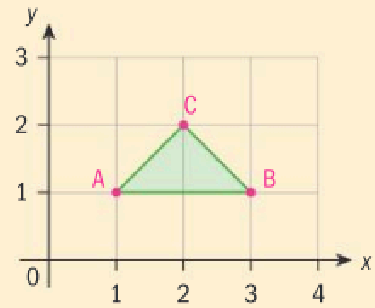
MATRIX TRANSFORMATIONS

3.9	Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$, reflection in the line $y = (\tan\theta)x$ $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, horizontal stretch / stretch parallel to x-axis with a scale factor of k $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$, vertical stretch / stretch parallel to y-axis with a scale factor of k $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, enlargement, with a scale factor of k, centre $(0, 0)$ $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, anticlockwise/counter-clockwise rotation of angle θ about the origin ($\theta > 0$) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, clockwise rotation of angle θ about the origin $(\theta > 0)$

Points in the plane can be represented by their position vectors. In the diagram below, for example, the position vector of A is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.



The position vectors of the vertices of the triangle ABC can be put in a single matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.

1 Find the product of $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.

Let the columns of the new matrix be the position vectors of the **image** of triangle ABC under the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

2 On a copy of the diagram above draw the triangle ABC and its image after the transformation.

What is the transformation?

3 In the same way use the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ to find the image of (1, 0) and (0, 1) under this transformation.

What do you notice about the image matrix?

4 Test your conjecture by considering the image of (1, 0) and (0, 1) under this transformation represented by the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

5 By considering the images of (1, 0) and (0, 1) suggest a matrix that represents an enlargement scale factor 2, centre (0, 0).

A matrix representing a linear transformation can be written $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ where (a, b) is the image of $(1, 0)$ and (c, d) the image of $(0, 1)$.

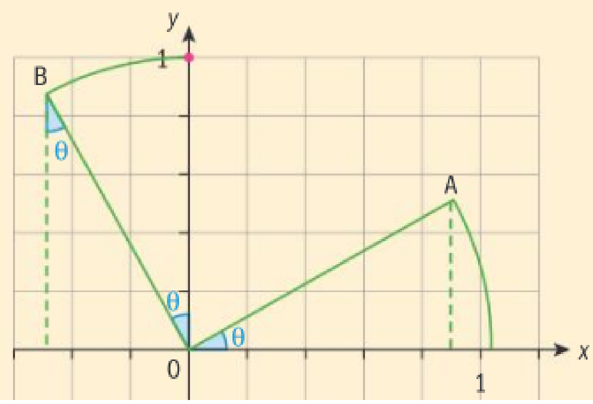
6 Find the matrix that represents a rotation of 90° clockwise about (0, 0).

7 Find the matrix that represents a stretch parallel to the x-axis with a scale factor of 2 and the y-axis invariant.

8 a In the diagram below A and B are the images of (1, 0) and (0, 1) under a counter clockwise rotation of θ about (0, 0). Use the diagram to show this rotation is represented by the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

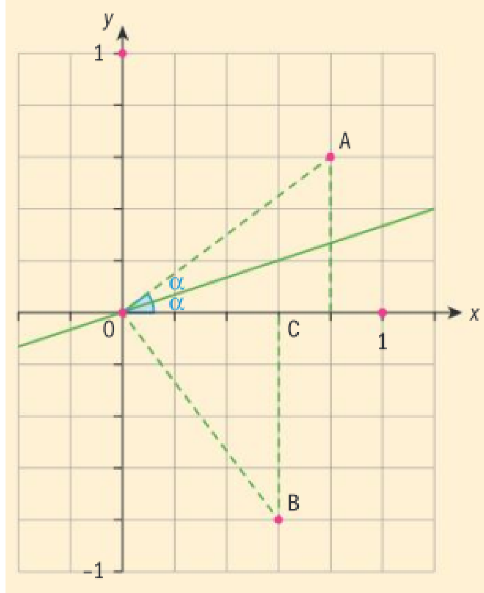
b What will be the matrix for a clockwise rotation of magnitude θ?

c Hence write down the matrix that represents a rotation of 60° clockwise about (0, 0).



9 **a** The line $y = mx$ can be written as $y = (\tan\alpha)x$, where α is the angle made with the x-axis.

In the diagram below A and B represent the images of (1, 0) and (0, 1) respectively under a reflection in the line $y = (\tan\alpha)x$.



a Explain why the image of (1, 0) has coordinates $(\cos 2\alpha, \sin 2\alpha)$.

b By finding $O\hat{B}C$ in terms of α find the image of (0, 1) under the transformation.

c Hence verify that the matrix that represents a reflection in the line $y = (\tan\alpha)x$ is $\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$;

d By first finding α, determine the matrix that represents a reflection in the line $y = \sqrt{3}x$.

10 By considering the images of (1, 0) and (0, 1) find the general matrices for:

a a one-way stretch, parallel to the x-axis, scale factor k

b an enlargement scale factor k, centre (0, 0).