

A Derivation of Basic Strategy for Blackjack

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1 Introduction

1.1 Background

Blackjack, also known as 21, is the most commonly played casino games in the world. Blackjack is played with 1 or more standard decks of 52 cards. The goal of blackjack is to get as close to 21 without going over.

1.2 Rules

Blackjack is played with at least 1 player and exactly 1 dealer per table. At each turn, the player has to bet a fixed amount. Then, the dealer deals 2 cards to the player (both face up) and 2 cards to himself (1 face up, 1 face down).

1.2.1 Card Values

- Numbered cards are worth their face value
- Face cards are worth 10 each
- An Ace can be either 11 or 1

1.2.2 Winning the Hand

- Get 21 points on first two cards (called a blackjack), without a dealer blackjack
- Reach a final score higher than the dealer without exceeding 21
- Let the dealer draw additional cards until his hand exceeds 21 (dealer bust).

1.2.3 Choices

- Hit - take another card
- Stand - stop taking cards
- Surrender (First turn only) - give up hand, sacrifice half of bet
- Split (First turn only) - Only applicable when both cards match. Split into two hands (doubling bet)
- Double Down (First turn only) - Double bet, take one more card and then stop.

1.3 Mathematics

Blackjack is a non-deterministic game, which means the outcome depends on one or more random events. Also, it is a game of hidden information because we do not know what the dealer has (or will have). The best we can do is to assign probability distributions and weight the decision making process accordingly.

It is a widely known fact that the casino has an edge in blackjack, even against players playing with an optimal strategy.¹ This edge is small, however, which makes the game "almost fair" for the player. The casino relies on a mathematical concept known as the Law of Large Numbers to ensure that they profit in the long run. This law states that the average value of a random event approaches the expected value after a large number of trials.

2 Assumptions

- Infinite sized deck
 - Each card is equally likely to be drawn
 - Impossible to count cards
- Dealer hits soft 17
- Blackjack pays 1.5 : 1

¹The only way for players to gain the edge against the casino is to count cards.

3 Solution

3.1 Approach

The first step to my solution is figuring out what the dealer will get based on the card that he shows. I will find the exact probability distribution of the possible hands that the dealer will end up with $\{17, 18, 19, 20, 21, \text{bust}\}$ for each of the starting cards. Once that is generated, it is possible to determine the probability that any given hand will beat the dealer. For example, let's say the dealer shows 3, and I have an 18:

Up Card	17	18	19	20	21	Bust
3	0.127	0.132	0.127	0.121	0.116	0.377

²

In this situation, my expected value against the dealer (D) is $p(D \text{ Bust}) + p(D < 18) - p(D > 18)$ which is $0.377 + 0.127 - (0.127 + 0.121 + 0.116) = 0.14$. This means I could expect to win 0.14 bets on average in this situation.

Once this step is complete, the next step is to implement a decision tree algorithm where the computer traverses all possible branches and returns the branch that maximizes expected value. This step must be broken up into two parts, however, since you have a different set of options on the first turn than you do on the rest of your turns.

3.2 Representation of the Game State

The only information that needs to be tracked along function calls are hand total, active ace³, and the dealer up card. With this information, it is possible to find the best possible play.

²The full probability distribution table can be found in the appendix

³Active ace is a flag that is true if there is an ace currently holding a value of 11, and false otherwise

3.3 Pseudocode

For simplicity, I will only show the function for game play after the first turn. This excludes three major (and profitable) choices: surrendering, splitting, and doubling down. The function that includes those options works in much the same way, however.

```
double optimalEV(int total, boolean activeAce, int dealerUpCard) {
    if(total > 21) {
        if(activeAce) //change A from 11 to 1
            return optimalEV(total-10,false,dealerUpCard);
        else
            return -1; //Bust - Lose 1 bet
    }
    double evHit = hit(total,activeAce,dealerUpCard);
    double evStand = stand(total,dealerUpCard);
    return max(evHit,evStand);
}

double hit(int handTotal, boolean activeAce, int dealerUpCard) {
    double expectation = 0.0;
    //normal valued card
    for(int t=2; t<10; t++)
        expectation += optimalEV(handTotal + t, activeAce, dealerUpCard);
    //10 or face card
    expectation += 4.0*optimalEV(handTotal + 10, activeAce, dealerUpCard);
    //ace -- handled different
    if(handTotal < 11)
        expectation += optimalEV(handTotal + 11, true, dealerUpCard);
    else
        expectation += optimalEV(handTotal + 1, activeAce, dealerUpCard);
    return expectation / 13.0; //average of all possibilities
}

double stand(int total, int dealerUpCard) {
    return p(D Bust) + p(D < total) - p(D > total); //array lookup
}
```

As you can see, we have a recursive definition of optimalEV (it calls hit, which in turn refers back to optimalEV with a different game state).

3.4 Results

After analyzing and weighting the expected values, I found an overall expected value (averaged over all possible starting hands) of -0.0047 bets per hand. This means that, on average, it would take 212 hands to lose a single bet. Of course, due to variance, you may win more or lose more depending on a random element. The variance of this strategy is 0.458. From these two variables, we can calculate the probability of profiting after any number of hands.

Number of Hands	Probability of Profiting
1	0.495888703
10	0.487001007
100	0.458958958
500	0.408874187
1000	0.372252233
5000	0.23308605
10000	0.151371798
50000	0.010599556

As you can see, the probability of profiting diminishes as the number of hands increase. This fundamental principal, known as the Law of Large Numbers, is the foundation by which casinos are built.

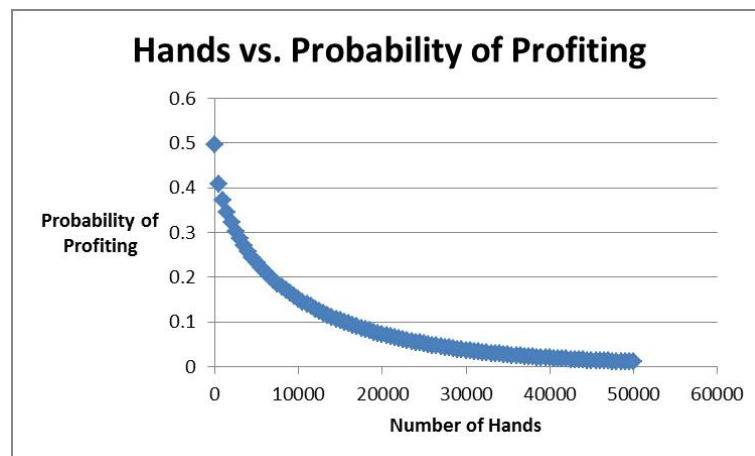


Figure 1: Key

4 Appendix

H	Hit
P	Split
DD	Double Down
S	Stand
L	Surrender

Figure 2: Key

Basic Strategy Blackjack -- Infinite Decks (First Play)										
	2	3	4	5	6	7	8	9	10	A
2-2	P	P	P	P	P	P	H	H	H	H
3-3	P	P	P	P	P	P	H	H	H	H
4-4	H	H	H	P	P	H	H	H	H	H
5-5	DD	DD	DD	DD	DD	DD	DD	DD	H	H
6-6	P	P	P	P	P	H	H	H	H	H
7-7	P	P	P	P	P	P	H	H	H	H
8-8	P	P	P	P	P	P	P	P	P	P
9-9	P	P	P	P	P	S	P	P	S	S
10-10	S	S	S	S	S	S	S	S	S	S
A-A	P	P	P	P	P	P	P	P	P	P
A-2	H	H	H	H	DD	H	H	H	H	H
A-3	H	H	H	DD	DD	H	H	H	H	H
A-4	H	H	DD	DD	DD	H	H	H	H	H
A-5	H	H	DD	DD	DD	H	H	H	H	H
A-6	H	DD	DD	DD	DD	H	H	H	H	H
A-7	DD	DD	DD	DD	DD	S	S	H	H	H
A-8	S	S	S	S	DD	S	S	S	S	S
A-9	S	S	S	S	S	S	S	S	S	S
A-10	S	S	S	S	S	S	S	S	S	S
5	H	H	H	H	H	H	H	H	H	H
6	H	H	H	H	H	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H
9	H	DD	DD	DD	DD	H	H	H	H	H
10	DD	DD	DD	DD	DD	DD	DD	DD	H	H
11	DD	DD	DD	DD	DD	DD	DD	DD	DD	DD
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	L	H
16	S	S	S	S	S	H	H	L	L	L
17	S	S	S	S	S	S	S	S	S	S
18	S	S	S	S	S	S	S	S	S	S
19	S	S	S	S	S	S	S	S	S	S
20	S	S	S	S	S	S	S	S	S	S

Figure 3: Optimal strategy for initial play

Basic Strategy Expectations (Normalized to 1 Bet)										
	2	3	4	5	6	7	8	9	10	A
2-2	-0.082	-0.014	0.069	0.156	0.277	0.007	-0.159	-0.241	-0.289	-0.280
3-3	-0.131	-0.051	0.040	0.131	0.254	-0.053	-0.217	-0.293	-0.338	-0.329
4-4	-0.023	0.007	0.040	0.087	0.211	0.082	-0.059	-0.211	-0.249	-0.246
5-5	0.360	0.410	0.464	0.513	0.581	0.392	0.288	0.143	0.025	0.048
6-6	-0.204	-0.111	-0.010	0.086	0.211	-0.213	-0.271	-0.340	-0.381	-0.374
7-7	-0.129	-0.041	0.056	0.149	0.274	-0.051	-0.371	-0.431	-0.466	-0.460
8-8	0.071	0.144	0.223	0.298	0.404	0.322	-0.020	-0.388	-0.479	-0.480
9-9	0.196	0.259	0.328	0.393	0.483	0.399	0.237	-0.077	-0.178	-0.198
10-10	0.637	0.647	0.659	0.669	0.686	0.774	0.793	0.758	0.554	0.620
A-A	0.854	0.903	0.962	1.028	1.131	1.049	0.928	0.756	0.633	0.649
A-2	0.048	0.075	0.105	0.134	0.212	0.122	0.055	-0.038	-0.105	-0.086
A-3	0.024	0.052	0.083	0.129	0.212	0.080	0.014	-0.075	-0.140	-0.121
A-4	0.002	0.031	0.065	0.129	0.212	0.037	-0.026	-0.113	-0.174	-0.157
A-5	-0.019	0.011	0.065	0.129	0.212	-0.005	-0.066	-0.149	-0.208	-0.191
A-6	0.000	0.056	0.122	0.183	0.266	0.054	-0.072	-0.150	-0.197	-0.205
A-7	0.118	0.176	0.238	0.295	0.374	0.399	0.107	-0.101	-0.144	-0.140
A-8	0.381	0.399	0.420	0.437	0.480	0.616	0.595	0.287	0.063	0.215
A-9	0.637	0.647	0.659	0.669	0.686	0.774	0.793	0.758	0.554	0.620
A-10	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.385	1.038
5	-0.125	-0.092	-0.057	-0.022	0.023	-0.119	-0.188	-0.267	-0.313	-0.305
6	-0.137	-0.104	-0.068	-0.033	0.012	-0.152	-0.217	-0.293	-0.338	-0.329
7	-0.108	-0.076	-0.041	-0.007	0.039	-0.069	-0.210	-0.286	-0.319	-0.335
8	-0.023	0.007	0.040	0.071	0.114	0.082	-0.059	-0.211	-0.249	-0.246
9	0.074	0.121	0.185	0.244	0.324	0.172	0.099	-0.053	-0.153	-0.106
10	0.360	0.410	0.464	0.513	0.581	0.392	0.288	0.143	0.025	0.048
11	0.472	0.519	0.569	0.616	0.678	0.463	0.352	0.227	0.179	0.136
12	-0.253	-0.233	-0.204	-0.164	-0.114	-0.213	-0.271	-0.340	-0.381	-0.374
13	-0.286	-0.246	-0.204	-0.164	-0.114	-0.269	-0.323	-0.387	-0.425	-0.418
14	-0.286	-0.246	-0.204	-0.164	-0.114	-0.321	-0.371	-0.431	-0.466	-0.460
15	-0.286	-0.246	-0.204	-0.164	-0.114	-0.370	-0.416	-0.472	-0.500	-0.499
16	-0.286	-0.246	-0.204	-0.164	-0.114	-0.415	-0.458	-0.500	-0.500	-0.500
17	-0.155	-0.119	-0.081	-0.046	0.003	-0.108	-0.381	-0.424	-0.418	-0.496
18	0.113	0.140	0.170	0.196	0.237	0.399	0.107	-0.184	-0.178	-0.198
19	0.381	0.399	0.420	0.437	0.466	0.616	0.595	0.287	0.063	0.215
20	0.637	0.647	0.659	0.669	0.686	0.774	0.793	0.758	0.554	0.620

Figure 4: Expected values for optimal initial play

Blackjack Basic Strategy (After First Move)										
	2	3	4	5	6	7	8	9	10	A
A-2	H	H	H	H	H	H	H	H	H	H
A-3	H	H	H	H	H	H	H	H	H	H
A-4	H	H	H	H	H	H	H	H	H	H
A-5	H	H	H	H	H	H	H	H	H	H
A-6	H	H	H	H	H	H	H	H	H	H
A-7	S	S	S	S	S	S	S	H	H	H
A-8	S	S	S	S	S	S	S	S	S	S
A-9	S	S	S	S	S	S	S	S	S	S
A-10	S	S	S	S	S	S	S	S	S	S
5	H	H	H	H	H	H	H	H	H	H
6	H	H	H	H	H	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H
9	H	H	H	H	H	H	H	H	H	H
10	H	H	H	H	H	H	H	H	H	H
11	H	H	H	H	H	H	H	H	H	H
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
16	S	S	S	S	S	H	H	H	H	H
17	S	S	S	S	S	S	S	S	S	S
18	S	S	S	S	S	S	S	S	S	S
19	S	S	S	S	S	S	S	S	S	S
20	S	S	S	S	S	S	S	S	S	S
21	S	S	S	S	S	S	S	S	S	S

Figure 5: Optimal strategy for simplified play

Blackjack Expectations (Optimal Play)										
	2	3	4	5	6	7	8	9	10	A
Soft 13	0.048	0.075	0.105	0.134	0.173	0.122	0.055	-0.038	-0.105	-0.086
Soft 14	0.024	0.052	0.083	0.113	0.153	0.08	0.014	-0.075	-0.14	-0.121
Soft 15	0.002	0.031	0.063	0.093	0.134	0.037	-0.026	-0.113	-0.174	-0.157
Soft 16	-0.019	0.011	0.044	0.075	0.116	-0.005	-0.066	-0.149	-0.208	-0.191
Soft 17	0	0.029	0.061	0.092	0.133	0.054	-0.072	-0.15	-0.197	-0.205
Soft 18	0.113	0.14	0.17	0.196	0.237	0.399	0.107	-0.101	-0.144	-0.14
Soft 19	0.381	0.399	0.42	0.437	0.466	0.616	0.595	0.287	0.063	0.215
Soft 20	0.637	0.647	0.659	0.669	0.686	0.774	0.793	0.758	0.554	0.62
Soft 21	0.881	0.884	0.888	0.892	0.897	0.927	0.931	0.939	0.961	0.912
5	-0.125	-0.092	-0.057	-0.022	0.023	-0.119	-0.188	-0.267	-0.313	-0.305
6	-0.137	-0.104	-0.068	-0.033	0.012	-0.152	-0.217	-0.293	-0.338	-0.329
7	-0.108	-0.076	-0.041	-0.007	0.039	-0.069	-0.21	-0.286	-0.319	-0.335
8	-0.023	0.007	0.04	0.071	0.114	0.082	-0.059	-0.211	-0.249	-0.246
9	0.074	0.101	0.13	0.158	0.197	0.172	0.099	-0.053	-0.153	-0.106
10	0.182	0.206	0.232	0.257	0.291	0.257	0.199	0.116	0.025	0.048
11	0.239	0.26	0.285	0.308	0.339	0.292	0.231	0.158	0.119	0.116
12	-0.253	-0.233	-0.204	-0.164	-0.114	-0.213	-0.271	-0.34	-0.381	-0.374
13	-0.286	-0.246	-0.204	-0.164	-0.114	-0.269	-0.323	-0.387	-0.425	-0.418
14	-0.286	-0.246	-0.204	-0.164	-0.114	-0.321	-0.371	-0.431	-0.466	-0.46
15	-0.286	-0.246	-0.204	-0.164	-0.114	-0.37	-0.416	-0.472	-0.504	-0.499
16	-0.286	-0.246	-0.204	-0.164	-0.114	-0.415	-0.458	-0.51	-0.54	-0.534
17	-0.155	-0.119	-0.081	-0.046	0.003	-0.108	-0.381	-0.424	-0.418	-0.496
18	0.113	0.14	0.17	0.196	0.237	0.399	0.107	-0.184	-0.178	-0.198
19	0.381	0.399	0.42	0.437	0.466	0.616	0.595	0.287	0.063	0.215
20	0.637	0.647	0.659	0.669	0.686	0.774	0.793	0.758	0.554	0.62
21	0.881	0.884	0.888	0.892	0.897	0.927	0.931	0.939	0.961	0.912

Figure 6: Expected values for optimal simplified play

Dealer Probabilities (Infinite Decks)							
Up Card	17	18	19	20	21	BJ	Bust
A	0.060	0.146	0.140	0.140	0.062	0.308	0.145
2	0.131	0.137	0.131	0.125	0.119	-	0.357
3	0.127	0.132	0.127	0.121	0.116	-	0.377
4	0.123	0.128	0.122	0.117	0.112	-	0.398
5	0.119	0.123	0.118	0.114	0.109	-	0.418
6	0.117	0.117	0.112	0.108	0.103	-	0.443
7	0.369	0.138	0.079	0.079	0.074	-	0.262
8	0.129	0.359	0.129	0.069	0.069	-	0.245
9	0.120	0.120	0.351	0.120	0.061	-	0.228
10	0.111	0.111	0.111	0.342	0.034	0.077	0.212
J	0.111	0.111	0.111	0.342	0.034	0.077	0.212
Q	0.111	0.111	0.111	0.342	0.034	0.077	0.212
K	0.111	0.111	0.111	0.342	0.034	0.077	0.212

Figure 7: Dealer's Probability Distribution