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# Cross Comparison of Empirical Equations for Calculating Potential Evapotranspiration with Data from Switzerland

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**Abstract.** Earlier studies (Singh and Xu, 1997; Xu and Singh, 2000, 2001) have evaluated and compared various popular empirical evapotranspiration equations that belonged to three categories: (1) mass-transfer based methods, (2) radiation based methods, and (3) temperature-based methods; and the best and worst equations of each category were determined for the study regions. In this study a cross comparison of the best or representative equation forms selected from each category was made. Five representative empirical potential evapotranspiration equations selected from the three categories, namely: Hargreaves and Blaney-Criddle (temperature-based), Makkink and Priestley-Taylor (radiation-based) and Rohwer (mass-transfer-based) were evaluated and compared with the Penman-Monteith equation using daily meteorological data from the Changins station in Switzerland. The calculations of the Penman-Monteith equation followed the procedure recommended by FAO (Allen *et al.*, 1998). The comparison was first made using the original constant values involved in each empirical equation and then made using the recalibrated constant values. The study showed that: (1) the original constant values involved in each empirical equation worked quite well for the study region, except that the value of $\alpha$ = 1.26 in Priestley-Taylor was found to be too high and the recalibration gave a value of $\alpha$ = 0.90 for the region. (2) Improvement was achieved for the Blaney-Criddle method by adding a transition period in determining the parameter $k$. (3) The differences of performance between the best equation forms selected from each category are smaller than the differences between different equations within each category as reported in earlier studies (Xu and Singh, 2000, 2001). Further examination of the performance resulted in the following rank of accuracy as compared with the Penman-Monteith estimates: Priestley-Taylor and Makkink (Radiation-based), Hargreaves and Blaney-Criddle (temperature-based) and Rohwer (Mass-transfer).

**Key words:** mass-transfer-based, potential evapotranspiration, radiation-based, Switzerland, temperature-based

## 1. Introduction

There exist a multitude of methods for the estimation of potential evapotranspiration $ET$ and free water evaporation E, which can be grouped into five categories: (1) water budget (e.g. Guitjens, 1982), (2) mass-transfer (e.g. Harbeck, 1962), (3) combination (e.g. Penman, 1948), (4) radiation (e.g. Priestley and Taylor, 1972), and (5) temperature-based (e.g. Thornthwaite, 1948; Blaney-Criddle, 1950). The availability of many equations for determining evaporation, the wide range of data types needed, and the wide range of expertise needed to use the various equations correctly make it difficult to select the most appropriate evaporation method for a given study.

An ongoing research programme has been underway since 1996, with the main objective of undertaking evaluation and generalisation of existing evaporation models. The research programme differs from other researches reported in the literature. At the first stage of the study, the most commonly used methods for estimating $E$ and $ET$ were evaluated and compared within each category and the best and good methods are ranked for every category. At the second stage of the research only the best models from each category are selected and a cross comparison is made. The results of the first stage study have been reported in Singh and Xu (1997), and Xu and Singh (2000, 2001) where evapotranspiration equations belonging to the categories of mass-transfer based, radiation-based and temperature-based, respectively, were evaluated and generalized. This paper reports some of the results of the second stage study, i.e., select one or two best equation forms from each category and do a cross comparison. Following the recommendation of FAO (see Allen *et al.*, 1994a, b, 1998), the Penman-Monteith equation was used as a comparison criterion for the selected empirical equations. Included in the study is a discussion of existing methods, evaluation and comparison of the selected models with the original values of the constants involved in each equation, and with locally calibrated values of the constants. Finally, the overall applicability of the selected methods is examined and their predictive ability for the study region is discussed.

## 2. Methods Description

### 2.1. PENMAN-MONTAITH METHOD

The FAO Penman-Monteith method for calculating reference (potential) evapotranspiration $ET$ can be expressed as (Allen *et al.*, 1998):

$$ET = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T_a+273} u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)}\,, \tag{1}$$

where

| | | |
|---|---|---|
| $ET$ | = | reference evapotranspiration (mm day$^{-1}$); |
| $R_n$ | = | net radiation at the crop surface (MJ m$^{-2}$ day$^{-1}$); |
| $G$ | = | soil heat flux density (MJ m$^{-2}$ day$^{-1}$); |
| T | = | mean daily air temperature at 2 m height (°C); |
| $u_2$ | = | wind speed at 2 m height (m s$^{-1}$); |
| $e_s$ | = | saturation vapour pressure (kPa); |
| $e_a$ | = | actual vapour pressure (kPa); |
| $e_s - e_a$ | = | saturation vapour pressure deficit (kPa); |
| $\Delta$ | = | slope vapour pressure curve (kPa °C$^{-1}$); |
| $\gamma$ | = | psychrometric constant (kPa °C$^{-1}$). |

Apart from the site location, the FAO Penman-Monteith equation requires air temperature, humidity, radiation and wind speed data for daily, weekly, ten-day or monthly calculations. The computation of all data required for the calculation of the reference evapotranspiration followed the method and procedure given in Chapter 3 of the FAO paper 56 (Allen *et al*., 1998). For the sake of completeness, some important equations are briefly summarized in what follows. It is important to verify the units in which the weather data are reported.

**Latent Heat of Vaporization (λ)**

$$\lambda = 2.501 - (2.361 \times 10^{-3}) T_a \ , \tag{2}$$

where

| | | |
|---|---|---|
| $\lambda$ | = | latent heat of vaporization (MJ kg$^{-1}$); |
| $T_a$ | = | air temperature (°C). |

**Atmospheric Pressure (*P*)**

$$P = 101.3 \left( \frac{293 - 0.0065z}{293} \right)^{5.26} \ , \tag{3}$$

where

$P$ = atmospheric pressure (kPa) at elevation $z$ (m).

**Saturation Vapour Pressure ($e_s$)**

$$e_s(T_a) = 0.611 \exp \left( \frac{17.27 T_a}{T_a + 237.3} \right) \ , \tag{4}$$

where

$e_a(T_d)$ = actual vapour pressure function (kPa);
$T_a$ = air temperature (°C).

**Actual Vapour Pressure ($e_a$)**

$$e_a(T_d) = 0.611 \exp\left(\frac{17.27 T_d}{T_d + 237.3}\right) , \tag{5}$$

where

$e_a(T_d)$ = actual vapour pressure function (kPa);
$T_d$ = dew point temperature (°C).

**Slope Vapour Pressure Curve ($\Delta$)**

$$\Delta = \frac{4098 e_s(T_a)}{(T_a + 237.3)^2} = \frac{2504 \exp\left(\frac{17.27 T_a}{T_a + 237.3}\right)}{(T_a + 237.3)^2} , \tag{6}$$

where

$\Delta$ = slope vapour pressure curve (kPa $C^{-1}$);
$T_a$ = air temperature (°C).

**Psychrometric Constant ($\gamma$)**

$$\gamma = \frac{C_p P}{\varepsilon \lambda} \times 10^{-3} = 0.00163 \frac{P}{\lambda} , \tag{7}$$

where

$\gamma$ = psychrometric constant (kPa $C^{-1}$);
$c_p$ = specific heat of moist air = 1.013 (kJ $kg^{-1}$ °$C^{-1}$);
$P$ = atmospheric pressure (kPa);
$\varepsilon$ = ratio molecular weight of water vapour/dry air = 0.622;
$\lambda$ = latent heat of vaporization (MJ $kg^{-1}$).

**Short Wave Radiation on a Clear-Sky Day ($R_{so}$)**

The calculation of $R_{so}$ is required for computing net long wave radiation. A good approximation for $R_{so}$ according to FAO (Allen *et al.*, 1998), for daily and hourly periods is:

$$R_{so} = (0.75 + 2 \times 10^{-5} z) R_a , \tag{8}$$

where

$R_{so}$ = short wave radiation on a clear-sky day (MJ $m^{-2}$ $d^{-1}$);
$z$ = station elevation (m);
$R_a$ = extraterrestrial radiation (MJ $m^{-2}$ $d^{-1}$).

**Extraterrestrial Radiation for Daily Periods ($R_a$)**

The extraterrestrial radiation, $R_a$, for each day of the year and for different latitudes is estimated from the solar constant, the solar declination and the time of the year by:

$$R_a = \frac{24(60)}{\pi} G_{sc} d_r [\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)] , \tag{9}$$

where

$R_a$ = extraterrestrial radiation (MJ $m^{-2}$ $day^{-1}$);
$G_{sc}$ = solar constant = 0.0820 MJ $m^{-2}$ $min^{-1}$;
$d_r$ = inverse relative distance Earth–Sun;
$\omega_s$ = sunset hour angle;
$\varphi$ = latitude (rad);
$\delta$ = solar decimation.

The equations for calculating $d_r$, $\omega_s$, $\varphi$ and $\delta$ are given in Chapter 3 of FAO paper 56 (Allen *et al.*, 1998).

**Net Solar or Net Shortwave Radiation ($R_{ns}$)**

The net shortwave radiation resulting from the balance between incoming and reflected solar radiation is given by:

$$R_{ns} = (1 - \alpha) R_s , \tag{10}$$

where

$R_{ns}$ = net solar or shortwave radiation (MJ $m^{-2}$ $day^{-1}$);
$\alpha$ = albedo or canopy reflection coefficient, which is 0.23 for the hypothetical grass reference crop (dimensionless);
$R_s$ = the incoming solar radiation (MJ $m^{-2}$ $day^{-1}$).

**Net Longwave Radiation ($R_{nl}$)**

The net outgoing longwave radiation is calculated by

$$R_{nl} = \sigma \left[ \frac{T^4_{\min,K} + T^4_{\min,K}}{2} \right] \left(0.34 - 0.14\sqrt{e_a}\right) \left(1.35 \frac{R_s}{R_{so}} - 0.35\right) , \qquad (11)$$

where

| | | |
|---|---|---|
| $R_{nl}$ | = | net outgoing longwave radiation (MJ $m^{-2}$ $day^{-1}$); |
| $\sigma$ | = | Stefan-Boltzmann constant ($4.903 \times 10^{-9}$ MJ $K^{-4}$ $m^{-2}$ $day^{-1}$); |
| $T_{\max,K}$ | = | maximum absolute temperature during the 24 hr period (K = °C + 273.16); |
| $T_{\min,K}$ | = | minimum absolute temperature during the 24 hr period (K = °C + 273.16); |
| $e_a$ | = | actual vapour pressure (kPa); |
| $R_s/R_{so}$ | = | relative shortwave radiation (limited to $\leq 1.0$); |
| $R_s$ | = | measured solar radiation (MJ $m^{-2}$ $day^{-1}$); |
| $R_{so}$ | = | calculated (Equation (8)) clear-sky radiation (MJ $m^{-2}$ $day^{-1}$). |

**Net Radiation ($R_n$)**

The net radiation ($R_n$) is the difference between the incoming net shortwave radiation ($R_{ns}$) and the outgoing net longwave radiation ($R_{nl}$):

$$R_n = R_{ns} - R_{nl} \qquad (12)$$

**Soil Heat Flux (G)**

For vegetation covered surfaces and calculation time steps are 24 hr or longer, a calculation procedure is proposed by FAO (Allen *et al*., 1998), based on the idea that the soil temperature follows air temperature is as follows,

$$G = c_s \frac{T_i - T_{i-1}}{\Delta t} \Delta z \qquad (13)$$

where

| | | |
|---|---|---|
| $G$ | = | soil heat flux (MJ $m^{-2}$ $day^{-1}$); |
| $c_s$ | = | soil heat capacity (MJ $m^{-3}$ $°C^{-1}$); |
| $T_i$ | = | air temperature at time $i$ (°C); |
| $T_{i-1}$ | = | air temperature at time $i - 1$ (°C); |
| $\Delta t$ | = | length of time interval (day); |
| $\Delta z$ | = | effective soil depth (m), which for a time interval of one or few days is about 0.10–0.20 m. |

Different equations are proposed by Allen *et al*., (1998) in calculating $G$ depending on the computation time periods.

### 2.2. TEMPERATURE-BASED METHODS

Those potential evapotranspiration ($ET$) estimation methods that require only temperature as an input variable are considered as temperature-based methods in this study. The temperature-based methods are some of the earliest methods for estimating $ET$. The relation of $ET$ to air temperature dates back to 1920s (Jensen *et al*., 1990). Most temperature-based equations take the form:

$$ET = c(T_a)^n \tag{14}$$

or

$$ET = c_1 d_l T_a (c_2 - c_3 h) \tag{15}$$

in which

$ET$ = potential evapotranspiration;

$T_a$ = air temperature;

$h$ = a humidity term;

$c_1$, $c_2$, $c_3$ and $c$ are constants;

$d_l$ = day-length.

Due to the wide-ranging inconsistency in meteorological data collection procedures and standards, many different evaporation equations have been used by different authors. Performance of the empirical equations usually varies from locations. In the comparative study of Jensen *et al*. (1990), it was concluded that at humid locations the FAO-24 Blaney-Criddle and Hargreaves methods ‘closely paralleled lysimeter $E_t$’. In a recent study, Xu and Singh (2001) evaluated and compared seven popular temperature-based potential evapotranspiration equations each representing a typical form, namely: Thornthwaite (1948), Linacre (1977), Blaney-Criddle (1950), Hargreaves and Samani (1985), Kharrufa (1985), Hamon (1961), and Romanenko (1961) methods. Meteorological data from two stations (Rawson Lake and Atikokan) in northwestern Ontario, Canada, were used in the study. It was concluded that with locally determined constant values, the Blaney-Criddle and Hargreaves methods gave better results than others, which consists with Jensen’s results. Therefore, these two methods were used in this study to represent the temperature-based methods. For the sake of completeness, these equations are briefly summarized in what follows. For a more complete discussion, the reader is referred to the cited references.

### 2.2.1. *Blaney-Criddle Method*

The Blaney-Criddle (1950) procedure for estimating $ET$ is well known in the western U.S.A. and has been used extensively elsewhere also (Singh, 1989). The usual form of the Blaney-Criddle equation converted to metric units is written as:

$$ET = kp(0.46T_a + 8.13), \tag{16}$$

where

| | | |
|---|---|---|
| $ET$ | = | potential evapotranspiration from a reference crop, in mm, for the period in which $p$ is expressed; |
| $T_a$ | = | mean temperature in °C; |
| $p$ | = | percentage of total daytime hours for the used period (daily or monthly) out of total daytime hours of the year (365 $\times$ 12); |
| $k$ | = | monthly consumptive use coefficient, depending on vegetation type, location and season and for the growing season (May to October), $k$ varies from 0.5 for orange tree to 1.2 for dense natural vegetation. |

Following the recommendation of Blaney and Criddle (1950), in the first stage of the comparative study, values of 0.85 and 0.45 were used for the growing season (April to September) and the non-growing season (October to March), respectively.

### 2.2.2. *Hargreaves Method*

Hargreaves and Samani (1982, 1985) proposed several improvements to the Hargreaves (1975) equation for estimating grass-related reference $ET$ (mm d$^{-1}$); one of them has the form:

$$ET = aR_a TD^{1/2}(T_a + 17.8)\ , \tag{17}$$

where

| | | |
|---|---|---|
| $a$ | = | 0.0023 is a parameter; |
| $TD$ | = | the difference between maximum and minimum daily temperature in °C; |
| $R_a$ | = | the extraterrestrial radiation expressed in equivalent evaporation units. For a given latitude and day $R_a$ is obtained from tables or may be calculated using Equation (9). The only variables for a given location and time period is the daily mean, max and min air temperature. Therefore, the Hargreaves method has become a temperature-based method. |

### 2.3. RADIATION-BASED METHODS

Empirical radiation-based equations for estimating potential evaporation generally are based on the energy balance (Jensen *et al.*, 1990). Most radiation-based equations take the form:

$$\lambda ET = C_r(wR_s) \qquad \text{or} \qquad \lambda ET = C_r(wR_n) \tag{18}$$

where

| | | |
|---|---|---|
| $\lambda$ | = | the latent heat of vaporisation; |
| $ET$ | = | the potential evapotranspiration; |
| $R_s$ | = | the total solar radiation; |
| $R_n$ | = | the net radiation; |
| $w$ | = | the temperature and altitude-dependent weighting factor; |
| $C_r$ | = | a coefficient depending on the relative humidity and wind speed. |

In the study of Jensen *et al*. (1990) a number of radiation-based equations together with other methods were evaluated. The study showed that the results vary from locations and at humid climate the Turc, Priestley and Taylor and FAO-24 radiation (modified Makkink) methods provided good estimates. Recently, a more complete comparison of radiation-based methods was performed by Xu and Singh (2000) using meteorological data from the Changins station in Switzerland. Eight popular radiation-based equations were evaluated and compared, namely: Turc (1961), Makkink (1957), Jensen and Haise (1963), Hargreaves (1975), Doorenbos and Pruitt (1977), McGuinness and Bordne (1972), Abtew (1996), and Priestley and Taylor (1972). The study concluded that with properly determined constant values, the Makkink and Priestley-Taylor equations provided better results in the study region. These two models are selected in this study to represent radiation-based methods and are discussed as follows.

#### 2.3.1. *Makkink Method*

For estimating potential evapotranspiration (mm $d^{-1}$) from grass Makkink (1957) proposed the equation

$$ET = 0.61 \frac{\Delta}{\Delta + \gamma} \frac{R_s}{\lambda} - 0.12 \tag{19}$$

where

| | | |
|---|---|---|
| $R_s$ | = | the total solar radiation in cal $cm^{-2}$ $day^{-1}$; |
| $\Delta$ | = | the slope of saturation vapour pressure curve (in mb/°C); |
| $\gamma$ | = | the psychrometric constant (in mb/°C); |
| $\lambda$ | = | latent heat (in calories per gram); |
| $P$ | = | atmospheric pressure (in millibar). |

These quantities are calculated using FAO-98 recommended procedure as described in Section 2.1 and units have to be converted to what are required in Equation (19).

On the basis of later investigation in the Netherlands and at Tåstrup, Hansen (1984) proposed the following form of the Makkink equation

$$ET = 0.7\frac{\Delta}{\Delta + \gamma}\frac{R_s}{\lambda} , \tag{20}$$

where all the notations have the same meaning and units as in Equation (19). This equation will be used in this study since the data used in determining the constant value are closer to the study region.

### 2.3.2. *Priestley and Taylor Method*

Priestley and Taylor (1972) proposed a simplified version of the combination equation (Penman, 1948) for use when surface areas generally were wet, which is a condition required for potential evapotranspiration, $ET$. The aerodynamic component was deleted and the energy component was multiplied by a coefficient, $\alpha = 1.26$, when the general surrounding areas were wet or under humid conditions.

$$ET = \alpha\frac{\Delta}{\Delta + \gamma}\frac{R_n}{\lambda} , \tag{21}$$

where $ET$ is in mm d$^{-1}$, $R_n$ is the net radiation in cal cm$^{-2}$ day$^{-1}$, which is calculated using Equation (12), other notations have the same meaning and units as in Equation (19).

## 2.4. MASS-TRANSFER BASED METHODS

The mass-transfer method is one of the oldest methods (Dalton, 1802; Meyer, 1915; Penman, 1948) and is still an attractive method in estimating free water surface evaporation $E$ because of its simplicity and reasonable accuracy. The mass-transfer methods are based on the Dalton equation which for free water surface can be written as:

$$E = C(e_s - e_a) , \tag{22}$$

where

$E$ = free water-surface evaporation;
$e_s$ = the saturation vapor pressure at the temperature of the water surface;
$e_a$ = the actual vapor pressure in the air;
$C$ = an empirically determined constant involving some function of windiness.

Many variations of the Daltonian equation have been proposed (e.g., Singh and Xu, 1997). Examples include Rohwer (1931).

$$E = 0.44(1 + 0.27u)(e_s - e_a) \, , \tag{23}$$

where

$E$ = in mm $d^{-1}$;
$u_2$ = the wind speed in m $s^{-1}$;
$e$ and $e_a$ are in mmHg.

Using experiment data from England, Penman (1948) concluded the best form of Equation (22) for practical use is

$$E = 0.35(1 + 0.98/100U_2)(e_s - e_a) \, , \tag{24}$$

where

$E$ = in mm $d^{-1}$;
$U_2$ = wind speed at 2 m high in miles $day^{-1}$;
$e_s$ and $e_a$ are in mmHg.

Harbeck (1962) developed a slightly different equation for estimating evaporation from reservoirs:

$$E = NU_2(e_s - e_a) \, , \tag{25}$$

where

$N$ = a coefficient related to the reservoir surface area.

The mass transport method offers the advantage of simplicity in calculation, once the empirical constants have been developed. Improvements in the empirical constants, such as those of Brutsaert and Yu (1968), will continue to make the method attractive for estimating $E$ from lakes or reservoirs. This method has also been used to estimate evaporation from bare soils (Conaway and Van Bavel, 1967; Ripple *et al.*, 1970; see also Rosenberg *et al.*, 1983) and potential evapotranspiration from vegetated surfaces (Pruitt and Aston, 1963; Blad and Rosenberg, 1976). Recently, Singh and Xu (1997) evaluated and compared 13 popular mass-transfer based evaporation equations. Unlike the studies of radiation and temperature-based methods, all 13 mass-transfer based equations gave almost equally good results, provided that the constant values were locally calibrated. In this study, Equation (23) was selected to represent the form of mass-transfer based equations.

*Table I.* Monthly averages of the main climatic parameters for station Changins in Switzerland (1990–1994)

| Month | Temperature (°C) | Wind speed ($m\ s^{-1}$) | Vapor pressure deficit (mb) | Humidity (%) | Radiation ($cal\ cm^{-2}\ d^{-1}$) |
|---|---|---|---|---|---|
| January | 2.19 | 2.44 | 1.52 | 80.50 | 88.16 |
| February | 2.77 | 2.45 | 1.79 | 78.25 | 159.33 |
| March | 7.47 | 2.61 | 3.50 | 69.22 | 266.14 |
| April | 8.98 | 2.71 | 4.29 | 67.76 | 346.34 |
| May | 14.30 | 2.60 | 5.89 | 68.19 | 457.92 |
| June | 16.70 | 2.54 | 6.37 | 70.74 | 462.61 |
| July | 20.14 | 2.42 | 9.08 | 66.08 | 513.29 |
| August | 20.53 | 2.40 | 9.90 | 63.84 | 445.13 |
| September | 15.28 | 2.18 | 4.91 | 75.05 | 295.72 |
| October | 10.03 | 2.35 | 2.45 | 81.16 | 169.04 |
| November | 6.04 | 2.21 | 1.69 | 82.65 | 84.70 |
| December | 2.75 | 3.08 | 1.52 | 80.37 | 71.98 |

## 3. Study Region and Data

The Changins climatological station in the state of Vaud in Switzerland was used in this study. This station is located at a latitude of 46°24′N and a longitude of 06°14′E. Several hydrometeorological variables, including air temperature, grass temperature, soil temperature at –5 cm, wind speed, relative humidity, solar radiation and vapor pressure among others, have been continuously recorded for the period 1990 to 1994. The monthly averages of the main climatic parameters are summarised in Table I.

## 4. Results and Discussions

### 4.1. COMPARISON OF THE METHODS WITH THEIR ORIGINAL CONSTANT VALUES

In the first stage of the comparative study, daily evapotranspiration from Penman-Monteith method (Equation (1)) and other five empirical methods, i.e., Equations (16), (17), (20), (21) and (23), respectively, was computed with their original constant values involved in each equation.

Monthly evapotranspiration values computed from five empirical methods were first compared with the Penman-Monteith values (Figure 1). A visual comparison shows that the value of $\alpha = 1.26$ in Priestley-Taylor Equation (23) seemed too high for the region, while other four empirical equations worked quite well with

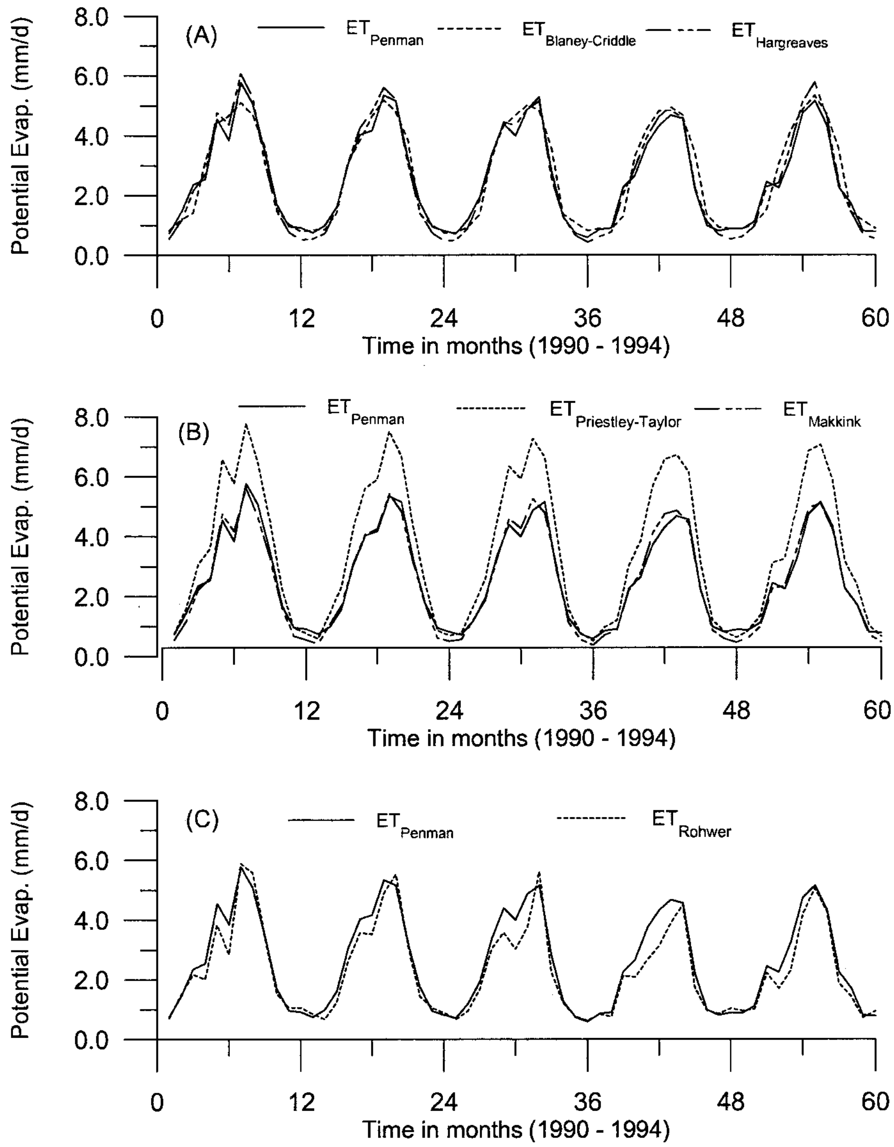


*Figure 1.* Plot of monthly potential evapotranspiration computed by the Penman-Monteith and five empirical methods with the original constant values involved in each equation.

original values of the constants. In order to have a quantitative evaluation, the correlations between the five empirical methods and the Penman-Monteith estimates were analyzed using a linear regression equation:

$$Y = mX + c \tag{26}$$

where $Y$ represents $ET$ computed by Penman-Monteith Equation (1) and $X$ is the $ET$ estimated from the above-mentioned five methods, and $m$ and $c$ are constants representing the slope and intercept of the regression equation, respectively. The resulted regression equations together with the cross-correlation ($R^2$) are presented in Figure 2. When the determination coefficient $R^2$ values are concerned, Priestley-Taylor, Makkink and Hargreaves have the highest value with $R^2 = 0.99$ and Blaney-Criddle has the lowest value with $R^2 = 0.92$. Concerning the slope and intercept of the regression equations, Blaney-Criddle method resulted in a slope close to unity and an intercept close to zero. As has been noticed in Figure 1, again Figure 2 shows that the Priestley-Taylor estimate is the worst when the regression equation's slope and intercept are concerned; this is because the constant value of $\alpha = 1.26$ is too high for the station.

In order to check seasonality of the estimation errors, mean monthly $ET$ values averaged over five years (1990–1994) from five empirical methods are computed and compared with that of Penman-Monteith estimates (Figure 3). It can be seen that: (1) Hargreaves and Makkink methods followed the same trend as that of Penman-Monteith method; (2) The Rohwer (mass-transfer) method underestimated evapotranspiration in April, May and June as well as the yearly value; (3) Blaney-Criddle estimates showed difference with that of Penman-Monteith method in three months, i.e., it overestimates evapotranspiration in April and September and underestimates in March. The reason is that there are two values of consumptive coefficient $k$ used in the calculation, i.e., $k = 0.85$ for growing season of April to September, and 0.45 for the rest months. Figure 3 reveals that it is necessary to define March, April and September as a transition period having a value of $k$ lies between 0.45 and 0.85. This proposal will be tested in the second stage of model evaluation when calibration of constant values is made. Using different $k$ values for every month will also improve the results, but it will result in too many free parameters as compared with other methods. (4) The value of $\alpha = 1.26$ in Priestley-Taylor is too high.

### 4.2. RECALIBRATION OF THE CONSTANT VALUES

The results presented in the previous section have shown that it is necessary to recalibrate the constants involved in some equations. In order to see if further improvement can be obtained the parameter values involved in all five empirical equations were recalibrated against the Penman-Monteith method using an 'automatic optimization' method as presented in Singh and Xu (1997). The optimality criterion adopted was the least square error. Let $E_{t,Pen}$ be the evapotranspiration computed by the Penman-Monteith, and $E_{t,comp}$ the computed evapotranspiration by five other methods which is a function of model parameters. The objective function, OF, to be minimized can be expressed as

$$OF = \sum (E_{t,Pen} - E_{t,comp})^2 = \text{minimum SSQ}\,, \tag{27}$$

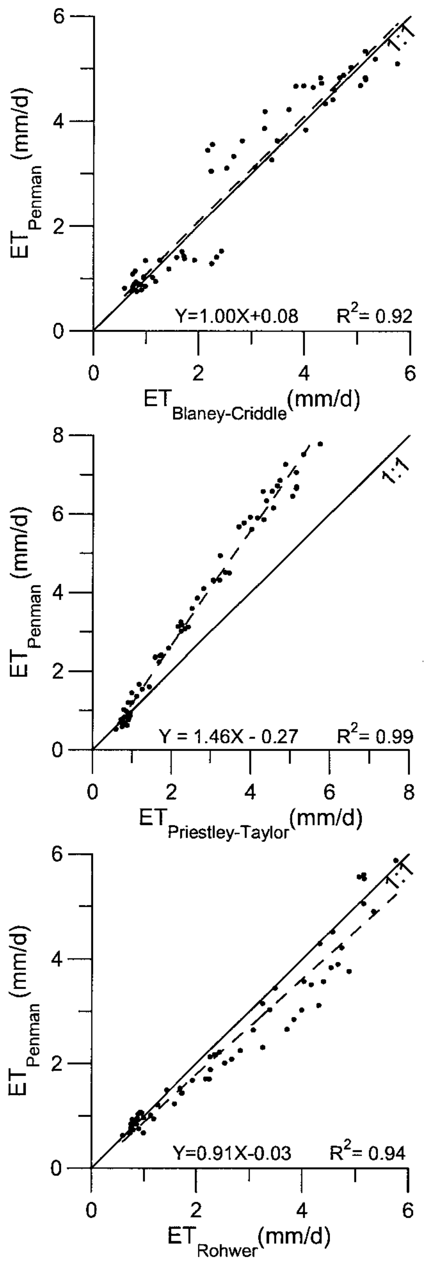


*Figure 2.* Penman-Monteith method estimated monthly potential evapotranspiration versus five empirical methods estimated values using the original constant values involved in each equation.

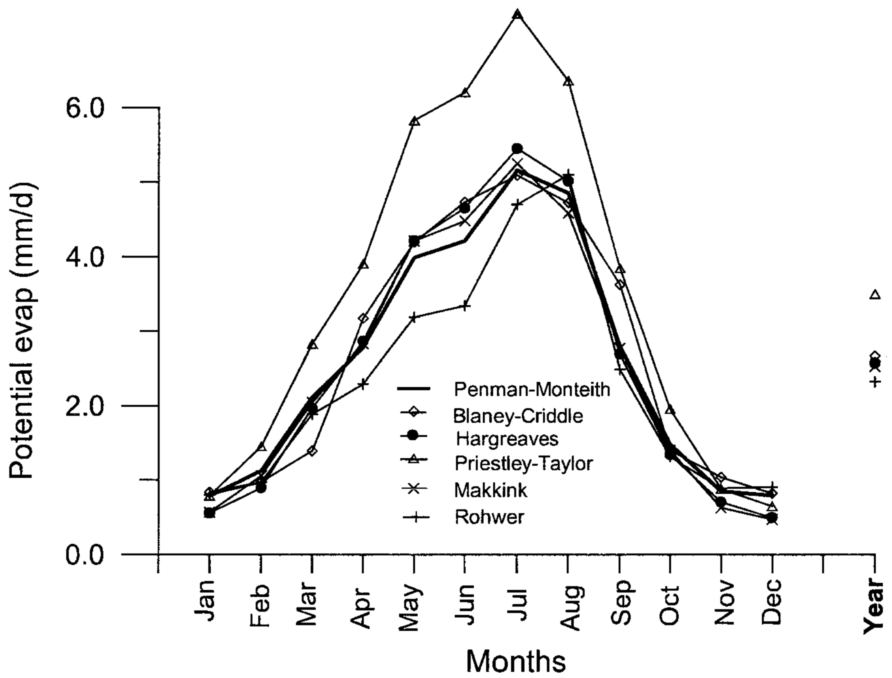


*Figure 3.* Comparison of mean monthly potential evapotranspiration computed by the Penman-Monteith and five empirical methods with the original constant values involved in each equation.

where summation is over the number of observations. Optimization of model parameters is the minimization of the objective function (Equation (27)).

As can be expected, the first improvement that could be made by the recalibration was to reduce the constant value of $\alpha = 1.26$ in the Priestley-Taylor equation to a new optimised value of 0.90. This value is consistent with the results of other studies taken in high latitude humid regions (e.g., Lafleur *et al.*, 1997; Kellner, 2001). Another parameter which could be improved slightly after the recalibration is parameter $a$ in the Rohwer equation, where the original value of 0.44 was changed to 0.47. No significant improvement could be found for the Hargreaves and Makkink equations and the original parameter values were retained for the study region. For the Blaney-Criddle equation, if the two periods were considered, i.e. a growing season from April to September and a non-growing season from October to March no further improvement could be obtained by recalibration, the value of $k$ = 0.85 and 0.45 for the two respective seasons could be used for the region. However, as discussed in the previous section, it is necessary to define March, April and September as transition periods. In this study a value of $k = (0.85 + 0.45)/2 = 0.65$ was used for these three months. For the sake of comparison, the original parameter values and the adopted values after recalibration for each empirical equation are shown in Table II.

*Table II.* Comparison of parameter values before and after recalibration

| Category | Equation form | Reference | Parameter values | |
|---|---|---|---|---|
| | | | Original | Recalibrated |
| Temperature-based | $ET = kp(0.46T_a + 8.13)$ | Blaney-Criddle (1950) | $k = 0.85$ (April to September)<br>$k = 0.45$ (October to March) | $k = 0.85$ (May to August)<br>$k = 0.45$ (October to February)<br>$k = 0.65$ (March, April, September) |
| | $ET = aR_aTD^{1/2}(T_a + 17.8)$ | Hargreaves (1982) | $a = 0.0023$ | $a = 0.0023$ |
| Radiation-based | $ET = a\frac{\Delta}{\Delta+\gamma}\frac{R_s}{\lambda}$ | Makkink (1957), Hansen (1984) | $a = 0.70$ | $a = 0.70$ |
| | $ET = \alpha\frac{\Delta}{\Delta+\gamma}\frac{R_n}{\lambda}$ | Priestley and Taylor (1972) | $\alpha = 1.26$ | $\alpha = 0.90$ |
| Mass-transfer | $E = a(1 + 0.27U_2)(e_s - e_a)$ | Rohwer (1931) | $a = 0.44$ | $a = 0.47$ |

Note: All symbols have the same meaning as mentioned in the text. Units of the variables see the text.

### 4.3. COMPARISON OF METHODS WITH RECALIBRATED CONSTANT VALUES

Potential evapotranspiration $ET$ computed from five empirical methods with the recalibrated parameter values were compared with the Penman-Monteith values (Figures 4, 5 and 6) in the same way as presented in Section 4.1. A comparison of Figures 1, 2 and 3 with Figures 4, 5 and 6 reveals that: (1) using the recalibrated constant value of $\alpha = 0.9$, a great improvement was achieved for the Priestley-Taylor method; (2) using a transition period for $k$ in the Blaney-Criddle equation improved the cross-correlation $R^2$ from 0.92 to 0.95 and the deviations of $ET$ estimates for months March and September were removed; (3) a slight improvement was obtained for the Rohwer methods using the recalibrated constant value. The results of Hargreaves and Makkink methods remain unchanged. From Figures 4, 5 and 6 it can be said that the Priestley-Taylor, Makkink, Hargreaves and Blaney-Criddle methods follow the same trend as that of Penman-Monteith method. The Rohwer (mass-transfer) method underestimates evapotranspiration in spring and earlier summer and shifts the maximum value from July to August.

## 5. Summary and Conclusions

Five empirical methods for calculating $ET$ namely Hargreaves and Blaney-Criddle (temperature-based), Makkink and Priestley-Taylor (radiation-based) and Rohwer (mass-transfer-based) were evaluated using meteorological data from Changins Station in Switzerland. The Penman-Monteith method as recommended by FAO (Allen *et al.*, 1998) was taken as a standard in evaluating the five methods. The comparison was made in two stages and in the first stage the original constant values involved in each equation were used when calculating $ET$. In the second stage the five methods were calibrated against Penman-Monteith method to determine best parameter (constants) values for the region. The results showed that the value of $\alpha = 1.26$ in Priestley-Taylor equation was too high for the study region and a value of $\alpha = 0.90$ has best fit as compared with Penman-Monteith method. A slight improvement was found for the Rohwer method when the original constant value of a = 0.44 changed to 0.47 after calibration. A further evaluation on the mean monthly evapotranspiration estimated by these six methods showed that the results of using two values for the consumptive coefficient $k$ in the Blaney-Criddle method, i.e. $k = 0.85$ for growing season of April to September, and 0.45 for the rest months, can significantly be improved by defining March, April and September as transition periods having a value of $k = (0.85 + 0.45)/2 = 0.65$.

It can be concluded from the study that using locally determined parameter values all five empirical methods gave acceptable estimates of yearly potential evapotranspiration as compared with that of Penman-Monteith estimates (Figure 6) for the region. Keep in mind that these five methods are the best (good) ones selected from each category. Further examination of the results of regression analysis between the Penman-Monteith estimates and other five methods resulted in the following rank of the performance: Priestley-Taylor and Makkink (Radiation-based),

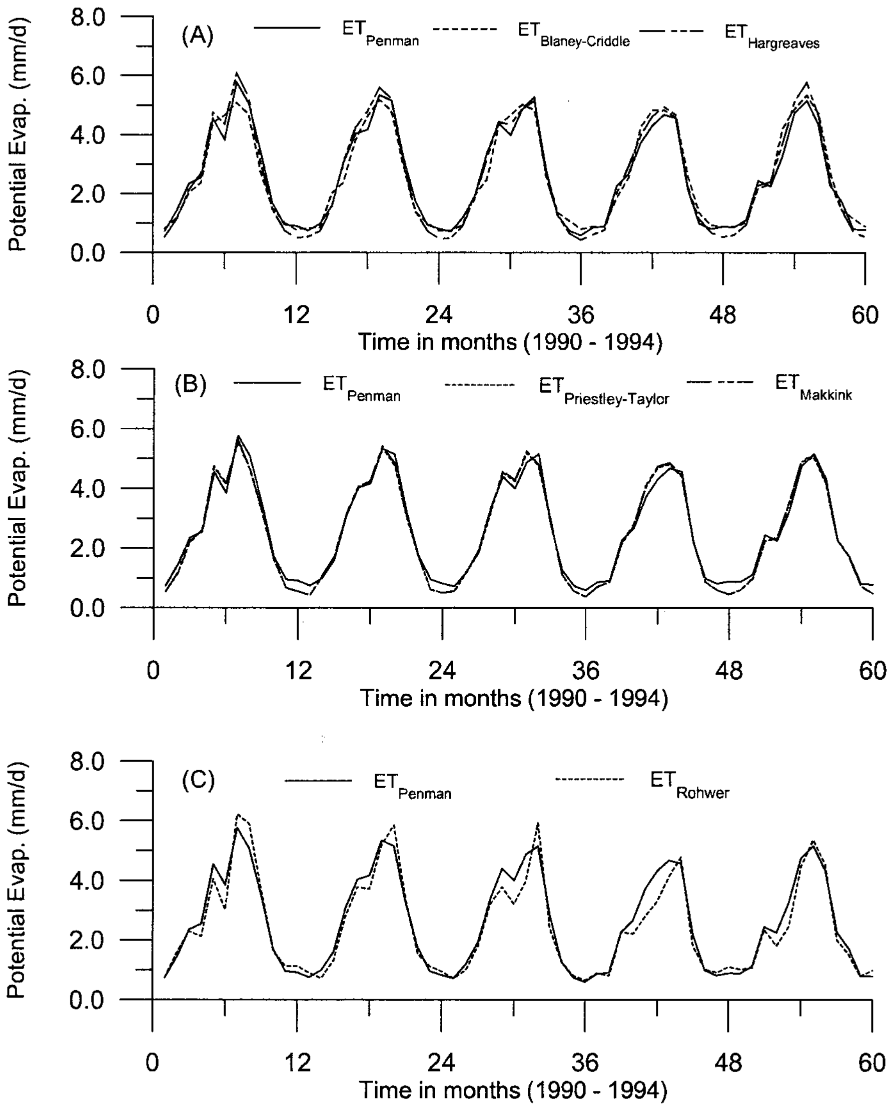


*Figure 4.* Plot of monthly potential evapotranspiration computed by the Penman-Monteith and five empirical methods with the recalibrated constant values involved in each equation.

Hargreaves and Blaney-Criddle (temperature-based) and Rohwer (Mass-transfer). It can also be concluded that the differences of performance between the best methods selected from each category are smaller than the differences between the different methods within each category as reported in earlier studies (e.g., Xu and Singh, 2000, 2001).

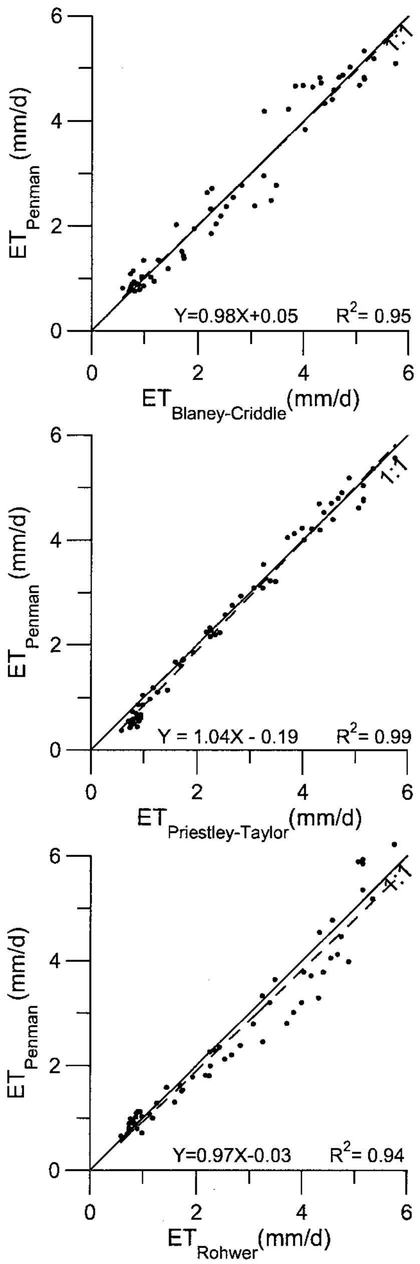


*Figure 5.* Penman-Monteith method estimated monthly potential evapotranspiration versus five empirical methods, estimated values using the recalibrated constant values involved in each equation.

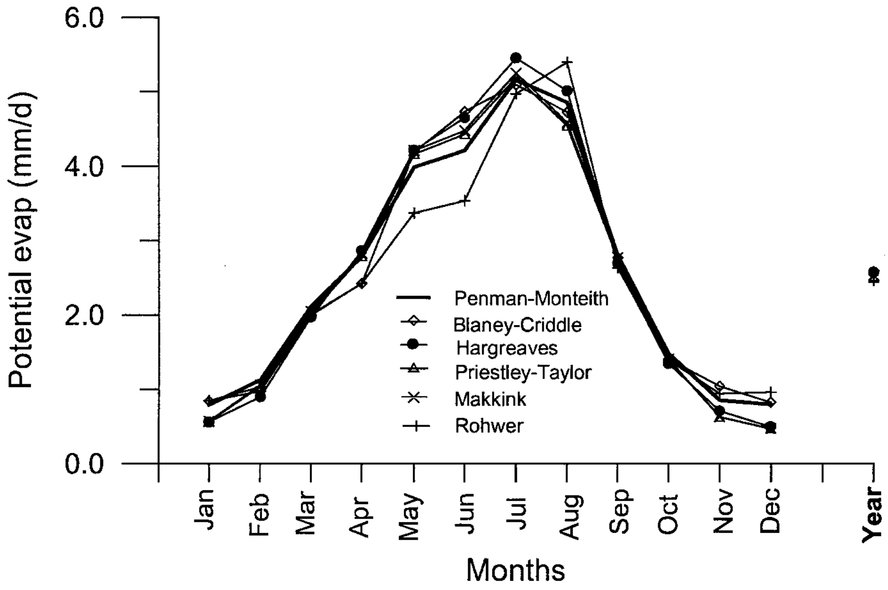


*Figure 6.* Comparison of mean monthly potential evapotranspiration computed by the Penman-Monteith and five empirical methods with the recalibrated constant values involved in each equation.

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