Modeling Foreign Exchange Markets

HackerRank

Problem Statement

Introduction

The **Foreign Exchange Market** or the **FX Market** is a global market for the trading of currencies and derivatives [1] on them. In terms of volume it is the largest market in the world with approximately $ 5.3 trillion worth of instruments trading daily.

An **FX Cross** represents an ordered pair of currencies- say (C1, C2). It is represented as C1/C2. C1 is denoted as the base currency while C2 is represented as the quoted currency. The **Spot** or **Exchange Rate** corresponding to a FX cross is the amount of quoted currency required to by a unit of the base currency. For example if EUR denote the Euro (€) and USD represents the American dollar($), saying EUR/USD is 1.12 means we require $ 1.12 to buy €1. Be careful regarding the convention when you solve the problems.

Triangle Equality

Let EUR, USD and CNH represent Euro, American Dollar and Chinese Yuan respectively. If EUR/USD is X, then USD/EUR is 1/X. Further if EUR/USD is X, and USD/CNH is Y, then EUR/CNH is XY. These equations come for the concept of Triangular Arbitrage [2]. These equations can be generalized to multiple FX Crosses. For example if C_1, C_2, ..., C_N represent N currencies and X_i represents the spot value of the i^{th} cross C_i/C_{i+1}, then the spot value of C_1/C_N will be equal to $X_1X_2 \ldots X_{N-1}$.

FX Volatility and FX Spot Modeling

The spot for a currency cross – say EUR/USD- is time varying and stochastic (i.e. the value the spot will take on a future date is uncertain). We try to model it via geometric Brownian motion [3] as follows. If X(t) represents the value of the cross at time t, then we assume it evolves as

$$\frac{dX(t)}{X(t)} = \sigma dW(t)$$

where W(t) is the (random) Weiner process [4]. The σ is called the volatility of the spot. The relative returns dX(t)/X(t) at time t is a random variable and hence so is X(T) - the value of the spot at T. A higher σ implies that the (random) variation of relative returns will be higher (in magnitude) than one with a smaller σ. This implies that the spot is expected to show more variation, hence the word volatility [5].

When this equation is integrated [6] we get,

$$X(t) = X(0)\exp\left(\sigma\sqrt{t}Z - (\sigma^2 t)/2\right)$$

where Z is a standard normal random variable(i.e. a normal/gaussian random variable with mean 0 and variance 1). This distribution is called the lognormal distribution [7]. Note that X(t) isn't simply $X(0)e^{\sigma\sqrt{t}Z}$ (as you would expect in a standard integration). The details of this are in [8], but it isn't strictly necessary for this exercise.

FX Derivatives

A derivative [1] is a financial instrument that derives its value from the performance of an underlying entity. For the purpose of this exercise assume the interest rates are 0.

A Call Option on EUR/USD represents the right but not an obligation to buy a predetermined amount(notional) of EUR and sell USD at a predetermined Spot(called Strike) on a predetermined date(called expiration) [8]. For example if the notional is N, strike is K and the expiration is T months, the call option would give the buyer the right to sell NK dollars for N euro (i.e. exchange rate of K), T months from now. The buyer will exercise this right (remember it is the right, but not the obligation) if and only if the EUR/USD exchange rate is higher than the strike.

So, in case $X \geq K$, the buyer can sell $\$$ NK and receive € N from the option seller. The € N can then be exchanged in the market to receive $\$$ NX (as the spot rate is X). Hence, the payoff to the buyer is $\$$ N(X-K).

In case $K > X$, the option is not exercised and has a payoff zero.

Hence, if X is the EUR/USD spot value the payoff of the call option is

$$P = N(X - K)^+$$

Because FX spots are time varying and random, this payoff is random. The option is valued by calculating the expected value of P. Thus is V is the option value/option price (there are technical differences between the two, but ignore them for now, we use them interchangeably),

$$V = E[N(X(T) - K)^+]$$

where the expectation is over X(T), which is a geometric random variable as explained earlier.

There are derivatives where the payoff is more complex and possibly dependent on spot of multiple crosses or even the path that the spot takes. There are different methods to calculate this expectation, or in general price options. A general method to calculate it is via Monte Carlo simulations [9].

FX Spot Correlations

In general the spots of two crosses might not move independently of each other. There are multiple reasons why this will be the case; the base currency could be the same, the quoted currency could be from the same region, etc. When we say the correlation between the two crosses in ρ, the standard normal variables driving the spot values will be correlated with value ρ. That is, supposing the spots X_1, X_2 of two crosses with correlation ρ, they will evolve as:

$$X_1(t) = X_1(0)e^{\sigma_1\sqrt{t}Z_1 - \frac{\sigma_1^2 t}{2}}$$

$$X_2(t) = X_2(0)e^{\sigma_2\sqrt{t}Z_2 - \frac{\sigma_2^2 t}{2}}$$

where Z_1 and Z_2 are jointly standard normal variables with correlation ρ. The correlation of the spots X_1 and X_2 is not necessarily ρ. This correlation would affect derivative prices whose payoff depends on these two crosses.

Questions:

Pricing Call Options on illiquid Crosses

Crosses which are not very actively traded are called illiquid. CNH/INR is one example. We do not have accurate values of its spot or volatility to price derivatives on them. We price them based on the spot values of liquidly traded crosses. Each cross will evolve as described in the preceding sections. Assume notional equal to 1 for the following questions.

1. First let us consider a vanilla call option on USD/EUR with, strike K, current value of the spot X, expiration T months, volatility of the cross σ. Given these parameters calculate the price of the option.

2. Let us assume CNH/USD and USD/INR have a correlation of ρ (Note: as mentioned in the previous section, this is **not** the correlation between X_1 and X_2). Let the volatilities of them be σ_1 and σ_2 respectively. We have a call option trading on CNH/INR with strike K and expiration T months. The current value of CNH/USD and USD/INR is $X_1(0)$ and $X_2(0)$ respectively. Given the value of K, T, $X_1(0)$, $X_2(0)$, ρ, σ_1 and σ_2, calculate the value/price of the call option.

3. Now we generalize the above problem to multiple currencies C_1, C_2, ..., C_M. X_i represents the spot value of the i^{th} cross C_i/Ci+1. Let X_i have volatility σi and the pairwise correlation between each crosses is ρ (i.e. this is the correlation between the standard normal variable in X_is and **not** between X_is). We have a call option on C_1/C_M, with strike K and expiration T. For this problem M is fixed to 10. Calculate the price of the call option given K, T, $X_1(0)$, $X_2(0)$, ... $X_9(0)$, ρ, σ_1, σ_2, .., σ_9, .

4. Consider the same setting as problem 3 above. However ρ is unknown. Instead we have the price of the option. Calculate the value of ρ which will give the option price. Such ρ which are back calculated from the prices are called market implied parameters. IF no such correlation is possible please output NA.

Pricing Exotic Options

Derivatives which are not simple call or put options and have complex payoffs are called exotic options [10]. In this section we will investigate pricing such derivatives.

Consider two crosses C1/C2 and C3/C4 with spots X1(t) and X2(t), volatility σ1 and σ2 and correlation ρ (again as a reminder, this in **not** the correlation between $X_1(t)$ and $X_2(t)$). At expiration T, notional(amount) N, and strikes K1 and K2, the payoff of the option is equal to

$$Payoff = N * I(X_1(T) > K_1) * (X_2(T) - K_2)^+$$

where I is the indicator function. Thus it is a call option on X_2, but conditioned on X_1 being above the barrier K_1 at time T. The crosses evolve as described in the preceding section.

1. Given σ_1, σ_2, ρ, T, $X_1(0)$, $X_2(0)$, K_1, K_2 and N calculate the price of the option. **K_1 can be large as compared to $X_1(0)$ here**

2. For each of the following subpart (a, b, c) how will the exotic option's price behave if the variable mentioned in it increases and everything else remains the same – increase, decrease, cannot be said. Please include an explanation in the documentation.

   a. σ_1

   b. σ_2

   c. ρ

Evaluation Criteria

The solutions will be evaluated in two stages:

1. Solutions for the final dataset and initial dataset. Please round your answers to the fourth decimal for the 1st question (pricing illiquid crosses - all four sub-questions). For the second, please round off to the neareast integer.

2. Subjective evaluation of the solutions (please include a documentation on your solution in the files you upload as mentioned in the output format section below). Extra points will be given to teams that implement special techniques to improve accuracy, reduce time/space complexity, reduce monte carlo noise(if the solution implements monte carlo simulations) etc.

Top teams will be selected based on the correctness of the solutions provided. These teams will then be evaluated based on the method used and analysis done.

References

1. https://en.wikipedia.org/wiki/Derivative_(finance)
2. https://en.wikipedia.org/wiki/Triangular_arbitrage
3. https://en.wikipedia.org/wiki/Wiener_process
4. https://en.wikipedia.org/wiki/Wiener_process
5. https://en.wikipedia.org/wiki/Volatility_(finance)
6. https://en.wikipedia.org/wiki/Geometric_Brownian_motion
7. https://en.wikipedia.org/wiki/Log-normal_distribution
8. https://en.wikipedia.org/wiki/Call_option
9. https://en.wikipedia.org/wiki/Monte_Carlo_method
10. https://en.wikipedia.org/wiki/Exotic_option

Additional resources

1. https://en.wikipedia.org/wiki/Variance_reduction
2. https://en.wikipedia.org/wiki/Black-Scholes_model
3. https://en.wikipedia.org/wiki/Multivariate_normal_distribution

Input Format

Initial Dataset

http://cdn.hackerrank.com/contests/gsquantify-2015/Quant_Data_Set_Sample.txt

Final Dataset

https://cdn.hackerrank.com/contests/gsquantify-2015/Quant_Data_Set_Final.txt

Number of Test Cases in Final Dataset

We have the following number of test cases per problem in the final dataset. The final dataset will be released 6 hours before the competition.

Pricing call options on illiquid crosses - Question 1: 11

Pricing Call Options on illiquid crosses - Question 2: 11

Pricing Call Options on illiquid crosses - Question 3: 11

Pricing Call Options on illiquid crosses - Question 4: 11

Pricing Exotic Options - Question 1: 14

Input Format

Two data sets will be uploaded as text files - initial dataset and final dataset. Each data sets will have multiple lines with comma separated numbers. The first number denotes the test case id (these are increasing from 1 to the total number of test case). The second number denotes the question. We have the following map:

1 - Pricing call options on illiquid crosses - Question 1

2 - Pricing Call Options on illiquid crosses - Question 2

3 - Pricing Call Options on illiquid crosses - Question 3

4 - Pricing Call Options on illiquid crosses - Question 4

5 - Pricing Exotic Options - Question 1

Note there is no dataset for Pricing Exotic options - Question 2.

The next set of (comma separated) numbers denote values of parameters for the particular questions. An option type 1 implies the option is a call option, if it is zero it is a put option(the data set will only have call option, and hence this value is always 1).The order of parameters for each questions is as follows:

1. **Pricing call options on illiquid crosses - Question 1:** Expiration, Option Type, X(0), Strike, σ
2. **Pricing call options on illiquid crosses - Question 2:** correlation, Expiration, Option Type, $X_1(0)$, $X_2(0)$, Strike, σ_1,σ_2
3. **Pricing call options on illiquid crosses - Question 3:** correlation, Expiration, Option Type, $X_1(0)$, $X_2(0)$,..., $X_9(0)$, Strike, σ_1, σ_2,..., σ_9
4. **Pricing call options on illiquid crosses - Question 4:** Expiration, Option Type, Price, $X_1(0)$, $X_2(0)$,..., $X_9(0)$, Strike, σ_1, σ_2,..., σ_9
5. **Pricing Exotic Options - Question 1:** σ_1, σ_2, ρ, Expiration, $X_1(0)$, $X_2(0)$, K_1, K_2,Notional

For example, a line with these numbers 1,1,0.1,1,0.01,0.01,0.001 would mean the following:

Test Case Id: 1

Question: Pricing Options on Illiquid Crosses

Expiration: 0.1

Option Type: 1

X(0): 0.01

Strike: 0.01

σ: 0.001

Output Format

Please upload the following in a single zip file:

1. Text file named “output.csv” (for the initial and final dataset), containing the solution of each test case in the initial and final dataset. Each line in the output should have 3 comma separated numbers in the following order- Test Case Id, Question Number (the mapping is defined in the Input format), Solution. If a solution doesn't exist, please put NA as the solution.

   In the final dataset at the end please include the solution to Pricing Exotic Options - Question 2 (the question on price behavior) as

   Pricing Exotic Options - Question 2,a : 1000, 6, Solution

   Pricing Exotic Options - Question 2,b : 1001, 6, Solution

Pricing Exotic Options - Question 2,c : 1002, 6, Solution,
where solution is 1 if the answer is increasing, 2 if the answer is decreasing, and 3 if it cannot be said.

There shouldn't be any blank lines between the solutions.

2. All source code files (quoting any references you may have used)
3. A documentation of your solution. This can be in PDF, PPT, PPTX, DOC or DOCX formats. Do include the following aspects in your model doc:

   a. Assumptions

   b. Any mathematical approximations with justifications

   d. Implementation and/or modeling choices and their justifications. For example if your implements monte carlo simulations you should include the number of sample points generated and its justification (this could for example include variance of the sample points, confidence intervals etc)

Sample Input

1,1,2,1,0.3,0.02,0.001
2,2,0.01,0.1,1,0.35,0.08,0.01,0.1,0.1
3,5,0.3,0.3,0.4,1,6,66,15.4,60,10000000
4,5,0.3,0.3,0.4,1,6,66,9.4,60,10000000
5,3,0.01,0.1,1.,3.,4.5,0.2,0.5,4.5,0.5,1.0,0.7,0.5,0.2,0.01,0.2,0.2,0.001,0.2,0.1,0.2,0.001,0.01

Sample Output

1,1,0.2800
2,2,0.0180
3,5,203502
4,5,12573052
5,3,0.8635

The above solutions are the actual answers to the sample inputs and you can test your implementation against it.

Furthermore if this is the output corresponding to the final data set, it should also include solution to question 2 in pricing exotic options. Hence if the answer to each of them is *cannot be determined* then the following lines will be appended in the end

1000,6,3
1001,6,3
1002,6,3

Explanation

In the sample input, the first test case- 1,1,2,1,0.3,0.02,0.001
denotes:

test case:1

question: Pricing call options on illiquid crosses - Question 1, with the following parameter values.

Expiration- 2, Option Type- Call, X(0)- 0.3, Strike- 0.02 and σ- 0.001. The price of the call option for these parameters is 0.2800. The first line in the output - 1,1,0.2800
- denotes:

test case id equal to 1,

question: Pricing call options on illiquid crosses - Question 1, and the final solution (i.e price) 0.2800