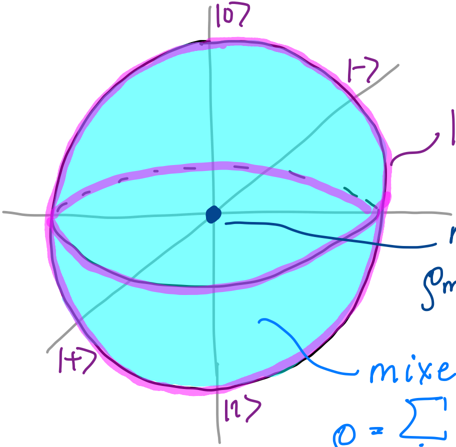
# CPEN 400Q / EECE 571Q Lecture 17
# Noisy device simulation, VQE, and error mitigation

Tuesday 15 March 2022

## Announcements

- Assignment 4 coming this week (will be due at end of term)
- Second-last quiz today

We introduced *density matrices* and *mixed states*.




pure states

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

maximally mixed state

$\rho_{max} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}\mathbb{1}$

mixed states

$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

# Last time

We saw how states, operations, and measurements look like on mixed vs. pure states.

| | Pure state | Pure state $\rho$ | Mixed state $\rho$ |
|---|---|---|---|
| States | $\lvert\psi\rangle$ | $\rho = \lvert\psi\rangle\langle\psi\rvert$ | $\rho = \sum_i p_i \lvert\psi_i\rangle\langle\psi_i\rvert$ |
| Ops. | $\lvert\psi'\rangle = U\lvert\psi\rangle$ | $\rho' = U\rho U^\dagger$ | $\rho' = U\rho U^\dagger$ |
| | | $\lvert\psi'\rangle\langle\psi'\rvert = U\lvert\psi\rangle\langle\psi\rvert U^\dagger$ | |
| Meas.* | $\lvert\langle\varphi_i\vert\psi\rangle\rvert^2$ | $\mathrm{Tr}(\lvert\varphi_i\rangle\langle\varphi_i\rvert\,\lvert\psi\rangle\langle\psi\rvert)$ | $\mathrm{Tr}(P_i\rho)$ |
| | $\langle\psi\vert B\vert\psi\rangle$ | $\mathrm{Tr}(B\rho)$ | $\mathrm{Tr}(B\rho)$ |

* where $\{\varphi_i\}$ form an orthonormal basis, and $\{P_i\}$ may be the associated set of projectors $P_i = \lvert\varphi_i\rangle\langle\varphi_i\rvert$ or more generally a POVM (in either case, $\sum_i P_i = I$).

# Learning outcomes

- Add simple noise to quantum circuits in PennyLane
- Perform simulations on noisy devices using PennyLane plugins
- Mitigate noise using zero-noise extrapolation

# Quantum channels and noise

Noise occurring in quantum systems is represented by **quantum channels**.

A quantum channel $\Phi$ *maps* states to other states.

$$\rho \;\; \rightarrow \;\; \rho' = \Phi(\rho)$$

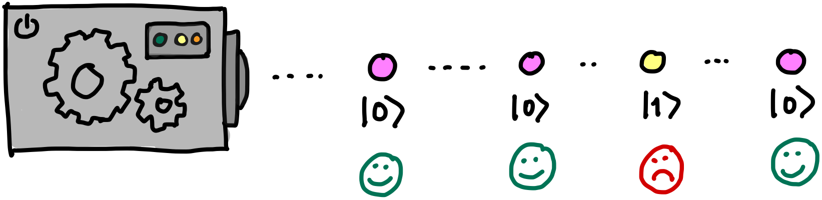
*More formally, quantum channels are linear CPTP (Completely Positive, Trace-Preserving) maps.*

Example: applying a unitary $U$ is a channel, $\mathcal{U}$.

$$\rho \;\; \rightarrow \;\; \rho' = \mathcal{U}(\rho) = U \rho U^\dagger$$

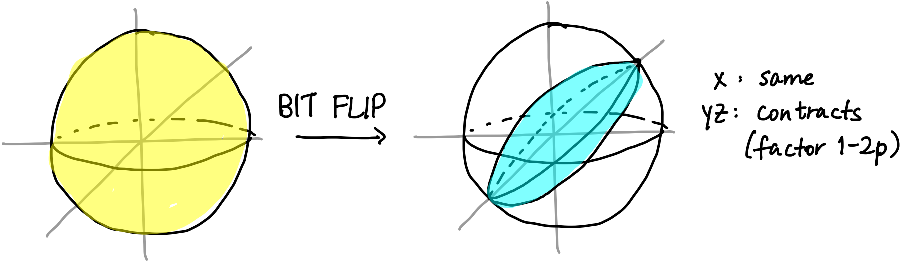
Suppose a "bit flip" (Pauli $X$) error occurs with probability $p$.

$$\mathcal{E}(\rho) = (1-p)\cdot\rho + p\cdot X\rho X$$



# The bit flip channel

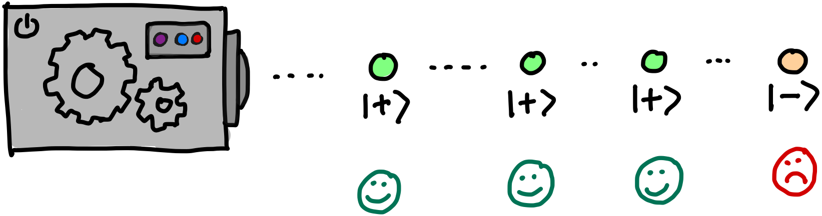
We can visualize the effects of such a channel by observing how it deforms the Bloch sphere.



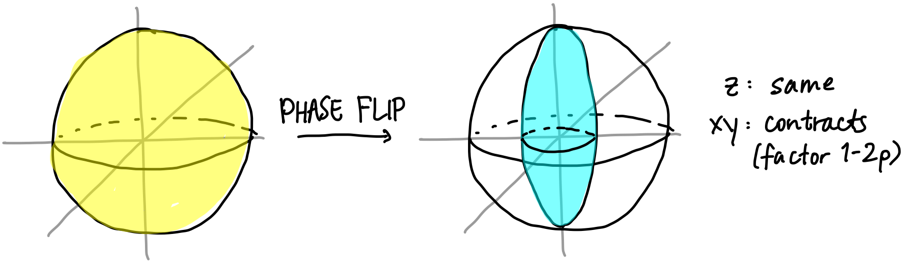
# The phase flip channel

Suppose a "phase flip" (Pauli $Z$) error occurs with probability $p$.

$$\mathcal{E}(\rho) = (1-p)\cdot\rho + p\cdot Z\rho Z$$



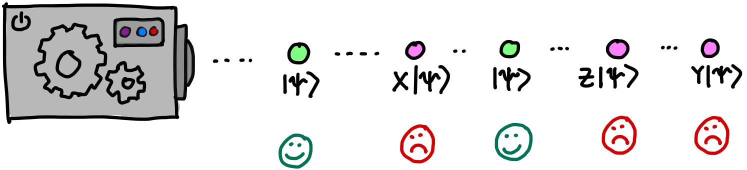
# The phase flip channel



# The depolarizing channel

Suppose each Pauli error occurs with probability $p/3$. This is called the *depolarizing channel*.

$$\mathcal{E}(\rho) = (1-p)\cdot\rho + \frac{p}{3}\cdot X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z$$



The depolarizing channel

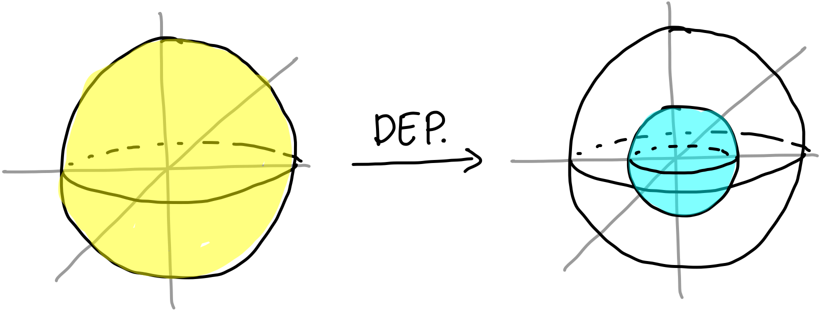
$$\mathcal{E}(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$$

can also be written as

$$\mathcal{E}(\rho) = (1-p)\rho + p \cdot \frac{I}{2}$$

Think of this as outputting $\rho$ w/probability $1-p$, and maximally mixed state with probability $p$.

# The depolarizing channel




... let's add some noise to quantum circuits.

How can we quantify "how much" error occurs? How close is $\sigma = \mathcal{E}(\rho)$ to $\rho$?

One common way is the **trace distance**:

$$T(\rho, \sigma) = \frac{1}{2}||\rho - \sigma||_1 = \frac{1}{2}\text{Tr}\left(\sqrt{(\rho - \sigma)^\dagger(\rho - \sigma)}\right)$$

Value of trace distance is bounded by $0 \leq T(\rho, \sigma) \leq 1$. *Lower* trace distance is better.

# Comparing density matrices

Another is the **fidelity**:

$$F(\rho,\sigma) = \left(\mathrm{Tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2$$

Value is bounded by $0 \leq F(\rho,\sigma) \leq 1$. *Higher* fidelity is better.

Can show that if $\rho = |\psi\rangle\langle\psi|$ is pure, then

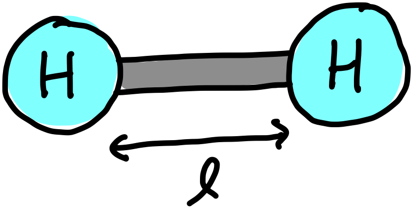
$$F(\rho,\sigma) = \langle\psi|\,\sigma|\psi\rangle$$

If $\sigma = |\varphi\rangle\langle\varphi|$ is also pure,

$$F(\rho,\sigma) = |\langle\varphi|\psi\rangle|^2$$

# VQE on (simulated) noisy hardware

Let's solve a (small) *quantum chemistry* problem: find the ground state energy of $H_2$.




$H_2$ is a molecule with 2 electrons.

The Hamiltonian for $H_2$ can be written using 4 qubits.

Qubits correspond to molecular orbitals that are either *occupied* ($|1\rangle$) or *unoccupied* ($|0\rangle$).

The ground state has the form:

$$|\psi_g\rangle = \cos(\theta/2)|1100\rangle - \sin(\theta/2)|0011\rangle$$

What gates should we use to prepare this state?

# Excitation operations

A **single excitation** has the form

$$G(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta/2) & -\sin(\theta/2) & 0 \\ 0 & \sin(\theta/2) & \cos(\theta/2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This has the action

$$\begin{aligned} G|01\rangle &= \cos(\theta/2)|01\rangle + \sin(\theta/2)|10\rangle \\ G|10\rangle &= \cos(\theta/2)|10\rangle - \sin(\theta/2)|01\rangle \end{aligned}$$

(In PennyLane: `qml.SingleExcitation`).

# Excitation operations

Single excitations can be visualized like so:

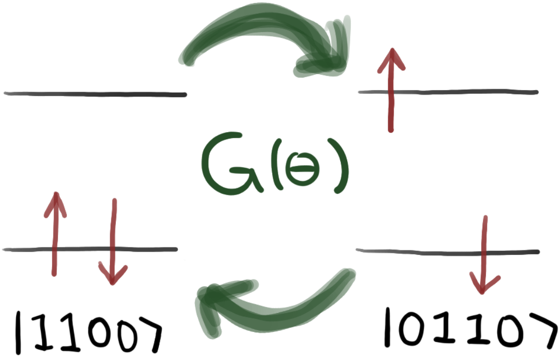
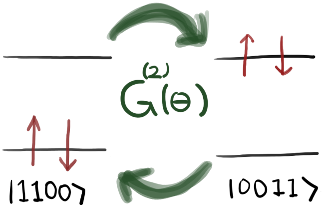



Image credit: Givens rotation demo, `https://pennylane.ai/qml/demos/tutorial_givens_rotations.html`

# Excitation operations

Similarly, there are double excitations:




This has the action

$$G^{(2)}|0011\rangle = \cos(\theta/2)|0011\rangle + \sin(\theta/2)|1100\rangle$$
$$G^{(2)}|1100\rangle = \cos(\theta/2)|1100\rangle - \sin(\theta/2)|0011\rangle$$

(In PennyLane: `qml.DoubleExcitation`).

Image credit: Givens rotation demo, `https://pennylane.ai/qml/demos/tutorial_givens_rotations.html`

So to produce

$$|\psi_g\rangle = \cos(\theta/2)|1100\rangle - \sin(\theta/2)|0011\rangle$$

we will

1. prepare the state $|1100\rangle$
2. apply a double excitation

Then we run VQE to find the optimal $\theta$.

We will run this on an ideal device, and a device with a simulated noise model based on a real processor using PennyLane-Qiskit.

# Comparing quantum computers

Clearly noise is problem. How do we measure the quality of noisy quantum devices?

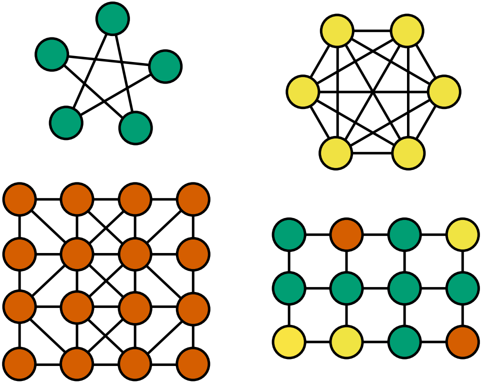


Image credit: quantum volume demo, `https://pennylane.ai/qml/demos/quantum_volume.html`

## Comparing quantum computers

It is challenging to characterize, benchmark, and to compare quantum computers: it's more than just number of qubits.

- error rates
- qubit connectivity
- software/compiler quality
- gate operation times
- size of problem it can solve
- size of meaningful problem it can solve
- ...

# Comparing supercomputers

It's more than just number of cores: comparison based on LINPACK benchmark (FLOPS while solving dense linear system).

https://www.top500.org/lists/top500/2021/11/

| Rank | System | Cores | Rmax (TFlop/s) | Rpeak (TFlop/s) | Power (kW) |
|---|---|---|---|---|---|
| 1 | **Supercomputer Fugaku** - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu<br>RIKEN Center for Computational Science<br>Japan | 7,630,848 | 442,010.0 | 537,212.0 | 29,899 |
| 2 | **Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM<br>DOE/SC/Oak Ridge National Laboratory<br>United States | 2,414,592 | 148,600.0 | 200,794.9 | 10,096 |
| 3 | **Sierra** - IBM Power System AC922, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband, IBM / NVIDIA / Mellanox<br>DOE/NNSA/LLNL<br>United States | 1,572,480 | 94,640.0 | 125,712.0 | 7,438 |

## Comparing quantum computers

Many competing metrics proposed by competing companies:

- quantum volume (IBM)
- CLOPS: circuit layer operations per second (IBM)
- algorithmic qubits (IonQ)
- Q-score (Atos)

# Error mitigation

All this goes to say: current quantum hardware is noisy. Noise comes from a variety of sources, and depends on the qubit technology and architecture.

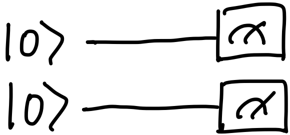
We need to do a combination of:

- Better-characterizing the behaviour of devices to learn how to improve their operation
- Processing the results to mitigate the effects of noise as much as possible

Today we will consider the latter.
(We will do the former another day, time permitting.)

# Measurement error mitigation

Errors can occur in the measurement process where states are read out incorrectly.

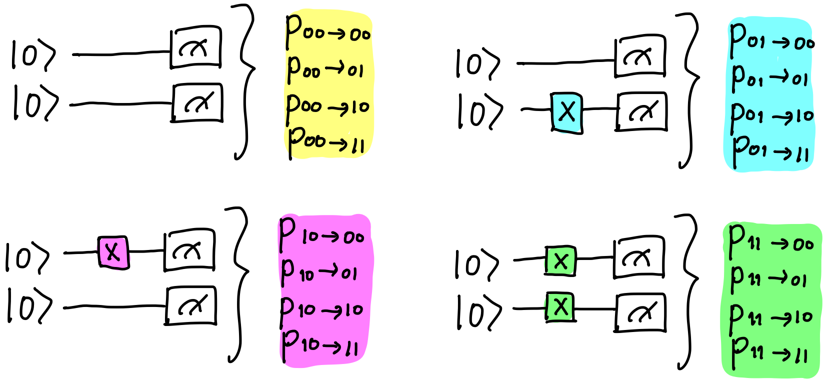



| | | |
|---|---|---|
| 00 : | 0.95 | $p_{00\to 00}$ |
| 01 : | 0.02 | $p_{00\to 01}$ |
| 10 : | 0.02 | $p_{00\to 10}$ |
| 11 : | 0.01 | $p_{00\to 11}$ |

These kinds of errors are quite straightforward to mitigate.

# Measurement error mitigation

Check what happens with all possible input states:



# Measurement error mitigation

We can put the results from our calibration circuits into a matrix:

$$M = \begin{bmatrix} p_{00\to 00} & p_{01\to 00} & p_{10\to 00} & p_{11\to 00} \\ p_{00\to 01} & p_{01\to 01} & p_{10\to 01} & p_{11\to 01} \\ p_{00\to 10} & p_{01\to 10} & p_{10\to 10} & p_{11\to 10} \\ p_{00\to 11} & p_{01\to 11} & p_{10\to 11} & p_{11\to 11} \end{bmatrix}$$

Can suppose that the probability vector $P_{noisy}$ we get at the end of a quantum algorithm is related to the ideal one, $P_{ideal}$, under multiplication by $M$ since that's what we see:

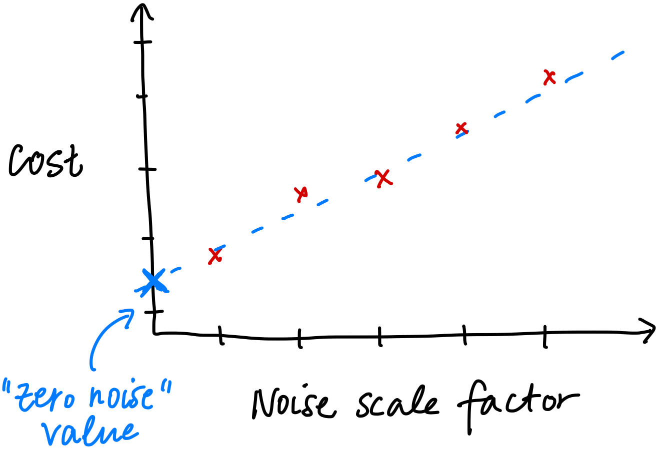
$$P_{noisy} = MP_{ideal}$$

So to get the ideal results:

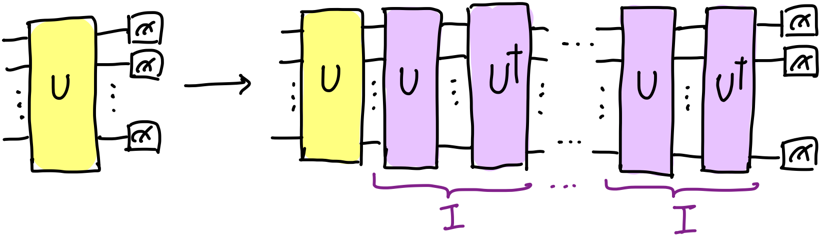
$$P_{ideal} = M^{-1}P_{noisy}$$

# Gate error mitigation: zero-noise extrapolation

Modify the circuits to systematically *increase* scale of the noise, then extrapolate down to the zero-noise limit.



# Unitary folding




Let's code up a very basic version of this in PennyLane.

More sophisticated version: Python package `mitiq`
`https://github.com/unitaryfund/mitiq`

# Next time

Content:

- Quantum approximate optimization algorithm

Action items:

1. Final project

Recommended reading:

- Nielsen and Chuang Ch. 8
- Qiskit tutorial on measurement error mitigation: `https://qiskit.org/textbook/ch-quantum-hardware/measurement-error-mitigation.html`
- `mitiq` documentation, for more fun error mitigation: `https://mitiq.readthedocs.io/en/stable/guide/guide.html`