REPORT

Rome, Expert consultation
Italy, on revision of FAO methodologies
28-31 May for crop water requirements
1990

ANNEX V
FAO PENMAN-MONTEITH FORMULA

FOOD AND AGRICULTURE ORGANIZATION
OF THE UNITED NATIONS

Annex V

Proposed calculation procedures for ET_o combination formula

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V.I PARAMETERS USED IN THE ET EQUATIONS

1. Conversion SI - C.G.S. system

In line with international standards SI units will be used uniformly for all parameters and equations and will replace the c.g.s. convention used previously. The following conversions factors may be useful in this:

Pressure:		1 mbar	--> 0.1 kPa (kiloPascal)
Radiation	:	1 cal cm^{-2} d^{-1}	--> 0.041868 MJ m^{-2} d^{-1}
		1 MJ m^{-2} d^{-1}	--> 23.884 cal cm^{-2} d^{-1}
			--> 0.408 mm d^{-1}
		1 mm d^{-1}	--> 2.45 MJ m^{-2} d^{-1}
			--> 58.6 cal cm^{-2} d^{-1}
		1 Wm^{-2}	--> 0.0864 MJ m^{-2} d^{-1}
			--> 2.064 cal cm^{-2} d^{-1}

2. Latent Heat of Vaporization (λ)

$$\lambda = 2.501 - (2.361 \times 10^{-3})\, T \qquad 1$$

where: λ : latent heat of vaporization [MJ kg^{-1}]
T : air temperature [°C]

Reference : Harrison 1963

As the value of the latent heat varies only slightly over normal temperature ranges a single value for lambda may be taken. For **T = 20 °C :**

$$\lambda = 2.45 \qquad (2)$$

3. Slope Vapour Pressure Curve (Δ)

$$\Delta = \frac{4098\, e_a}{(T + 237.3)^2} \qquad 3$$

Δ : slope vapour pressure curve [kPa°C^{-1}]
T : air temperature [°C]
e_a : saturation vapour pressure at temperature T [kPa]

Reference : Tetens (1930), Murray (1967). Derived from equation (10).

4. Psychrometric Constant (γ)

$$\gamma = \frac{c_p P}{\varepsilon \lambda} \, x \, 10^{-3} = 0.00163 \frac{P}{\lambda} \qquad \mathbf{4}$$

γ	:	psychrometric constant [kPa °C^{-1}]
c_p	:	specific heat of moist air = 1.013 [kJ kg^{-1} °C^{-1}]
P	:	atmospheric pressure [kPa]
ε	:	ratio molecul. weight water vapour/dry air = 0.622
λ	:	latent heat [MJ kg^{-1}]

Reference: Brunt (1952)

5. Atmospheric Pressure (P)

$$P = P_o \left(\frac{T_{ko} - \alpha(z - z_o)}{T_{ko}} \right)^{\frac{g}{\alpha R}} \qquad \mathbf{5}$$

P	:	atmospheric pressure at elevation z [kPa]
P_o	:	atmospheric pressure at sea level [kPa]
z	:	elevation [m]
z_o	:	elevation at reference level [m]
g	:	gravitational acceleration = 9.8 [m s^{-2}]
R	:	specific gas constant = 287 [J kg^{-1} K^{-1}]
T_{ko}	:	reference temperature [K] at elev. z_o _ 273 + T [°C]
α	:	Constant lapse rate saturated air = 0.0065 [K m^{-1}]

Reference: Burman *et al*. (1987)

When assuming:

P_o = 101.3 [kPa] at z_o = 0
T_{ko} = 293 [K] for T = 20 [°C]

$$P = 101.3 \left(\frac{293 - 0.0065\, z}{293} \right)^{5.26} \qquad \mathbf{6}$$

6. Atmospheric density (ρ)

$$\rho = \frac{1000\,P}{T_{kv}\,R} = 3.486\,\frac{P}{T_{kv}} \qquad 7$$

ρ	:	atmospheric density [kg m^{-3}]
P	:	atmospheric pressure at elevation z [kPa]
R	:	specific gas constant = 287 [J kg^{-1} K^{-1}]
T_{kv}	:	virtual temperature [K]

$$T_{kv} = T_k \left(1 - 0.378\,\frac{e_d}{P}\right)^{-1} \qquad 8$$

T_k	:	absolute temperature [K] _ 273 + T [°C]
e_d	:	vapour pressure at dew point [kPa]
P	:	atm pressure at elevation z [kPa]

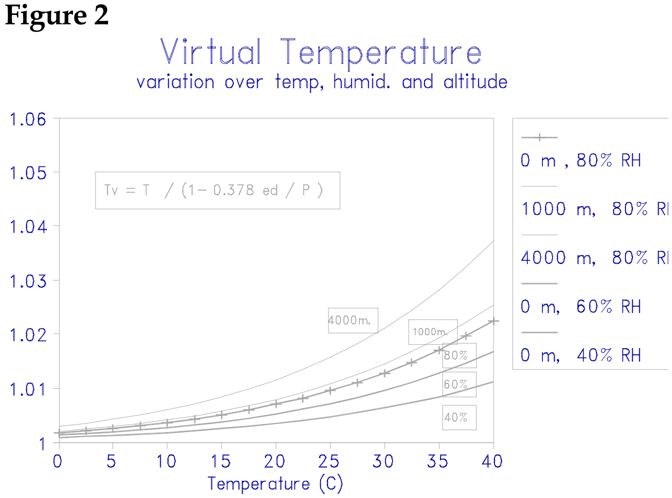
For average conditions:

e_d	=	1 - 5 [kPa]
P	=	80 - 100 [kPa]

$$T_{kv} \approx 1.01(T + 273) \qquad 9$$

Figure 1 shows the very minor changes of T_{kv} for different temperature, humidity and altitude values.

Figure 2



7. Saturation Vapour Pressure (e_a)

$$e_a = 0.611 \; \exp\left(\frac{17.27 \; T}{T + 237.3}\right) \qquad 10$$

e_a : saturation vapour pressure [kPa]
T : temperature [°C]

Reference: Tetens, 1930.

8. Actual Vapour Pressure (e_d)

Defined as the Saturation Vapour Pressure at **Dewpoint temperature (e_d)** the Actual vapour pressure or average daily vapour pressure can be determined from:

a. **Hygrometer** measurements (RH) where: Average daily vapour pressure is best determined from two relative humidity measurements daily at T_{max} (early afternoon) and T_{min} (early morning)

$$e_d = \frac{e_{d(T_{\min})} + e_{d(T_{\max})}}{2} = \frac{1}{2} e_{a(T_{\min})} \cdot \frac{RH_{\max}}{100} + \frac{1}{2} e_{a(T_{\max})} \frac{RH_{\min}}{100} \qquad 11$$

where e_d : average daily vapour pressure

at early morning:

RH_{max} : maximum daily relative humidity [%]
T_{min} : minimum daily temperature [°C]
$e_{a(Tmin)}$: saturation vapour pressure at T_{min} [kPa]
$e_{d(Tmin)}$: actual vapour pressure at T_{min} [kPa]

at early afternoon (around 14.00):

RH_{min} : minimum daily relative humidity [%]
T_{max} : maximum daily temperature [°C]
$e_{a(Tmax)}$: saturation vapour pressure at T_{max} [kPa]
$e_{d(Tmax)}$: actual vapour pressure at T_{max} [kPa]

Mean daily Relative Humidity is defined as follows:

$$RH_{mean} = \frac{RH_{\max} + RH_{\min}}{2} \qquad (12)$$

where RH_{mean} : mean daily relative humidity.

As actual vapour pressure does not vary greatly over one day, e_d (T_{max}) _ e_d (T_{min}).

Thus RH_{mean} can be determined from average daily vapour pressure and T_{max} and T_{min} as follows:

$$RH_{mean} = e_d \left(\frac{50}{e_{a(T_{min})}} + \frac{50}{e_{a(T_{max})}} \right) \qquad \textbf{(13)}$$

Similarly average daily vapour pressure can be derived from RH_{mean} as follows:

$$e_d = RH_{mean} / \left(\frac{50}{e_{a(T_{min})}} + \frac{50}{e_{a(T_{max})}} \right) \qquad \textbf{(14)}$$

NOTE: To calculate RH_{mean} or e_d from T_{mean} is discouraged in view of the obvious deviating results.

b. **Psychrometer** measurements (dry and wet bulb thermometers) :

$$e_d = e_{a(Twet)} - \gamma_{asp} \, (T_{dry} - T_{wet}) \, P \qquad \textbf{15}$$

where: γ_{asp} = 0.00066 for Assmann aspiration at 5 m/s [C^{-1}]
= 0.0008 for natural ventilation at 1 m/s [C^{-1}]
= 0.0012 for indoor ventilation at 0 m/s [C^{-1}]
T_{dry} : dry bulb temperature [°C]
T_{wet} : wet bulb temperature [°C]
P : atmospheric pressure [kPa]
$e_{a(Twet)}$: saturation vapour pressure at wet bulb temperature [kPa]

Reference: Bosen (1958)

c. If humidity data are lacking an estimate of vapour pressure can be made by assuming **minimum temperature** equal to dewpoint temperature. However particularly for arid regions a further calibration is required to adjust minimum temperatures to dewpoint temperatures:

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9. Vapour Pressure Deficit (VPD)

To be determined according to the following relationship:

$$VPD = e_a - e_d = \frac{e_{a(Tmax)} + e_{a(Tmin)}}{2} - e_d \qquad \textbf{17}$$

where:	VPD	=	vapour pressure deficit [kPa]
	$e_{a(Tmax)}$	=	saturation vapour pressure at T_{max} [kPa]: equation 10
	$e_{a(Tmin)}$	=	saturation vapour pressure at T_{min} [kPa]: equation 10
	e_d	=	actual vapour pressure [kPa]: equation 15, 11 or 14

10. Extraterrestrial Radiation (R_a)

$$R_a = \frac{24x60}{\pi} G_{sc}\, d_r\, \_(\omega_s \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\omega_s) \qquad 18$$

$$R_a = 37.6\, d_r \left(\omega_s \sin\varphi \sin\delta + \cos\varphi \cos\delta \sin\omega_s \_ \right) \qquad 19$$

R_a	:	extraterrestrial radiation [MJ m^{-2} d^{-1}]
G_{sc}	:	solar constant [MJ m^{-2} min^{-1}] = 0.0820
d_r	:	relative distance Earth - Sun
δ	:	solar declination [rad]
_	:	latitude [rad]
ω_s	:	sunset hour angle [rad]

$$\omega_s = \arccos(- \tan\varphi \tan\delta) \qquad 20$$

$$d_r = 1 + 0.033 \cos(\frac{2\pi}{365}\_J) = 1 + 0.033 \cos(0.0172J)$$

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$$\delta = 0.409 \sin(\frac{2\pi}{365} J - 1.39)\_ = 0.409 \sin(0.0172J - 1.39) \qquad 22$$

J : number of the day in the year [ ]

Reference: Duffie and Beckman (1980)

For monthly values J can be determined by:

$$J = integer\left(30.42\, M - 15.23 \right) \qquad 23$$

M : Month number (1 - 12) [ ]

Reference: Gommes (1983).

For daily values, J can be determined by:

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If M < 3, then J = J+2.
If leap year and M > 2, then J = J+1

D : day of the month

Reference: Craig (1984).

Note : For the winter months on latitudes above 55 degrees the equations have a limited validity and reference should be made to the Smithsonian tables to assess possible deviations.

11. Daylight Hours (N)

$$N = \frac{24}{\pi}\,\omega_s = 7.64\,\omega_s \qquad 25$$

N : maximum day light hours [h]

12. Wind speed (U_2)

To adjust windspeed data obtained from instruments placed at elevations different from the standard height of 2 m for which the ET equation has been calibrated, the following formula may be used:

$$U_2 = U_z \frac{\ln[\frac{z_2 - d}{z_o}]}{\ln[\frac{z - d}{z_o}]} \qquad 26$$

For a standardized reference crop with **crop height 0.12 m** and height at **windspeed measurements at 2.00 m**, the conversion factor can be determined as follows:

$$\frac{U_2}{U_z} = 4.87 / \ln(67.8\,z - 5.42) \qquad 27$$

where: U_z : windspeed measurement at height z [ms^{-1}]
U_2 : windspeed measurement at 2 m height [$m\ s^{-1}$]
z : height windspeed measurements [m]
z_2 : standard height windspeed measurements [m] = 2 [m]
d : zero plane displacement of wind profile [m] = 0.08 (see equation 37)
z_o : roughness parameter for momentum [m] = 0.015 (see equation 38)

Reference: Allen *et al*. (1989)

13. Day Wind

Wind speed measurements concern normally daily averages over 24 hours. To determine day time wind (07.00 - 19.00 hrs) the following relationship can be used:

$$U_d = \frac{2U\,(U_d/U_n)}{(1+U_d/U_n)} \qquad \mathbf{28}$$

where: U_d : windspeed during day time (07.00 - 19.00 hrs) [$m\ s^{-1}$]
U_n : windspeed during night time (19.00 - 07.00) [$m\ s^{-1}$]
U : average windspeed over 24 hours [$m\ s^{-1}$]

For average conditions:

$$U_d/U_n \approx 2$$

$$U_d = 1.33\,U \qquad \mathbf{29}$$

V.II PENMAN MONTEITH EQUATION

The original form of the Penman-Monteith equation can be written as:

$$\lambda ET_o = \frac{\Delta(R_n - G) + \rho c_p (e_a - e_d)/r_a}{\Delta + \gamma(1 + r_c/r_a)} \qquad \mathbf{(30)}$$

where: λET_o : latent heat flux of evaporation [$kJ\ m^{-2}\ s^{-1}$]
R_n : net radiation flux at surface [$kJ\ m^{-2}\ s^{-1}$]
G : soil heat flux [$kJ\ m^{-2}\ s^{-1}$]
ρ : atmospheric density [$kg\ m^{-3}$]
c_p : specific heat moist air [$kJ\ kg^{-1}\ °C^{-1}$]
(e_a-e_d) : vapour pressure deficit [kPa]
r_c : crop canopy resistance [$s\ m^{-1}$]
r_a : aerodynamic resistance [$s\ m^{-1}$]
Δ : slope vapor pressure curve [$kPa\ °C^{-1}$]
γ : psychrometric constant [$kPa\ °C^{-1}$]
λ : latent heat of vaporization [$MJ\ kg^{-1}$]

Reference : Monteith (1965, 1981)

To facilitate the analysis of the combination equation we define the aerodynamic and radiation term as:

$$ET_o = ET_{rad} + ET_{aero} \tag{31}$$

where: ET_o : reference evapotranspiration of standard crop canopy [mm d^{-1}]
ET_{rad} : radiation term [mm d^{-1}]
ET_{aero} : aerodynamic term [mm d^{-1}]

In the section below the various parameters are further analysed and defined.

1. RESISTANCE FACTORS

1.1 Crop Canopy Resistance (r_c)

$$r_c = \frac{R_l}{0.5\ LAI} = \frac{200}{LAI} \tag{32}$$

where: R_l : average daily (24 hours) stomata resistance of a single leaf [s m^{-1}] ≈ **100**
LAI : leaf area index []

Reference : Allen *et al*. (1989)

For **Clipped GRASS**:

$$LAI = 24\ h_c \tag{33}$$

h_c : crop height [m] = 0.05 - 0.15 [m]

For **ALFALFA and other Field Crops**:

$$LAI = 5.5 + 1.5 \ln(h_c) \tag{34}$$

h_c : crop height [m] = 0.10 - 0.50 [m]

Reference : Allen *et al*. (1989)

For the **REFERENCE CROP** (grass) is defined:

h_c = 0.12 [m]
LAI = 24 x 0.12 = 2.88

The canopy resistance [sm^{-1}] becomes:

$$r_c = \frac{200}{2.88} \approx 70 \tag{35}$$

1.2 Aerodynamic Resistance (r_a)

$$r_a = \frac{\ln\left(\frac{z_m - d}{z_{om}}\right) . \ln\left(\frac{z_h - d}{z_{oh}}\right)}{k^2 U_z} \quad \textbf{(36)}$$

r_a	:	aerodynamic resistance [s m^{-1}]
z_m	:	height windspeed measurements [m]
z_h	:	height temperature and humidity measurements [m]
k	:	von Karman constant = 0.41 []
U_z	:	windspeed measurement at height z_m [m s^{-1}]

Reference : Allen *et al*. (1989)

d : zero plane displacement of wind profile [m]

$$d = \frac{2}{3}\ h_c = 0.08 \quad \textbf{(37)}$$

Reference : Monteith (1981).

z_{om} : roughness parameter for momentum [m]

$$z_{om} = 0.123\ h_c = 0.015 \quad \textbf{(38)}$$

z_{oh} : roughness parameter for heat and water vapour [m]

$$z_{oh} = 0.1\ z_{om} = 0.0123\ h_c = \_0.0015 \quad \textbf{(39)}$$

Reference : Brutsaert (1975)

For a standardized height for windspeed, temperature and humidity measurements at 2.00 m, and a standardized crop height of 0.12 m, the aerodynamic resistance can be estimated as follows:

$$r_a = \frac{208}{U_2} \quad \textbf{(40)}$$

r_a	:	aerodynamic resistance [s m^{-1}]
U_2	:	windspeed measurement at 2 m. height [m s^{-1}]
208	:	coefficient [ ] for temp/humidity measurements at **2 m height**, recommended WMO standard for agricultural stations
199	:	coefficient [ ] for temp/humidity measurements at **a.5 m height**, still common practice in many manually-operated stations with temperature sensors at eye level

1.3 Modified Psychrometric Constant (γ*)

$$\gamma^* = \gamma\left(1 + \frac{r_c}{r_a}\right) \quad \textbf{(41)}$$

γ^*	:	modified psychrometric constant [kPa °C^{-1}]
γ	:	psychrometric constant [kPa °C^{-1}]
r_c	:	crop canopy resistance [s m^{-1}]
r_a	:	aerodynamic resistance [s m^{-1}]

Reference : Monteith (1965).

Applying equations (35) and (40) for the **Reference Crop** the modified psychrometric constant (γ^*) can thus be determined according to the following equation:

$$\gamma^* \approx \gamma\,(1 + 0.34\, U_2) \quad \textbf{(42)}$$

2. AERODYNAMIC TERM

From the original Penman-Monteith equation (eq. 30), the aerodynamic term can be written as:

$$ET_{aero} = \frac{86.4}{\lambda}\ \frac{1}{\Delta + \gamma^*}\ \frac{\rho\, c_p}{r_a}\,(e_a - e_d) \quad \textbf{(43)}$$

where:	ET_{aero}	:	aerodynamic term [mm d^{-1}]
	86.4	:	conversion factor to [mm d^{-1}]
	γ^*	:	modified psychrometric constant [kPa °C^{-1}], see equations 41 & 42

When introducing (see equation 4)

$$c_p = \gamma\, \frac{0.622\, \lambda}{P} \cdot 10^3 \quad \textbf{(44)}$$

where:	c_p	:	specific heat moist air [kJ kg^{-1} °C^{-1}]
	P	:	atmospheric pressure [kPa]
	λ	:	latent heat of vaporization [MJ kg^{-1}]
	10^3	:	conversion factor [MJ] to [kJ]

equation (43) can be rewritten as:

$$ET_{aero} = \frac{\gamma}{\Delta + \gamma^*}\, \rho\, \frac{0.622\_\lambda}{P}\, \frac{86400}{\lambda}\, \frac{(e_a - e_d)}{r_a} \quad \textbf{(45)}$$

With the ideal gas law (equation 7):

$$\rho = \frac{1000\, P}{T_{kv}\, R} = 3.486\, \frac{P}{T_{kv}} \quad \textbf{(46)}$$

where:	ρ	:	atmospheric density [kg m^{-3}]
	P	:	atm pressure at elevation z [kPa]
	R	:	specific gas constant = 287 [J kg^{-1} K^{-1}]

T_{kv}	:	virtual temperature [K] = 1.01 (T + 273) (see equation 9)
r_a	:	aerodynamic resistance = 208/U_2 (see equation 40).

The aerodynamic term in the Combination equation can be rewritten now:

$$ET_{aero} = \frac{\gamma}{\Delta + \gamma^*} \frac{0.622}{P} \frac{3.486\,P}{1.01(T+273)}\, 86400 \frac{U_2}{208} (e_a - e_d) \quad \textbf{(47)}$$

$$ET_{aero} = \frac{\gamma}{\Delta + \gamma(1+0.34\,U_2)} \frac{900}{(T+273)} U_2 (e_a - e_d) \quad \textbf{(48)}$$

where:	ET_{aero}	:	aerodynamic term of ET_o [mm d^{-1}]
	U_2	:	windspeed [m s^{-1}]
	(e_a-e_d)	:	vapour pressure deficit [kPa]
	T	:	air temperature [°C]
	900	:	conversion factor

Note: In view of the approximate nature of the roughness parameters, a coefficient value of 900 is selected rather than the more precise value 892 calculated for temp/humidity measurements at 2.00 m height and 932 for 1.50 height (see equation 35).

3. RADIATION TERM

$$ET_{rad} = \frac{\Delta}{\Delta + \gamma^*} (R_n - G) \frac{1}{\lambda} - \frac{0.408\,\Delta(R_n - G)}{\Delta + \gamma(1+0.34\,U_2)} \quad \textbf{(49)}$$

where:	ET_{rad}	:	radiation term [mm d^{-1}]
	R_n	:	net radiation [MJ m^{-2} d^{-1}]
	G	:	soil heat flux [MJ m^{-2} d^{-1}]
	λ	:	latent heat of vaporization [MJ kg^{-1}] = 2.45

3.1 Net Radiation (R_n)

$$R_n = R_{ns} \downarrow - R_{nl} \uparrow \quad \textbf{(50)}$$

where:	R_n	:	net radiation [MJ m^{-2} d^{-1}]
	R_{ns}	:	net incoming shortwave radiation [MJ m^{-2} d^{-1}]
	R_{nl}	:	net outgoing longwave radiation [MJ m^{-2} d^{-1}]

3.1.1 Net Shortwave Radiation R_{ns}

The net shortwave radiation is the radiation received effectively by the crop canopy taking into account losses due to reflection :

$$R_{ns} = (1 - \alpha)\, R_s \approx 0.77\, R_s \quad \textbf{(51)}$$

where:	α	:	albedo or canopy reflection coefficient = **0.23** overall average for grass
	R_s	:	incoming solar radiation [MJ m^{-2} d^{-1}]

Solar radiation can be measured in the more advanced agro-meteorological stations with various radiometers and pyranometers. They require however careful calibration and maintenance. Although electronic weather stations equipped with global pyranometers are becoming more widespread, measured solar radiation data may not be available from many agro-meteorological stations.

Angstrom Values

Shortwave radiation will in many cases be estimated from measured sunshine hours according to the following empirical relationship:

$$R_s = \left(a_s + b_s \frac{n}{N} \right) R_a \qquad \textbf{(52)}$$

where:	a_s	:	fraction of extraterrestrial radiation (Ra) on overcast days ≈ **0.25** for average climate
	$a_s + b_s$	:	fraction of radiation on clear days ≈ 0.75
	b_s	≈	**0.50** for average climate
	n/N	:	relative sunshine fraction [ ]
	n	:	bright sunshine hours per day [hr]
	N	:	total daylength [hr]
	R_a	:	extraterrestrial radiation [MJ m^{-2} d^{-1}]: see equation (19)

Available local radiation data can be used to carry out a regression analysis to determine the Angstrom coefficients a_s and b_s according to the following relationships:

$$R_{so} = (a_s + b_s) R_a \approx (0.75) R_a \qquad \textbf{(53)}$$

$$R_{sc} = a_s \, R_a \tag{54}$$

where:
- R_{so} : measured shortwave radiation during bright sunshine [MJ m^{-2} d^{-1}]
- R_{sc} : measured shortwave radiation for completely overcast sky [MJ m^{-2} d^{-1}]
- R_a : extraterrestrial radiation [MJ m^{-2} d^{-1}]: see equation (19)

Depending on atmospheric conditions (humidity, dust) and solar declination (latitude and month) the Angstrom values (a_s & b_s) will vary.

When no actual solar radiation data are available and no calibration has been carried out for improved a_s and b_s parameters the following values are recommended for average climates:

a_s = 0.25 **b_s = 0.50**

For **α = 0.23** : reference crop (grass)

Net shortwave radiation can thus be estimated according to the following general equation:

$$R_{ns} = 0.77\,(0.25 + 0.50\,\frac{n}{N})\,R_a \tag{55}$$

3.1.2 Net Longwave Radiation (R_{nl})

The thermal radiation from vegetation and soil to the atmosphere and the reflected radiation from atmosphere and clouds can be represented by the radiation law as follows:

$$R_{nl} = -\,R_{ld}\downarrow + R_{lu}\uparrow = f\,(\varepsilon_{vs}\,(\varepsilon_a - 1))\,\sigma\,T_k^4 \approx f\,(\varepsilon_a - \varepsilon_{vs})\,\sigma\_T_k^4 \tag{56}$$

where:
- R_{nl} : net longwave radiation [MJ m^{-2} d^{-1}]
- $R_{lu}\uparrow$: outgoing thermal radiation emitted by the vegatation and soil into the atmosphere (upward flux) [MJ m^{-2} d^{-1}]
- $R_{ld}\downarrow$: incoming (thermal) radiation emitted by the atmosphere and cloud cover to the earth surface (downward flux) [MJ m^{-2} d^{-1}]
- f : adjustment for cloud cover [ ]
- ε_a : effective emissivity of the atmosphere [ ]
- ε_{vs} : emissivity by vegetation (0.99 - 0.94) and soil (0.98 - 0.80) [ ] ≈ **0.98**
- σ : Stefan-Boltzmann constant = **4.90×10^{-9}** [MJ m^{-2} K^{-4} d^{-1}]
- T_k : mean air temperature [K]

Cloudiness Factor (f)

1. When solar radiation data is available the net thermal radiation can be estimated using the following expression to determine the cloudiness factor :

$$f = \frac{R_{nl}}{R_{nlo}} = \left(a_c \frac{R_s}{R_{so}} + b_c \right) \qquad \textbf{(57)}$$

where: f : cloudiness factor [ ]
R_{nl} : net longwave radiation [MJ m^{-2} d^{-1}]
R_{nlo} : net longwave radiation for clear skies [MJ m^{-2} d^{-1}]
R_s : measured shortwave solar radiation [MJ m^{-2} d^{-1}]
R_{so} : shortwave solar radation for clear skies [MJ m^{-2} d^{-1}]
$a_c + b_c$: cloudiness factor for clear skies [ ] = 1.0
a_c ≈ 1.35 (arid) - 1.0 (humid areas)
thus b_c ≈ -0.35 - 0.0

Reference : Wright and Jensen (1972), Jensen *et al*. (1990).

a_c and b_c parameters are calibration values to be determined through specialized local studies, measuring long wave radiation values.

The following indicative values are recommended :

a_c = 1.35 **b_c = - 0.35**

Reference: FAO-No. 24

2. For measured sunshine hours the longwave fraction for partly cloudy skies can be determined by combining equations 52 and 53 into expression 57 :

$$f = \frac{R_{nl}}{R_{nlo}} = (a_c \frac{b_s}{a_s + \_b_s}) \frac{n}{N} + (b_c + \frac{a_s}{a_s + b_s} a_c) \qquad (58)$$

$$f = \frac{R_{nl}}{R_{nlo}} = \left(0.9 \frac{n}{N} + 0.1 \right) \qquad \textbf{(59)}$$

For a_c = 1.35 b_c = - 0.35
a_s = 0.25 b_s = 0.50
Reference: FAO-No 24

Net Emissivity ε'

$$\varepsilon' = (\varepsilon_a - \varepsilon_{vs}) = \left(a_l + b_l \sqrt{e_d} \right) \_ \left(0.34 - 0.14 \sqrt{e_d} \right) \qquad \textbf{(60)}$$

where: ε' : net emissivity
e_d : vapour pressure at dew point [kPa]
a_l : correlation coefficient [] ≈ 0.34 - 0.44
b_l : correlation coefficient [] ≈ -0.14 - -0.25

Reference : Brunt (1932), Jensen et al. (1990)

For average atmospheric conditions the following indicative values can be taken:

a_l = 0.34 **b_l = -0.14**

Reference: FAO-No. 24

When humidity measurements are not available, the minimum temperature may be taken as dew point temperature to estimate average vapour pressure.

Alternatively net emissivity can be estimated from average temperature according to the following equation:

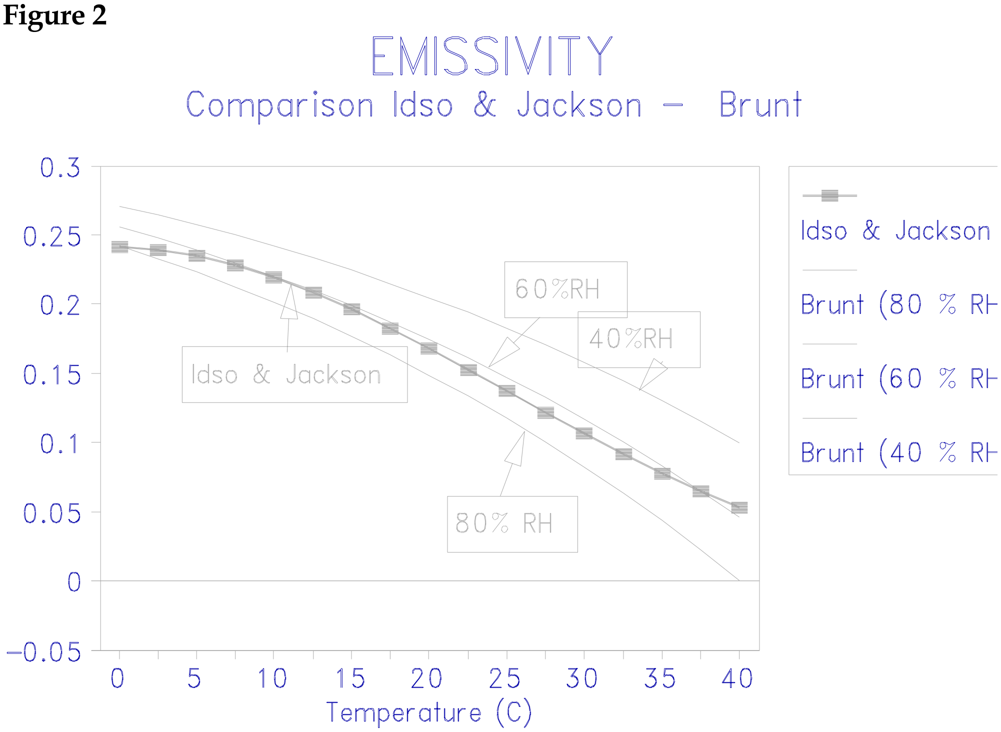
$$\varepsilon' = 0.261 \exp(-7.77 * 10^{-4} T^{2}) - 0.02 \qquad \textbf{(61)}$$

where: T : mean daily temperature [°C]

Reference : Idso and Jackson (1969)

Figure 2 shows a comparison of the Idso & Jackson (equation 61) and Brunt (equation 60) for $a_l = 0.34$ and $b_l = -0.14$ at 80%, 60% and 40% relative humidity levels.

Figure 2



Summarizing the determination of the net thermal radiation the following general relationship can be given:

$$R_{nl} = \left(a_c \frac{R_s}{R_{so}} + b_c \right) \cdot \left(a_1 + b_1 \sqrt{e_d} \right) \_ \sigma \left(T_{kx}^4 + T_{kn}^4 \right) \frac{1}{2} \qquad \textbf{(62)}$$

where: R_{nl} : net longwave radiation [$MJm^{-2}\ d^{-1}$]
σ : Stefan-Boltzmann constant [= 4.90 . 10^{-9} [$MJm^{-2}\ K^{-4}\ d^{-1}$]
T_{kx} : maximum day temperature [K]
T_{kn} : minimum day temperature [K]

For general purposes when only sunshine and humidity data are available, net thermal radiation can be estimated by the following equation :

$$R_{nl} = 2.45\ 10^{-9} \left(0.9 \frac{n}{N} + 0.1 \right) \left(0.34 - 0.14 \sqrt{e_d} \right) \left(T_{kx}^4 + T_{kn}^4 \right) \qquad \textbf{(63)}$$

3.2 SOIL HEAT FLUX (G)

Heat is stored in and released from the soil. To estimate the soil heat flux for a given period the following equation may be used:

$$G = c_s\, d_s \left(\frac{T_n - T_{n-1}}{\Delta t} \right) \qquad \textbf{(64)}$$

where: G : soil heat flux [$MJ\ m^{-2}\ d^{-1}$]
T_n : temperature [°C] on day (or month) n
T_{n-1} : temperature [°C]in preceding day (or month) n-1
Δt : length period n [d]
c_s : volumetric heat capacity [$MJ\ m^{-3}\ °C^{-1}$] ≈ 2.1 [$MJ\ m^{-3}\ °C^{-1}$] for average moist soil
d_s : estimated effective soil depth [m]

Reference : v. Wijk and de Vries (1963)

For **DAILY** temperature fluctuations (effective soil depth 0.18 m) the formula would become:

$$G = 0.38 \left(T_{day\,n} - T_{day\,n-1} \right) \qquad \textbf{(65)}$$

Reference : Wright and Jensen (1972)

For **MONTHLY** temperature (effective soil depth 2.0 m) fluctuations this would be :

$$G = 0.07 \left(T_{month\,n+1} - T_{month\_n-1} \right) \qquad \textbf{(66)}$$

or if an estimation for the temperature n+1 is not available for the next month

$$G = 0.14 \left(T_{month\,n} - T_{month\,n-1} \right) \qquad \textbf{(67)}$$

Reference : Jensen et al. 1990

Since the magnitude of daily soil heat flux over 10 - 30 day periods is relatively small, it normally can be neglected and thus

$$G = 0 \tag{68}$$

V.III RECOMMENDED COMBINATION FORMULA FOR REFERENCE EVAPOTRANSPIRATION (ET_o)

Defining reference evapotranspiration (ET_o) as the **rate of evapotranspiration from a hypothetic crop with an assumed crop height of 12 cm, a fixed canopy resistance of 70 sm^{-1} and an albedo of 0.23, closely resembling the evapotranspiration from an extensive surface of green grass of uniform height, actively growing, completely shading the ground and not short of water**, the estimation of the ET_o can be determined with the combination formula based on the Penman-Monteith approach. When combining the derivations found for the aerodynamic and radiation terms as presented above, the combination formula can be noted as:

$$ET_o = \frac{0.408\,\Delta(R_n - G) + \gamma \frac{900}{T+273} U_2 (e_a - e_d)}{\Delta + \gamma(1 + 0.34 U_2)} \tag{69}$$

where:

ET_o	:	reference crop evapotranspiration [mm d^{-1}]
R_n	:	net radiation at crop surface [MJ m^{-2} d^{-1}]
G	:	soil heat flux [MJ m^{-2} d^{-1}]
T	:	average temperature [°C]
U_2	:	windspeed measured at 2m height [m s^{-1}]
(e_a-e_d)	:	vapour pressure deficit [kPa]: equation (17)
Δ	:	slope vapour pressure curve [kPa °C^{-1}]: see equation (3)
γ	:	psychrometric constant [kPa °C^{-1}]: equation (4)
900	:	conversion factor

When no measured radiation data are available, the net radiation can be estimated as follows:

$$R_n = R_{ns} - R_{nl} \tag{50}$$

$$R_{ns} = 0.77(0.25 + 0.50\frac{n}{N})\, R_a \tag{55}$$

$$R_{nl} = 2.45 \,.\, 10^{-9} (0.9\frac{n}{N} + 0.1)(0.34 - 0.14\sqrt{ed}\,)(T^4_{kx} + T^4_{kn}) \tag{63}$$

$$G = 0.14\,(T_{month\,n} - T_{month\,n-1}) \approx 0 \tag{68}$$

where:

R_n	:	net radiation [MJ m^{-2} d^{-1}]
R_{ns}	:	net shortwave radiation [MJ m^{-2} d^{-1}]
R_{nl}	:	net longwave radiation [MJ m^{-2} d^{-1}]
R_a	:	extraterrestrial radiation [MJ m^{-2} d^{-1}]. See Equation 19.
n/N	:	relative sunshine fraction [ ]

T_{kx}	:	maximum temperature [K]
T_{kn}	:	minimum temperature [K]
e_d	:	actual vapour pressure [kPa]
G	:	soil heat flux [MJ m^{-2} d^{-1}] :see Equation 67

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