Unit 3:Geometry and Trigonometry

VECTORS

AIHL 3.11 - 3.13

Vector Equations of a line

Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$

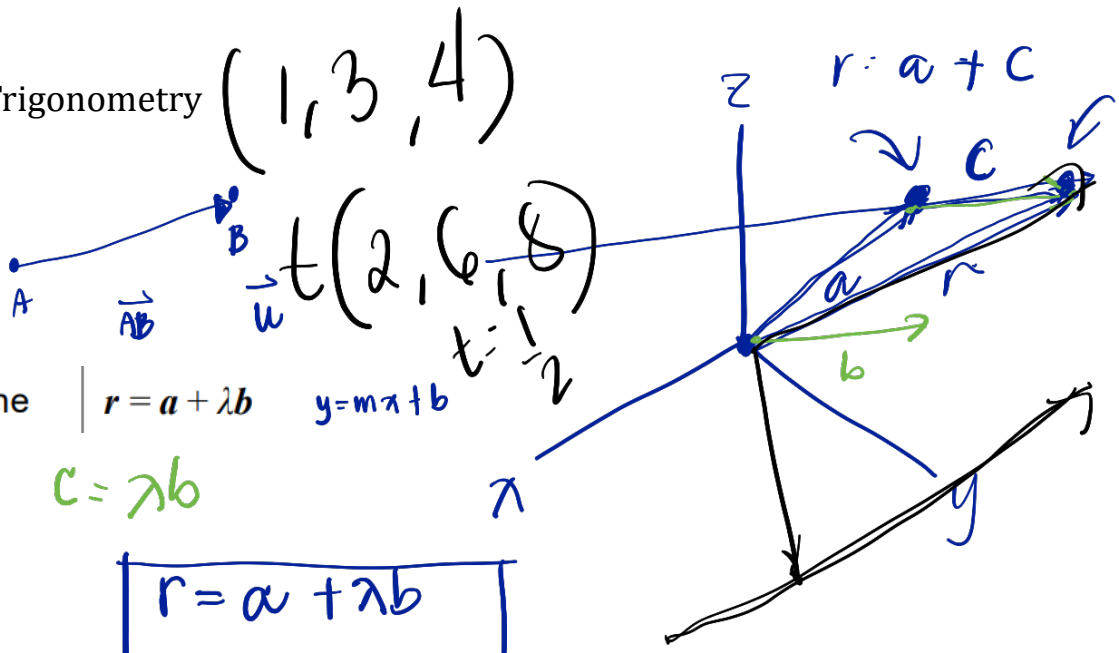
$y = mx + b$

$(1, 3, 4)$

$t(2, 6, 8)$

$t = \frac{1}{2}$

$\vec{AB}$ $\vec{u}$

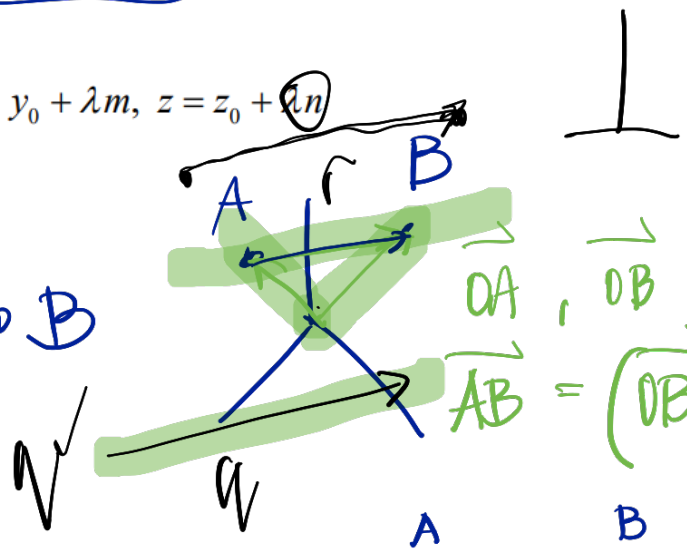


$c = \lambda b$

$r = a + \lambda b$

- a : position vector
- b : direction vector
- λ : scalar value

Parametric form of the equation of a line	$x = x_0 + \lambda l,\ y = y_0 + \lambda m,\ z = z_0 + \lambda n$



Direction Vector is the Vector from A to B

$\vec{OA}$, $\vec{OB}$

$\vec{AB} = (\vec{OB} - \vec{OA})$

Example 1. Write down the equation of the line that passes through the points (2, -1, 3) and (1, 4, -3). Write all the three forms of the equation of the line.

$r = a + \lambda b$

$r = (2, -1, 3) + \lambda(1-2, 4+1, -3-3)$

$r = (2, -1, 3) + \lambda(-1, 5, -6)$

$r = (2i - j + 3k) + \lambda(-i + 5j - 6k)$

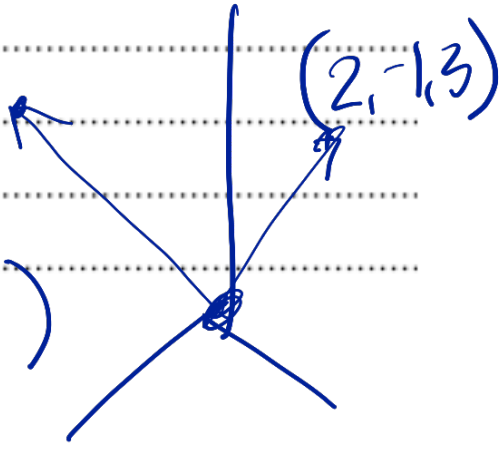
$\vec{v} = (4, 5)$

$\vec{v} = 4i + 5j$

$r = (2-\lambda)i + (-1+5\lambda)j + (3-6\lambda)k$

$\begin{cases} x = 2 - \lambda \\ y = -1 + 5\lambda \\ z = 3 - 6\lambda \end{cases}$

$(2-\lambda, -1+5\lambda, 3-6\lambda)$



Example 2. Determine if the that line (from example 1) passes through the point (0, 3, 8) and is parallel to the line given by $x = 10 + 3t, y = 12t, z = -3 - t$ passes through the xz-plane. If it does, give the coordinate of that point.

$(0,3,8)$

$x = 10 + 3t$

$y = 12t$

$z = -3 - t$

$r = (0,-3,8) + t(-3,12,-1)$

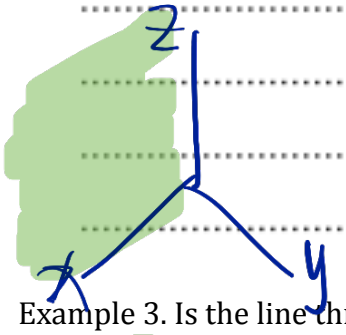
$r = (3t, -3+12t, 8-t)$

to $y = 0$

$0 = -3 + 12t$

$t = \frac{1}{4}$

$r\left(\frac{3}{4}, 0, \frac{31}{4}\right)$



direction vector

Example 3. Is the line through the points (2, 0, 9) and (-4, 1, -5) parallel, orthogonal or neither to the line given by $r = (5, 1 - 9t, -8 - 4t)$?

$\vec{v} \parallel \vec{w}$ if $\vec{v} = k\vec{w}$

$\vec{v} \cdot \vec{w} = 0$

perpendicular

$v = \begin{pmatrix} 1 \\ 9 \\ 3 \end{pmatrix}$ $w = (0,-3,5) = \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix}$

$v \cdot w = (1)(0) + (9)(-3) + (3)(5)$

$= -3$

$r = (5, 1-9t, -8-4t)$

$r_1 = (6,-1,14)$

$r_2 = (0,-9,-4)$

$r_1 \cdot r_2 = (6)(0) + (-1)(-9) + (14)(-4)$

$= -47$

HOME PRACTICE

Example 4. Point A has coordinates $(-4, -12, 1)$ and point B has coordinates $(2, -4, -4)$. The line L passes through A and B.

a. Show that $\vec{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 2-(-4) \\ -4-(-12) \\ -4-1 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

b. Find the vector equation for L.

$$L = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}\lambda \quad \text{OR} \quad L = (-4i - 12j + k) + (6i + 8j - 5k)\lambda$$

$$L = (-4 + 6\lambda)i + (-12 + 8\lambda)j + (1 - 5\lambda)k$$

c. Point $C(k, 12, -k)$ is on L. Show that $k = 14$.

$$\begin{pmatrix} K \\ 12 \\ -K \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}\lambda$$

$12 = -12 + 8\lambda$

$24 = 8\lambda$

$3 = \lambda$

if $\lambda = 3$

$K = -4 + 6\lambda$ $\quad -K = 1 + -5\lambda$

$K = -4 + 6(3)$ $\quad -k = 1 + -5(3)$

$K = 14$ $\quad -k = -14$

$k = 14$

d. Find $\vec{OB} \cdot \vec{AB}$

$$\vec{OB} \cdot \vec{AB} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$$

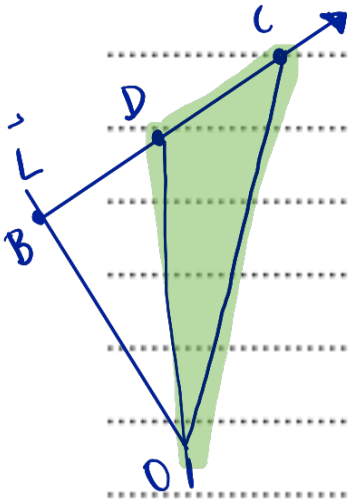
$$= 12 - 32 + 20$$

$$= 0$$

e. Write down the values of angle OBA.

$OBA = 90°$

f. Point D is also on L and has coordinates $(8, 4, -9)$. Find the area of triangle OCD.



$A_{OCD} = \frac{1}{2}|\vec{OB}| \times |CD| = \frac{1}{2} \times 6 \times \sqrt{125}$

$= 15\sqrt{5}$

$CD = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ $|\vec{OB}| = 6$

$|CD| = \sqrt{125}$