FINANCIAL MATHEMATICS

Week 22 – January 22 to 26, 2024

Objectives

- To differentiate between simple and compound interest
- To solve problems involving simple and compound interest:
  - using the formula
  - using technology (GDC)

setrenit

Interest

- money paid regularly at a particular rate for the use of money lent, or for delaying the repayment of a debt.

naol

loan

- a form of debt incurred by an individual or other entity

atgrepnece

Percentage

- is a way of expressing a number as a fraction of 100
- is used to calculate interest on money

nanifec

Finance (Financial Math)

- describes the application
of mathematics and mathematical modeling
to solve financial problems.

(CFI Team. "Financial Mathematics." *Corporate Finance Institute*, 26 May 2020, corporatefinanceinstitute.com/resources/data-science/financial-mathematics/.)

SIMPLE INTEREST

January 22, 2024

Terms

- Principal or Capital – the money that you initially put in a savings institution or a bank
- Account Balance – the total amount of money either saved or owed at a given point in time
- Interest – the percentage of the principal or account balance paid to you (for a savings account) or paid by you (for a loan)
- Simple Interest – a fixed percentage of the principal and hence is a type of arithmetic sequence

SIMPLE INTEREST

- The total amount of simple intrest I earned on a principal P over n years at an interest rate of $r\%$ per year can be calculated using the formula:

$$I = Prn \text{ (simple interest earned)}$$

- The total savings account balance is given by:

$$A = P + (Prn) \text{ (total balance including the simple interest earned)}$$

Example 1

An amount of $5000 is invested at a simple interest rate of 3% per annum (p.a., meaning each year) for a period of 8 years.

a. Calculate the interest received after 8 years.
b. Find the account balance after 8 years.
c. Find the time it will take for the account balance to double.

a \$1200	Substituting in the formula $I = P \times r \times n$: $I = 5000 \times 0.03 \times 8 = 1200$
b \$6200	The account balance is found by adding the interest to the capital: $5000 + 1200 = 6200$
c 34 years	The account balance must be double its initial principal: $2 \times 5000 = 10\,000$. Substituting and solving with technology: $10\,000 = 5000 + 5000 \times 0.03 \times n$ $n = 33.3$ (3 s.f.) As the interest is calculated at the end of the year, the principal will not exceed double until the 34th year.

Example 2

You paid a total of $1500 in simple interest on a car loan over 5 years with a 2.3% interest rate p.a. Find, to the nearest dollar, the original value of the car.

$P = \$13\,043$	Using $I = P \times r \times n$: $1500 = P \times 0.023 \times 5$ $P = 13\,043$ Note that for a loan, simple interest is paid to the bank. It is not added to the account balance as savings interest is.

Let’s Try

First five☺

1.

Find the simple interest payable on an investment of $3000 at 11% per annum (p.a.) for 2 years.

Answer: $660

2.

Find the simple interest payable on an investment of €5500 at $7\frac{1}{2}\%$ p.a. for 5 years. (1 d.p)

Answer: €2062.5

BEYOND

3.

Find the simple interest payable on an investment of £15000 at 5.75% p.a. for $4\frac{1}{2}$ years. (2 d.p)

Answer: £3881.25

4.

Find the simple interest payable on an investment of $25000 at 6.2% p.a. for 10 years.

Answer: $15500

5.

Find the simple interest payable on a loan of €2000 at 1.4% p.a. over 9 months.

Answer: €21

6.

Find the simple interest payable on a loan of $8500 at 8% p.a. over 3 months.

Answer: $170

BEYOND

7.

Find the simple interest payable on a loan of ¥2 750 000 at 7% p.a. over 18 months.

Answer: ¥288 750

8.

Find the simple interest payable on a loan of $60 000 at 3.25% p.a. over 10 months.

Answer: $1625

9.

Cameron borrows €15 000 from a bank to renovate his house. He borrows the money at 9% p.a. **simple interest** over 8 years. What are his **monthly** repayments? (2 d.p)

Answer: €268.75

BEYOND

COMPOUND INTEREST

January 22, 2024

COMPOUND INTEREST

- If the interest paid compound interest, then the interest is added to the original amount and the new value is used to calculate the interest for the next period.
- If you invest a **present** value *PV*, the rate is *r%* compounded annually, and the number of years *n*, then the future value *FV* is found by the formula:

$$FV = PV(1 + \frac{r}{100})^n$$

Example 1

Pater invests AUD 2000 (Australian dollars) in a bank that offers simple interest at a rate of 3% per annum.

Paul invests AUD 2000 in another bank that offers 2.9% interest compounded annually.

a. Calculate how much money they each have in the bank after 10 years.
b. Determine after how many years they will have at least AUD 5000 in their accounts.

a Peter:

3% of 2000 = AUD 60

$60 \times 10 = 600$

So at the end of 10 years, Peter has AUD 600 + his original AUD 2000 = AUD 2600

Paul:

Using the formula, Paul has

$$FV = 2000\left(1+\frac{2.9}{100}\right)^{10}$$

FV = AUD 2661.85

b Peter:

3% of 2000 = AUD 60

So, Peter earns AUD 60 interest each year.

In order to have AUD 5000, Peter will have to make 5000 – 2000 = AUD 3000 interest.

So, it will take $\frac{3000}{60} = 50$ years before he has AUD 5000 in the bank.

Paul:

Using the formula:

$$5000 = 2000\left(1+\frac{2.9}{100}\right)^{n}$$

So $n = 32.05$

It will take Paul just over 32 years to have AUD 5000 in the bank.

Let’s Try Again

First five☺

1.

What will an investment of $3000 at 9% p.a. compound interest amount to after 5 years? Write your answer in 2 d.p.

Answer: $4615.87

2.

How much compound interest is earned by investing 20 000 dirhams for 6 years at 4.5% p.a.? Round off your answer to 2 d.p.

Answer: 6045.20 dirhams

3.

Pal wants to invest £400 in a bank account for 3 years. Which bank account should she choose? Explain your answer.

Account A	Account B
3% simple interest per annum	2.5% compound interest per annum

Account A:

$0.03 \times 400 = £12$

$12 \times 3 = £36$

$400 + 36 = £436$

Account B:

$400 \times 1.025^3 = £430.76$ (2 d.p.)

Pal should choose bank account A.

COMPOUND INTEREST
(COMPOUNDING PERIOD)

January 25, 2024

Compounding Period

- The compounding period is the time period between interest payments. (For example, for interest compounded quarterly, the compounding period is three months.)

- If *PV* is the present value, *FV* the future value, *r* the interest rate, *n* the number of years and *k* the compounding frequency or number of times interest is paid in a year (i.e., k = 1 for yearly, k = 2 for half-yearly, k = 4 for quarterly, k = 12 for monthly), then the general formula for finding the future value is:

$$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$$

Example 1

Rafael invests BRL 5000 (Brazilian real) in a bank offering 2.5% interest compounded annually.

a Calculate the amount of money he has after five years.

After the five years, Rafael withdraws all his money and puts it in another bank that offers 2.5% interest per annum compounded monthly.

b Calculate the amount of money that he has in the bank after three more years.

a $FV = 5000\left(1 + \frac{2.5}{1 \times 100}\right)^{1\times 5}$ $\quad PV = 5000,\ r = 2.5,\ k = 1,\ n = 5$

$= \text{BRL } 5657.04$

b $FV = 5657.04\left(1 + \frac{2.5}{12 \times 100}\right)^{12\times 3}$ $\quad PV = 5657.04,\ r = 2.5,\ k = 12,\ n = 3$

$= \text{BRL } 6097.16$

Example 1 (GDC use)

Rafael invests BRL 5000 (Brazilian real) in a bank offering 2.5% interest compounded annually.

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b Calculate the amount of money that he has in the bank after three more years.

F2: CMPD

a N = 5

I% = 2.5

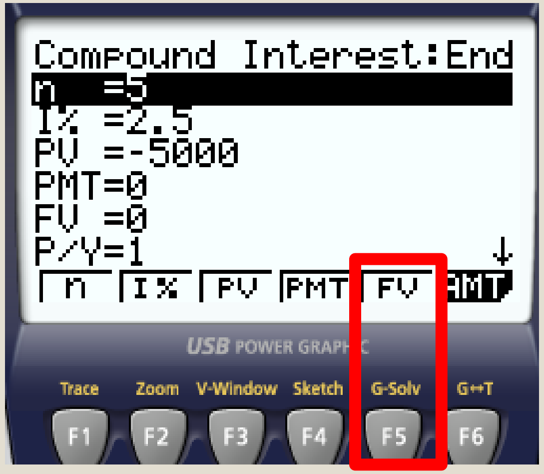
PV = −5000

PMT = 0

FV =

PpY = 1

CpY = 1



N: Number of years
I%: Interest rate in %
PV: Present Value (usually negative because you have given it to the bank)
PMT: Periodic Monthly Transfer (or a fixed payment made each period)
FV: Future Value
PpY: Periods per Year (when dealing with years, this is always 1)
CpY: Compounding Periods (depending on how interest is paid.

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F2: CMPD

b N = 3

I% = 2.5

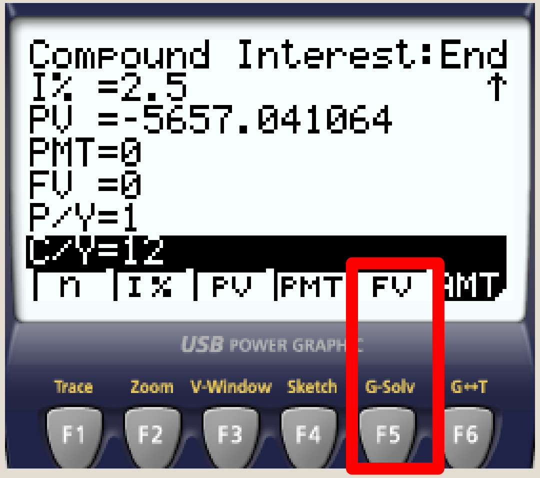
PV = −5657.04

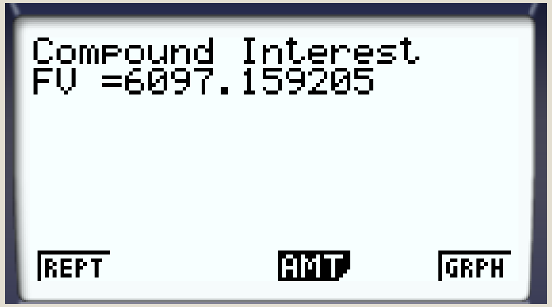
PMT = 0

FV =

PpY = 1

CpY = 12





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PpY: Periods per Year (when dealing with years, this is always 1)
CpY: Compounding Periods (depending on how interest is paid.

Example 2

Alexis invests RUB 80 000 (Russian ruble) in a bank that offers interest at 3% per annum compounded quarterly.

a Calculate how much money Alexis has in the bank after six years.

b Calculate how long it takes for his original amount of money to double.

a $FV = 80\,000\left(1+\frac{3}{4\times 100}\right)^{4\times 6}$

$= \text{RUB } 95\,713.08$

N = 6

I% = 3

PV = −80 000

PMT = 0

FV =

PpY = 1

CpY = 4

Move the cursor back to FV and press enter.

b $160\,000 = 80\,000\left(1+\frac{3}{4\times 100}\right)^{4\times n}$

Using the Finance app:

$n = 23.19$

So it would take 23 years for his money to double.

Double the original amount is RUB 160 000

N =

I% = 3

PV = −80 000

PMT = 0

FV = 160 000

PpY = 1

CpY = 4

Move the cursor back to N and press enter.

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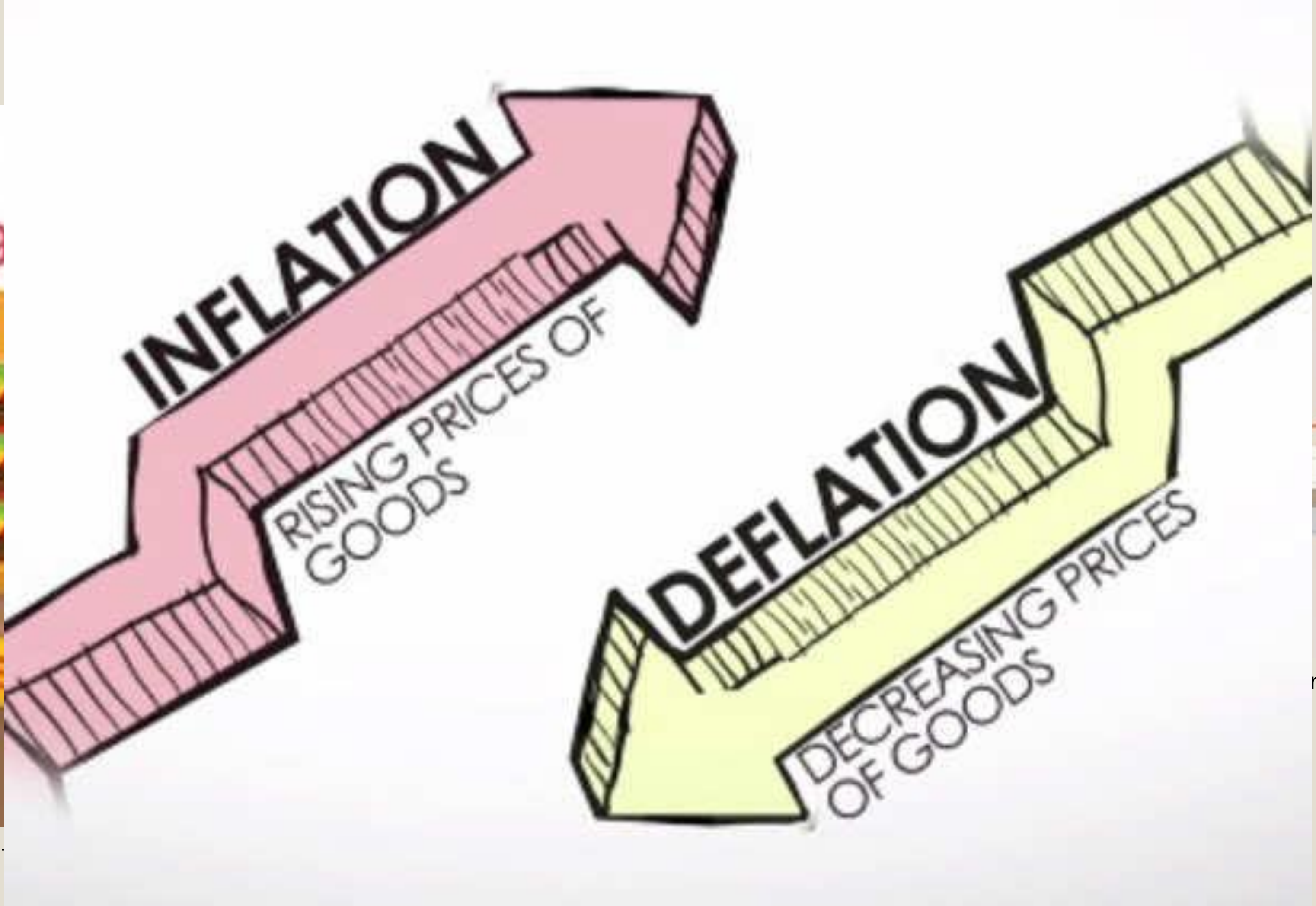

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Inflation

Inflation measures the rate that prices for goods increase over time and, as a result, how much less your money can buy.

This means that if inflation is at $i\%$ an investment which receives $r\%$ interest compounded annually will actually have its **real value** increased by only $(r - i)\%$.

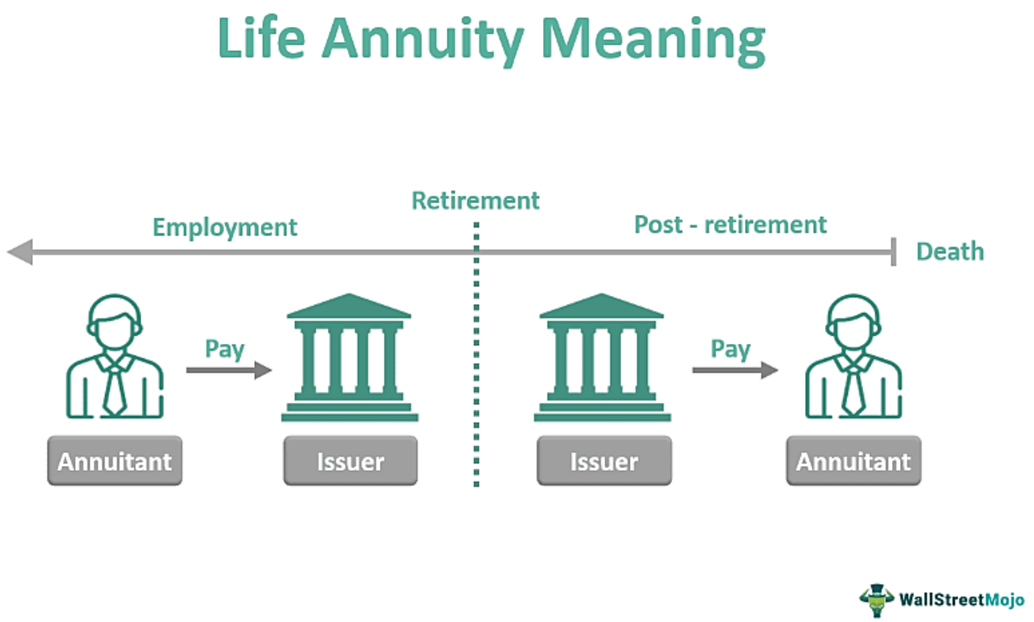
Therefore when adjusting for inflation to find the real value of an investment replace r by $(r - i)$ in the compound interest formula.

Example

Kathryn would like to buy a new house in five year's time.
The average price of houses in the area she is considering is €120 000.
She has €110 000 in an investment account, which is earning 5.2% interest per year compounded yearly. House inflation is expected to be 3.1% per year. Will she be able to afford an average price house in five year's time?

Investment will be worth $110\,000\left(1+\frac{5.2-3.1}{100}\right)^5$ $= €122\,045$ So she will be able to afford it	This can also be done using the financial application on your GDC entering the interest as 2.1%.

ANNUITY



Life Annuity Meaning
Retirement
Employment
Post - retirement
Death
Pay
Pay
Annuitant
Issuer
Issuer
Annuitant
WallStreetMojo

Annuity

When a constant investment, *P*, is made for *n* periods always compounded with the same interest rate, *r*%, it is called an annuity.

The formula for working out an annuity is

$$FV = A\frac{(1+r)^n - 1}{r}$$

where FV is the future value, A is the amount invested each year, r is the interest rate and n is the number of years.

Example 1

Jin decides to save for a yacht. He would like to have TRY 1 000 000 (Turkish Lira) at the end of 10 years. He saves every year in an annuity that pays 4% interest. Calculate how much money he has to save each year.

$$FV = A\frac{(1+r)^n - 1}{r}$$

N = 10

I% = 4

PV = 0

PMT =

FV = 1 000 000

PpY = 1

CpY = 1

Move the cursor to PMT and press Enter (or ALPHA ENTER)

This gives a PMT of TRY 83 290.94.

So, he will have to save TRY 83 290.94 every year for the next 10 years.

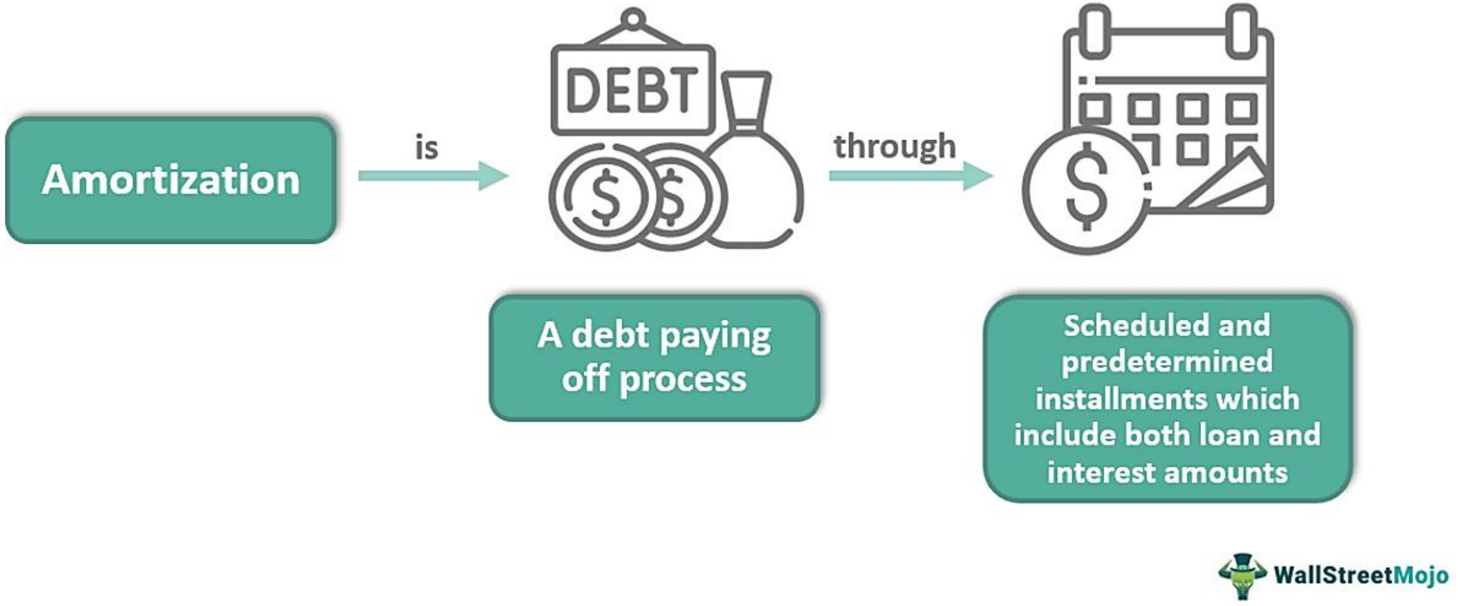
Example 2

Jimin has been left an annuity of €5000 in a will. The annuity is for five years at 8% per annum to be paid out monthly.

Find the monthly payments.

N = 60 I% = 8/12 PV = 5000 PMT = FV = 0 PpY = 1 CpY = 1 Move the cursor to PMT and press Enter (or ALPHA ENTER) This gives PMT = €101.38.	Because the money is paid each month, you have to multiply the number of years by 12. The interest rate per month is 8/12 = 0.666...%

AMORTIZATION



Amortization

When a constant payment, *P*, to repay a loan is made for a certain number of *n* periods always compounded with the same interest rate, *r%*, it is called amortization.

The formula to find the payments is

$$A = PV \frac{r(1+r)^n}{(1+r)^n - 1}$$

where A is the amount, PV is the present value, r is the rate and n is the number of periods.

Fortunately, you do not need to remember this formula because you can use your calculator in the exams.

You can use your GDC to work out payments, and so on.

Example

Namjoon takes out a loan of €35 000 for a car. The loan is for 10 years at 1% interest per month.

Find how much he has to repay each month.

N = 120

I% = 1

PV = 35 000

PMT =

FV = 0

PpY = 1

CpY = 1

Using your GDC, this gives PMT = €502.15.

YOUR TURN!

Jan. 29 (Monday) – Revision
Jan. 31 (Wed) – Quiz
(Arithmetic/Geometric/Financial Math)