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Chapter 10.11 p. 170-173: ex.12

a)

Ex 12

Let  $X_1, \dots, X_n \sim \text{Poisson}(\lambda)$

$$L(\lambda) = e^{-n\lambda} \cdot \frac{\lambda^n}{n!} = e^{-n\lambda} \frac{\lambda^n}{n!}$$

$$\log(L(\lambda)) = -n\lambda + n \cdot \log(\lambda) - \log(n!)$$

MLE:

$$\frac{d}{d\lambda} = \frac{n - \lambda}{\lambda} = \frac{n}{\lambda} - 1 = \frac{\sum x_i}{\lambda} - 1$$

$$\frac{\sum x_i}{\lambda} - 1 = 0$$

$$\hat{\lambda} = \frac{\sum x_i}{n}$$

$$W = \frac{\hat{\lambda} - \lambda_0}{\sqrt{\text{Var}(\hat{\lambda})}}$$

$$\frac{d^2}{d\lambda^2} = -\frac{n}{\lambda^2}$$

$$\text{Var}(\hat{\lambda}) = \frac{1}{n \cdot \lambda_0^2}$$

$$\hat{\lambda}(\text{MLE}) = \lambda_0$$

So,

$$W = \frac{\lambda_0 - \lambda_0}{\sqrt{\text{Var}(\hat{\lambda})}} = 0$$

if  $|W| > z_{\frac{\alpha}{2}}$  reject

b)

```
1 n <- 20
2 lambda0 <- 1
3 alpha <- 0.05
4
5 vec <- vector()
6
7 for (i in 1:10000){
8   x <- rpois(n, lambda0)
9   w <- (mean(x) - lambda0) / sqrt(mean(x) / n)
10  if (w > 1.96) {
11    vec[i] <- 1
12  }else {
13    vec[i] <- 0
14  }
15 }
```

## Chapter 9.14 p.146-148: ex. 9

a)

```
set.seed(123)

mu <- 5
n <- 100
x <- rnorm(n, mean = mu, sd = 1)

theta_hat <- exp(mean(x))

nonparametric_bootstrap <- function(x, theta_hat, B) {
  theta_star <- rep(NA, B)
  for (i in 1:B) {
    x_star <- sample(x, replace = TRUE)
    theta_star[i] <- exp(mean(x_star))
  }
  se <- sd(theta_star)
  ci <- c(theta_hat - qnorm(0.975)*se, theta_hat + qnorm(0.975)*se)
  list(se = se, ci = ci)
}

nonparam_boot <- nonparametric_bootstrap(x, theta_hat, B = 1000)
nonparam_boot$se
nonparam_boot$ci
```

b)

```
set.seed(123)

mu <- 5
n <- 100
x <- rnorm(n, mean = mu, sd = 1)

theta_hat <- exp(mean(x))

parametric_bootstrap <- function(x, theta_hat, B) {
  theta_star <- rep(NA, B)
  for (i in 1:B) {
    x_star <- rnorm(n, mean = mean(x), sd = 1)
    theta_star[i] <- exp(mean(x_star))
  }
  theta_star
}

param_boot_dist <- parametric_bootstrap(x, theta_hat, B = 1000)

nonparametric_bootstrap <- function(x, theta_hat, B) {
  theta_star <- rep(NA, B)
  for (i in 1:B) {
    x_star <- sample(x, replace = TRUE)
    theta_star[i] <- exp(mean(x_star))
  }
  theta_star
}

nonparam_boot_dist <- nonparametric_bootstrap(x, theta_hat, B = 1000)

par(mfrow=c(1,3))
hist(param_boot_dist, main = "Parametric Bootstrap", xlab = "theta")
hist(nonparam_boot_dist, main = "Nonparametric Bootstrap", xlab = "theta")
```

