LOGISTIC MODELS

WEEK 12 – OCTOBER 18, 2023

OBJECTIVES

- TO DEFINE A LOGISTIC MODEL
- TO SKETCH THE GRAPH OF A LOGISTIC FUNCTION
- TO SOLVE PROBLEMS INVOLVING LOGISTIC FUNCTIONS AND MODELS

The progression of the world record for men's high jump at the beginning of each year since 1912 is shown on the scatter diagram.

How could you describe the basic features of the progression of the world record?

LOGISTIC FUNCTION

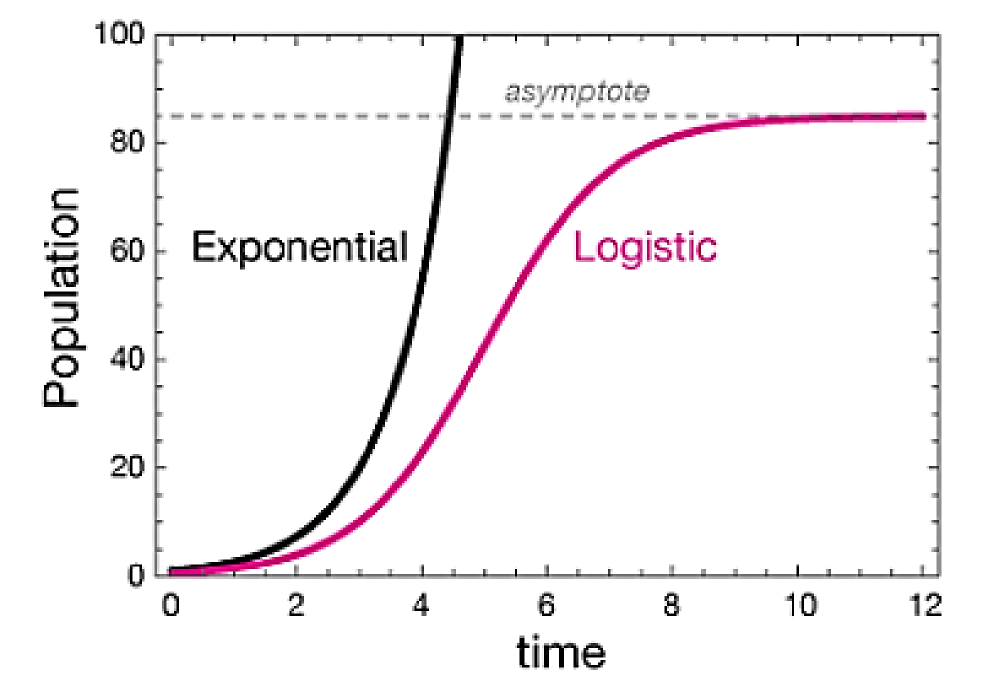
- This is used to model situations where there is a restriction on the growth.

LOGISTIC FUNCTION OR NOT?

- the increase in height of a person or seeding
- saving 10 dollars every month until you reach the desired amount
- hours worked compared to the amount of money earned
- spreading rumors and disease in a limited population

LOGISTIC FUNCTION

- The logistic function models the exponential growth of a population, but also considers factors (like the carrying capacity of land).
- Ex: A certain region simply won't support unlimited growth because as one population grows, its resources diminish.
- So, **a logistic function puts a *limit* on growth**.



https://xaktly.com/LogisticFunctions.html

LOGISTIC FUNCTION

The equation of a logistic function has the standard form $f(x) = \dfrac{L}{1 + Ce^{-kx}}$,

where L, C and k are the parameters of the function and e is Euler's number.

Horizontal asymptote: $f(x) = L$

the limit on the size of a population

y-intercept: $\dfrac{L}{1+C}$

Sketch the graph of the given functions. State the horizontal asymptote and the y-intercept.

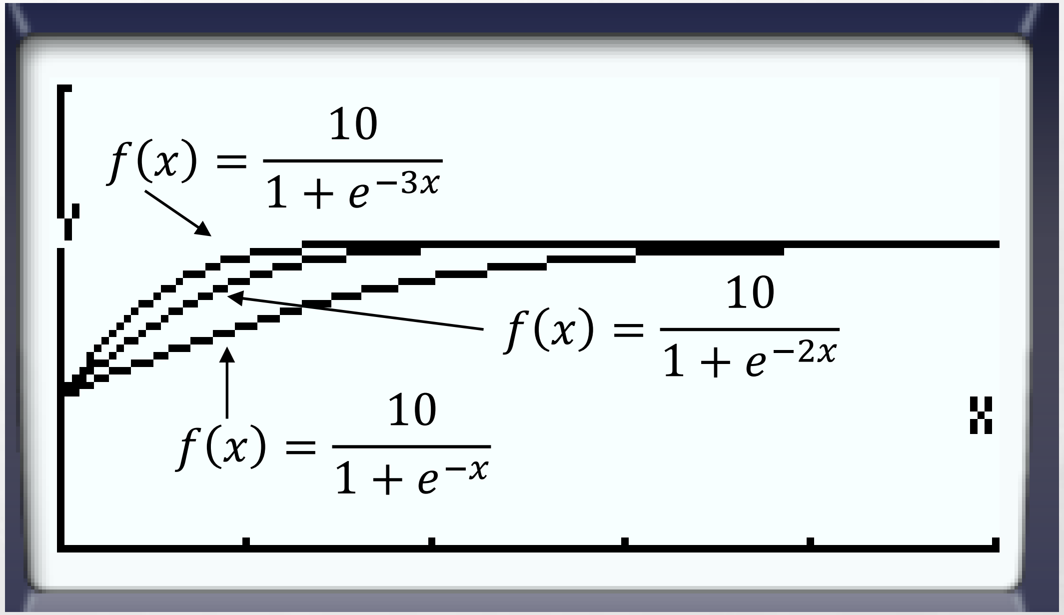
Function: $f(x) = \frac{L}{1+Ce^{-kx}}$	**Horizontal Asymptote:** $f(x) = L$	**Y-intercept:** $\frac{L}{1+C}$
$f(x) = \frac{10}{1+e^{-x}}$	$f(x)\ or\ y = 10$	$y = 5$
$f(x) = \frac{20}{1+e^{-x}}$	$f(x)\ or\ y = 20$	$y = 10$
$f(x) = \frac{30}{1+e^{-x}}$	$f(x)\ or\ y = 30$	$y = 15$

Sketch the graph of the given functions. State the horizontal asymptote and the y-intercept.

1. $f(x) = \frac{10}{1+e^{-x}}$

2. $f(x) = \frac{10}{1+e^{-2x}}$

3. $f(x) = \frac{10}{1+e^{-3x}}$



What is the relation of the parameter multiplying x to the graph of the function?

LOGISTIC FUNCTION

- Logistic functions have a varying rate of change.
- The most usual shape of a logistic function starts with the data being almost constant (close to zero rate of change), then the data show a rapid increase, and finally become almost constant again (close to zero rate of change again).

MORE EXAMPLES

1 The function that models the percentage of people globally with access to broadband Internet with respect to time is given by the logistic function $P(t) = \dfrac{100}{1+10e^{-0.5t}}$, $t \geq 0$.

a Sketch the graph of the function.

b Write down the range of values of the function and interpret their significance.

c Use the model to predict how long it will take for half the people to have access to broadband Internet.

d Calculate the percentage of people who will have access to broadband Internet in 20 years.

2 Data for the population of the Virgin Islands from 1950 to 2015 have produced the following logistic function model.

$P(t) = \dfrac{107000}{1 + 4e^{-0.135t}}$, where t is the number of years that have passed since 1950.

a Estimate the population of the Virgin Islands in 1950.

b Estimate the population of the Virgin Islands in 2015.

c Assuming that this model will also be valid in the future, state the largest population that the Virgin Islands will ever be able to accommodate.