

CPEN 400Q / EECE 571Q Lecture 17

Noisy device simulation, VQE, and error mitigation

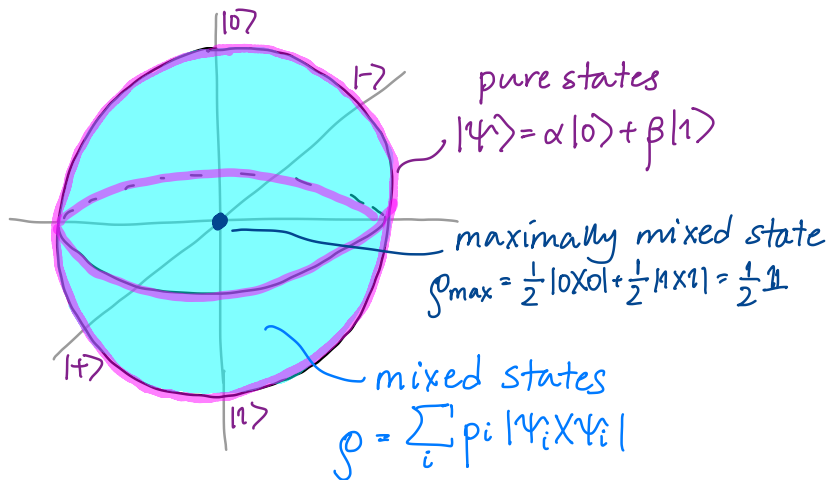
Tuesday 15 March 2022

Announcements

- Assignment 4 coming this week (will be due at end of term)
- Second-last quiz today

Last time

We introduced *density matrices* and *mixed states*.



Last time

We saw how states, operations, and measurements look like on mixed vs. pure states.

	Pure state	Pure state ρ	Mixed state ρ
States			
Ops.			
Meas.*			

* where $\{\varphi_i\}$ form an orthonormal basis, and $\{P_i\}$ may be the associated set of projectors $P_i = |\varphi_i\rangle\langle\varphi_i|$ or more generally a POVM (in either case, $\sum_i P_i = I$).

- Add simple noise to quantum circuits in PennyLane
- Perform simulations on noisy devices using PennyLane plugins
- Mitigate noise using zero-noise extrapolation

Quantum channels and noise

Quantum channels

Noise occurring in quantum systems is represented by **quantum channels**.

A quantum channel Φ *maps* states to other states.

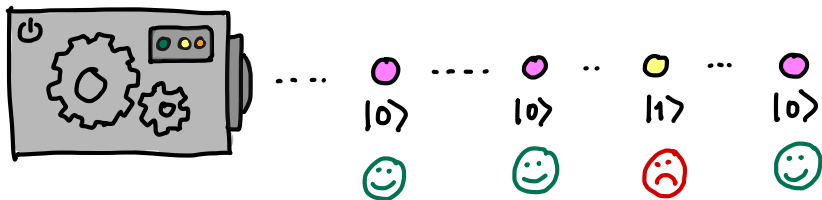
More formally, quantum channels are linear CPTP (Completely Positive, Trace-Preserving) maps.

Example: applying a unitary U is a channel, \mathcal{U} .

The bit flip channel

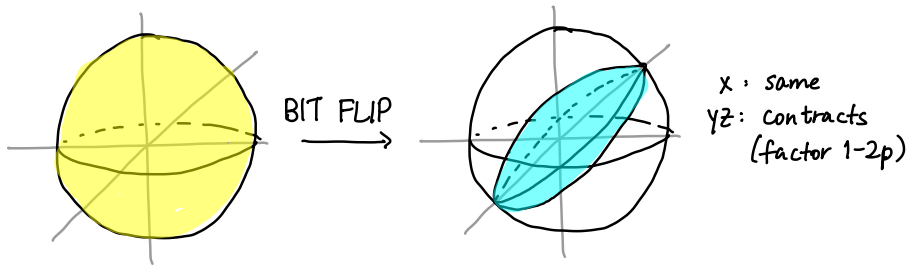
Suppose a “bit flip” (Pauli X) error occurs with probability p .

$$\mathcal{E}(\rho) = (1-p) \cdot \rho + p \cdot X \rho X$$



The bit flip channel

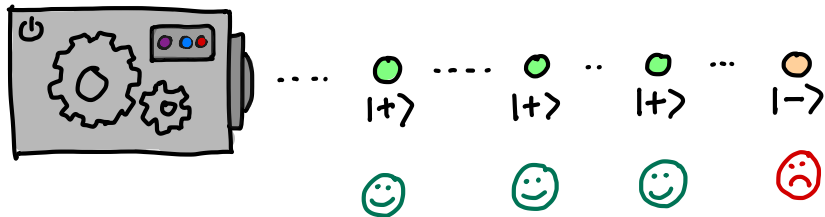
We can visualize the effects of such a channel by observing how it deforms the Bloch sphere.



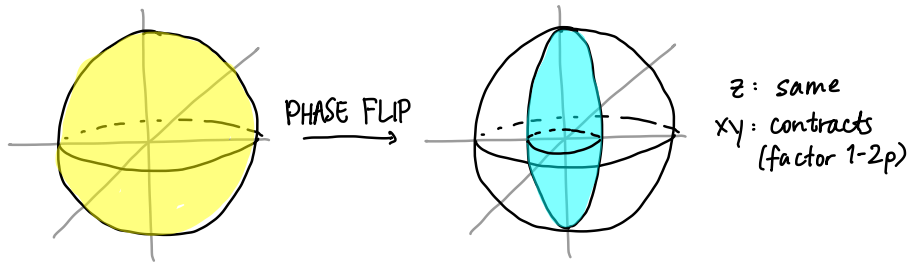
The phase flip channel

Suppose a “phase flip” (Pauli Z) error occurs with probability p .

$$\mathcal{E}(\rho) = (1-p) \cdot \rho + p \cdot Z \rho Z$$



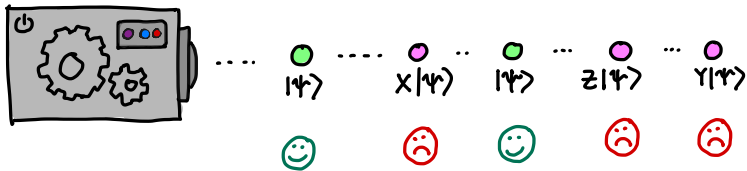
The phase flip channel



The depolarizing channel

Suppose each Pauli error occurs with probability $p/3$. This is called the *depolarizing channel*.

$$\mathcal{E}(\rho) = (1-p) \cdot \rho + \frac{p}{3} \cdot X \rho X + \frac{p}{3} Y \rho Y + \frac{p}{3} Z \rho Z$$



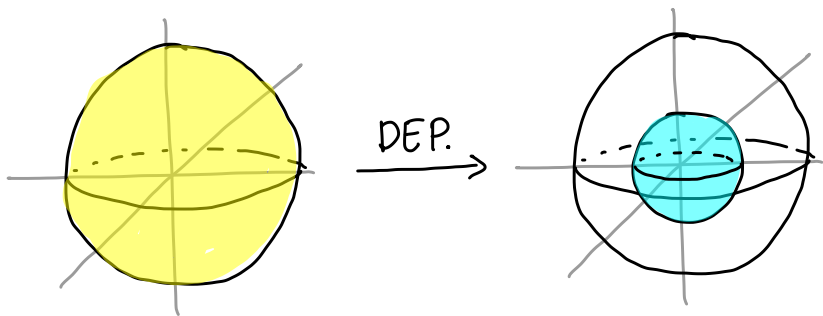
The depolarizing channel

The depolarizing channel

can also be written as

Think of this as outputting ρ w/probability $1 - p$, and maximally mixed state with probability p .

The depolarizing channel



... let's add some noise to quantum circuits.

Comparing density matrices

How can we quantify “how much” error occurs? How close is $\sigma = \mathcal{E}(\rho)$ to ρ ?

One common way is the **trace distance**:

Value of trace distance is bounded by $0 \leq T(\rho, \sigma) \leq 1$. *Lower* trace distance is better.

Comparing density matrices

Another is the **fidelity**:

Value is bounded by $0 \leq F(\rho, \sigma) \leq 1$. *Higher* fidelity is better.

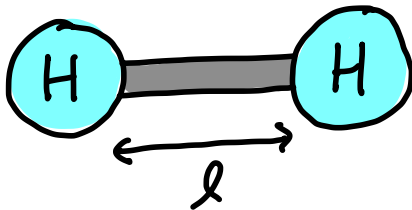
Can show that if $\rho = |\psi\rangle\langle\psi|$ is pure, then

If $\sigma = |\varphi\rangle\langle\varphi|$ is also pure,

VQE on (simulated) noisy hardware

VQE on a noisy device

Let's solve a (small) *quantum chemistry* problem: find the ground state energy of H_2 .



H_2 is a molecule with 2 electrons.

The Hamiltonian for H_2 can be written using 4 qubits.

Qubits correspond to molecular orbitals that are either *occupied* ($|1\rangle$) or *unoccupied* ($|0\rangle$).

The ground state has the form:

What gates should we use to prepare this state?

A **single excitation** has the form

$$G(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta/2) & -\sin(\theta/2) & 0 \\ 0 & \sin(\theta/2) & \cos(\theta/2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This has the action

(In PennyLane: `qml.SingleExcitation`).

Excitation operations

Single excitations can be visualized like so:

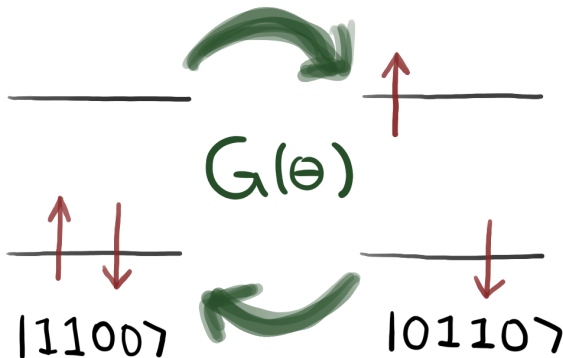
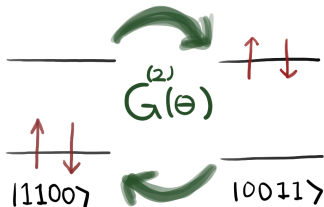


Image credit: Givens rotation demo, https://pennylane.ai/qml/demos/tutorial_givens_rotations.html

Excitation operations

Similarly, there are double excitations:



This has the action

(In PennyLane: `qml.DoubleExcitation`).

Image credit: Givens rotation demo, https://pennylane.ai/qml/demos/tutorial_givens_rotations.html

VQE on a noisy device

So to produce

$$|\psi_g\rangle = \cos(\theta/2)|1100\rangle - \sin(\theta/2)|0011\rangle$$

we will

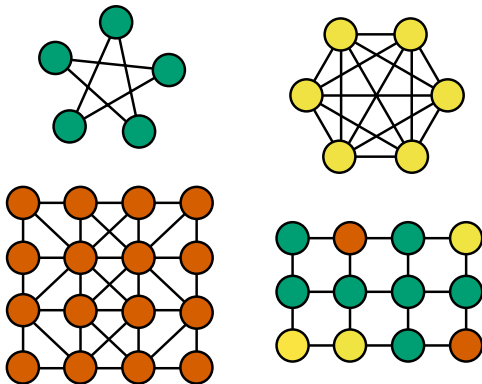
1. prepare the state $|1100\rangle$
2. apply a double excitation

Then we run VQE to find the optimal θ .

We will run this on an ideal device, and a device with a simulated noise model based on a real processor using PennyLane-Qiskit.

Comparing quantum computers

Clearly noise is problem. How do we measure the quality of noisy quantum devices?




Comparing quantum computers

It is challenging to characterize, benchmark, and to compare quantum computers: it's more than just number of qubits.

- error rates
- qubit connectivity
- software/compiler quality
- gate operation times
- size of problem it can solve
- size of meaningful problem it can solve
- ...

Comparing supercomputers

It's more than just number of cores: comparison based on LINPACK benchmark (FLOPS while solving dense linear system).



The image shows a screenshot of a web browser displaying the Top500 website. The address bar shows the URL: <https://www.top500.org/lists/top500/2021/11/>. Below the browser window, a table lists the top 3 supercomputers based on the LINPACK benchmark.

Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	Supercomputer Fugaku - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu RIKEN Center for Computational Science Japan	7,630,848	442,010.0	537,212.0	29,899
2	Summit - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM DOE/SC/Oak Ridge National Laboratory United States	2,414,592	148,600.0	200,794.9	10,096
3	Sierra - IBM Power System AC922, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM / NVIDIA / Mellanox DOE/NNSA/LLNL United States	1,572,480	94,640.0	125,712.0	7,438

Comparing quantum computers

Many competing metrics proposed by competing companies:

- quantum volume (IBM)
- CLOPS: circuit layer operations per second (IBM)
- algorithmic qubits (IonQ)
- Q-score (Atos)

All this goes to say: current quantum hardware is noisy. Noise comes from a variety of sources, and depends on the qubit technology and architecture.

We need to do a combination of:

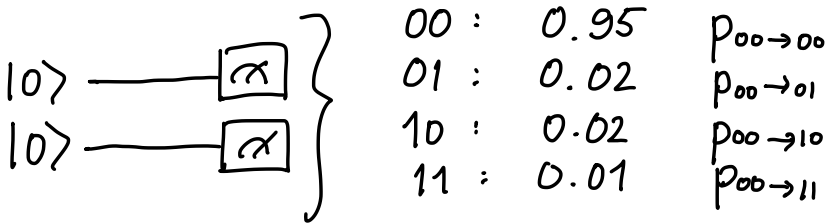
- Better-characterizing the behaviour of devices to learn how to improve their operation
- Processing the results to mitigate the effects of noise as much as possible

Today we will consider the latter.

(We will do the former another day, time permitting.)

Measurement error mitigation

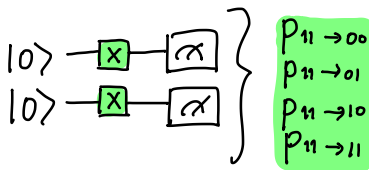
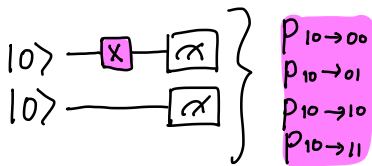
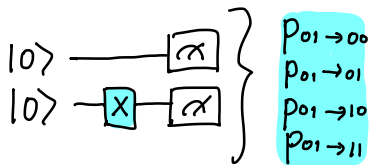
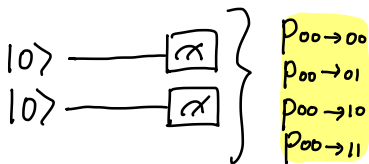
Errors can occur in the measurement process where states are read out incorrectly.



These kinds of errors are quite straightforward to mitigate.

Measurement error mitigation

Check what happens with all possible input states:



Measurement error mitigation

We can put the results from our calibration circuits into a matrix:

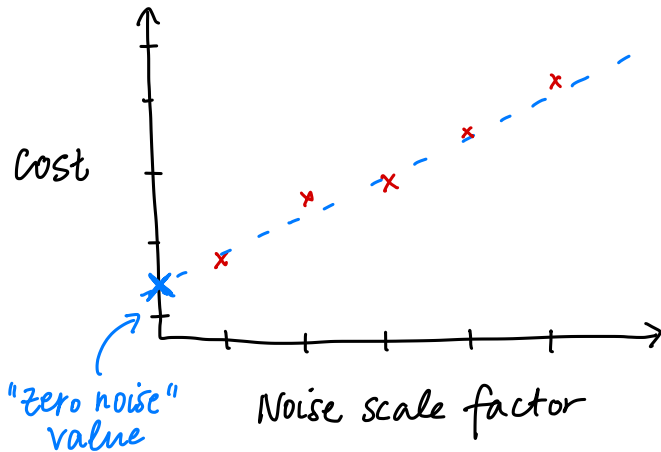
$$M = \begin{bmatrix} p_{00 \rightarrow 00} & p_{01 \rightarrow 00} & p_{10 \rightarrow 00} & p_{11 \rightarrow 00} \\ p_{00 \rightarrow 01} & p_{01 \rightarrow 01} & p_{10 \rightarrow 01} & p_{11 \rightarrow 01} \\ p_{00 \rightarrow 10} & p_{01 \rightarrow 10} & p_{10 \rightarrow 10} & p_{11 \rightarrow 10} \\ p_{00 \rightarrow 11} & p_{01 \rightarrow 11} & p_{10 \rightarrow 11} & p_{11 \rightarrow 11} \end{bmatrix}$$

Can suppose that the probability vector P_{noisy} we get at the end of a quantum algorithm is related to the ideal one, P_{ideal} , under multiplication by M since that's what we see:

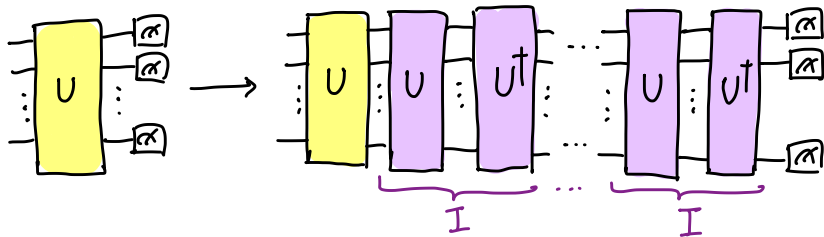
So to get the ideal results:

Gate error mitigation: zero-noise extrapolation

Modify the circuits to systematically *increase* scale of the noise, then extrapolate down to the zero-noise limit.



Unitary folding



Let's code up a very basic version of this in PennyLane.

More sophisticated version: Python package `mitiq`
<https://github.com/unitaryfund/mitiq>

Next time

Content:

- Quantum approximate optimization algorithm

Action items:

1. Final project

Recommended reading:

- Nielsen and Chuang Ch. 8
- Qiskit tutorial on measurement error mitigation:
<https://qiskit.org/textbook/ch-quantum-hardware/measurement-error-mitigation.html>
- mitiq documentation, for more fun error mitigation: <https://mitiq.readthedocs.io/en/stable/guide/guide.html>