The Outer Solar System

Yingbo Ma, Chris Rackauckas

August 12, 2021

0.1 Data

The chosen units are: masses relative to the sun, so that the sun has mass 1. We have taken $m_0 = 1.00000597682$ to take account of the inner planets. Distances are in astronomical units , times in earth days, and the gravitational constant is thus $G = 2.95912208286 \cdot 10^{-4}$.

planet	mass	
Jupiter	$m_1 = 0.000954786104043$	-3.5023653 -3.8169847 -1.5
Saturn	$m_2 = 0.000285583733151$	9.0755314 -3.0458353 -1.6
Uranus	$m_3 = 0.0000437273164546$	8.3101420 -16.2901086 -7.2
Neptune	$m_4 = 0.0000517759138449$	11.4707666 -25.7294829 -10.8
Pluto	$ m_5 = 1/(1.3 \cdot 10^8)$	-15.5387357 -25.2225594 -3.1

The data is taken from the book "Geometric Numerical Integration" by E. Hairer, C. Lubich and G. Wanner.

```
using Plots, OrdinaryDiffEq, DiffEqPhysics, RecursiveArrayTools
gr()

G = 2.95912208286e-4
M = [1.00000597682, 0.000954786104043, 0.000285583733151, 0.0000437273164546,
0.0000517759138449, 1/1.3e8]
planets = ["Sun", "Jupiter", "Saturn", "Uranus", "Neptune", "Pluto"]

pos_x = [0.0,-3.5023653,9.0755314,8.3101420,11.4707666,-15.5387357]
pos_y = [0.0,-3.8169847,-3.0458353,-16.2901086,-25.7294829,-25.2225594]
pos_z = [0.0,-1.5507963,-1.6483708,-7.2521278,-10.8169456,-3.1902382]
pos = ArrayPartition(pos_x,pos_y,pos_z)

vel_x = [0.0,0.00565429,0.00168318,0.00354178,0.00288930,0.00276725]
vel_y = [0.0,-0.00412490,0.00483525,0.00137102,0.00114527,-0.00170702]
vel_z = [0.0,-0.00190589,0.00192462,0.00055029,0.00039677,-0.00136504]
vel = ArrayPartition(vel_x,vel_y,vel_z)

tspan = (0.,200_000)
```

```
(0.0, 200000)
```

The N-body problem's Hamiltonian is

$$H(p,q) = \frac{1}{2}\sum_{i=0}^{N}\frac{p_i^T p_i}{m_i} - G\sum_{i=1}^{N}\sum_{j=0}^{i-1}\frac{m_i m_j}{\|q_i - q_j\|}$$

Here, we want to solve for the motion of the five outer planets relative to the sun, namely, Jupiter, Saturn, Uranus, Neptune and Pluto.

```
const ∑ = sum
const N = 6
potential(p, t, x, y, z, M) = -G*∑(i->∑(j->(M[i]*M[j])/sqrt((x[i]-x[j])^2 +
(y[i]-y[j])^2 + (z[i]-z[j])^2), 1:i-1), 2:N)
```

```
potential (generic function with 1 method)
```

0.2 Hamiltonian System

`NBodyProblem` constructs a second order ODE problem under the hood. We know that a Hamiltonian system has the form of

$$\dot{p} = -H_q(p,q) \quad \dot{q} = H_p(p,q)$$

For an N-body system, we can symplify this as:

$$\dot{p} = -\nabla V(q) \quad \dot{q} = M^{-1}p.$$

Thus $\dot{q}$ is defined by the masses. We only need to define $\dot{p}$, and this is done internally by taking the gradient of V. Therefore, we only need to pass the potential function and the rest is taken care of.

```
nprob = NBodyProblem(potential, M, pos, vel, tspan)
sol = solve(nprob,Yoshida6(), dt=100);
```

```
Error: UndefVarError: NBodyProblem not defined
```

```
orbitplot(sol,body_names=planets)
```

```
Error: UndefVarError: sol not defined
```

0.3 Appendix

These tutorials are a part of the SciMLTutorials.jl repository, found at: https://github.com/SciML/SciML
For more information on high-performance scientific machine learning, check out the SciML Open Source Software Organization https://sciml.ai.

To locally run this tutorial, do the following commands:

```
using SciMLTutorials
SciMLTutorials.weave_file("tutorials/models","07-outer_solar_system.jmd")
```

Computer Information:

```
Julia Version 1.6.2
Commit 1b93d53fc4 (2021-07-14 15:36 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
```

```
  CPU: AMD EPYC 7502 32-Core Processor
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-11.0.1 (ORCJIT, znver2)
Environment:
  JULIA_DEPOT_PATH = /root/.cache/julia-buildkite-plugin/depots/a6029d3a-f78b-41ea-bc9
  JULIA_NUM_THREADS = 16
```

Package Information:

```
      Status `/var/lib/buildkite-agent/builds/7-amdci4-julia-csail-mit-edu/julialang/s
  [479239e8] Catalyst v6.12.1
  [459566f4] DiffEqCallbacks v2.16.1
  [f3b72e0c] DiffEqDevTools v2.27.2
  [055956cb] DiffEqPhysics v3.9.0
  [0c46a032] DifferentialEquations v6.17.1
  [31c24e10] Distributions v0.24.18
  [587475ba] Flux v0.12.1
  [f6369f11] ForwardDiff v0.10.18
  [23fbe1c1] Latexify v0.15.5
  [961ee093] ModelingToolkit v5.17.3
  [2774e3e8] NLsolve v4.5.1
  [315f7962] NeuralPDE v3.10.1
  [429524aa] Optim v1.3.0
  [1dea7af3] OrdinaryDiffEq v5.56.0
  [91a5bcdd] Plots v1.15.2
  [731186ca] RecursiveArrayTools v2.11.4
  [30cb0354] SciMLTutorials v0.9.0
  [789caeaf] StochasticDiffEq v6.34.1
  [37e2e46d] LinearAlgebra
  [2f01184e] SparseArrays
```

And the full manifest:

```
      Status `/var/lib/buildkite-agent/builds/7-amdci4-julia-csail-mit-edu/julialang/s
  [c3fe647b] AbstractAlgebra v0.16.0
  [621f4979] AbstractFFTs v1.0.1
  [1520ce14] AbstractTrees v0.3.4
  [79e6a3ab] Adapt v3.3.0
  [ec485272] ArnoldiMethod v0.1.0
  [4fba245c] ArrayInterface v3.1.15
  [4c555306] ArrayLayouts v0.7.0
  [13072b0f] AxisAlgorithms v1.0.0
  [ab4f0b2a] BFloat16s v0.1.0
  [aae01518] BandedMatrices v0.16.9
  [764a87c0] BoundaryValueDiffEq v2.7.1
  [fa961155] CEnum v0.4.1
```

```
[00ebfdb7] CSTParser v2.5.0
[052768ef] CUDA v2.6.3
[479239e8] Catalyst v6.12.1
[082447d4] ChainRules v0.7.65
[d360d2e6] ChainRulesCore v0.9.44
[b630d9fa] CheapThreads v0.2.5
[944b1d66] CodecZlib v0.7.0
[35d6a980] ColorSchemes v3.12.1
[3da002f7] ColorTypes v0.11.0
[5ae59095] Colors v0.12.8
[861a8166] Combinatorics v1.0.2
[a80b9123] CommonMark v0.8.1
[38540f10] CommonSolve v0.2.0
[bbf7d656] CommonSubexpressions v0.3.0
[34da2185] Compat v3.30.0
[8f4d0f93] Conda v1.5.2
[88cd18e8] ConsoleProgressMonitor v0.1.2
[187b0558] ConstructionBase v1.2.1
[d38c429a] Contour v0.5.7
[a8cc5b0e] Crayons v4.0.4
[8a292aeb] Cuba v2.2.0
[667455a9] Cubature v1.5.1
[9a962f9c] DataAPI v1.6.0
[82cc6244] DataInterpolations v3.3.1
[864edb3b] DataStructures v0.18.9
[e2d170a0] DataValueInterfaces v1.0.0
[bcd4f6db] DelayDiffEq v5.31.0
[2b5f629d] DiffEqBase v6.62.2
[459566f4] DiffEqCallbacks v2.16.1
[f3b72e0c] DiffEqDevTools v2.27.2
[5a0ffddc] DiffEqFinancial v2.4.0
[aae7a2af] DiffEqFlux v1.37.0
[c894b116] DiffEqJump v6.14.2
[77a26b50] DiffEqNoiseProcess v5.7.3
[055956cb] DiffEqPhysics v3.9.0
[41bf760c] DiffEqSensitivity v6.45.0
[163ba53b] DiffResults v1.0.3
[b552c78f] DiffRules v1.0.2
[0c46a032] DifferentialEquations v6.17.1
[c619ae07] DimensionalPlotRecipes v1.2.0
[b4f34e82] Distances v0.10.3
[31c24e10] Distributions v0.24.18
[ced4e74d] DistributionsAD v0.6.26
[ffbed154] DocStringExtensions v0.8.4
[e30172f5] Documenter v0.26.3
[d4d017d3] ExponentialUtilities v1.8.4
[e2ba6199] ExprTools v0.1.3
[c87230d0] FFMPEG v0.4.0
[7a1cc6ca] FFTW v1.4.1
```

```
[7034ab61] FastBroadcast v0.1.8
[9aa1b823] FastClosures v0.3.2
[1a297f60] FillArrays v0.11.7
[6a86dc24] FiniteDiff v2.8.0
[53c48c17] FixedPointNumbers v0.8.4
[587475ba] Flux v0.12.1
[59287772] Formatting v0.4.2
[f6369f11] ForwardDiff v0.10.18
[069b7b12] FunctionWrappers v1.1.2
[d9f16b24] Functors v0.2.1
[0c68f7d7] GPUArrays v6.4.1
[61eb1bfa] GPUCompiler v0.10.0
[28b8d3ca] GR v0.57.4
[a75be94c] GalacticOptim v1.2.0
[5c1252a2] GeometryBasics v0.3.12
[af5da776] GlobalSensitivity v1.0.0
[42e2da0e] Grisu v1.0.2
[19dc6840] HCubature v1.5.0
[cd3eb016] HTTP v0.9.9
[eafb193a] Highlights v0.4.5
[0e44f5e4] Hwloc v2.0.0
[7073ff75] IJulia v1.23.2
[b5f81e59] IOCapture v0.1.1
[7869d1d1] IRTools v0.4.2
[615f187c] IfElse v0.1.0
[d25df0c9] Inflate v0.1.2
[83e8ac13] IniFile v0.5.0
[a98d9a8b] Interpolations v0.13.2
[c8e1da08] IterTools v1.3.0
[42fd0dbc] IterativeSolvers v0.9.1
[82899510] IteratorInterfaceExtensions v1.0.0
[692b3bcd] JLLWrappers v1.3.0
[682c06a0] JSON v0.21.1
[98e50ef6] JuliaFormatter v0.13.7
[e5e0dc1b] Juno v0.8.4
[5ab0869b] KernelDensity v0.6.3
[929cbde3] LLVM v3.7.1
[b964fa9f] LaTeXStrings v1.2.1
[2ee39098] LabelledArrays v1.6.1
[23fbe1c1] Latexify v0.15.5
[a5e1c1ea] LatinHypercubeSampling v1.8.0
[73f95e8e] LatticeRules v0.0.1
[1d6d02ad] LeftChildRightSiblingTrees v0.1.2
[093fc24a] LightGraphs v1.3.5
[d3d80556] LineSearches v7.1.1
[2ab3a3ac] LogExpFunctions v0.2.4
[e6f89c97] LoggingExtras v0.4.6
[bdcacae8] LoopVectorization v0.12.23
[1914dd2f] MacroTools v0.5.6
```

```
[739be429] MbedTLS v1.0.3
[442fdcdd] Measures v0.3.1
[e89f7d12] Media v0.5.0
[c03570c3] Memoize v0.4.4
[e1d29d7a] Missings v1.0.0
[961ee093] ModelingToolkit v5.17.3
[4886b29c] MonteCarloIntegration v0.0.2
[46d2c3a1] MuladdMacro v0.2.2
[f9640e96] MultiScaleArrays v1.8.1
[ffc61752] Mustache v1.0.10
[d41bc354] NLSolversBase v7.8.0
[2774e3e8] NLsolve v4.5.1
[872c559c] NNlib v0.7.19
[77ba4419] NaNMath v0.3.5
[315f7962] NeuralPDE v3.10.1
[8913a72c] NonlinearSolve v0.3.8
[6fe1bfb0] OffsetArrays v1.9.0
[429524aa] Optim v1.3.0
[bac558e1] OrderedCollections v1.4.1
[1dea7af3] OrdinaryDiffEq v5.56.0
[90014a1f] PDMats v0.11.0
[65888b18] ParameterizedFunctions v5.10.0
[d96e819e] Parameters v0.12.2
[69de0a69] Parsers v1.1.0
[ccf2f8ad] PlotThemes v2.0.1
[995b91a9] PlotUtils v1.0.10
[91a5bcdd] Plots v1.15.2
[e409e4f3] PoissonRandom v0.4.0
[f517fe37] Polyester v0.3.1
[85a6dd25] PositiveFactorizations v0.2.4
[21216c6a] Preferences v1.2.2
[33c8b6b6] ProgressLogging v0.1.4
[92933f4c] ProgressMeter v1.6.2
[1fd47b50] QuadGK v2.4.1
[67601950] Quadrature v1.8.1
[8a4e6c94] QuasiMonteCarlo v0.2.2
[74087812] Random123 v1.3.1
[fb686558] RandomExtensions v0.4.3
[e6cf234a] RandomNumbers v1.4.0
[c84ed2f1] Ratios v0.4.0
[3cdcf5f2] RecipesBase v1.1.1
[01d81517] RecipesPipeline v0.3.2
[731186ca] RecursiveArrayTools v2.11.4
[f2c3362d] RecursiveFactorization v0.1.12
[189a3867] Reexport v1.0.0
[ae029012] Requires v1.1.3
[ae5879a3] ResettableStacks v1.1.0
[37e2e3b7] ReverseDiff v1.9.0
[79098fc4] Rmath v0.7.0
```

```
[47965b36] RootedTrees v1.0.0
[7e49a35a] RuntimeGeneratedFunctions v0.5.2
[476501e8] SLEEFPirates v0.6.20
[1bc83da4] SafeTestsets v0.0.1
[0bca4576] SciMLBase v1.13.4
[30cb0354] SciMLTutorials v0.9.0
[6c6a2e73] Scratch v1.0.3
[efcf1570] Setfield v0.7.0
[992d4aef] Showoff v1.0.3
[699a6c99] SimpleTraits v0.9.3
[ed01d8cd] Sobol v1.5.0
[b85f4697] SoftGlobalScope v1.1.0
[a2af1166] SortingAlgorithms v1.0.0
[47a9eef4] SparseDiffTools v1.13.2
[276daf66] SpecialFunctions v1.4.1
[860ef19b] StableRNGs v1.0.0
[aedffcd0] Static v0.2.4
[90137ffa] StaticArrays v1.2.0
[82ae8749] StatsAPI v1.0.0
[2913bbd2] StatsBase v0.33.8
[4c63d2b9] StatsFuns v0.9.8
[9672c7b4] SteadyStateDiffEq v1.6.2
[789caeaf] StochasticDiffEq v6.34.1
[7792a7ef] StrideArraysCore v0.1.11
[09ab397b] StructArrays v0.5.1
[c3572dad] Sundials v4.4.3
[d1185830] SymbolicUtils v0.11.2
[0c5d862f] Symbolics v0.1.25
[3783bdb8] TableTraits v1.0.1
[bd369af6] Tables v1.4.2
[5d786b92] TerminalLoggers v0.1.3
[8290d209] ThreadingUtilities v0.4.4
[a759f4b9] TimerOutputs v0.5.9
[0796e94c] Tokenize v0.5.16
[9f7883ad] Tracker v0.2.16
[3bb67fe8] TranscodingStreams v0.9.5
[592b5752] Trapz v2.0.2
[a2a6695c] TreeViews v0.3.0
[5c2747f8] URIs v1.3.0
[3a884ed6] UnPack v1.0.2
[1986cc42] Unitful v1.7.0
[3d5dd08c] VectorizationBase v0.20.11
[81def892] VersionParsing v1.2.0
[19fa3120] VertexSafeGraphs v0.1.2
[44d3d7a6] Weave v0.10.8
[efce3f68] WoodburyMatrices v0.5.3
[ddb6d928] YAML v0.4.6
[c2297ded] ZMQ v1.2.1
[a5390f91] ZipFile v0.9.3
```

```
[e88e6eb3] Zygote v0.6.11
[700de1a5] ZygoteRules v0.2.1
[6e34b625] Bzip2_jll v1.0.6+5
[83423d85] Cairo_jll v1.16.0+6
[3bed1096] Cuba_jll v4.2.1+0
[7bc98958] Cubature_jll v1.0.4+0
[5ae413db] EarCut_jll v2.1.5+1
[2e619515] Expat_jll v2.2.10+0
[b22a6f82] FFMPEG_jll v4.3.1+4
[f5851436] FFTW_jll v3.3.9+7
[a3f928ae] Fontconfig_jll v2.13.1+14
[d7e528f0] FreeType2_jll v2.10.1+5
[559328eb] FriBidi_jll v1.0.5+6
[0656b61e] GLFW_jll v3.3.4+0
[d2c73de3] GR_jll v0.57.2+0
[78b55507] Gettext_jll v0.21.0+0
[7746bdde] Glib_jll v2.68.1+0
[e33a78d0] Hwloc_jll v2.4.1+0
[1d5cc7b8] IntelOpenMP_jll v2018.0.3+2
[aacddb02] JpegTurbo_jll v2.0.1+3
[c1c5ebd0] LAME_jll v3.100.0+3
[dd4b983a] LZO_jll v2.10.1+0
[dd192d2f] LibVPX_jll v1.9.0+1
[e9f186c6] Libffi_jll v3.2.2+0
[d4300ac3] Libgcrypt_jll v1.8.7+0
[7e76a0d4] Libglvnd_jll v1.3.0+3
[7add5ba3] Libgpg_error_jll v1.42.0+0
[94ce4f54] Libiconv_jll v1.16.1+0
[4b2f31a3] Libmount_jll v2.35.0+0
[89763e89] Libtiff_jll v4.1.0+2
[38a345b3] Libuuid_jll v2.36.0+0
[856f044c] MKL_jll v2021.1.1+1
[e7412a2a] Ogg_jll v1.3.4+2
[458c3c95] OpenSSL_jll v1.1.1+6
[efe28fd5] OpenSpecFun_jll v0.5.4+0
[91d4177d] Opus_jll v1.3.1+3
[2f80f16e] PCRE_jll v8.44.0+0
[30392449] Pixman_jll v0.40.1+0
[ea2cea3b] Qt5Base_jll v5.15.2+0
[f50d1b31] Rmath_jll v0.3.0+0
[fb77eaff] Sundials_jll v5.2.0+1
[a2964d1f] Wayland_jll v1.17.0+4
[2381bf8a] Wayland_protocols_jll v1.18.0+4
[02c8fc9c] XML2_jll v2.9.12+0
[aed1982a] XSLT_jll v1.1.34+0
[4f6342f7] Xorg_libX11_jll v1.6.9+4
[0c0b7dd1] Xorg_libXau_jll v1.0.9+4
[935fb764] Xorg_libXcursor_jll v1.2.0+4
[a3789734] Xorg_libXdmcp_jll v1.1.3+4
```

```
[1082639a] Xorg_libXext_jll v1.3.4+4
[d091e8ba] Xorg_libXfixes_jll v5.0.3+4
[a51aa0fd] Xorg_libXi_jll v1.7.10+4
[d1454406] Xorg_libXinerama_jll v1.1.4+4
[ec84b674] Xorg_libXrandr_jll v1.5.2+4
[ea2f1a96] Xorg_libXrender_jll v0.9.10+4
[14d82f49] Xorg_libpthread_stubs_jll v0.1.0+3
[c7cfdc94] Xorg_libxcb_jll v1.13.0+3
[cc61e674] Xorg_libxkbfile_jll v1.1.0+4
[12413925] Xorg_xcb_util_image_jll v0.4.0+1
[2def613f] Xorg_xcb_util_jll v0.4.0+1
[975044d2] Xorg_xcb_util_keysyms_jll v0.4.0+1
[0d47668e] Xorg_xcb_util_renderutil_jll v0.3.9+1
[c22f9ab0] Xorg_xcb_util_wm_jll v0.4.1+1
[35661453] Xorg_xkbcomp_jll v1.4.2+4
[33bec58e] Xorg_xkeyboard_config_jll v2.27.0+4
[c5fb5394] Xorg_xtrans_jll v1.4.0+3
[8f1865be] ZeroMQ_jll v4.3.2+6
[3161d3a3] Zstd_jll v1.5.0+0
[0ac62f75] libass_jll v0.14.0+4
[f638f0a6] libfdk_aac_jll v0.1.6+4
[b53b4c65] libpng_jll v1.6.38+0
[a9144af2] libsodium_jll v1.0.20+0
[f27f6e37] libvorbis_jll v1.3.6+6
[1270edf5] x264_jll v2020.7.14+2
[dfaa095f] x265_jll v3.0.0+3
[d8fb68d0] xkbcommon_jll v0.9.1+5
[0dad84c5] ArgTools
[56f22d72] Artifacts
[2a0f44e3] Base64
[ade2ca70] Dates
[8bb1440f] DelimitedFiles
[8ba89e20] Distributed
[f43a241f] Downloads
[7b1f6079] FileWatching
[9fa8497b] Future
[b77e0a4c] InteractiveUtils
[4af54fe1] LazyArtifacts
[b27032c2] LibCURL
[76f85450] LibGit2
[8f399da3] Libdl
[37e2e46d] LinearAlgebra
[56ddb016] Logging
[d6f4376e] Markdown
[a63ad114] Mmap
[ca575930] NetworkOptions
[44cfe95a] Pkg
[de0858da] Printf
[9abbd945] Profile
```

```
[3fa0cd96] REPL
[9a3f8284] Random
[ea8e919c] SHA
[9e88b42a] Serialization
[1a1011a3] SharedArrays
[6462fe0b] Sockets
[2f01184e] SparseArrays
[10745b16] Statistics
[4607b0f0] SuiteSparse
[fa267f1f] TOML
[a4e569a6] Tar
[8dfed614] Test
[cf7118a7] UUIDs
[4ec0a83e] Unicode
[e66e0078] CompilerSupportLibraries_jll
[deac9b47] LibCURL_jll
[29816b5a] LibSSH2_jll
[c8ffd9c3] MbedTLS_jll
[14a3606d] MozillaCACerts_jll
[4536629a] OpenBLAS_jll
[bea87d4a] SuiteSparse_jll
[83775a58] Zlib_jll
[8e850ede] nghttp2_jll
[3f19e933] p7zip_jll
```