Homework 2

Student: Torekhan Erdauit

Chapter 3.8 p.58-61: ex. 1, 4, 5, 11

Exercise 1:

(1) $F_n = 2 \cdot F_{n-1}$ – if double

$F_n = \frac{1}{2} \cdot F_{n-1}$ – if half.

$$E[F_n] = \left(\frac{1}{2} \cdot 2 \cdot F_{n-1}\right) + \left(\frac{1}{2} \cdot \frac{1}{2} \cdot F_{n-1}\right) =$$

$$= \frac{5}{4} F_{n-1}$$

recursion.

$$= \frac{5}{4} \cdot E[F_{n-1}] = \boxed{\left(\frac{5}{4}\right)^n \cdot C}$$

Exercise 4:

4) $E[X_n] = E[X_1 + \ldots X_n] =$

$$= E[X_1] + E[X_2] + \ldots + E[X_n] =$$

$$= p \cdot (-1) + (1-p) \cdot 1 = -p + 1 - p = 1-2p$$

$$= n \cdot (1-2p)$$

$$V[X_n] = E[X_n^2] - E[X_n]^2$$

$$V[X_i] = E[X_i^2] - E[X_i]^2$$

$$= 1 - (1 - 4p + 4p^2) = 4p(1-p)$$

$$V[X_n] = 4p(1-p) \cdot n$$

Exercise 5:

5

$$E(X) = \sum_{k=1}^{\infty} k \cdot p(1-p)^{k-1} =$$

$$= p \cdot (1-p)^0 + 2p(1-p)^1$$

$$= p + (2p - 2p^2)$$

if $p = \frac{1}{2}$, we obtain that:

$$E[X] = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \ldots$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{4}{16} + \ldots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

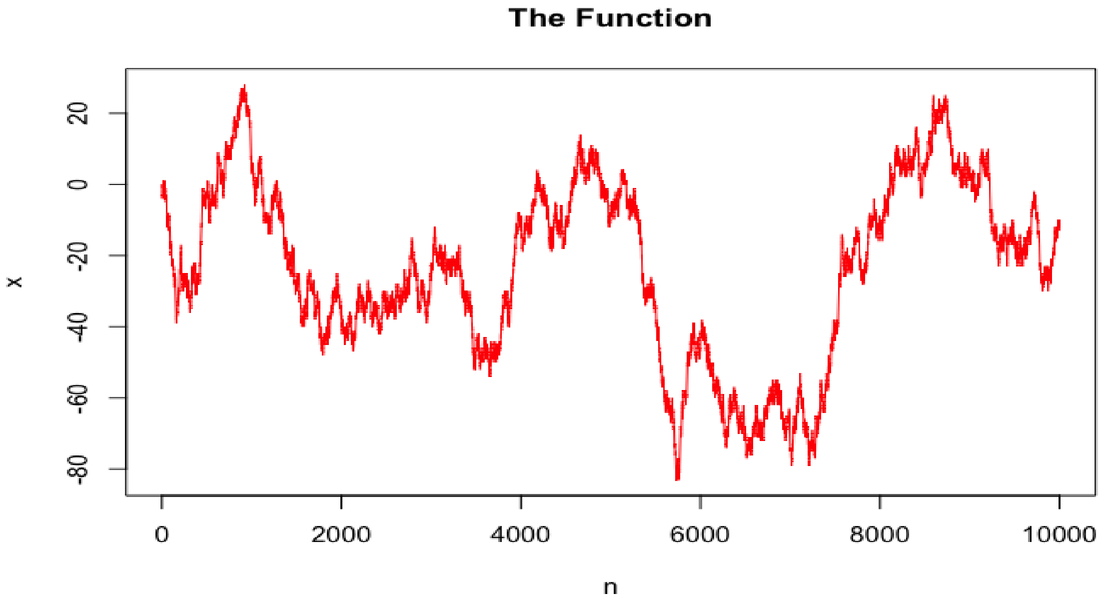
$a = 1$

$r = \frac{1}{2}$

$$S_n = \frac{1}{1 - \frac{1}{2}} = 2.$$

Exercise 11 (Comp experiment):

```
y <- sample(c(-1, 1), size = 10000, replace=TRUE)
x <- cumsum(y)
n <- seq(1, 10000)
plot(n, x, main="The Function",type="l", col="red")
```



Explanation: cumsum() is used to evaluate the cumulative sum of vector (0, 10000), in this case we can see that all values between std deviation

Chapter 4.5 p.68-69: ex. 2

Exercise 2:

2) $X \sim Poisson(\lambda)$

$P(X \geq 2\lambda) \leq \frac{1}{\lambda}$

Chebyshev inequality:

$$P(|X - E[X]| \geq k) \leq \frac{Var(X)}{k^2}$$

in the Poisson Distribution:

$E[X] = \lambda$

$V(X) = \lambda$

$$P(|X - \lambda| \geq k) \leq \frac{\lambda}{k^2}.$$

$$P((X \geq 2\lambda)) \leq \frac{\lambda}{\lambda^2}$$

$$P(X \geq 2\lambda) \leq \frac{1}{\lambda}$$

Chapter 5.8 p.82-84: ex. 6, 8

Exercise 6:

6

$\mu = 68$

$\sigma = 2,6$

$n = 100$

$P(X \geq 68)$

$$P\left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \geq \frac{68-68}{2,6/10}\right) = P(Z \geq 0) =$$

$= \frac{1}{2}$ (According to the table).

Exercise 8:

(8) $n = 100$

$X_i \sim Poisson(1)$

$Y = \sum_{i=1}^{n} X_i$

$P(Y < 90)$

$Mean = \underset{mean}{Poisson(1)} \cdot 100 = \underline{100}$

$Var(Y) = Var(Y_1) + Var(Y_2) + \dots$

$= 1 + 1 + 1 + \dots = 100.$

$\sigma = \sqrt{100} = \sqrt{Var(Y)} = \underline{10}$

Using Central Limit Theorem:

$$P\left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} < \frac{90 - 100}{10/\sqrt{100}}\right) = P(Z < -10)$$

= <u>kinda small</u>