A Two-Stage Event-Based Demand Response Management Algorithm for Microgrids

Abstract—**Demand response has become one of the key enabling technologies for smart grids. With the increasing demand response incentives set by utilities, more customers are subscribing to the various demand response schemes. However, with growing customer participation, the problem of determining the solutions of optimal load curtailment for customers becomes computationally complex (even with hundreds of customers). This paper proposes efficient algorithms for event-based demand response management for microgrids. In these systems, it is important to optimally curtail loads as fast as possible to maintain microgrid stability, considering a combination of active and reactive power. An efficient two-stage algorithm is proposed to determine the optimal loads to be curtailed during islanded operation. The first stage relies on a greedy approach that is capable of determining a close-to-optimal load curtailment scheme rapidly to maintain microgrid stability. The second stage relies on a one-dimensional projection algorithm that can further improve the optimality solution of the first stage, when more response time is permitted. The algorithms are corroborated extensively by simulations with up to thousands of customers.**

Keywords—Algorithms, Smart Grids, Demand Response Management, Microgrids

NOMENCLATURE

$\mathcal{N}$	Set of customers
k	Index of a customer
$u_k(t)$	Valuation of the k-th customer if his power demand is not curtailed at time t
$c_k(t)$	Compensation paid to k-th customer if his power demand is curtailed at time t
$x_k(t)$	Binary decision variable if the k-th customer's power demand is retained (i.e., not curtailed) at time t
$X(t)$	Set of customers whose power demand are retained (i.e., not curtailed) at time t
$S_k(t)$	(Apparent) power demand of the k-th customer at time t, represented by a complex number
$P_k(t)$	Active power demand of the k-th customer at time t, represented by a real number
$Q_k(t)$	Reactive power demand of the k-th customer at time t, represented by a real number
$C(t)$	Total (apparent) generation power capacity at time t
θ	Maximum phase angle between any pair of power demands
$T_k^{\rm off}$	Minimum duration to maintain the k-th customer's power off, once his power is curtailed

I. INTRODUCTION

Distributed Generation (DG) is one of the key enabling technologies for smart grids. As the number of installed DGs increase in the system, microgrid implementation becomes an attractive and valuable option. Microgrids typically are medium-to-low voltage networks with integrated DG, capable of operating in grid connected or islanded mode. Designing a smart grid with the capability of operating in an islanded mode can enhance system reliability and power quality. Nevertheless, there is a high probability that a microgrid once initiated will be short of power, consequently resulting in significant voltage and frequency deviations, and leading to microgrid instability.

Demand Response (DR) programs can be broadly classified into three classes: economic demand response, emergency demand response, and ancillary services demand response. Emergency demand response [3] is utilized when there is insufficient supply of power to meet the available demand, especially for microgrids. Demand response is a key feature for smart grids and can be used to alter loads during contingency conditions. Demand response has proven to have many benefits including decrease in price variations [16], increased reliability [13], congestion management [11] and security enhancement [17]. In [9], an *event-based demand response* algorithm has been used to improve microgrid operation lifetime by modifying the load consumptions. The method relies on an optimization model that selects the best combination of remedial actions, including load curtailment and load transfer to neighboring substation in order to reduce loading during contingencies. In [6], a discrete event based simulation framework is developed to examine whether the capacity of the existing power system can meet the demand of plug-in hybrid vehicles. The power system limited generation and transmission capacities are considered to be the major constraints [6]. In [12], an emergency demand response model was developed to maximize DR benefits while satisfying the reserved capacity constrains for an interconnected power system. In [14], simultaneous implementation of the unit commitment algorithm and emergency demand response has been considered and tested on an interconnected power system.

Sudden islanding of microgrids can cause high imbalances between the local generation and demand and thus, management strategies are needed to ensure the microgrid endurance during its autonomous operation [7]. Innovative demand response strategies for microgrids can contribute to improve the microgrid stability especially during emergency conditions [7]. The emergency demand response method, proposed in [7], [15], is based on local frequency measurement to switch on/off a group of loads. However, such methods do not take into account customer utility and operator costs.

Prior studies, focusing on microgrid, only considered systems with small number of loads, and thus optimizing the operation of the microgrid within a short time frame during emergency conditions (within milliseconds) is possible. However, with growing customer participation, the problem of determining the solutions of optimal load shedding for customers becomes computationally complex (even with hundreds of customers). This paper proposes a two-stage event-based demand response algorithm for microgrids with a large number of customers. In order to assure microgrid stable operation as a result of sudden demand imbalance, the first stage utilizes a greedy algorithm, capable of obtaining a close to optimal solution, to determine the load to be curtailed in microseconds base. This is designed considering (1) the bids (representing the valuations of retaining the power demand) from customers, or (2) the compensation (stipulated as a penalty from the operators) paid to customers. The second stage relies on a one-dimensional projection algorithm that can further improve the optimality solution of the first stage, when more response time is permitted. The proposed event-based approach is tested by simulations for microgrids with up to thousands of customers.

This paper is structured as follows. Section II provides the model definitions and notations needed. Then efficient algorithms are presented in Section III. Section IV evaluates the algorithms by simulations with a large number of customers. Finally, the conclusion is provided in Section V.

II. Model

To model the decision mechanism of determining load curtailment, two major scenarios are considered in this paper. The first scenario considers customers submitting bids (i.e., valuations) to prevent their power demand from being curtailed. The objective is to maximize the total valuation of satisfiable customers. The second scenario considers paying customers compensations (or penalties) for their power interruption, which may be stipulated as a part of the usage contract. The objective is to minimize the total compensation paid to unsatisfiable customers.

The two problems involving a decision at a particular time are formulated as follows.

1) *Valuation-maximizing power allocation problem*:

$$(\textsc{maxPA}) \qquad \max_{x_k \in \{0,1\}} \sum_{k \in \mathcal{N}} u_k x_k \tag{1}$$

$$\text{subject to} \qquad \Big| \sum_{k \in \mathcal{N}} S_k x_k \Big| \le C. \tag{2}$$

where $\mathcal{N}$ is the set of customers, u_k is the valuation of k-th customer, $S_k = P_k + \mathbf{i} Q_k \in \mathbb{C}$ is the *complex-valued* apparent power demand for k-th customer, $C \in \mathbb{R}_+$ is a real-valued capacity of total apparent generation power. x_k is a binary decision variable if the k-th customer's power demand is retained.

2) *Compensation-minimizing power allocation problem*:

$$(\textsc{minPA}) \qquad \min_{x_k \in \{0,1\}} \sum_{k \in \mathcal{N}} c_k (1 - x_k) \tag{3}$$

$$\text{subject to} \qquad \Big| \sum_{k \in \mathcal{N}} S_k x_k \Big| \le C. \tag{4}$$

where c_k is the compensation paid k-th customer. $(1 - x_k)$ is a binary decision variable if the k-th customer's power demand is curtailed.

While the preceding optimization problems are not entirely new, there lack efficient algorithms to compute the decisions with provable guarantees for a large number of customers. The reason is that these problems encompass a classical NP-hard problem (known as knapsack problem [10]). Specifically, MAXPA is equivalent to the classical knapsack problem when setting zero reactive power, namely, $Q_k = 0$ for all $k \in \mathcal{N}$. There is no known efficient algorithms that can compute the exact optimal solutions for any NP-hard problem. Furthermore, the incorporation of active and reactive power creates a harder problem than the classical knapsack problem [2]. Nonetheless, this paper provides efficient algorithms that compute solutions that are close to the optimal solution with a precise theoretical guarantee on the optimality gap. The efficient algorithms give scalable running time with the number of customers, enabling fast decisions for a large number of customers.

The generation capacity $C(t)$ could be dynamically varying, and the decisions of load curtailment $(x_k(t))_{k \in \mathcal{N}}$ need to be determined from time to time. It is assumed that the decisions are triggered at every discrete timeslot t.

A. Close-to-optimal Solutions

Since the exact optimal solutions are computationally hard, this paper focuses on feasible solutions that are close to the optimal solutions. Furthermore, these close-to-optimal solutions are assured with a precise worst-case guarantee on the optimality gap. Some notations are defined as follows to characterize the optimality gap.

Definition 2.1: For clarity, MAXPA is considered. Let x_k^* be an optimal solution to MAXPA and $\textsc{Opt} \triangleq \sum_{k \in \mathcal{N}} u_k x_k^*$ be the corresponding total valuation. An approximation solution with $\alpha \in [0, 1]$ worst-case guarantee to MAXPA is a feasible solution $(\hat{x}_k)_{k \in \mathcal{N}} \in \{0, 1\}^n$ satisfying

$$\sum_{k \in \mathcal{N}} u_k \hat{x}_k \ge \alpha \cdot \textsc{Opt} \tag{5}$$

$$\text{and} \qquad \Big| \sum_{k \in \mathcal{N}} S_k \hat{x}_k \Big| \le C. \tag{6}$$

The worst-case guarantee is called *approximation ratio*, which characterizes the ratio between the optimal solution and the approximation solution. When $\alpha = 1$, this implies exact optimal solutions. In the subsequent sections, efficient algorithms are presented with definite worst-case guarantees.

First, a greedy approach is proposed that is capable of determining a close-to-optimal solution rapidly. The worst-case guarantee of this greedy approach depends on the maximum phase angle between any pair of power demand. The smaller phase angle produces a smaller optimality gap.

Second, a one-dimensional projection algorithm is presented that projects the power demands represented by two-dimensional vector in complex plane to one-dimensional vectors, and utilizes a well-known approximation algorithm from knapsack problem to solve the problem. This approach requires more computations, and a longer running time. However, the worst-case guarantee is superior to that of the greedy approach.

Finally, a hybrid approach is proposed that harnesses both approaches as a two-stage process, where the greedy approach is firstly utilized rapidly to maintain microgrid stability, and then the one-dimensional projection algorithm is used to improve the decisions, when more response time is permitted.

B. Power-off Protection

It may be undesirable to curtail the load from a customer frequently, as this may cause considerable damages to the electric equipment. To ensure the protection in demand response management, a minimum power-off protection constraint is considered.

Definition 2.2: Minimum power-off protection constraint (MPOP) is a requirement that when a customer's power is curtailed, his power must remain off for a certain minimum duration ($T_k^{\rm off}$) for protection. Namely, if t and t' such that $1 \le t' - t \le T_k^{\rm off}$, then

$$x_k(t') = 0 \qquad \text{if } x_k(t) < x_k(t-1) \tag{7}$$

Setting $T_k^{\rm off} = 0$ will disable the power-off protection constraint. MPOP can be applied to both MAXPA and MINPA for a period of duration T.

1) *Protection-ensured Valuation-maximizing power allocation problem* (PEMAXPA):

$$\text{(PEMAXPA)} \qquad \max_{x_k(t)\in\{0,1\}} \sum_{t=1}^{T} \sum_{k\in\mathcal{N}} u_k(t) x_k(t)$$
$$\text{subject to} \quad \Big| \sum_{k\in\mathcal{N}} S_k(t) x_k(t) \Big| \le C(t) \text{ for all } t,$$
$$x_k(t') = 0 \text{ if } x_k(t) < x_k(t-1)$$
$$\text{for all } t' \text{ such that } 1 \le t' - t \le T_k^{\rm off}$$

2) *Protection-ensured Compensation-minimizing power allocation problem* (PEMINPA):

$$\text{(PEMINPA)} \qquad \min_{x_k(t)\in\{0,1\}} \sum_{t=1}^{T} \sum_{k\in\mathcal{N}} c_k(t)(1 - x_k(t))$$
$$\text{subject to} \quad \Big| \sum_{k\in\mathcal{N}} S_k(t) x_k(t) \Big| \le C(t) \text{ for all } t,$$
$$x_k(t') = 0 \text{ if } x_k(t) < x_k(t-1)$$
$$\text{for all } t' \text{ such that } 1 \le t' - t \le T_k^{\rm off}$$

The two-stage algorithm is also applied heuristically to the settings considering minimum power-off protection constraint.

III. EFFICIENT ALGORITHMS

First, note that the problems are invariant, when the arguments of all complex-valued demands are rotated by the same angle (see Fig. 1 for an illustration).

Without loss of generality, this paper assumes that one of the demands, say S_1 is aligned along the positive real axis, and define a class of sub-problems, by restricting the maximum phase angle θ (i.e., the argument) that any other demand makes with S_1 (see Fig. 1 for an illustration). Note that that in practice $\theta \le \frac{\pi}{2}$, because there are regulations that require electric equipment to conform with a certain maximum power factor (e.g., $\frac{P_k}{|S_k|} \ge 0.8$ [1]). For the clarity of presentation, this paper assumes that $P_k \ge 0$ and $Q_k \ge 0$.

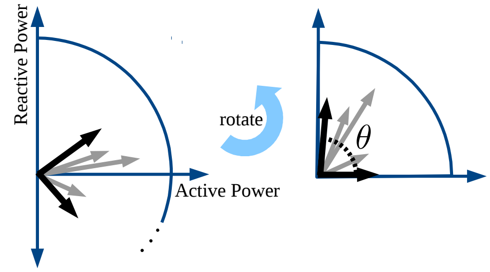

Fig. 1: Each vector represents a power demand S_k. The figure shows that the demands are rotated by the same angle. θ is the maximum angle between any pair of demands.

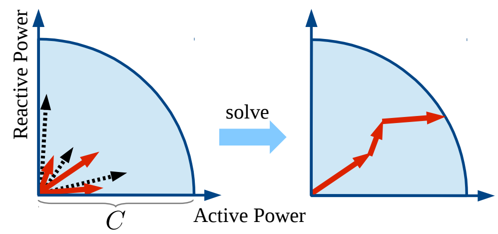

Fig. 2: The red vectors (thick arrows) represent a feasible solution to MAXPA such that the total magnitude of the red demands lies within the radius C.

An allocation $(x_k)_{k\in\mathcal{N}}$ can be equivalently represented by the set of satisfied customers $X \triangleq \{k \in \mathcal{N} \mid x_k = 1\}$. For a subset $X \subseteq \mathcal{N}$, denote $u(X) \triangleq \sum_{k\in X} u_k$.

A. Greedy Algorithms

In this section, three simple greedy algorithms are presented. For brevity, consider only the valuation-maximizing problem (MAXPA). Furthermore, without loss of generality, assume $|S_k| \le C$ for all k.

1) *Greedy Valuation Algorithm* (GVA): First, sort customers in $\mathcal{N} = \{1, ..., n\}$ by their valuations in a non-increasing order (where ties are broken arbitrarily):
$$\text{if } k \le k', \text{ then } u_k \ge u_{k'} \tag{8}$$
Then, select the demands in that order until the capacity constraint ($\big|\sum_{k\in\mathcal{N}} S_k x_k\big| \le C$) is not satisfied.

2) *Greedy Demand Algorithm* (GDA): Similar to GVA, but sort the customers by the magnitude of their demands in a non-decreasing order:
$$\text{if } k \le k', \text{ then } |S_k| \le |S_{k'}| \tag{9}$$
Then, select the demands in that order until the capacity constraint ($\big|\sum_{k\in\mathcal{N}} S_k x_k\big| \le C$) is not satisfied.

3) *Greedy Ratio Algorithm* (GRA): Similar to GVA and GDA, but sort the customers by the efficiency ($\frac{u_k}{|S_k|}$) in a non-increasing order:
$$\text{if } k \le k', \text{ then } \frac{u_k}{|S_k|} \ge \frac{u_{k'}}{|S_{k'}|} \tag{10}$$

Then, select the greedy solution X in that order until the capacity constraint ($\left|\sum_{k\in X} S_k x_k\right| \le C$) is not satisfied. Lastly, return the maximum valuation of two candidate solutions: either greedy solution X, or the maximum valuation of a single customer $\arg\max_{k\in\mathcal{N}}\{u_k\}$. Fig. 3 presents a flowchart of GRA.

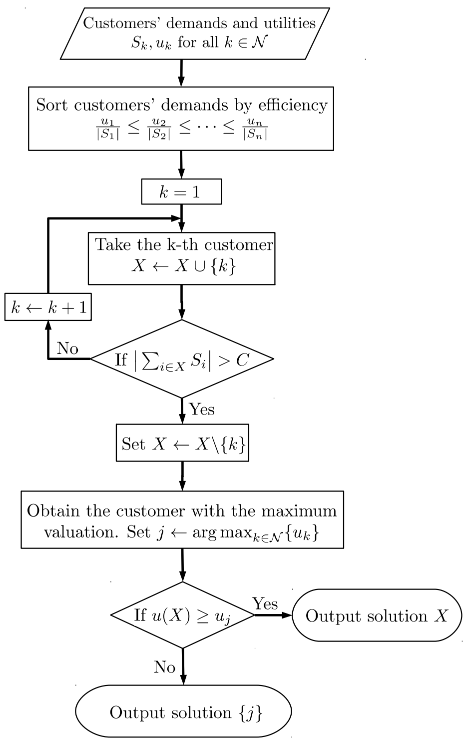

Fig. 3: Flow chart for Greedy Ratio Algorithm (GRA).

GVA and GDA are common strategies of load curtailment in practice. However, GRA possesses a worst-case guarantee, and shows a good empirical ratio with the optimal solutions in simulations.

Theorem 1: Algorithm GRA produces a feasible solution within a worst-case guarantee of $\frac{1}{2}\sqrt{\frac{\cos\theta+1}{2}}$ of the optimal solution of MAXPA.

The worst-case guarantee of GRA depends on θ; the smaller the θ, the better guarantee it provides. The basic idea of the proof is discussed in the Appendix. Algorithm GRA can be applied to MINPA by defining the efficiency as $(\frac{c_k}{|S_k|})$. For MINPA, the worst-case guarantee does not apply, but still shows a good empirical ratio to the optimal solutions by simulations.

B. *One-dimensional Projection Algorithm*

In this section, a one-dimensional projection algorithm (1DPA) is presented. Efficient approximation algorithms for classical knapsack problem have been proposed in the literature [10]. However, the problems concerned in this paper belong to a two-dimensional generalization of the one-dimensional classical knapsack problem, for which the immediate application of these algorithms for knapsack problem cannot be applied. A plausible approach is to reduce the two-dimensional problems onto a one-dimensional instance by a proper projection of the power demand vectors to a certain reference vector.

First, consider MAXPA. The basic idea is to project all power demand vectors onto the $\frac{\pi}{4}$ line (see Fig. 4 for an illustration). Namely, create a new vector along the $\frac{\pi}{4}$ line with magnitude $\tilde{S}_k = \frac{P_k+Q_k}{\sqrt{2}}$ for each $S_k = P_k + \mathbf{i}Q_k$. Then apply the well-known efficient approximation algorithm for the classical knapsack problem [10] (denoted by $\mathrm{Alg}^{\mathrm{kp}}$) on the set $\{\tilde{S}_k\}_{k\in\mathcal{N}}$ and capacity $\frac{C}{\sqrt{2}}$. Lastly, return the maximum valuation of two candidate solutions: either the solution by $\mathrm{Alg}^{\mathrm{kp}}$, or the maximum valuation of a single customer $\arg\max_{k\in\mathcal{N}}\{u_k\}$. Fig. 5 presents a flowchart of 1DPA.

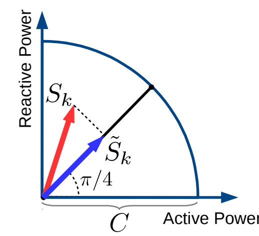

Fig. 4: Each power demand S_k is projected onto the $\frac{\pi}{4}$ line in 1DPA.

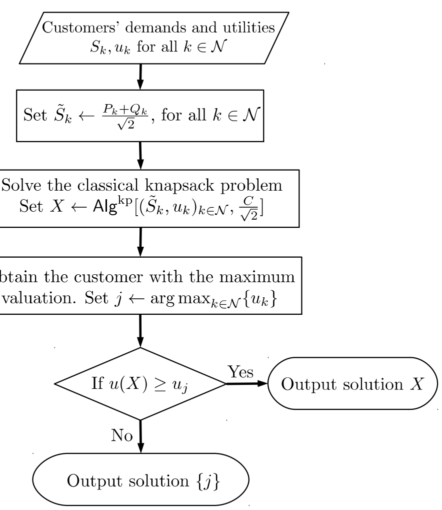

Fig. 5: Flow chart for One-dimensional Projection Algorithm (1DPA).

Theorem 2: Algorithm 1DPA produces a feasible solution with a worst-case guarantee $\frac{1}{2}$ with the optimal solution of MAXPA.

Because $\frac{1}{2}\sqrt{\frac{\cos\theta+1}{2}} \le \frac{1}{2}$, 1DPA gives a superior worst-case guarantee than that of GRA. The basic idea of the proof is discussed in the Appendix. Algorithm 1DPA can be applied to MINPA by defining the efficiency as $(\frac{c_k}{|S_k|})$.

Algorithm 1DPA can be applied to MINPA by applying a similar one-dimensional projection approach to reduce the two-dimensional problem into a one-dimensional min-knapsack problem [5].

C. Two-stage Hybrid Approach

GRA can be executed rapidly, as it relies on a greedy approach. However, the worst-case guarantee of 1DPA outperforms that of GRA. Note that a superior worst-case guarantee does not necessarily imply superior empirical ratio with the optimal solutions in practice. It is possible that some instances of problems may give GRA a superior solution to that of 1DPA. Therefore, one can obtain an improved solution by selecting the one with the maximum valuation (or the minimum compensation cost) of the candidate solutions from GRA and 1DPA.

Therefore, a two-stage algorithm (2SA) is proposed to leverage both approaches. The first stage uses GRA that is capable of determining a close-to-optimal load curtailment scheme rapidly to maintain microgrid stability. The second stage relies on 1DPA that can further improve the optimality solution of the first stage, when more response time is permitted.

D. Power-off Protection

The aforementioned algorithms can be applied heuristically to the settings with minimum power-off protection constraint (MPOP). At each time t, the decision is made with respect to the set of feasible customers $\hat{\mathcal{N}}(t)$, who are not restricted by minimum power-off protection constraint. Namely, define

$$\hat{\mathcal{N}}(t) = \Big\{ k \in \mathcal{N} \quad | \quad \text{there exists no } t' \text{ such that } x_k(t') = 0 \text{ and } t - t' \leq T_k^{\rm off} \Big\} \tag{11}$$

Then, GRA, 1DPA and 2SA are applied to $\hat{\mathcal{N}}(t)$ similarly.

IV. Simulations

This section presents the simulation results for corroborating the performance of the proposed algorithms in diverse settings.

A. Settings

The simulations are based on a microgrid setting with a number of customers, each having a power demand and valuation generated according to a probability preference model. The customers may suffer from a reduction of generation capacity occasionally. The appearances and levels of reduction vary according to a stochastic process model.

The model is designed to resemble a real-world scenario, where the microgrid has a total capacity C up to 2MVA connecting to residential customers (with relatively small demand) and industrial customers (with considerably large demand).

The algorithms (GVA, GDA, GRA, 1DPA) are applied to both valuation-maximizing and compensation-minimizing problems. The simulations were evaluated using 2 Quad core Intel Xeon CPU E5520s 2.21 GHz processors. The algorithms were implemented using C and Python programming languages and a standard random number generation library. Each customer's reactive power demand equals to $Q_k = \tan(\phi) \cdot P_k$, where ϕ is the phase angle between the reactive and apparent powers.

B. Scenarios

Diverse scenarios are considered in the following aspects.

i) Power demands:

1) *Full (F) power*: The power demand of each customer can have both non-negative active power ($P_k \geq 0$) and non-negative reactive power ($Q_k \geq 0$).
2) *Active (A) power only*: The power demand of each customer can have non-negative active power ($P_k \geq 0$) but zero reactive power ($Q_k = 0$).

ii) Valuation/Compensation-demand correlation:

1) *Correlated (C)*: The valuation/compensation of each customer is a function of the power demand:
$$u_k(|S_k|) = a(|S_k|)^2 + b|S_k| + c \tag{12}$$
where $a > 0, b, c \geq 0$ are constants.
2) *Uncorrelated (U)*: The valuation/compensation of each customer is independent of the power demand and is generated randomly from $[0, a(|S_k|)^2 + b|S_k| + c]$.

iii) Customer types

1) *Industrial (I) customers*: Customers have big active power demands ranging from 300KW up to 1MW.
2) *Residential (R) customers*: Customers have small active power demands ranging from 500W to 5KW.
3) *Mixed (M) customers*: Customers have a mix of both industrial and residential customers

In the following, a scenario is represented by the acronyms of the types. For example, scenario ACI stands for the one with industrial customers, zero reactive power demands and valuations/compensations-demand correlation.

C. Results

A summary of the simulation results for each the scenario is shown in Table I, which indicate most superior algorithm in the respective scenario. Specifically, fours scenarios are highlighted in Figs. 6-7

		F		A	
		C	U	C	U
MAXPA	R	GRA	GRA	1DPA	1DPA
	M	GDA	1DPA	1DPA	1DPA
MINPA	R	GRA	GRA	1DPA	1DPA
	M	GDA	1DPA	1DPA	1DPA

TABLE I: The results of simulations in each scenario. Each cell of the table indicates the most superior algorithm in the respective scenario.

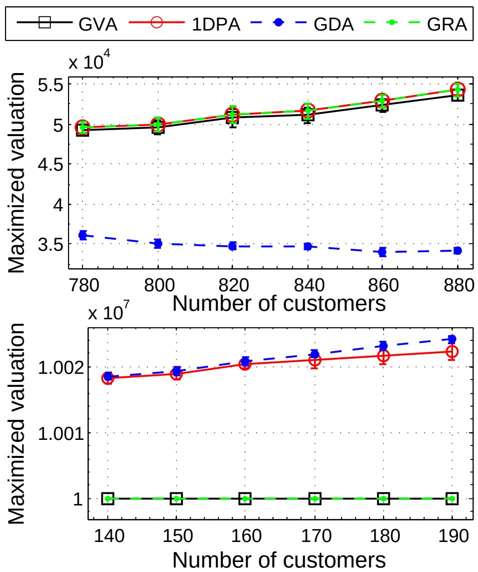

Fig. 6: GVA, GDA, GRA, 1DPA apply to MAXPA for scenarios AUR and FCM on top and bottom respectively.

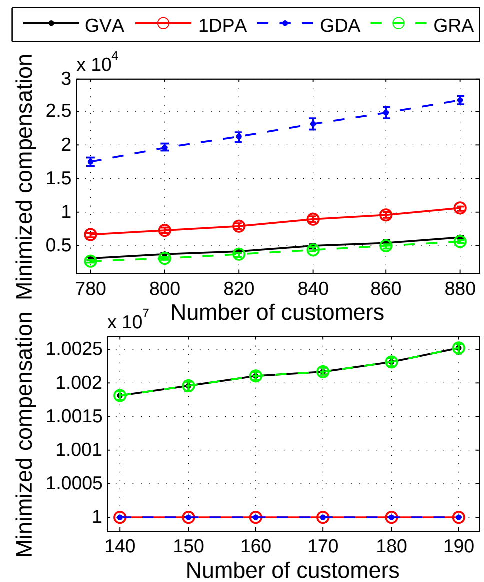

Fig. 7: GVA, GDA, GRA, 1DPA apply to MINPA for scenarios FUR and ACM on top and bottom respectively.

1) Empirical Ratios with Optimal Solutions:

When customers' reactive demand Q_k is not taken into account, MAXPA and MINPA for 1DPA become a simple classical knapsack problem. In fact, in that case the maximized valuation and minimized compensation by 1DPA is considerably close to the possible optimal value. This can be seen from Fig. 6 and Table I, which show that 1DPA algorithm outperforms all greedy algorithms mentioned in this paper.

On the other hand, when customers' reactive demand is considered, MAXPA and MINPA become computationally complex problems for 1DPA. In this case GVA, GDA, GRA, and 1DPA's performance varies for these cases when the customers are only residential and when there is a mix of residential and industrial customers as indicated in Fig. 7 and Table I.

To sum up, the aforementioned simulations show that two key factors, namely the correlation between valuation/compensation and demand, and customers' apparent power, have a substantial impact on the performance of GVA, GDA, GRA, and 1DPA.

The importance of the load (i.e., the priority and the benefit to the system) are among various fundamental factors that should be carefully considered when planning a load curtailment algorithm [4]. The conventional load curtailment strategies used in power grids (i.e., curtailing loads simply according to the ascending order of priorities [8], [18]), are basically GVA, when valuations reflect the priorities.

For the case when the microgrid customers have strictly defined priorities and customers with high priorities are of critical importance and should never be curtailed, the GVA is an effective algorithm. Nevertheless, the analysis of the results shows that when the loads have the same or no priority like that in the simulations, in such cases the GVA does not produce high valuation or low compensation cost solutions, which can be seen in Table I.

2) Running Time:

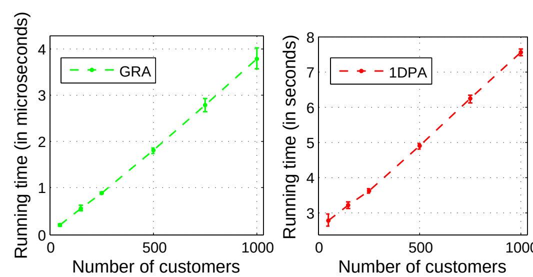

Fig. 8: The running time of 1DPA and GRA.

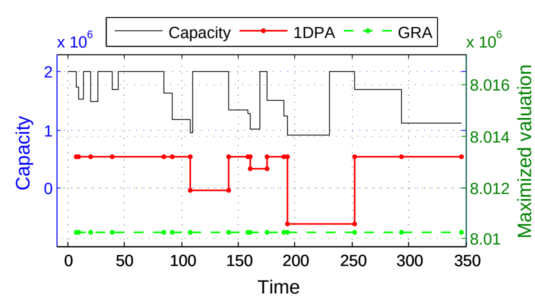

Fig. 9: 1DPA and GRA apply to MAXPA without power-off protection constraint for Scenario AUM.

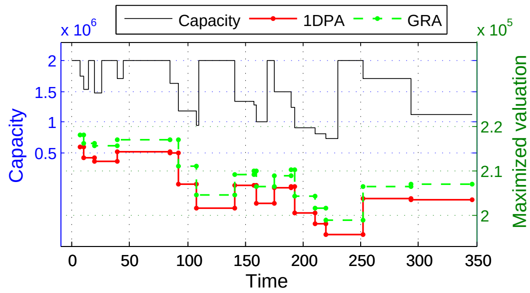

Fig. 10: 1DPA and GRA apply to MAXPA with power-off protection constraint for Scenario FUR.

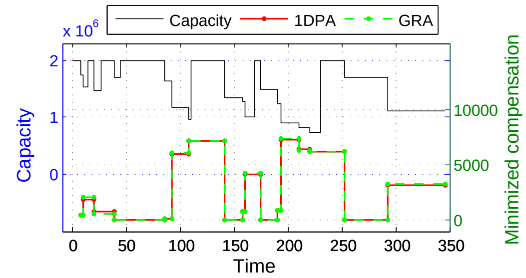

Fig. 12: 1DPA and GRA apply to MINPA with power-off protection constraint for Scenario AUR.

One of the major parameters that evaluates an event-based demand response management algorithm is its running time. The algorithm should be sufficiently fast. Despite a considerable body of literature on this topic, however, they only considered microgrids with a significantly small number of customers, as compared to the number of customers considered in this paper. The running time with respect to a large number of customers is not studied.

Although in most cases 1DPA's solutions are remarkably close to the optimal, the running time is more than that of GRA, especially in the scenarios with mixed customers. Generally, the more the number of customers, the larger is the running time, as seen from Fig. 8.

Therefore, this paper proposes a two-stage hybrid approach to take advantage of both 1DPA and GRA.

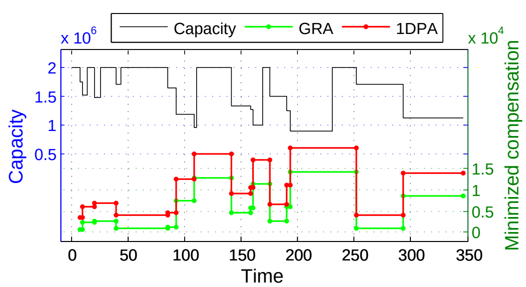

Fig. 11: 1DPA and GRA apply to MINPA without power-off protection constraint for Scenario FUR.

3) Dynamic Capacity:

In addition to the aforementioned scenarios, simulations are performed considering the case when the microgrid's capacity is varying over time due to possible events (e.g., failure, or resumption), whereas the set of customers is fixed.

The aforementioned scenario is applied to both MAXPA and MINPA with and without power-off protection constraint. The events, namely Failure and Resumption, occur according to an exponential distribution with a n expected rate 200. When the microgrid is in the Failure state, the capacity decreases randomly from $5\% - 35\%$, whereas when in the Resumption state the microgrid's capacity is fully resumed (i.e., C =2MVA), except the times when an events occurred the remaining time the microgrid is in the Steady state (i.e., it remains at the same state as was in the previous). Whether an event is the Failure or Resumption state is determined according to a Markov chain with the following settings:

1) Steady $\rightarrow$ Failure with a probability of 65%
2) Steady $\rightarrow$ Resumption with a probability of 35%

The value of $T_k^{\rm off}$ for each customer is generated randomly from a range $[0, |S_k|]$. The results show that without power-off protection constraint for the scenarios with active power only, 1DPA outperforms GRA for both MAXPA and MINPA. In Fig. 9, GRA's maximized valuation is lower than the one of 1DPA and is constant. This is because GRA simply fails to consider all the residential customers with small power demands as well as the remaining industrial customers whose power demand is relatively smaller compared to the industrial customers having considerably big power demand. For the other scenarios performance of 1DPA and GRA varies considerably based on the values of $T_k^{\rm off}$ as well as on customers' power demands' range, as seen from Figs. 10,11,12.

V. Conclusion

This paper presented efficient algorithms for event demand-based response in microgrid with provable approximation guarantee. A two-stage method was suggested that combines the advantages of two different approaches: the one-dimensional projection approach (1DPA) which guarantees a good approximation ratio but is computationally more demanding; and the greedy ratio approach (GRA) which is computationally very efficient, but has worse approximation guarantee. This paper compares these methods with the methods currently used in practice, such as greedy utility, or greedy demand. The simulation results show the superiority of the suggested methods under various practical settings. The proposed approach can be applied to microgrid with a large number of customers.

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VI. Appendix

In the Appendix, the basic ideas of the proofs of Theorems 1-2 are sketched.

A. Greedy Ratio Algorithm

The basic idea of the proof of Theorem 1 is to compare the solution $X \cup \{j\}$ obtained by GRA with the optimal solution. By the greedy order, each customer has a better or equal efficiency ratio than the corresponding optimum customer, when both the greedy and optimal solutions are considered according to the order $\frac{u_k}{|S_k|} \geq \frac{u_{k'}}{|S_{k'}|}$, when $k \leq k'$. One can easily derive form this that the ratio of the total valuation of the set of customers in $X \cup \{j\}$ to the total valuation in any optimal set is bounded from below by the ratio of the sum of the absolutes of the demands in these two sets. It remains thus to bound this ratio from below. This essentially amounts to relating the sum of the absolutes of a set vectors in $\mathbb{C}$ to the absolute of the sum of these vectors, and can be shown by induction on the number of vectors.

Fig. 13 illustrates a tight extreme case when the optimum packs more power demands (in length) than that of GRA. One can show by induction that the two sides corresponding to X^* (optimal sides) are equivalent (in the extreme case). Rescale all power demands such that each of the two optimal sides is equal to 1, then apply basic trigonometry $\sum_{k \in X \cup \{j\}} |S_k| = \sqrt{(1+\cos\theta)^2 + \sin^2\theta} = \sqrt{2 + 2\cos\theta}$ Therefore, $\frac{\sum_{k \in X \cup \{j\}} |S_k|}{\sum_{k \in X^*} |S_k|} = \frac{\sqrt{2+2\cos\theta}}{2} = \sqrt{\frac{\cos\theta + 1}{2}}$ In this case, it follows that $u(X \cup \{j\}) \geq \sqrt{\frac{\cos\theta+1}{2}} \cdot \textsc{Opt}$. Finally, since the algorithm returns the maximum valuation of either X or $\{j\}$, the solution objective is at least half of $X \cup \{j\}$.

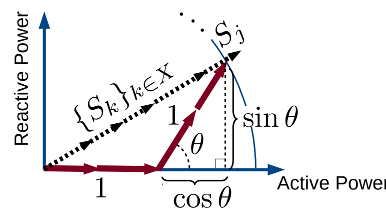

Fig. 13: The figure depicts an extreme case when the ratio $\frac{\sum_{k \in X \cup \{j\}} |S_k|}{\sum_{k \in X^*} |S_k|}$ is minimal, where X^* is an optimal solution. The dotted arrows represent solution $X \cup \{j\}$ (which is infeasible), whereas the red solid arrows represent the optimal solution X^*. These two solutions constitute a triangle.

B. One-dimensional Projection Algorithm

The basic idea of the proof of Theorem 2 is to divide the feasibility region into two parts: $\mathcal{D}_1$ and $\mathcal{D}_2$ (see Fig. 14). There are three cases. (Case 1) If an optimal solution X^* resides in $\mathcal{D}_1$ (i.e., $\sum_{k \in X^*} S_k(t) \in \mathcal{D}_1$), then it is equivalent to the classical knapsack problem with capacity $\frac{C}{\sqrt{2}}$, and Alg^{kp} can find a close-to-optimal solution.

(Case 2 and 3) On the other hand, if an optimal solution X^* resides in $\mathcal{D}_2$ (i.e., $\sum_{k \in X^*} S_k(t) \in \mathcal{D}_2$), then one can show that it is either a singleton power demand with the maximum valuation, or a sum of power demands with at least half of valuation lies in $\mathcal{D}_1$ (which is equivalent to the classical knapsack problem with capacity $\frac{C}{\sqrt{2}}$).

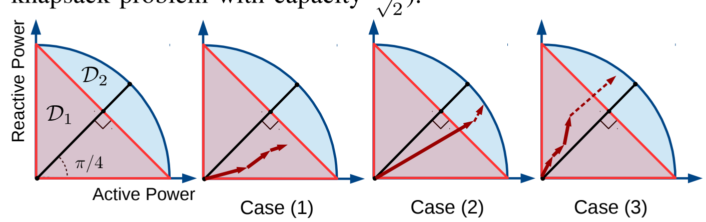

Fig. 14: The feasibility region is divided into $\mathcal{D}_1$ and $\mathcal{D}_2$.