Homework 1

Student: Torekhan Erdauit
ID: 21B030934

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1. Chapter 1.10 (Probability)

Exercise 5:

5) Ω - infinite, but discrete

$\Omega = \{ \omega = \{ \omega_1, \omega_2, \omega_3, \dots \omega_n \}, n \geq 2, \omega_n = H,$
$\omega_{n-1} = H, \omega_i \in \{T\}$ for $i < n-1 \}$

k - tosses, znaum k-th toss must be $\underline{H}$
$k-1$ - also $\underline{H}$

$$\underbrace{\binom{k-1}{1} \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{k-2}}_{\text{permutation, that 1 head}} \cdot \underbrace{\frac{1}{2}}_{\text{last}} = \underline{\binom{k-1}{1} \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{k-1}}$$

Exercise 13:

13. Ω - infinite, but discrete

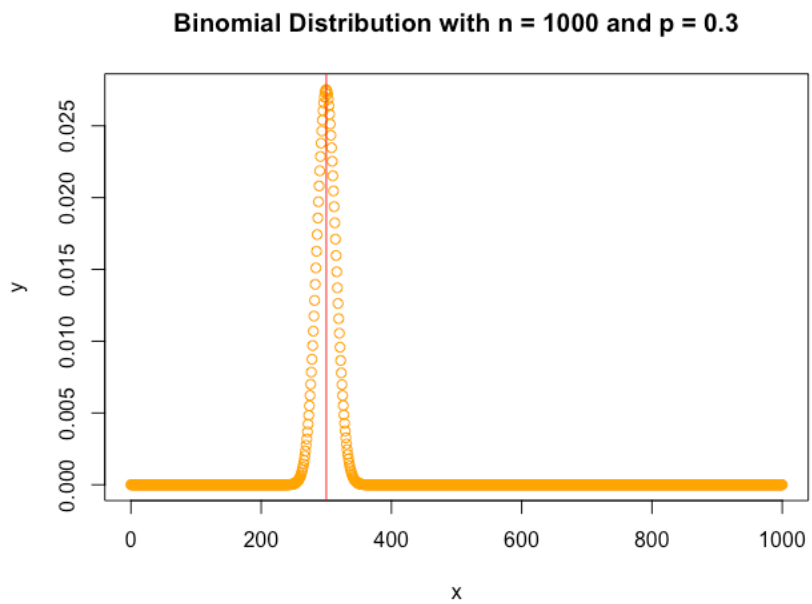
$\Omega = \{ \omega = \{ \omega_1, \omega_2, \dots \omega_n \}, n \geq 2, (\omega_n \in \{H\}, \omega_i \in \{T\}),$
or $(\omega_n \in \{T\}, \omega_i \in \{H\})$ for $i < n-1 \}$

$\boxed{HHT}$ or $\boxed{TTH}$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0{,}25$$

Exercise 22:

```
x <- seq(0,1000, by = 1)

y <- dbinom(x,1000,0.3)

a <- 1000 * 0.3

plot(x,y,  main = "Binomial Distribution with n = 1000 and p = 0.3", col = "orange")
abline (v = a, col = "red")
```



Conclusion: As we can see, n * p in this case will be the center of the binomial distribution

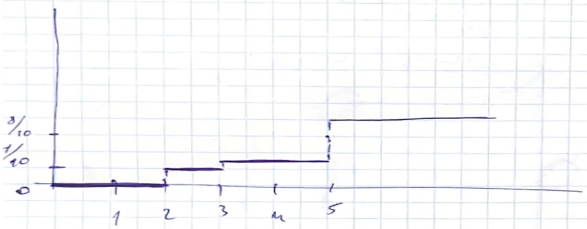
2. Chapter 2.14 (Random Variables)

Exercise 2:

$P(X=2) = P(X=3) = 1/10$

$P(X=5) = 8/10$.

$$F(x) = \begin{cases} 0, & \text{if } x < 2 \\ 1/10, & \text{if } 2 \le x < 3 \\ 2/10, & \text{if } 3 \le x < 5 \\ 1, & \text{if } x \ge 5 \end{cases}$$



a) $P(2 < x \le 4{,}8) = F(4{,}8) - F(2) =$

$= \frac{2}{10} - \frac{1}{10} = \boxed{\frac{1}{10}}$

b) $P(2 \le x \le 4{,}8) = P(2) + P(2 < x \le 4{,}8)$

$= P(X = x) + P(X > 2 \text{ and } x \le 4{,}8) =$

$= P(X = x) + P(X > 2) + P(x \le 4{,}8) =$

$= \cancel{F(2)} - F(2^-) + F(4{,}8) - \cancel{F(2)} =$

$= F(4{,}8) - F(2^-) = \frac{2}{10} - 0 = \frac{2}{10} = \boxed{0{,}2}$

Exercise 11a:

p = prob of H

X – # of heads

Y = # of tails

$n = 1$

	$Y=0$	$Y=1$
$X=0$	0	$1-p$
$X=1$	p	0

$$P(Y=0, X=1) = P(Y=0) + P(X=1) = 1-p+p = \boxed{1}$$

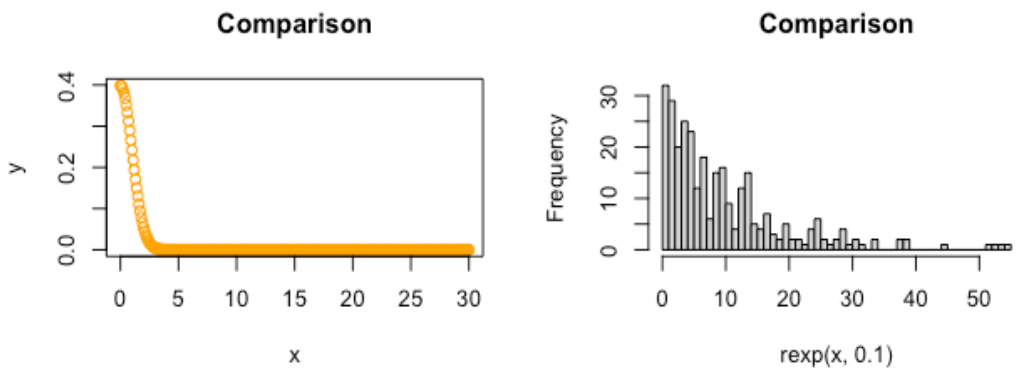
$$P(Y=0) \cdot P(X=1) = (1-p) \cdot p = \boxed{p - p^2}$$

not independent

Exercise 13b:

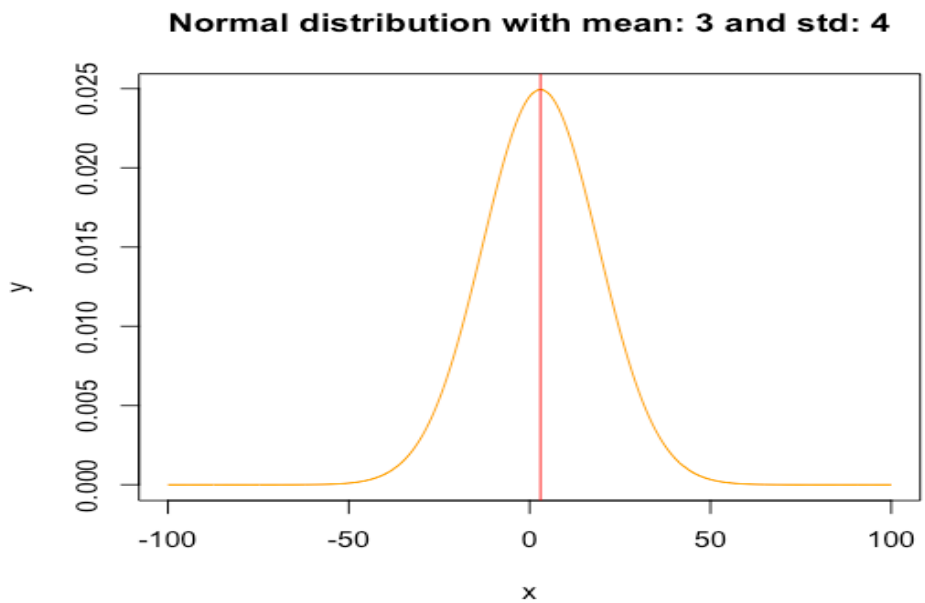
```
x <- seq(0, 30, by = 0.1)
y <- dnorm(x, mean = 0, sd = 1.0)
plot(x,y, main = "Comparison", col = "orange")

x <- seq(0, 30, by = .1)
hist(rexp(x, .1), breaks=40, main="Comparison" )
```

Following plots:



Conclusion: In this case we can see that our plots +- in the same level and they looks similar.

Exercise 18:



a) Find P (X < 7)

```
> #Find P(X < 7)
>
> pnorm(7, mean = 3, sd = 4, lower.tail = TRUE)
[1] 0.8413447
```

b) Find P (X > -2)

```
> #Find P(X > -2)
>
> pnorm(-2, mean = 3, sd = 4, lower.tail = FALSE)
[1] 0.8943502
```

c) Find x such that P (X > x) = 0.05

```
> #Find x such that P(X > x) = 0.05
>
> qnorm(p = 0.05, mean = 3, sd = 4, lower.tail = FALSE)
[1] 9.579415
```

d) Find P (0 ≤ X < 4)

```
> #Find P(0 ≤ X < 4)
>
> pnorm(q = 4, mean = 3, sd = 4, lower.tail = TRUE) - pnorm(q = 0, mean = 3, sd = 4, lower.tail = TRUE)
[1] 0.372079
```

e) Find x such that P (|X| > |x|) = .05.

```
> #Find x such that P(|X| > |x|) = .05.
>
> #In this case our normal distribution is kinda symmetric, so we can say that our point can be in 95% of our
> #density. If we take left or right sides, the percentage will be 0.975
> #  0.025 | 0.950 | 0.025    ->   0.975
>
> qnorm(p = 0.975, mean = 3, sd = 4, lower.tail = TRUE)
[1] 10.83986
```