Properties of Logarithms

Week 12 – October 16, 2023

Objectives

1

Rewrite exponential equations into logarithmic form and vice versa

2

Apply the properties of logarithms

3

Solve problems involving logarithms

Investigation 12

a Consider the exponential equation $2^x = 8$. How could you describe the solution in words? What is the exact solution?

The number 2 must be multiplied by itself 3 times to get 8. So, $x = 3$.

b Consider the exponential equation $2^x = 5$. How could you describe the solution in words? Why can't you determine the exact numerical solution without the use of technology? Sketch the graph $f(x) = 2^x$ and the horizontal line $y = 5$ in order to solve the equation.

The value of x is between 2 and 3. It is not a whole number ($x = 2.321\ldots$).

c Consider the exponential equation $2^x = -2$. Does this equation have a solution?

There is no solution. Any positive number raised to an exponent results to a positive answer.

d Consider the equation $a^x = b$. Can you describe the solution in words?

x is a real number, a and b > 0, a $\neq$ 1.

e Do all exponential equations have a solution? How can you find the solution to an exponential equation?

No. Question (c) is an example of an exponential equation with no solution.

Exponential to Logarithmic Form and Vice Versa

- $a^x = b \leftrightarrow log_a b = x$, a and b > 0, $a \neq 1$

- a is called the base
- x is the exponent
- b is the argument

- $log_{10}x \rightarrow \log x$
- $log_{e}x \rightarrow \ln x$

- **Euler's ("oiler") number** ($e = 2.71828 \ldots$) is an important constant that is found in many contexts and is the base for natural logarithms.
- Euler's number is used from explaining exponential growth to radioactive decay.
- In finance, Euler's number is used to calculate how wealth can grow due to compound interest.

On your GDC, $\log_{10}$ is called "LOG" and $\log_e$ is called "LN".

Two Fundamental exponential equations:

- $10^{x} = c \leftrightarrow \log c = x$
- $e^{x} = c \leftrightarrow \ln c = x$

Two Fundamental logarithmic equations:

- $\log x = c \leftrightarrow 10^c = x$
- $\ln x = c \leftrightarrow e^c = x$

Examples

- Find the exact values of x in each of the following equations.

1. $10^x = 5$

2. $e^{2x} = 12$

3. $\log x = 3$

4. $3lnx = 7$

1 $x = \log 5$

2 $2x = \ln 12 \Rightarrow x = \frac{1}{2}\ln 12$

3 $x = 10^3 = 1000$

4 $\ln x = \frac{7}{3} \Rightarrow x = e^{\frac{7}{3}}$

Investigation 13

1 Consider the equations $10^x = 1$ and $e^x = 1$ or even the general case $a^x = 1$.

Find the value of x. $x = 0.$

What is the solution in terms of logs? $log_a 1 = 0$

2 Consider the equations $10^x = 10$ and $e^x = e$ or even the general case $a^x = a$.

Find the value of x. $x = 1.$

What is the solution in terms of logs? $log_a a = 1$

3 Consider the equations $10^x = 10^n$ and $e^x = e^n$ or even the general case $a^x = a^n$, where x is the unknown variable and n is a constant parameter.

Find the value of x. $x = n.$

What is the solution in terms of logs? $log_a a^n = x$

4 Use your GDC to copy and complete the following table, giving your answers to three significant figures.

log 2	0.301 ...	log 3	0.477 ...	log 6	0.778 ...
log 3	0.477 ...	log 4	0.602 ...	log 12	1.079 ...
ln 5	1.609 ...	ln 7	1.945 ...	ln 35	3.555 ...

What do you notice? What can you conjecture about $\log_a x + \log_a y$? $log_a xy$

5 Use your GDC to copy and complete the following table, giving your answers to three significant figures.

log 12	1.079 ...	log 2	0.301 ...	log 6	0.778 ...
log 15	1.176 ...	log 3	0.477 ...	log 5	0.698 ...
ln 11	2.397 ...	ln 7	1.945 ...	$\ln \frac{11}{7}$	0.451 ...

What do you notice? What can you conjecture about $\log_a x - \log_a y$? $log_a \frac{x}{y}$

6 Use your GDC to copy and complete the following table, giving your answers to three significant figures.

log (3^2)	0.954 ...	log 3	0.477 ...	2log 3	0.954 ...
$\ln\sqrt{2}$	0.346 ...	ln 2	0.693 ...	$\frac{1}{2}\ln\sqrt{2}$	0.346 ...

What do you notice? What can you conjecture about $\log_a(x^n)$? $nlog_a x$

$\log\ 1 = 0, \ln 1 = 0$ and in general, $\log_a 1 = 0$

$\log\ 10 = 1,\ \ln e = 1$ and in general, $\log_a a = 1$

$\log\ 10^n = n, \ln e^n = n$ and in general, $\log_a a^n = n$

Laws of logarithms:

- $\log_a x + \log_a y = \log_a xy$
- $\log_a x - \log_a y = \log_a \dfrac{x}{y}$
- $\log_a (x)^n = n \log_a x$

Write each of the following as a single logarithm:

a $3\log x$ **b** $\dfrac{\log x}{2}$ **c** $2\log x + \log y$ **d** $\log x - 2\log y$

e $-\ln x$ **f** $1 + \ln x$ **g** $\ln x + \ln y - \ln z$

a $3\log x = \log(x^3)$

b $\dfrac{\log x}{2} = \dfrac{1}{2}\log x = \log\left(x^{\frac{1}{2}}\right) = \log\sqrt{x}$

c $2\log x + \log y = \log(x^2) + \log y = \log(x^2 y)$

d $\log x - 2\log y = \log x - \log(y^2) = \log\left(\dfrac{x}{y^2}\right)$

e $-\ln x = -1\ln x = \ln(x^{-1}) = \ln\left(\dfrac{1}{x}\right)$

f $1 + \ln x = \ln e + \ln x = \ln(ex)$

g $\ln x + \ln y - \ln z = \ln(xy) - \ln z = \ln\left(\dfrac{xy}{z}\right)$

Thank you!