CUBIC **FUNCTIONS** AND MODELS

OBJECTIVES

- × Solve problems involving cubic functions
- × Develop and fit the cubic model
- × Comment on the appropriateness and reasonableness of the cubic model

ONE MINUTE...

× What is a cubic function?

Cubic functions are polynomial functions where the highest power of x is 3.

For example, $f(x) = ax^3 + bx^2 + cx + d$, $a \ne 0$ and $a, b, c, d \in \mathbb{R}$ is a cubic function.

The fundamental (simplest) cubic function is $f(x) = x^3$.

Reflect State what type of function $f(x) = ax^3 + bx^2 + cx + d$ would be if a = 0.

Here is a picture of part of a roller coaster.

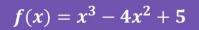
It is in the shape of the graph of a cubic function.

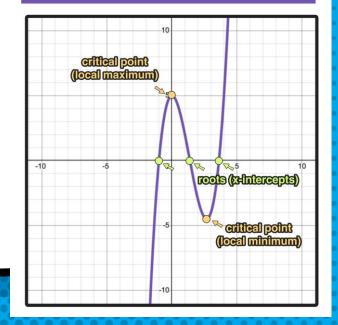
Reflect How are graphs of cubic functions different from graphs of quadratic functions?

Are there any similarities between the graphs of quadratic and cubic functions?



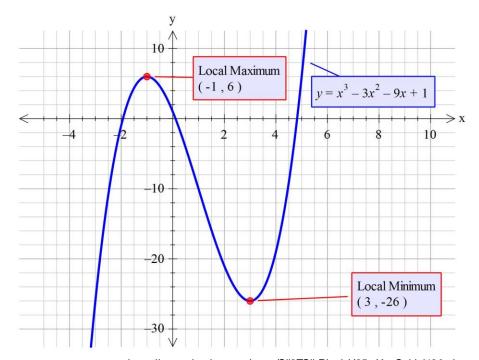
- A cubic function is of the form $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are constants and a $\neq 0$.
- A cubic function may have 1 real root and 2 complex roots.
- It may also have 3 real roots and 0 complex roots.
- A cubic function is maximum or minimum at the critical points.





HINT

A point where the gradient of the curve changes from positive to negative or vice versa is called a local maximum or minimum point. The word local is used because often in the case of cubics there is no actual maximum or minimum.

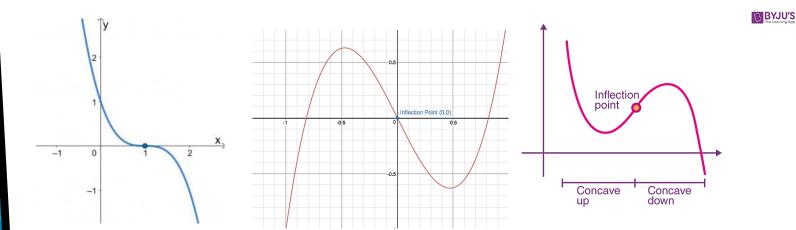


 ${\it https://useruploads.socratic.org/Sij8TSjhRhmixY85srXz_Cubic\%20minmax.jpg}$

You can sketch (or draw) the graph of a cubic function by hand with the help of your GDC. In many cases the **domain** of the function that you need to sketch or draw will be explicitly given. In such cases you should only sketch (or draw) the part of the function that is in the given domain.

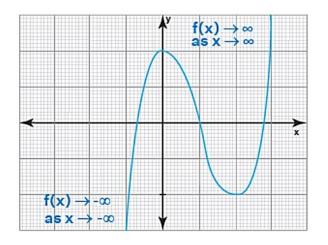
Make sure that any *x*- and *y*-intercepts are in the correct place on the graph. Also, the local maximum and minimum values need to be in the correct place.

You can also use the table of values on the GDC to plot the coordinates of some more points that lie on the curve if necessary.

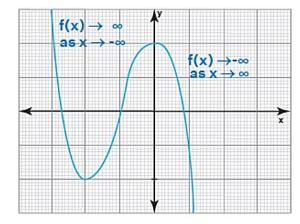


Not all cubic functions have a maximum and a minimum turning point. Some have neither. Also, cubic functions can have one, two or three real roots and have rotational symmetry about a point called the **point of inflexion**.

When the leading coefficient > 0



When the leading coefficient < 0



https://www.cuemath.com/calculus/cubic-function/

FIND THE ROOTS AND THE POINT OF INFLECTION OF THE FOLLOWING CUBIC FUNCTIONS

1.
$$f(x) = x^3 - 6x^2 + 11x - 6$$

2.
$$f(x) = x^3 - 5x^2 + 8x - 4$$

3.
$$f(x) = x^3 - 3x^2 + 3x - 1$$

4.
$$f(x) = x^3 + x^2 + x - 3$$

Answers (Roots):

1.
$$x = 1,2,3$$

2.
$$x = 1$$
 and $x = 2$ (2 times)

3.
$$x = 1$$
 (3 times)

4.
$$x = 1$$
 (one real root)

https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-cubicequations-2009-1.pdf