PROBLEM 1

FIND THE TRANSFER FUNCTION, $G(s) = \frac{\theta_2(s)}{T(s)}$, FOR THE ROTATIONAL MECHANICAL SYSTEM SHOWN IN FIGURE 2.26

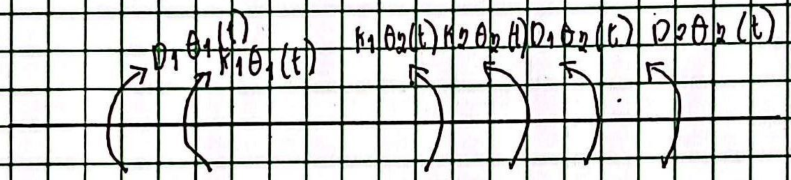
FBD 1:

$$J_1\ddot{\theta}_1(t) + K_1\theta_1(t) + D_1\dot{\theta}_1(t) = T(t) + K_1\theta_2(t) + D_1\dot{\theta}_2(t)$$

OBTAINING THE LAPLACE TRANSFORM

$$J_1 s^2\theta_1(s) + K\theta_1(s) + D_1\theta_1(s) = T(s) + (K_1 + D_1 s)\theta_2(s) \rightarrow \textbf{EQ. 1}$$

FBD 2



$$K_1\theta_2(t) + K_2\theta_2(t) + D_1\dot{\theta}_2(t) + D_2\dot{\theta}_2(t) = D_1\dot{\theta}_1(t) + K_1\theta_1(t)$$

OBTAINING LAPLACE TRANSFORM

$$(K_1 + K_2)\theta_2(s) + (D_1(s) + D_2 s)\theta_2(s) = D_1 s\theta_1(s) + K_1\theta_1(s)$$

$$(K_1 + K_2 + D_1 s + D_2 s)\theta_2(s) = (D_1 s + K_1)\theta_1(s) \rightarrow \textbf{EQ. 2}$$

SUBSTITUTE EQ. 2 IN EQ. 1

$$(J_1 s^2 + K_1 + D_1 s)\theta_1 s = T(s) + (K_1 + D_1 s)\left(\frac{D_1 s + K_1}{K_1 + K_2 + D_1 s + D_2 s}\right)\theta_1 s$$

$$\left[(J_1 s^2 + K_1 + D_1 s) - \frac{(K_1 + D_1 s)(D_1 s + K_1)}{(K_1 + K_2 + D_1 s + D_2 s)}\right]\theta_1(s) = T(s)$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{K_1 + K_2 + D_1 s + D_2 s}{s^3(J_1 D_1 + J_1 D_2) + s^2(J_1 K_1 + J_1 K_2 + D_1 D_2) + s(D_1 K_2 + D_2 K_1) K_1 K_2}}$$

SUBSTITUTE EQ. 1 IN EQ. 2

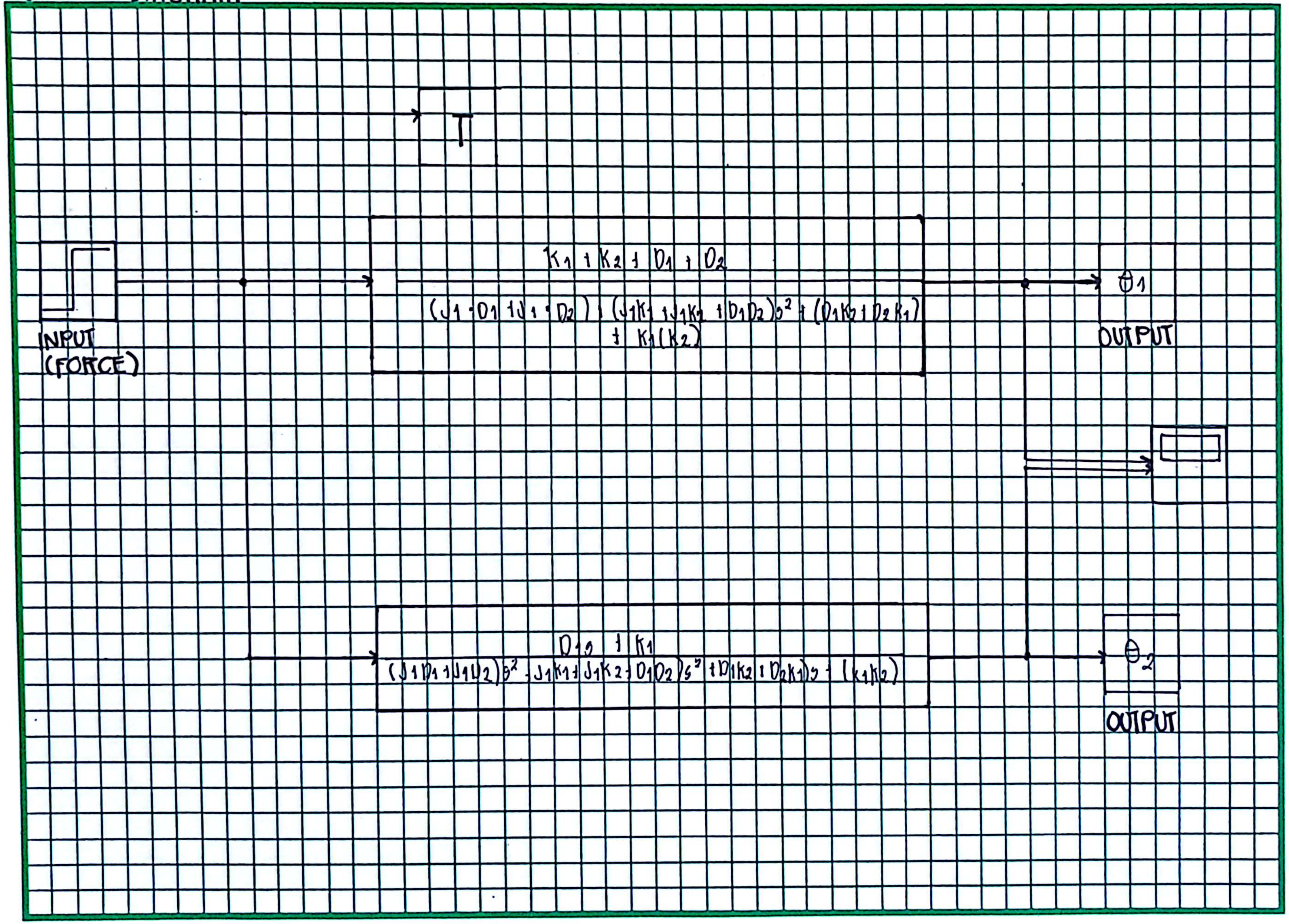
$$(K_1 + K_2 + D_1 s + D_2 s)\,\theta_2(s) = (D_1 s + K_1)\left[\frac{T(s) + (K_1 + D_1 s)}{(J_1 s^2 + K_1 + D_1 s)}\right]\theta_2(s)$$

$$\frac{(J_1 s^2 + K_1 + D_1 s)(K_1 + K_2 + D_1 s + D_2 s)\,\theta_2(s)}{D_1(s) + K_1} = T(s) + (K_1 + D_1 s)\,\theta_2(s)$$

$$\left[\frac{(J_1 s^2 + K_1 + D_1 s)(K_1 + K_2 + D_1 s + D_2 s)}{D_1 s + K_1} - K_1 + D_1 s\right]\theta_2(s) = T(s)$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{D_1 s + K_1}{s^3(J_1 D_1 + J_2 D_2) + s^2(J_1 K_1 + J_1 K_2 + D_1 D_2) + s(D_1 K_2 + D_2 K_1) + K_1 K_2}}$$

BLOCK DIAGRAM



PROBLEM 2

FOR THE ROTATIONAL SYSTEM SHOWN IN FIGURE P2.21, FIND THE TRANSFER FUNCTION $G(s) = \frac{\theta_2(s)}{T(s)}$

FOR $G(s) = \frac{\theta_1(s)}{T(s)}$

$$J_{eq} = J_1\left(\frac{N_1}{N_1}\right)^2 + J_2\left(\frac{N_1}{N_2}\right)^2 + J_3\left(\frac{N_3}{N_4}\cdot\frac{N_1}{N_2}\right)^2$$
$$J_1\left(\frac{4}{4}\right)^2 + J_2\left(\frac{4}{12}\right)^2 + J_3\left(\frac{4}{16}\cdot\frac{4}{12}\right)^2$$
$$J_{eq} = J_1(1) + J_2(1/9) + J_3(1/144)$$

$$D_{eq} = D_1\left(\frac{N_1}{N_1}\right)^2 + D_2\left(\frac{N_1}{N_2}\right)^2 + D_3\left(\frac{N_3}{N_4}\cdot\frac{N_1}{N_2}\right)^2$$
$$= D_1\left(\frac{4}{4}\right)^2 + D_2\left(\frac{4}{12}\right)^2 + D_3\left(\frac{4}{16}\cdot\frac{4}{12}\right)^2$$
$$D_{eq} = D_1(1) + D_2\left(\frac{1}{9}\right) + D_3\left(\frac{1}{144}\right)$$

$$K_{eq} = K_3\left(\frac{N_3}{N_4}\cdot\frac{N_1}{N_2}\right)^2$$
$$= K_3\left(\frac{4}{16}\cdot\frac{4}{12}\right)^2$$
$$K_{eq} = K_3\left(\frac{1}{144}\right)$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{1}{[(J_1)(1) + J_2(1/9) + J_3(1/144) + D_1(1) + D_2(1/9) + D_3(1/144) + K_3(1/144)]}}$$

FOR $G(s) = \frac{\theta_2(s)}{T(s)}$

$$J_{eq} = J_1\left(\frac{N_2}{N_1}\right)^2 + J_2\left(\frac{N_2}{N_2}\cdot\frac{N_3}{N_3}\right)^2 + J_3\left(\frac{N_3}{N_4}\right)^2$$
$$J_1\left(\frac{12}{4}\right)^2 + J_2\left(\frac{12}{12}\cdot\frac{4}{4}\right)^2 + J_3\left(\frac{4}{12}\right)^2$$

$$J_{eq} = J_1(9) + J_2(1) + J_3(1/9)$$

$$D_{eq} = D_1\left(\frac{N_2}{N_1}\right)^2 + D_2\left(\frac{N_2}{N_2}\cdot\frac{N_3}{N_3}\right)^2 + D_3\left(\frac{N_3}{N_4}\right)^2$$

$$= D_1\left(\frac{12}{4}\right)^2 + D_2\left(\frac{12}{12}\cdot\frac{4}{4}\right)^2 + D_3\left(\frac{4}{12}\right)^2$$

$$D_{eq} = D_1(9) + D_2(1) + D_3(1/9)$$

$$K_{eq} = K_1\left(\frac{N_3}{N_4}\right)^2$$

$$= K_1\left(\frac{4}{12}\right)^2$$

$$K_{eq} = K_1\left(\frac{1}{9}\right)$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{\quad 3 \quad}{J_1(9) + J_2(1) + J_3(1/9) + D_1(9) + D_2(1) + D_3(1/9) + K_1(1/9)}}$$

PROBLEM 3

$$\frac{V_c(s)}{V_i(s)}$$

KVL @ LOOP $i_1(t)$

$$\mathcal{L}\{ v_1(t) = i_1(t) + i_1(t) - i_2(t) \}$$

$$V_i(s) = 2I_1(s) - I_2(s) \longrightarrow 1$$

KVL @ LOOP $i_2(t)$

$$\mathcal{L}\left\{ 0 = i_2(t) + \frac{di_2(t)}{dt} + \frac{1}{2}\int i_2(t)dt - i_1(t) \right\}$$

$$0 = I_2(s) + sI_2(s) + \frac{I_2(s)}{s} - I_1(s)$$

$$0 = I_2(s)\left[\frac{2s^2+2s+1}{2s}\right] - I_1(s)$$

CRAMERS RULE

$$I_2(s) = \frac{\begin{vmatrix} 2 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & \frac{2s^2+2s+1}{2s} \end{vmatrix}} = \frac{0 - [-V_i(s)]}{\frac{2s^2+2s+1}{s} - 1} = \frac{V_i(s)}{2s^2+s+1}$$

$$\frac{I_2(s)}{V_i(s)} = \frac{1}{2s^2+s+1} \quad ; \quad V_c(s) = \frac{I_2(s)}{Cs} = \frac{I_{2s}}{2s}$$

$$I_2(s) = 2s\,V_c(s)$$

$$\frac{2s\,V_c(s)}{V_{i_s}} = \frac{1}{2s^2+s+1}$$

$$V_i(s) \rightarrow \boxed{\frac{1/2}{s(s^2+s+1)}} \rightarrow V_c(s)$$