Reminders

- Week 31 – April 1 to 3: Right Triangle Trigonometry (SohCahToa/ChoShaCao/Angles of Elevation and Depression)
- April 8 to 19: Lebaran Break
- Week 32 – April 22 to 26: Arc Length and Area of a Sector (in radian and degree measurements) + Quiz Revision
- Week 33 – April 29 – May 3: Non-Right triangle Trigonometry (Sine/Cosine Rules) + **Quiz on May 2 (Friday)**
- Week 34 – May 6 to 10: Triangulation and Area of Triangle
- Week 35 – May 13 to 17: Introduction to Vectors and Revision for the finals
- Week 36 – May 20 – 24: Final Exams

Trigonometric Ratios in a Right Triangle

Week 31

OBJECTIVES

- Use the sine, cosine, and tangent ratios and angles of elevation and depression in solving the unknown sides and angles of a right triangle
- Construct labelled diagrams from written statements

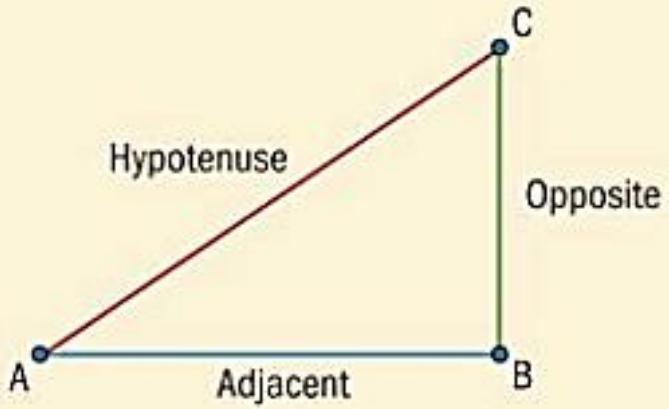
Trigonometric ratios in a right triangle

The ratios of the sides of a right-angled triangle are called **trigonometric ratios**. The three most common trigonometric ratios are the **sine (sin)**, **cosine (cos)**, and **tangent (tan)**. These are defined for acute angle A in the right-angled triangle below:

$$\sin(\hat{A}) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos(\hat{A}) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

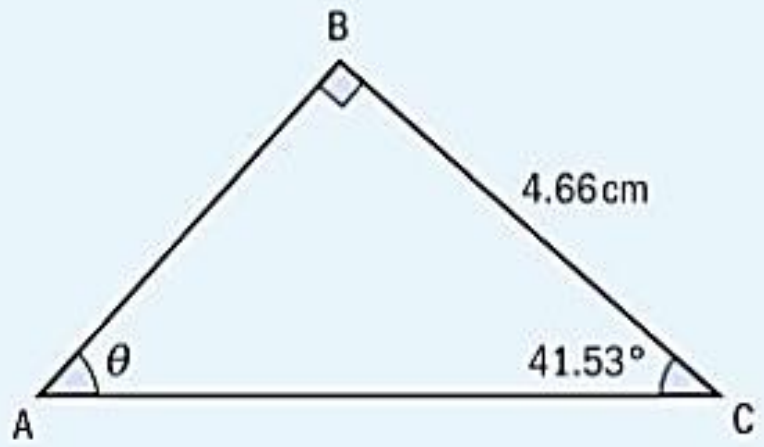
$$\tan(\hat{A}) = \frac{\text{Opposite}}{\text{Adjacent}}$$



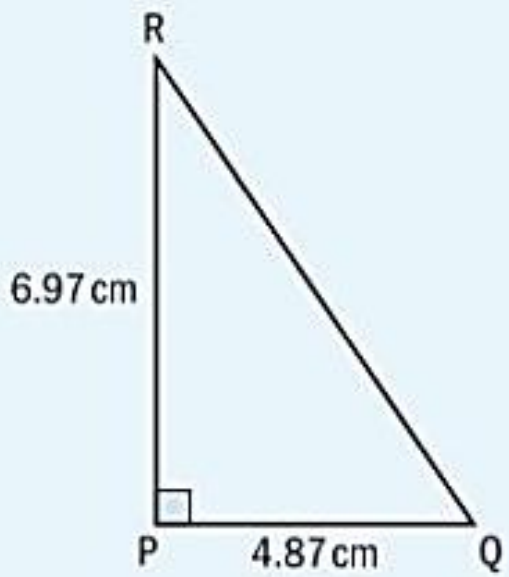
In the definitions above, "opposite" refers to the length of the side opposite angle $\hat{A}$, "adjacent" refers to the length of the side adjacent to angle $\hat{A}$, and "hypotenuse" refers to the length of the hypotenuse (the side opposite the right angle).

For each triangle, solve for the unknown angles and sides.

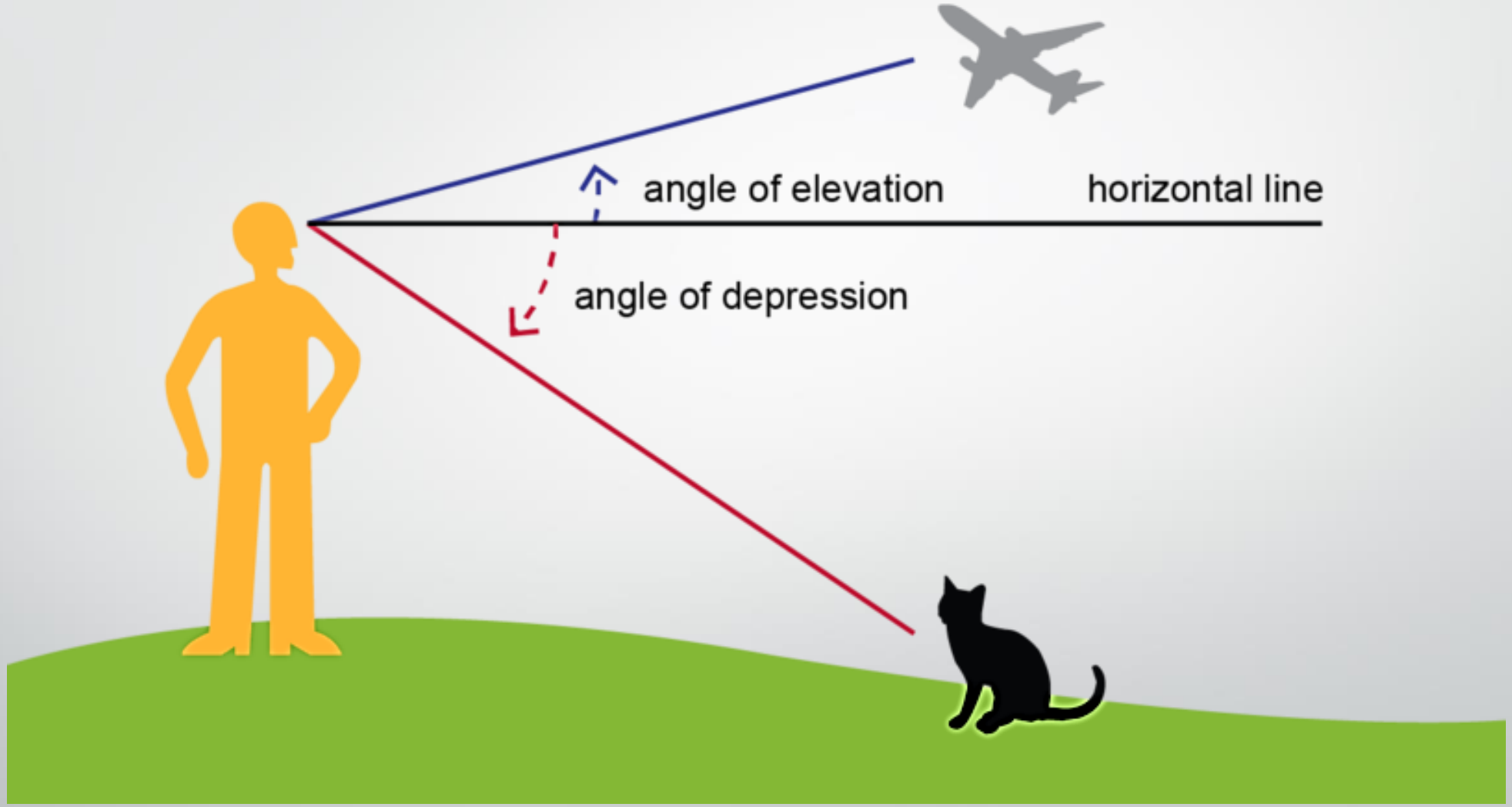
a



b



a In ΔABC:	
$\theta = 90° - 41.53° = 48.47°$	
$\cos(41.53°) = \frac{BC}{AC}$	[BC] is adjacent to angle $\hat{C}$, use the cosine ratio to find hypotenuse AC. Ensure technology is set to degree mode. Round answers to 3 s.f.
$AC = \frac{4.66}{\cos(41.53°)} \Rightarrow AC = 6.22\,(3\text{ s.f.})$	
$AB = \sqrt{AC^2 - BC^2} = 4.13\text{ cm}\,(3\text{ s.f.})$	Use Pythagoras' theorem to find AB.
	Use the exact value of AC stored in your calculator (not the rounded value) when performing your calculations.
b In ΔPQR:	Use Pythagoras' theorem to find RQ.
$RQ = \sqrt{PR^2 + PQ^2} = 8.50(3\text{ s.f.})$	
$\tan(P\hat{Q}R) = \frac{PR}{PQ} = \frac{6.97}{4.87}$	When solving for an unknown angle, determine a trig ratio with two known sides (use exact lengths when possible). PR is opposite side to $P\hat{Q}R$ and PQ is adjacent, so choose tangent.
$P\hat{Q}R = \tan^{-1}\left(\frac{6.97}{4.87}\right) = 55.1°(3\text{ s.f.})$	Then use an **inverse trig function**, in this case $\tan^{-1}(x)$, to find the angle.
$P\hat{R}Q = 90 - 55.1 = 34.9°$	


angle of elevation
horizontal line
angle of depression

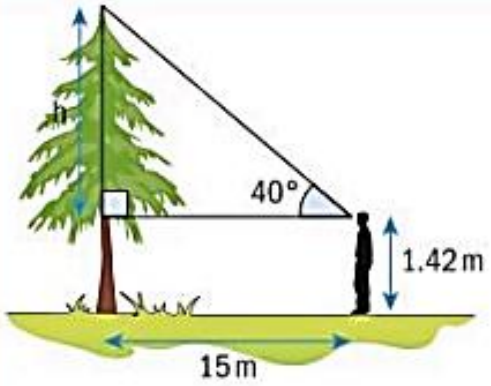
Emma stands 15 m away from a tree. She measures the angle of elevation to the top of the tree as 40° and her height to eye level as 142 cm.

a Find the height of the tree.

b Frank, whose height is 1.8 m to eye level, is standing on the other side of the tree. His kite is stuck at the very top of the tree. He knows the length of the kite string is 16 m. What is the angle of elevation as Frank looks up at his kite?

a $\tan 40° = \frac{h}{15}$

$h = 15\tan 40°$



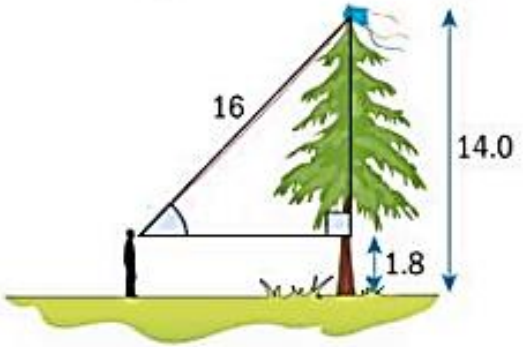
Height of tree $= 15 \tan 40° + 1.42$

$= 14.0\,\text{m}$ (3 s.f.)

Add Emma's height to eye level.

b $\sin\theta = \frac{15\tan 40° + 1.42 - 1.8}{16} = \frac{12.206\ldots}{16}$

Subtract Frank's height from the height of the tree to find the opposite side length. Use the sine ratio as opposite and hypotenuse lengths are known.



$\theta = \sin^{-1}\left(\frac{12.206}{16}\right) = 49.7°$ (3 s.f.)

Use inverse sine ($\sin^{-1}$) to find the angle.

Seat work

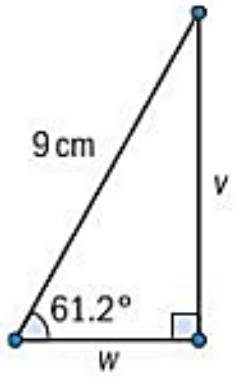
- See MB (Tasks tab) dated April 1

More Examples

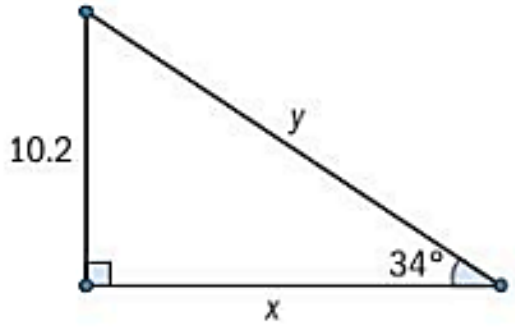
Exercise 1D

1 Determine all unknown sides and angles for each of the right triangles below:

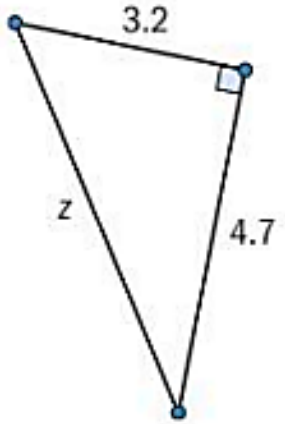
a



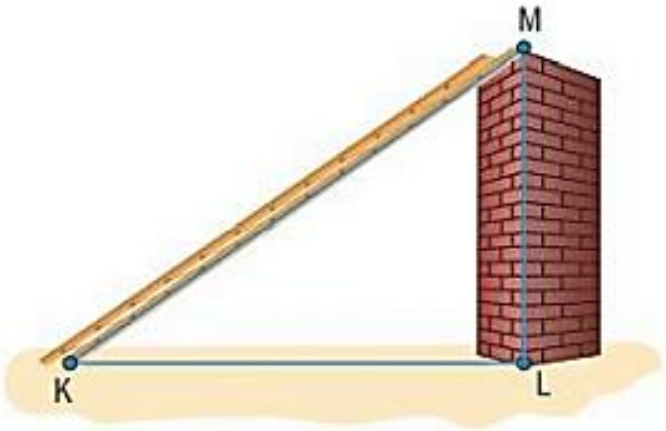
b



c



2 A ladder [KM] is 8.5 m long. It currently leans against a vertical wall so that $L\hat{K}M = 30°$.



i Find the distance KL.

ii Find how far up the wall the ladder reaches.

iii The instructions for use of the ladder state that the angle it makes with the ground should not exceed 55°. Find the maximum height that the ladder can reach up the wall.

3 A hiker, whose eye is 1.6 m above ground level, stands 50 m from the base of a vertical cliff. The angle between the line connecting her eye and the top of the cliff and a horizontal line is 58°.

a Draw a diagram representing the situation.

b Find the height of the cliff.

4 The angle of depression from the top of a vertical cliff to a boat in the sea is 17°. The boat is 450 m from the shore.

a Draw a diagram.

b Find the height of the cliff. Give your answer rounded to the nearest metre.