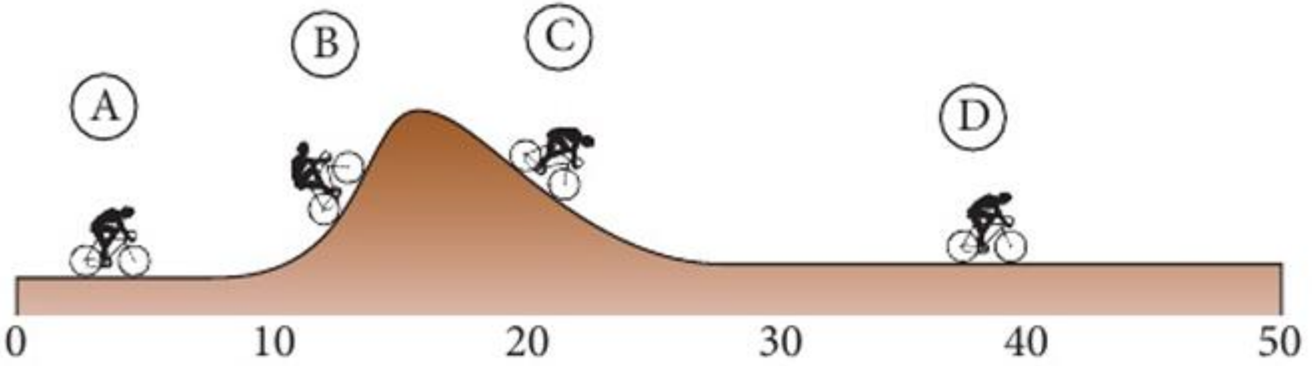
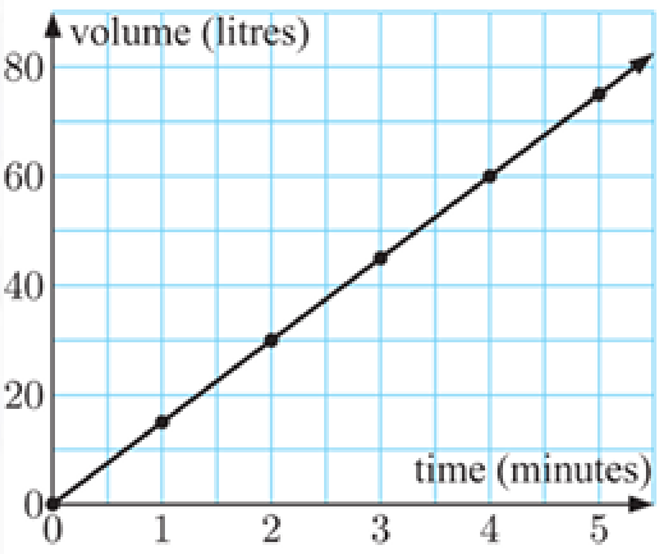
Calculus



RATE OF CHANGE

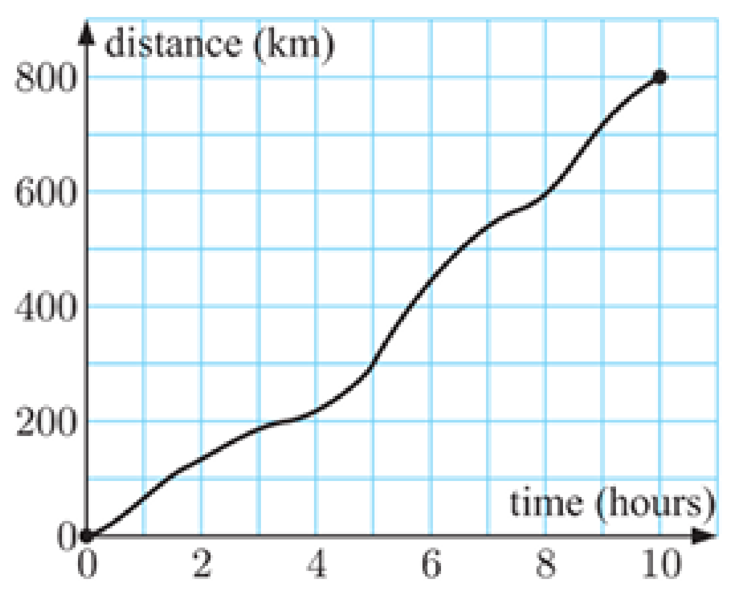
refers to how one quantity changes in relation to another
measures the ratio of the change in one variable to the change in another



$$\text{rate of flow} = \frac{15 - 0}{1 - 0}$$
$$= 15 \text{ litres per minute}$$

RATE OF CHANGE

refers to how one quantity changes in relation to another
measures the ratio of the change in one variable to the change in another



i the first 5 hours

average speed from $t = 0$ to $t = 5$ h

$$= \frac{(300 - 0)\text{ km}}{(5 - 0)\text{ h}}$$

$$= 60 \text{ km per hour}$$

ii the final 5 hours.

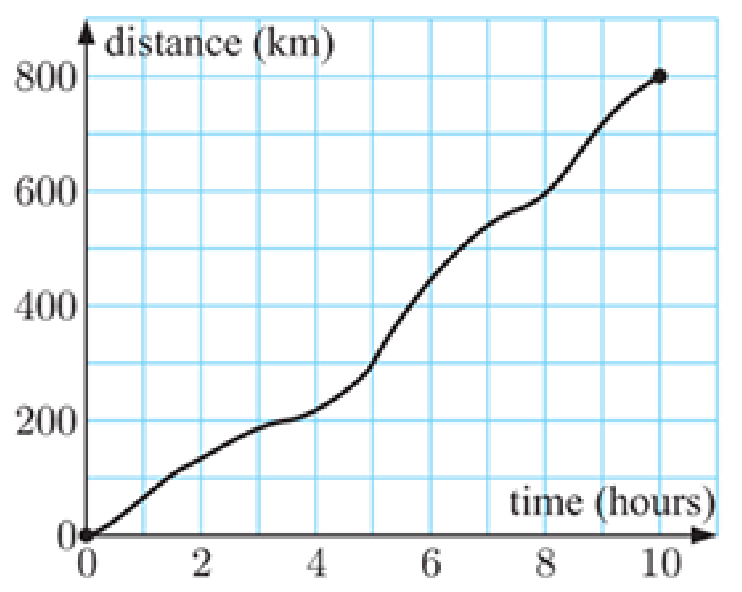
average speed from $t = 5$ h to $t = 10$ h

$$= \frac{(800 - 300)\text{ km}}{(10 - 5)\text{ h}}$$

$$= \frac{500}{5} \text{ km per hour}$$

$$= 100 \text{ km per hour}$$

AVERAGE RATE OF CHANGE



the first 5 hours

average speed from $t = 0$ to $t = 5$ h

$$= \frac{(300 - 0)\text{ km}}{(5 - 0)\text{ h}}$$

$= 60$ km per hour

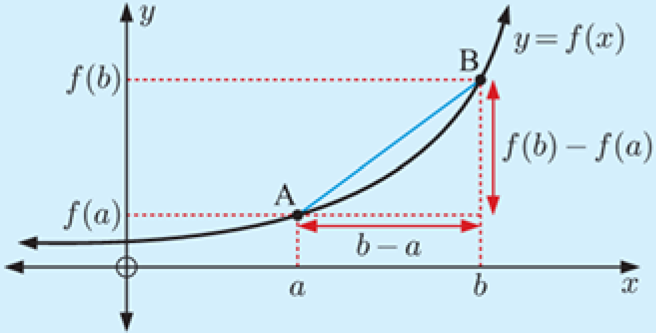
Measures the overall change in a function over a given interval

AVERAGE RATE OF CHANGE

Measures the overall change in a function over a given interval

The **average rate of change** in $f(x)$ from $x = a$ to $x = b$ is $\dfrac{f(b) - f(a)}{b - a}$.

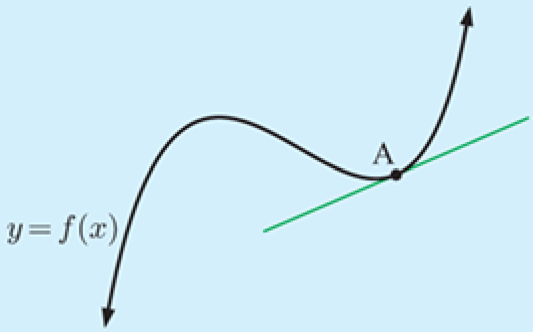
This is the **gradient of the chord [AB]**.



INSTANTANEOUS RATE OF CHANGE

Measures how a function changes at a single point (specific instant)

The **instantaneous rate of change** in $f(x)$ at any point A on the curve is the **gradient of the tangent** at A.

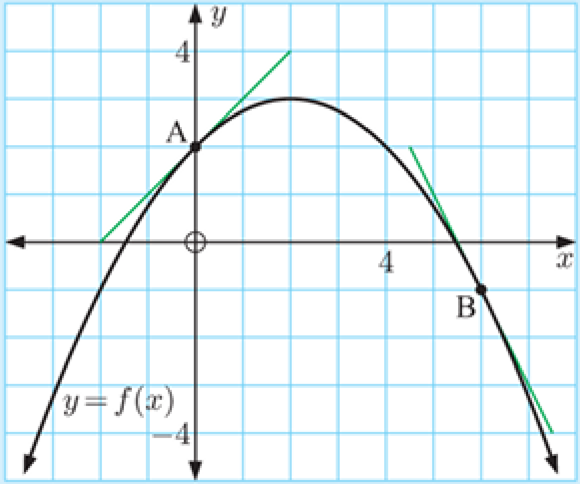


INSTANTANEOUS RATE OF CHANGE

Measures how a function changes at a single point (specific instant)

Use the tangents drawn to find the instantaneous rate of change in $y = f(x)$ at:

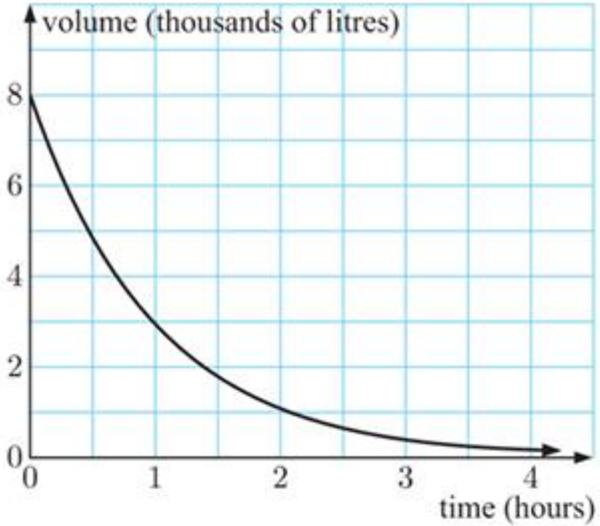
a A **b** B



INSTANTANEOUS RATE OF CHANGE

Water is leaking from a tank. The volume of water left in the tank over time is shown on the graph alongside.

- **a** How much water was in the tank originally?
- **b** How much water was in the tank after 1 hour?
- **c** How quickly was the tank losing water initially?
- **d** How quickly was the tank losing water after 1 hour?
- **e** Describe what happens to the rate at which the water is leaking.



INSTANTANEOUS RATE OF CHANGE

a Initially at time 0 hours, the volume is 8 thousands of litres.
So, there were 8000 L in the tank originally.

b At time 1 hour, the volume is 3 thousands of litres.
So, after 1 hour there were 3000 L in the tank.

c The tangent at time $t = 0$ hours passes through $(0, 8)$ and $(0.75, 0)$.

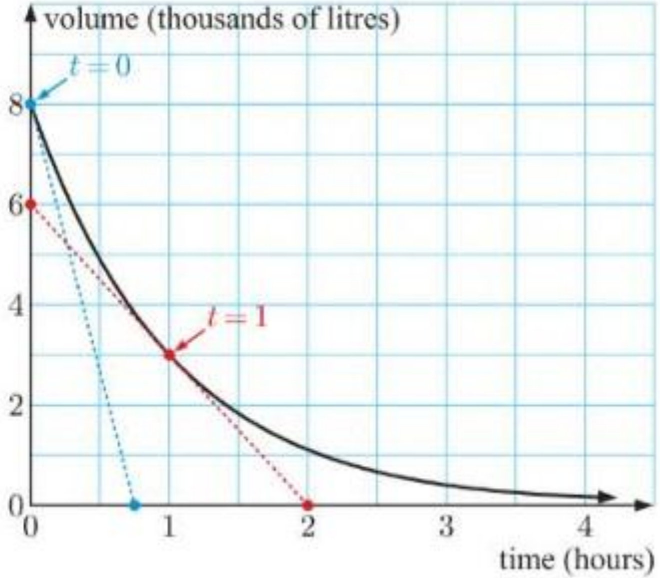
$\therefore$ initial rate of water loss

$= $ gradient of tangent at time 0 hours

$$\approx \frac{(0-8) \text{ thousand L}}{(0.75-0) \text{ hours}}$$

≈ -10.667 thousand L per hour

$\therefore$ initially the rate of water loss was about 10 700 L per hour.



INSTANTANEOUS RATE OF CHANGE

d The tangent at time $t = 1$ passes through $(0, 6)$ and $(2, 0)$.

$\therefore$ rate of water loss after 1 hour = gradient of tangent at time 1 hour

$$\approx \frac{(0-6) \text{ thousand L}}{(2-0) \text{ hours}}$$

$$\approx -3 \text{ thousand L per hour}$$

$\therefore$ after 1 hour the rate of water loss was about 3000 L per hour.

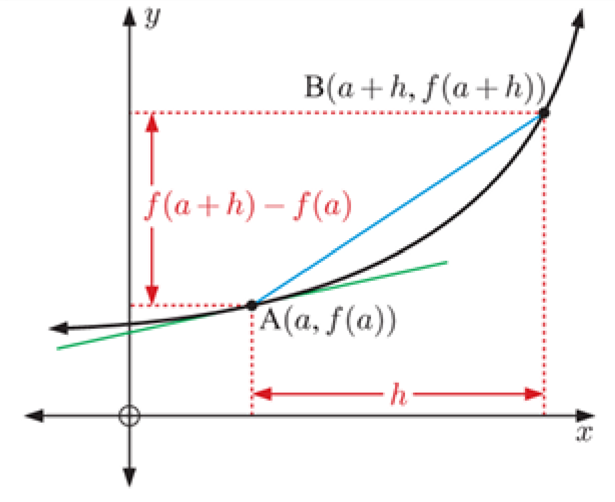
e The rate at which the tank is leaking water is decreasing.

Imagine that you are given 50 cm of wire and are told to use the wire to form a rectangle with the largest possible area. What dimensions should the rectangle have?

LIMITS

If $f(x)$ is as close as we like to some real number A for all x sufficiently close to (but not equal to) a, then we say that $f(x)$ has a **limit** of A as x approaches a, and we write $\lim_{x \to a} f(x) = A$.

In this case, $f(x)$ is said to **converge** to A as x approaches a.



LIMITS

Evaluate:

a $\lim_{x \to 2} x^2$ **b** $\lim_{x \to 0} \frac{x^2 + 3x}{x}$ **c** $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

a x^2 can be made as close as we like to 4 by making x sufficiently close to 2.

$$\therefore \quad \lim_{x \to 2} x^2 = 4.$$

b

$$\begin{aligned} & \lim_{x \to 0} \frac{x^2 + 3x}{x} \\ &= \lim_{x \to 0} \frac{x(x+3)}{x} \\ &= \lim_{x \to 0} (x+3) && \{\text{since } x \neq 0\} \\ &= 3 && \{\text{as } x \to 0,\ x+3 \to 3\} \end{aligned}$$

c

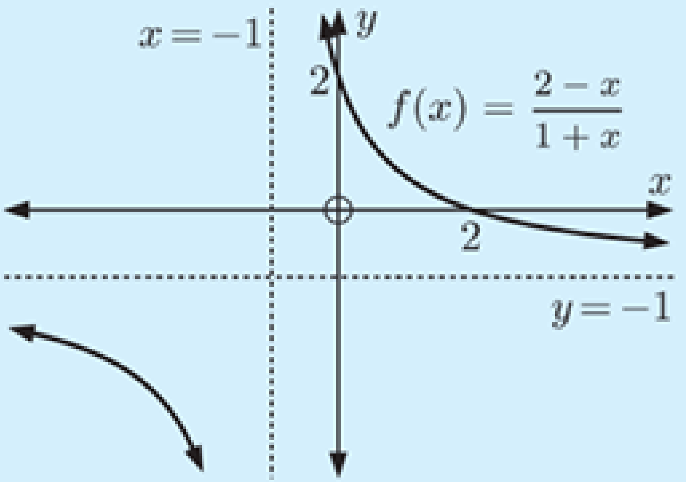
$$\begin{aligned} & \lim_{x \to 3} \frac{x^2 - 9}{x - 3} \\ &= \lim_{x \to 3} \frac{(x+3)(x-3)}{x-3} \\ &= \lim_{x \to 3} (x+3) && \{\text{since } x \neq 3\} \\ &= 6 && \{\text{as } x \to 3,\ x+3 \to 6\} \end{aligned}$$

Use tables and/or graphs [GDC] in finding the limits of functions

LIMITS AT INFINITY

a Discuss the behaviour of $f(x) = \dfrac{2-x}{1+x}$ near its asymptotes, and hence deduce their equations.

a As $x \to -1^-$, $f(x) \to -\infty$
As $x \to -1^+$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -1^-$
As $x \to \infty$, $f(x) \to -1^+$
The vertical asymptote is $x = -1$.
The horizontal asymptote is $y = -1$.

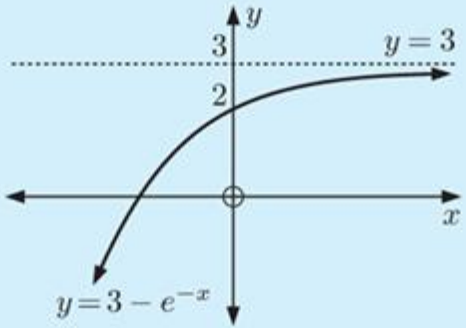


b If they exist, state the values of $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$.

b $\lim\limits_{x\to-\infty} f(x) = -1$ and $\lim\limits_{x\to\infty} f(x) = -1$.

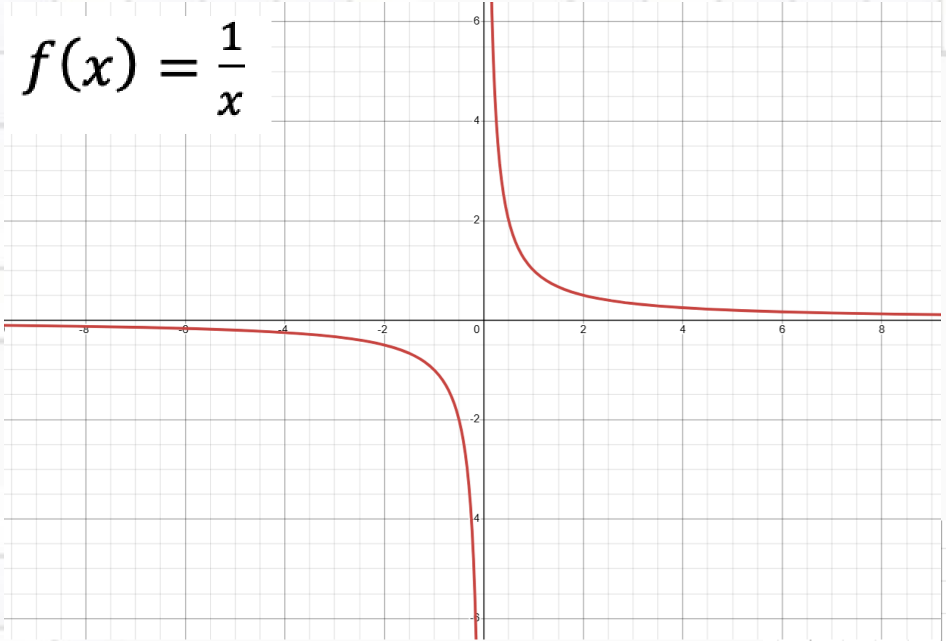
LIMITS AT INFINITY

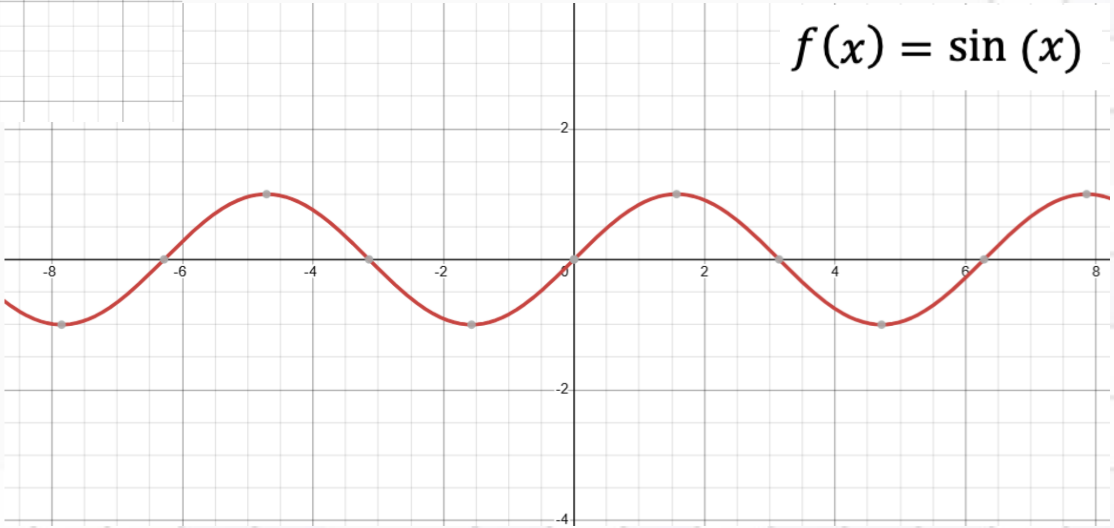
Find, if possible: **a** $\lim_{x \to -\infty} (3 - e^{-x})$ **b** $\lim_{x \to \infty} (3 - e^{-x})$.



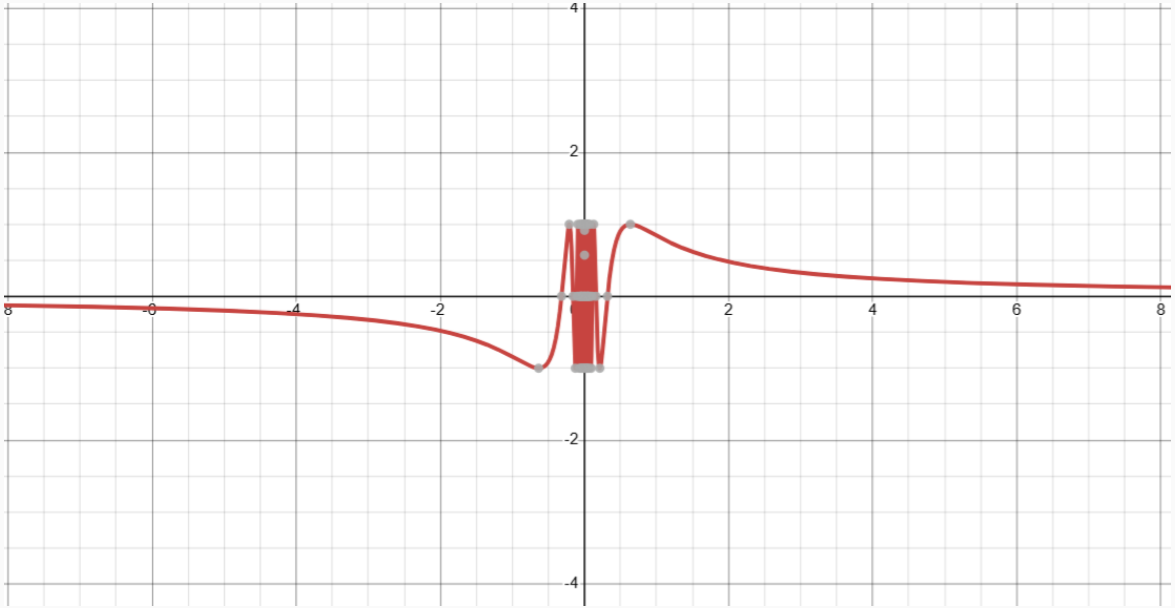
a As $x \to -\infty$, $3 - e^{-x} \to -\infty$.
Since $3 - e^{-x}$ does not approach a finite value, $\lim_{x \to -\infty} (3 - e^{-x})$ does not exist.

b As $x \to \infty$, $3 - e^{-x} \to 3^-$
$\therefore \lim_{x \to \infty} (3 - e^{-x}) = 3$

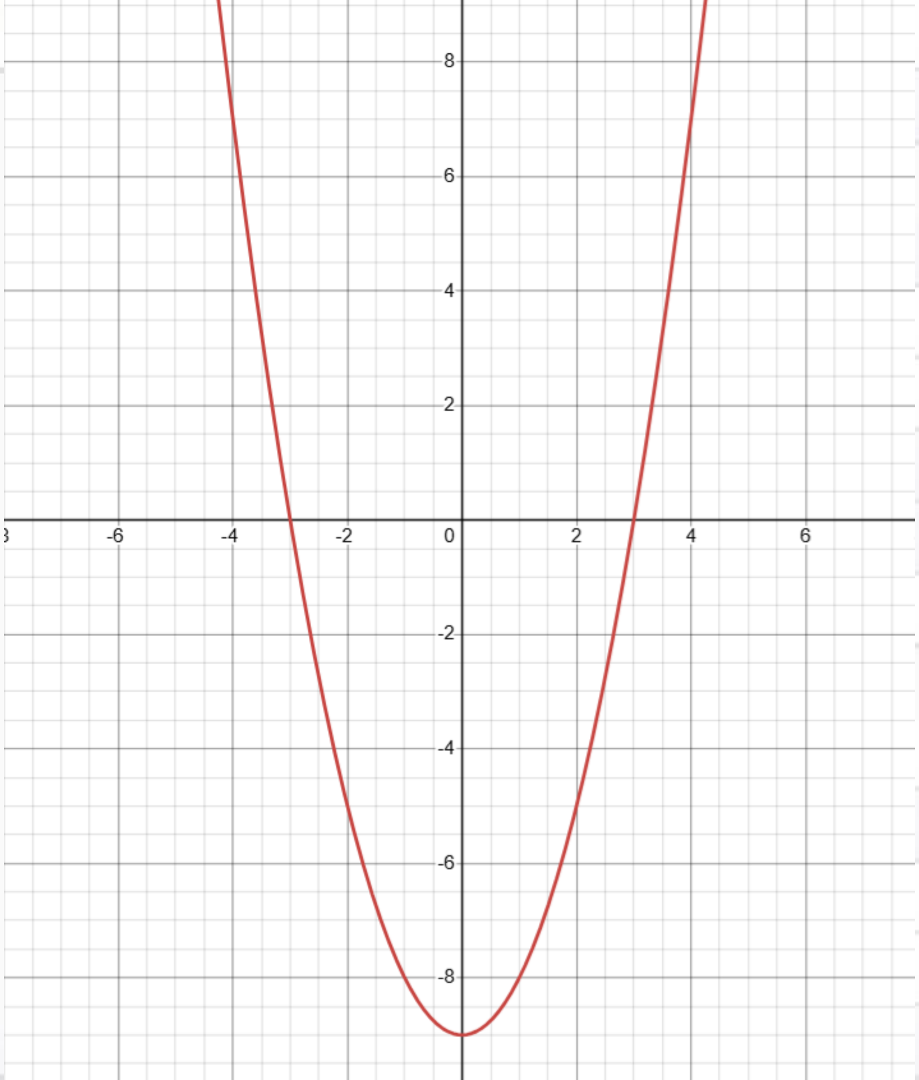

$f(x) = \frac{1}{x}$


$f(x) = \sin(x)$

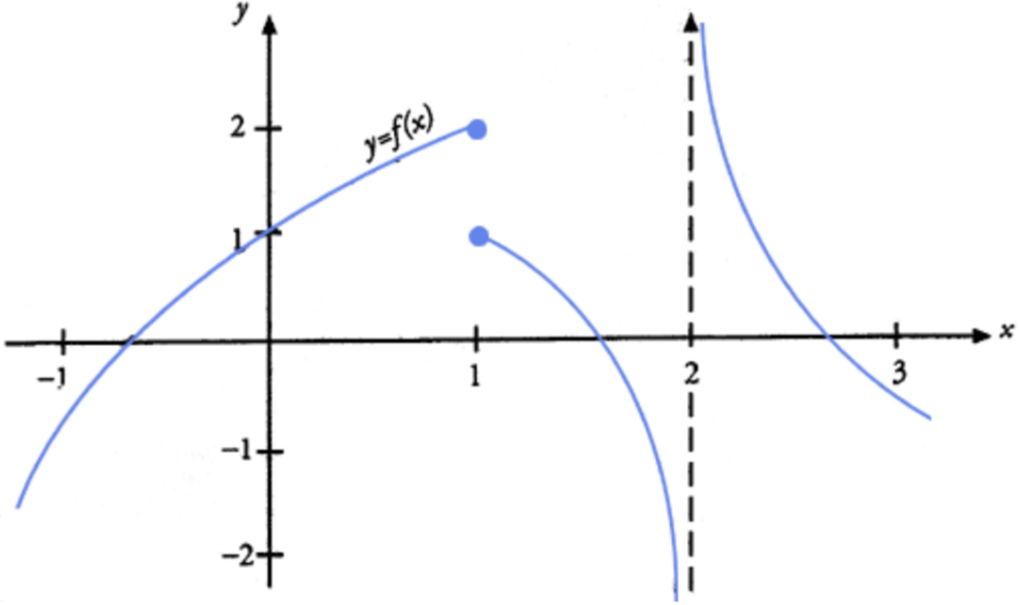
$$f(x) = \sin\left(\frac{1}{x}\right)$$



$$\lim_{x\to 2}\frac{x^3-2x^2-9x+18}{x-2}$$

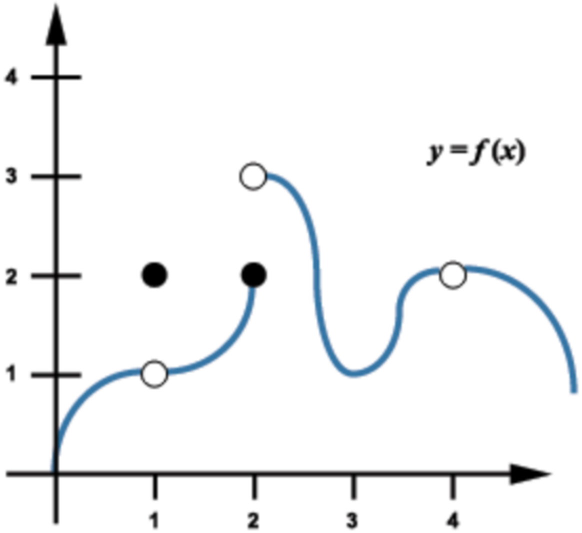


$\lim_{x\to 0} f(x)$ $\lim_{x\to 1} f(x)$ $\lim_{x\to 1^-} f(x)$ $\lim_{x\to 2^-} f(x)$



Using the graph of f as shown o

1. The value of $\lim_{x\to 0} f(x)$
2. The value of $\lim_{x\to 1} f(x)$
3. The value of $\lim_{x\to 1^-} f(x)$
4. The value of $\lim_{x\to 2^-} f(x)$



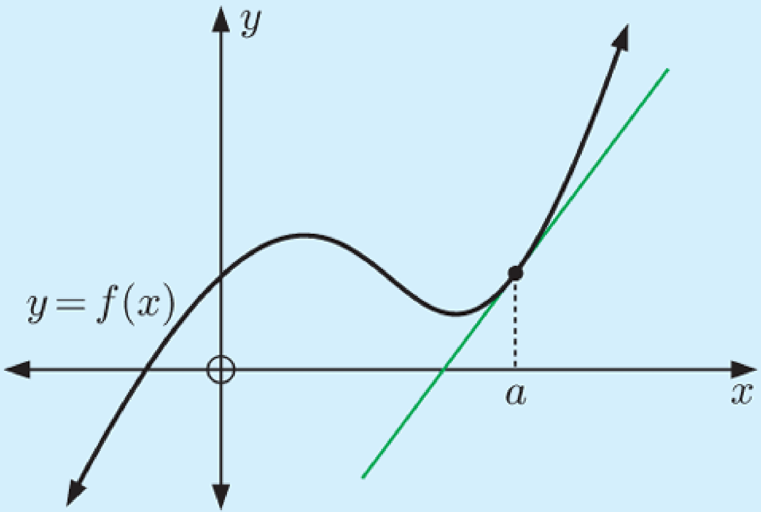
Using the graph of f as shown on

1. The value of $\lim_{x \to 1} f(x)$
2. The value of $\lim_{x \to 2} f(x)$
3. The value of $\lim_{x \to 2^+} f(x)$
4. The value of $\lim_{x \to 3} f(x)$
5. The value of $\lim_{x \to 4} f(x)$

GRADIENT OF A TANGENT

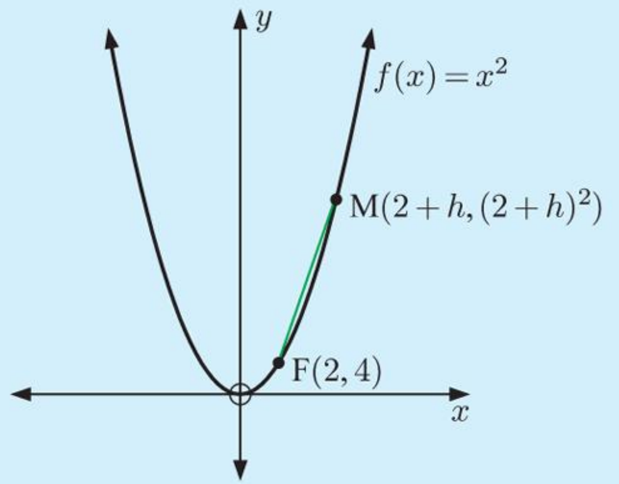
The **gradient of the tangent** to the curve $y = f(x)$ at the point where $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$



GRADIENT OF A TANGENT

Find the gradient of the tangent to $f(x) = x^2$ at the point $(2, 4)$.



The gradient of the tangent at F

$$
\begin{aligned}
&= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h \to 0} \frac{(2+h)^2 - 4}{h} \\
&= \lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h} \\
&= \lim_{h \to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} \\
&= \lim_{h \to 0} (4+h) \qquad \{\text{as } h \neq 0\} \\
&= 4
\end{aligned}
$$

GRADIENT OF A TANGENT

Find the gradient of the tangent to $y = \dfrac{1}{x}$ at the point where: $x = 1$

$$\begin{aligned}\frac{1}{x+h} - \frac{1}{x} &= \frac{x-(x+h)}{x(x+h)} \\ &= \frac{-h}{x(x+h)}\end{aligned}$$

DIFFERENTIATION FROM THE FIRST PRINCIPLE

find $f'(x)$ from first principles: $f(x) = 3 - x$

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{[3 - (x+h)] - [3 - x]}{h} \\
&= \lim_{h \to 0} \frac{\cancel{3} - \cancel{x} - h - \cancel{3} + \cancel{x}}{h} \\
&= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}} \\
&= \lim_{h \to 0} -1 \qquad \{\text{as } \; h \neq 0\} \\
&= -1
\end{aligned}
$$

DIFFERENTIATION FROM THE FIRST PRINCIPLE

Find $\frac{dy}{dx}$ from first principles: $y = 2x^2 + x$

$$
\begin{aligned}
y &= f(x) = 2x^2 + x \\
\frac{dy}{dx} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{[2(x+h)^2 + (x+h)] - [2x^2 + x]}{h} \\
&= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + \cancel{x} + h - 2x^2 - \cancel{x}}{h} \\
&= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + h - \cancel{2x^2}}{h} \\
&= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h} \\
&= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 1)}{\cancel{h}} \\
&= \lim_{h \to 0} (4x + 2h + 1) \qquad \{\text{as} \quad h \neq 0\} \\
&= 4x + 1
\end{aligned}
$$

Thanks!

Do you have any questions?
youremail@freepik.com
+34 654 321 432
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