Modeling Foreign Exchange Markets

Pricing Call Options on Illiquid Crosses

Part 1

Expected payoff of the option is given as-

$$V = E[N(X(T) - K)^{+}]$$

So putting all the values from the question and observing that it has the form of **Log-Normal Distribution** we have the expectation value (of the lognormal distribution) represented as-

$$E[X] = e^{\mu + 1/2\sigma^2}$$

Where mu and sigma are the mean and variance of the normal variable. While putting in the values of mu and sigma, this value came out to be equal to the initial spot of the cross. Intuitively, this can be explained since the given brownian motion to model the derivative price had no drift due to which it would move along the centre point. As a result of this, without a drift, the expected payoff depends on the initial value and the strike (for a European call option).

Part 2

Monte Carlo simulation methods are applied to find the expected payoff of the call option. Given the correlation between the two normal standard variables, the covariance would be the same (since both of them have variance 1) and thus the covariance and mean matrices can be constructed. Using these, a sample is drawn from a multivariate normal distribution. The accuracy of the result can be controlled by increasing the number of iterations for which the simulation is run.

Part 3

The question is same as part 2 above. The number of spots for the required cross are increased to 9. As a result, the dimensions of the covariance matrix becomes 9X9. This part becomes more computationally expensive as compared to part 2. However, the method of simulation remains the same in this case as well. We reduced the total number of iterations carried out by a factor of 10 as compared to part 2 to compensate for the increase in the dimensions of the samples drawn from the distribution.

Part 4

First we tried to study the values for certain input test cases. We found that with increasing correlation, the price of the estimate kept increasing. This implied that we could use a technique like binary search to quickly calculate the required correlation.

Given the price, we used an initial estimate for the correlation as 0.5 and then used binary search to reduce this value. These iterations were carried out till the change in the correlation between two subsequent iterations was greater than 0.0001. Once the change was reduced to below that value, the iterations were stopped and the final value was presented.

Pricing Exotic Options

Part 1

In case of exotic options whose payoff is more complex than a simple call option, given the correlation between the two normal standard variables, the covariance would be the same (since both of them have variance 1) and thus the covariance and mean matrices can be constructed. Using these, a sample is drawn from a multivariate normal distribution. It would have been possible to reduce the noise by sampling over only those points for which X_1 takes value larger than strike K_1. For such values the Indicator variable would give 1 and this would effectively eliminate those values for which the payoff is zero (due to X1). However, we were unable to completely understand how to crack that part.

Part 2

To study the effect of changing vol1, vol2 and the correlation, we carried out the simulations again over a linear space for all the three variables

a. **$\sigma_{1,}$-** By taking some trivial constant values for covariance such as 0 and varying also keeping the value for other sigma constant and varying the value of σ_1 we observed that the value of the price of the option increases on increasing the value of σ_1. Further it can be explained on the basis of the fact that an increase in the value of σ_1 increases the volatility of X_1 hence it becomes more probable that it breaches the value of K_1 and hence more values of X_2 will be considered and less values of X_2 will be nullified on account of the σ_1
b. **σ_2** - Quite similar to σ_1, σ_2 also manifest a similar trend i.e. on increasing the value of σ_2 the value of price of the option increases. It can be explained on the basis of the fact that σ_2 determines the volatility of X_2. More volatile X_2 is more will be the expectation value of the variable, hence more will be the price of the option.

c. ρ **-** Just like the last two cases the value of ρ **is also directly proportional** to that of the value of price of the option. It was observed on simulation involving constant values for σ_1 and σ_2 and varying values of rho. It can be explained as more the value of rho more is the correlation between the X_1 and X_2, hence both will take high values together as compared to the case with low correlation. In that case, one of the two can be high and the other can be low thus giving less contribution to the overall payoff. Example: If X1 is high and X2 is low or vice versa, the contribution is less. However if both are high together then the contribution is high.