-amdci4-julia-csail-mit-edu/julialang/s
[479239e8] Catalyst v6.12.1
[459566f4] DiffEqCallbacks v2.16.1
[f3b72e0c] DiffEqDevTools v2.27.2
[055956cb] DiffEqPhysics v3.9.0
[0c46a032] DifferentialEquations v6.17.1
[31c24e10] Distributions v0.24.18
[587475ba] Flux v0.12.1
[f6369f11] ForwardDiff v0.10.18
[23fbe1c1] Latexify v0.15.5
[961ee093] ModelingToolkit v5.17.3
[2774e3e8] NLSolve v4.5.1
[315f7962] NeuralPDE v3.10.1
[429524aa] Optim v1.3.0
[1dea7af3] OrdinaryDiffEq v5.56.0
[91a5bcd] Plots v1.15.2
[731186ca] RecursiveArrayTools v2.11.4
[30cb0354] SciMLTutorials v0.9.0
[789caeaf] StochasticDiffEq v6.34.1
[37e2e46d] LinearAlgebra
[2f01184e] SparseArrays
```

And the full manifest:

```
Status `~/var/lib/buildkite-agent/builds/1-amdci4-julia-csail-mit-edu/julialang/s
[c3fe647b] AbstractAlgebra v0.16.0
[621f4979] AbstractFFTs v1.0.1
[1520ce14] AbstractTrees v0.3.4
[79e6a3ab] Adapt v3.3.0
[ec485272] ArnoldiMethod v0.1.0
[4fba245c] ArrayInterface v3.1.15
[4c555306] ArrayLayouts v0.7.0
[13072b0f] AxisAlgorithms v1.0.0
[ab4f0b2a] BFloat16s v0.1.0
[aae01518] BandedMatrices v0.16.9
[764a87c0] BoundaryValueDiffEq v2.7.1
[fa961155] CEnum v0.4.1
```

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[861a8166] Combinatorics v1.0.2  
[a80b9123] CommonMark v0.8.1  
[38540f10] CommonSolve v0.2.0  
[bbf7d656] CommonSubexpressions v0.3.0  
[34da2185] Compat v3.30.0  
[8f4d0f93] Conda v1.5.2  
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[a8cc5b0e] Crayons v4.0.4  
[8a292aeb] Cuba v2.2.0  
[667455a9] Cubature v1.5.1  
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[c894b116] DiffEqJump v6.14.2  
[77a26b50] DiffEqNoiseProcess v5.7.3  
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[699a6c99] SimpleTraits v0.9.3  
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[78b55507] Gettext\_jll v0.21.0+0  
[7746bdde] Glib\_jll v2.68.1+0  
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[e9f186c6] Libffi\_jll v3.2.2+0  
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[7e76a0d4] Libglvnd\_jll v1.3.0+3  
[7add5ba3] Libgpg\_error\_jll v1.42.0+0  
[94ce4f54] Libiconv\_jll v1.16.1+0  
[4b2f31a3] Libmount\_jll v2.35.0+0  
[89763e89] Libtiff\_jll v4.1.0+2  
[38a345b3] Libuuid\_jll v2.36.0+0  
[856f044c] MKL\_jll v2021.1.1+1  
[e7412a2a] Ogg\_jll v1.3.4+2  
[458c3c95] OpenSSL\_jll v1.1.1+6  
[efe28fd5] OpenSpecFun\_jll v0.5.4+0  
[91d4177d] Opus\_jll v1.3.1+3  
[2f80f16e] PCRE\_jll v8.44.0+0  
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[a2964d1f] Wayland\_jll v1.17.0+4  
[2381bf8a] Wayland\_protocols\_jll v1.18.0+4  
[02c8fc9c] XML2\_jll v2.9.12+0  
[aed1982a] XSLT\_jll v1.1.34+0  
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[0c0b7dd1] Xorg\_libXau\_jll v1.0.9+4  
[935fb764] Xorg\_libXcursor\_jll v1.2.0+4  
[a3789734] Xorg\_libXdmcp\_jll v1.1.3+4



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 [ea2f1a96] Xorg\_libXrender\_jll v0.9.10+4  
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 [cc61e674] Xorg\_libxkbfile\_jll v1.1.0+4  
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 [c22f9ab0] Xorg\_xcb\_util\_wm\_jll v0.4.1+1  
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 [33bec58e] Xorg\_xkeyboard\_config\_jll v2.27.0+4  
 [c5fb5394] Xorg\_xtrans\_jll v1.4.0+3  
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 [3161d3a3] Zstd\_jll v1.5.0+0  
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 [f638f0a6] libfdk\_aac\_jll v0.1.6+4  
 [b53b4c65] libpng\_jll v1.6.38+0  
 [a9144af2] libsodium\_jll v1.0.20+0  
 [f27f6e37] libvorbis\_jll v1.3.6+6  
 [1270edf5] x264\_jll v2020.7.14+2  
 [dfaa095f] x265\_jll v3.0.0+3  
 [d8fb68d0] xkbcommon\_jll v0.9.1+5  
 [0dad84c5] ArgTools  
 [56f22d72] Artifacts  
 [2a0f44e3] Base64  
 [ade2ca70] Dates  
 [8bb1440f] DelimitedFiles  
 [8ba89e20] Distributed  
 [f43a241f] Downloads  
 [7b1f6079] FileWatching  
 [9fa8497b] Future  
 [b77e0a4c] InteractiveUtils  
 [4af54fe1] LazyArtifacts  
 [b27032c2] LibCURL  
 [76f85450] LibGit2  
 [8f399da3] Libdl  
 [37e2e46d] LinearAlgebra  
 [56ddb016] Logging  
 [d6f4376e] Markdown  
 [a63ad114] Mmap  
 [ca575930] NetworkOptions  
 [44cfe95a] Pkg  
 [de0858da] Printf  
 [9abbd945] Profile

[3fa0cd96] REPL  
[9a3f8284] Random  
[ea8e919c] SHA  
[9e88b42a] Serialization  
[1a1011a3] SharedArrays  
[6462fe0b] Sockets  
[2f01184e] SparseArrays  
[10745b16] Statistics  
[4607b0f0] SuiteSparse  
[fa267f1f] TOML  
[a4e569a6] Tar  
[8dfed614] Test  
[cf7118a7] UUIDs  
[4ec0a83e] Unicode  
[e66e0078] CompilerSupportLibraries\_jll  
[deac9b47] LibCURL\_jll  
[29816b5a] LibSSH2\_jll  
[c8ffd9c3] MbedTLS\_jll  
[14a3606d] MozillaCACerts\_jll  
[4536629a] OpenBLAS\_jll  
[bea87d4a] SuiteSparse\_jll  
[83775a58] Zlib\_jll  
[8e850ede] nghttp2\_jll  
[3f19e933] p7zip\_jll