Poisson Distribution Distribution of Rare Events

- Events occur uniformly. For a small interval the probability of the event occurring is proportional to the size of the interval.
- The probability of more than one occurrence in the small interval is negligible. Events must not occur simultaneously.
- 3. Each occurrence must be **independent** of others.
- 4. The event must occur at random.

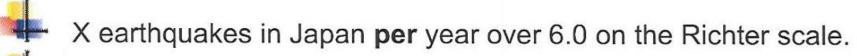
The parameter for the Poisson random variable is λ (lambda), it is the rate and the average or mean number of occurrences over the whole interval.

Setting $\mu = \lambda t$, we define the Poisson probability distribution as:

$$P(X = x) = \frac{e^{-\mu} \times \mu^x}{x!}$$
, $x = 0, 1, 2, ...$

The Poisson random variable is the number of such occurrences in the fixed interval.

Examples:







An example 1:

Cars have been observed to pass a given point on a back road at a rate of 0.5 cars per hour. Find the probability that no cars pass this point in a two-hour period.

From the information we have $\lambda = 0.5$. We define the random variable X as the number of cars that pass the given point in a two-hour period.

This means our parameter $\mu = \lambda \times 2 = 0.5 \times 2 = 1$ So our Poisson distribution is $P(X = x) = \frac{e^{-1} \times 1^x}{x!}$, x = 0, 1, 2, ...

So
$$P(X=0) = \frac{e^{-1} \times 1^0}{0!} = 0.3679$$
. Check using GDC, Distribution

D: poissonpdf(

4 steps:

i.	Justify that the scenario fits the conditions for a Poisson
	distribution.
1 11	Determine the 'hear' rate 1

iii. Determine the base rate λ .

Define the random variable X.

iv. Determine the parameter, μ , that corresponds to the random variable in step iii.

Example 2:

Faults occur on a piece of string at an average of one very three metres.

Bobbins, each containing 5 metres of this string, are to be used. What is the probability that a randomly selected bobbin will contain:

- i. two faults
- ii. At least 2 faults.

$$\lambda = \frac{1}{3}$$
 (one in 3 metres)
 $\mu = \lambda \times 5 = \frac{1}{3} \times 5 = \frac{5}{3}$

$$P(X=2) = \frac{e^{-\frac{5}{3}} \times \left(\frac{5}{3}\right)^2}{2!} \approx 0.2623$$

Example 2:

Faults occur on a piece of string at an average of one very three metres. Bobbins, each containing 5 metres of this string, are to be used.

What is the probability that a randomly selected bobbin will contain:

- i. two faults
- ii. At least 2 faults.
- ii. $P(X \ge 2)$ has unlimited calculations so we use the fact that the sum of the probabilities = 1
 - $P(X \ge 2) = 1 P(X < 2)$ = 1 P(X = 0) P(X = 1) = 1 P(X = 0) P(X = 1)
 - $=1 \frac{e^{-\frac{5}{3}} \times \left(\frac{5}{3}\right)^0}{0!} \frac{e^{-\frac{5}{3}} \times \left(\frac{5}{3}\right)^1}{1!} = 0.4963$

Example 3:

A radioactive sourse emits particles at an average rate of one very 12 seconds. Find the probability that at most 5 particles are emitted in one minute.

- Check conditions for Poisson
- ii. We are given that $\lambda = \frac{1}{12}$ (one in 12 seconds)
- iii. Random Variable *X* is the number of particles emitted after 1 minute (or 60 seconds)
- iv. Parameter $\mu = \lambda \times 60 = \frac{1}{12} \times 60 = 5$

Example 3:

A radioactive sourse emits particles at an average rate of one very 12 seconds. Find the probability that at most 5 particles are emitted in one minute.

minute.
$$P(X \le 5) = \frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!} + \frac{e^{-5} \times 5^2}{2!} + \cdots + \frac{e^{-5} \times 5^5}{5!} \approx 0.6160.$$

Check: poissCdf(5, 0, 5)

Mean and Variance of the POISSON DISTRIBUTION.

For a random variable X having a Poisson distribution with parameter μ , then:

$$E(X) = \mu$$
 and $Var(X) = \mu$.

Example 4:

A data entry operative finds that, on average, they make two mistakes every three screens. Assuming that the number of errors per screen follows a Poisson distribution, what are the chances that there will be two mistakes on the next screen they enter?

$$E(X) = \frac{2}{3} \text{ and } \mu = \frac{2}{3}$$

$$P(X = 2) = \frac{e^{-\frac{2}{3}} \times (\frac{2}{3})^2}{2!} \approx 0.1141$$

Example 5

The number of flaws in metal sheets 100 cm by 150 cm is to follow a Poisson distribution. On inspecting a large number of these metal sheets, it is found that 20% of these sheets contain at least one flaw.

- i. Find the average number of flaws
 ii. Find the probability of observing one flaw in a metal sheet selected at random.
- Let X be the random variable of the number of flaws per 100 cm by 150 cm metal sheet.

Example 5

The number of flaws in metal sheets 100 cm by 150 cm is to follow a Poisson distribution. On inspecting a large number of these metal sheets, it is found that 20% of these sheets contain at least one flaw.

Let X be the random variable of the number of flaws per 100 cm by 150 cm metal sheet.

We have $X \sim Pn(\mu)$ where μ is to be determined.

Knowing $P(X \ge 1) = 0.2$, we know the opposite P(X = 0) = 0.8

$$P(X = 0) = \frac{e^{-\mu} \times \mu^0}{0!}$$
 or $e^{-\mu} = 0.8$

 $\mu = 0.2231$ the average number of flaws per sheet is 0.2232

Example 5

The number of flaws in metal sheets 100 cm by 150 cm is to follow a Poisson distribution. On inspecting a large number of these metal sheets, it is found that 20% of these sheets contain at least one flaw.

ii. $P(X = 1) = \frac{e^{-0.2231} \times 0.2231^{1}}{1!} \approx 0.1785$