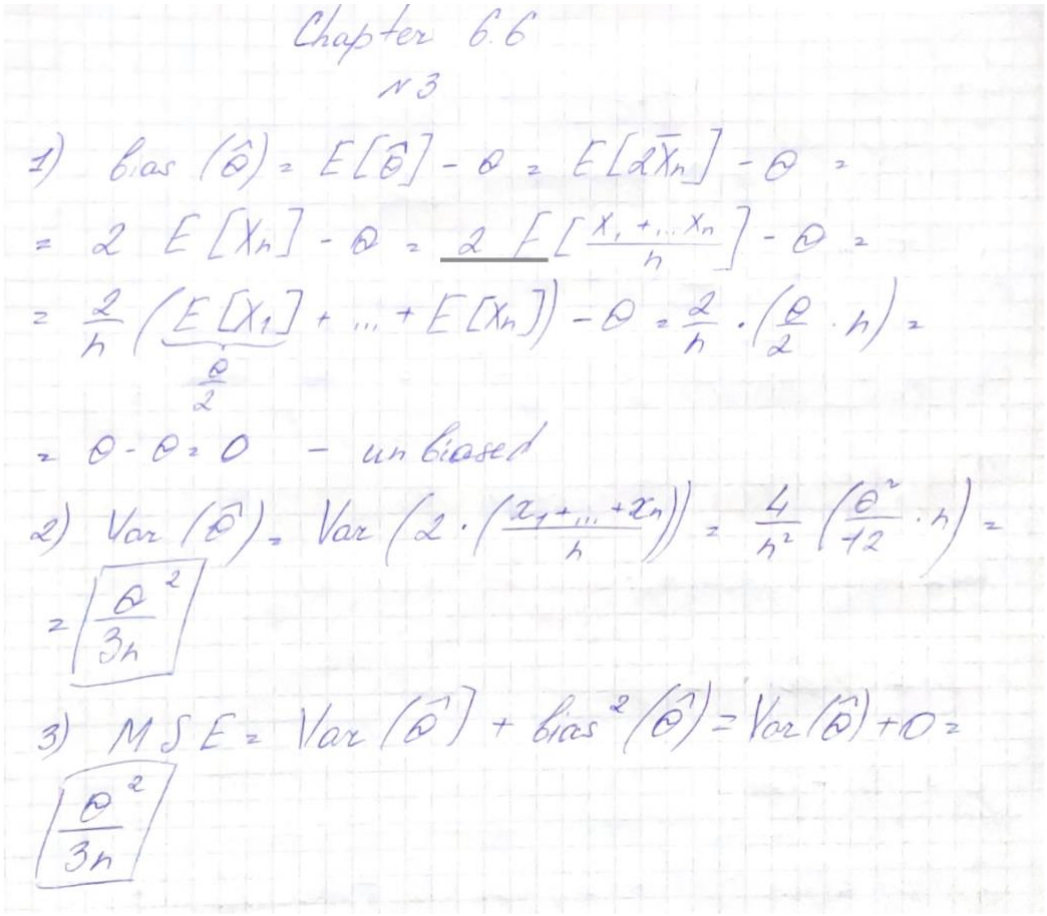
HOMEWORK 3

Student: Torekhan Erdauit

Exercise: 3



Chapter 6.6

№3

1) $bias(\hat{\theta}) = E[\hat{\theta}] - \theta = E[2\bar{X}_n] - \theta =$

$= 2E[X_n] - \theta = 2E\left[\frac{X_1 + \dots X_n}{n}\right] - \theta =$

$= \frac{2}{n}\left(\underbrace{E[X_1]}_{\frac{\theta}{2}} + \dots + E[X_n]\right) - \theta = \frac{2}{n} \cdot \left(\frac{\theta}{2} \cdot n\right) =$

$= \theta - \theta = 0$ – unbiased

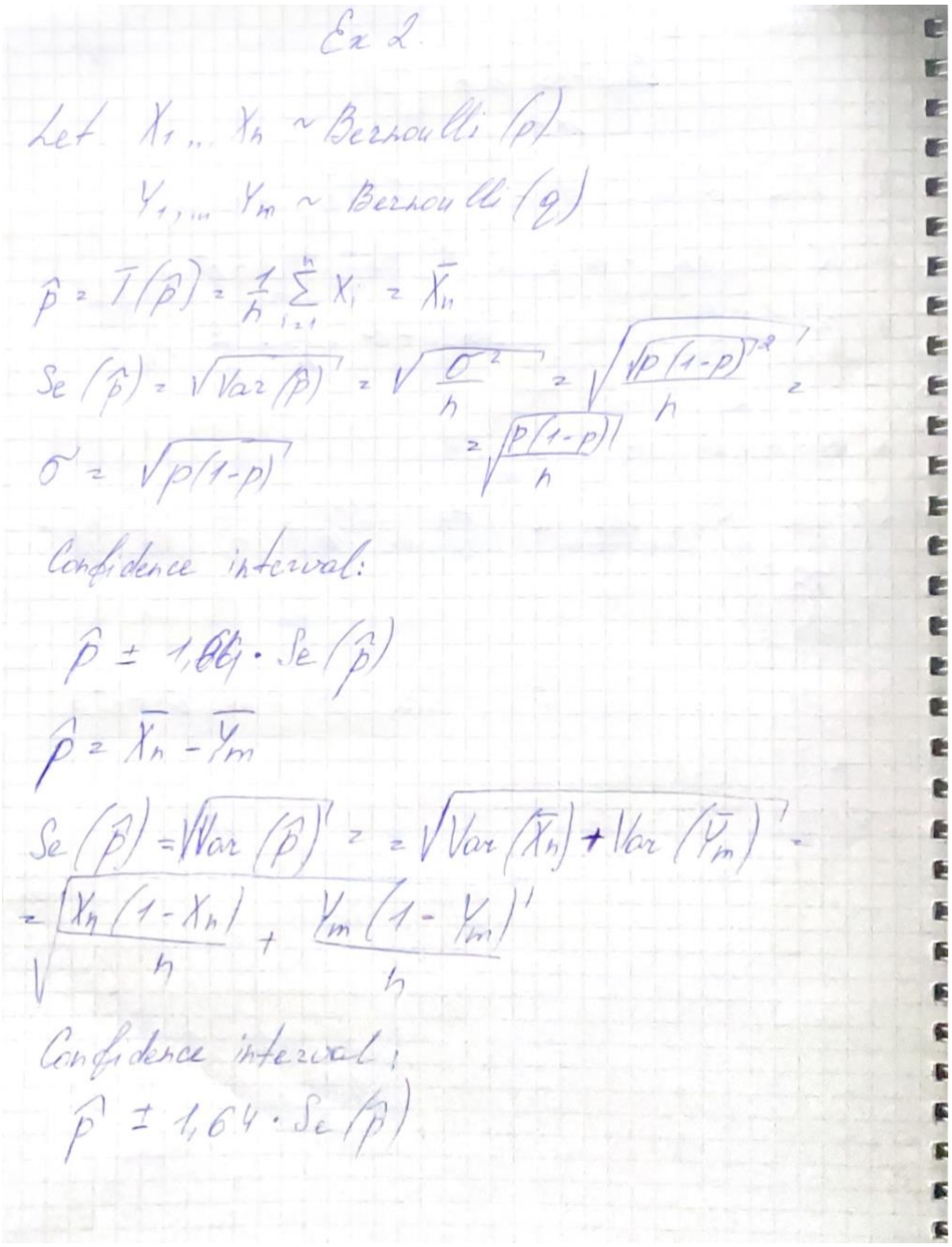
2) $Var(\hat{\theta}) = Var\left(2 \cdot \left(\frac{x_1 + \dots + x_n}{n}\right)\right) = \frac{4}{n^2}\left(\frac{\theta^2}{12} \cdot n\right) =$

$= \boxed{\frac{\theta^2}{3n}}$

3) $MSE = Var(\hat{\theta}) + bias^2(\hat{\theta}) = Var(\hat{\theta}) + 0 =$

$\boxed{\frac{\theta^2}{3n}}$

Exercise: 2



Ex 2

Let $X_1 \dots X_n \sim Bernoulli(p)$

$Y_1, \dots Y_m \sim Bernoulli(q)$

$\hat{p} = T(\hat{p}) = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}_n$

$Se(\hat{p}) = \sqrt{Var(\hat{p})} = \sqrt{\frac{\sigma^2}{n}} = \sqrt{\frac{p(1-p)}{n}} =$

$\sigma = \sqrt{p(1-p)}$ $= \sqrt{\frac{p(1-p)}{n}}$

Confidence interval:

$\hat{p} \pm 1{,}64 \cdot Se(\hat{p})$

$\hat{p} = \bar{X}_n - \bar{Y}_m$

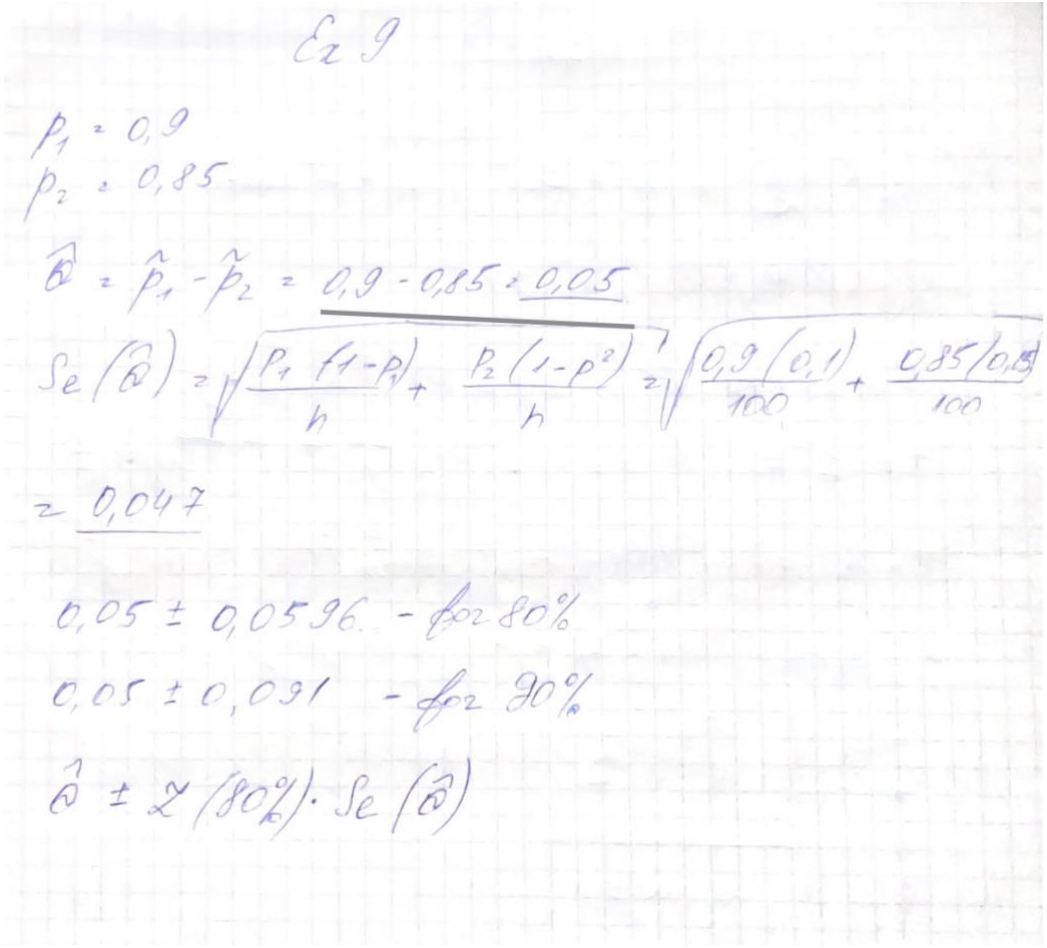
$Se(\hat{p}) = \sqrt{Var(\hat{p})} = \sqrt{Var(\bar{X}_n) + Var(\bar{Y}_m)} =$

$= \sqrt{\frac{X_n(1-X_n)}{n} + \frac{Y_m(1-Y_m)}{n}}$

Confidence interval:

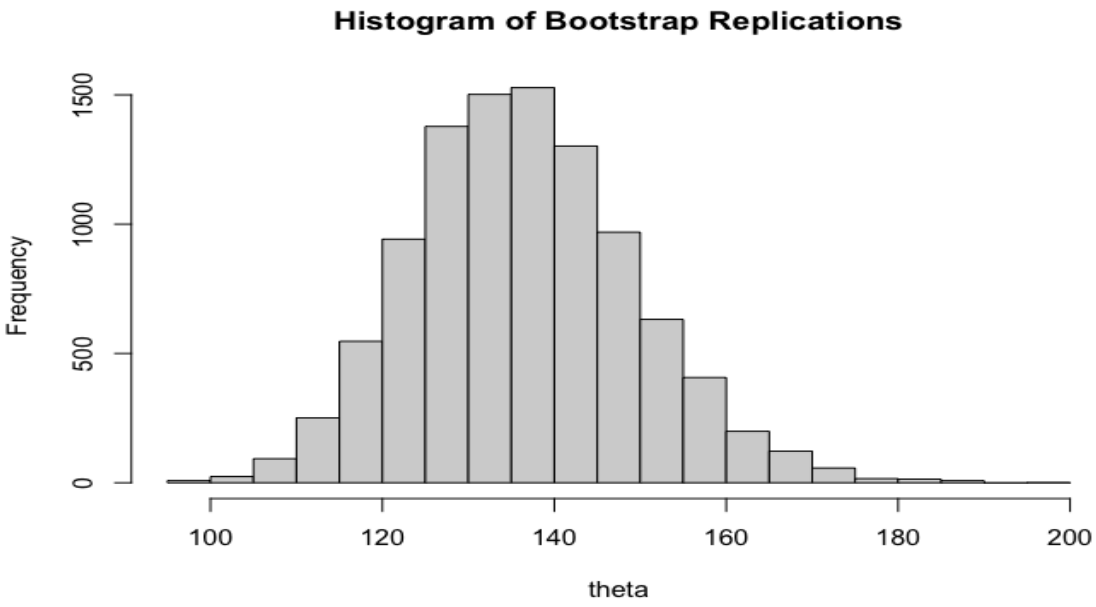
$\hat{p} \pm 1{,}64 \cdot Se(\hat{p})$

Exercise: 9



Exercise: 6

```r
mu <- 5
n <- 100
X <- rnorm(n, mean = mu, sd = 1)

theta_func <- function(x) exp(mean(x))

B <- 10000

theta_boot <- replicate(B, theta_func(sample(X, replace = TRUE)))

hist(theta_boot, main = "Histogram of Bootstrap Replications", xlab = "theta")

curve(dexp(x, rate = 1), from = 0, to = 50, add = TRUE, col = "blue")
```


Histogram of Bootstrap Replications
Frequency
0
500
1000
1500
100
120
140
160
180
200
theta