Parallel and Perpendicular Lines

Week 29

International Mindedness

Some rivers and canals are the border between two or more countries, like the Akanyaru River in Africa. One bank of the river is in Rwanda and the other in Burundi.

How do countries decide which parts of a river or canal belong to them? Could this create conflict between them? How can they find the midline of a river or canal?

What you should know

By the end of this subtopic you should be able to:

- find the coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) using

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- find the gradient of a line: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- use the equations of a straight line:

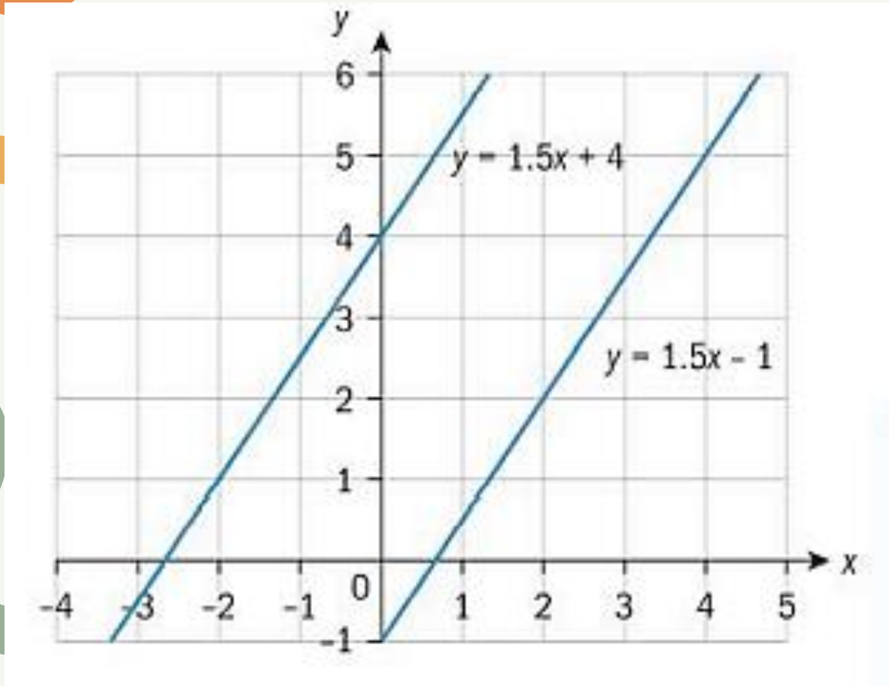
$$y = mx + c\,, ax + by + d = 0 \text{ and } y - y_1 = m\left(x - x_1\right)$$

  to find the gradient of a perpendicular bisector using

$$m_{\text{segment}} \times m_{\text{perpendicular bisector}} = -1$$

- put all this together to find the equation of a perpendicular bisector of a line segment with endpoints (x_1, y_1) and (x_2, y_2).

Parallel Lines

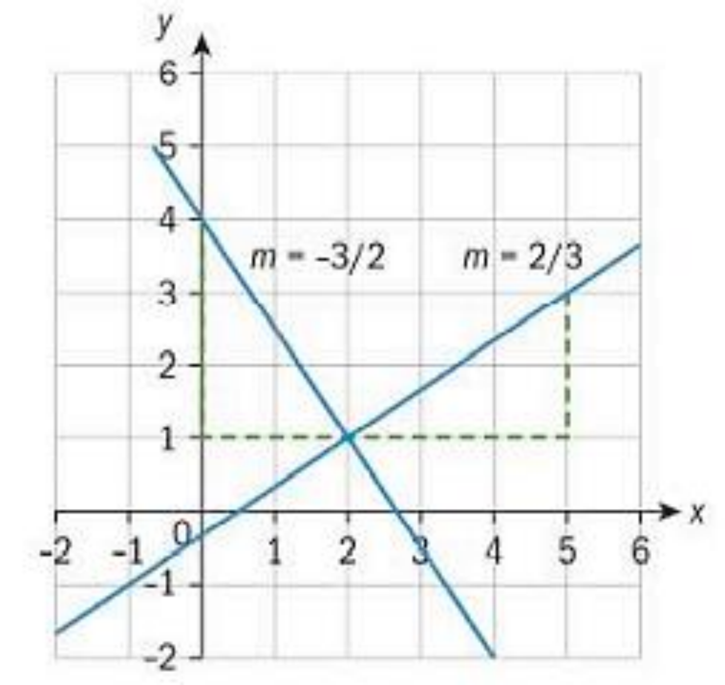


Line L_1 has gradient m_1 and line L_2 has gradient m_2.

If L_1 is parallel to L_2, written $L_1 \parallel L_2$, then $m_1 = m_2$.

The converse is also true: If $m_1 = m_2$, then $L_1 \parallel L_2$.

Perpendicular Lines



Line L_1 has gradient m_1 and line L_2 has gradient m_2.

If L_1 is perpendicular to L_2, written $L_1 \perp L_2$, then m_1 x m_2 = -1.

The converse is also true: If m_1 x m_2, = -1, then $L_1 \perp L_2$.

Examples

What is the gradient of the line parallel to:

1. $y = -x + 6$
2. $x - 3y = 5$

What is the gradient of the line perpendicular to:

1. $y = -3x + 11$
2. $y = \frac{2(x-1)}{3}$

The trajectory lines L_1 and L_2 of two cargo ships are given. In each case determine whether the lines are parallel, perpendicular, or neither.

a. L_1 passing through points (2,4) and (-1,-2); and L_2 passing through points (0,-4) and (2,0).

b. L_1 passing through points (0.5,-1) and (-7,-1.5); and L_2 passing through points (-4,0) and (3,2).

Perpendicular Bisector

Plot these points on the Cartesian plane (using Geogebra or Desmos):

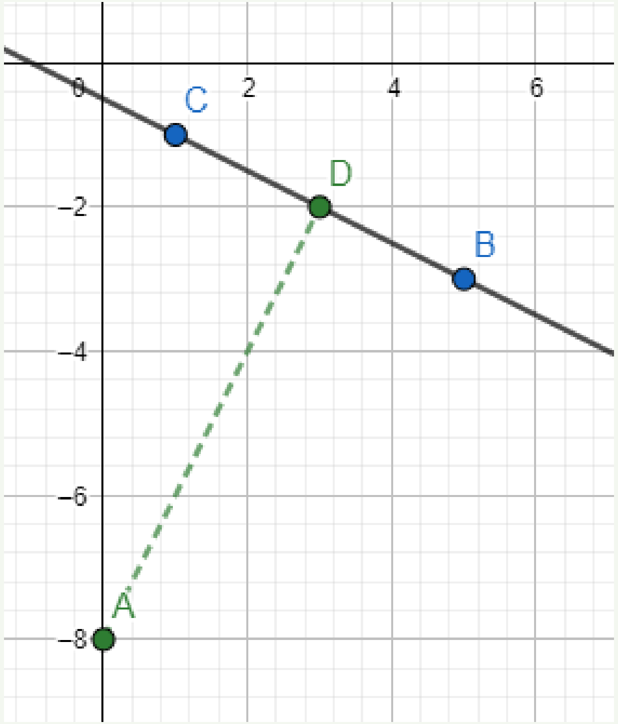
Plot the points A (2,2) and B (4,6).

1. What is the midpoint of [AB]?
2. From the midpoint of [AB], draw the line perpendicular to it. What must be the gradient of this line?
3. Plot point D (2, 4.5) on the same plane.
4. How far is point D from A? How far is it from B?
5. Do the same for the points E (1,5), F (0, 5.5), G (5,3).
6. Will all points lying on [EG] be equidistant from points A and B?
7. [EG] is the perpendicular bisector of [AB]. What is a perpendicular bisector?

How many perpendicular bisector/s is/are there in a given line segment?

How many perpendicular line/s is/are there in a given line segment?

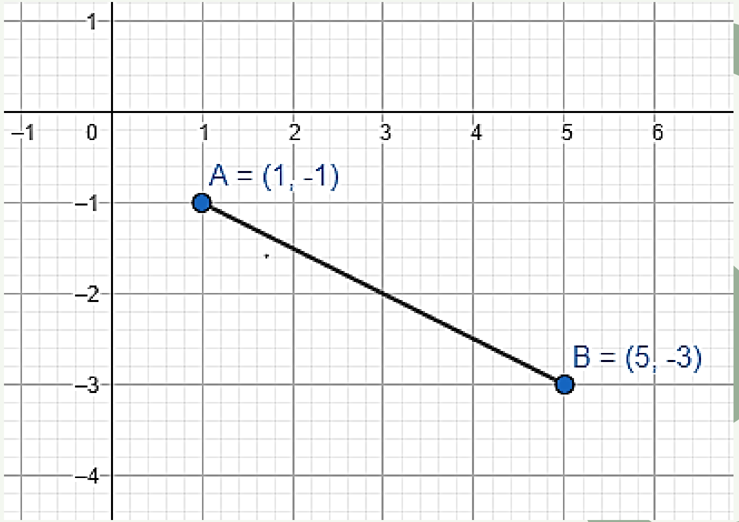
The perpendicular bisector of the line segment joining two points is the line, line segment, or ray that passes through the midpoint of the line segment and is perpendicular to it.



The <u>shortest distance</u> from a point A to a straight line (BC) is the segment [AD], where D lies on (BC) and [AD] ⊥ (BC).

Examples

Example 1: Find the equation of the perpendicular bisector of the line segment whose endpoints are A(1,-1) and B (5,-3).



Step 1: Midpoint of [AB]

Step 2: Gradient of [AB]

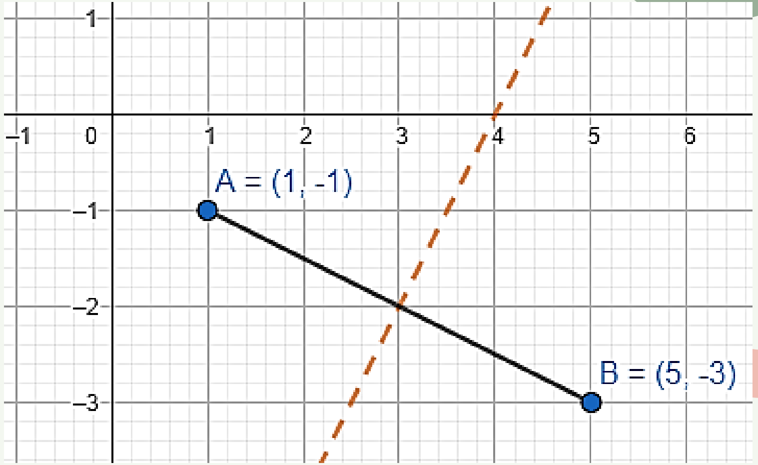
Step 3: Gradient of the $\perp$ bisector

Step 4: Find b in $y = mx + b.$

Use the midpoint of [AB].

(or use the point-gradient form and substitute the midpoint)

Step 5: Write the equation of the $\perp$ bisector.

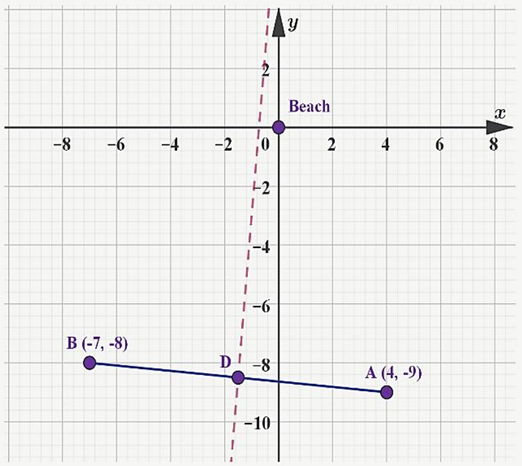


Example 2: Find the equation of the perpendicular bisector of the line segment joining A (2,2) and B (4,6).

Step 1: Midpoint of [AB]
Step 2: Gradient of [AB]
Step 3: Gradient of the $\perp$ bisector
Step 4: Find b in $y = mx + b.$
Use the midpoint of [AB].
(or use the point-gradient form and substitute the midpoint)
Step 5: Write the equation of the $\perp$ bisector.

In a thunderstorm, you hear the thunder after you see the lightning strike. Two friends who live near a beach were watching the same thunderstorm and heard the thunder at the same time. This means they must be equidistant from where the lightning struck.

If Anuman's house is 9 km south and 4 km east of the beach, and Boonsri's house is 8 km south and 7 km west of the beach, where did the lightning strike?



Since both heard the thunder at the same time, it must have happened at a place equidistant from both houses.

Sketch the given information.

$m = \dfrac{-8-(-9)}{-7-4} = -\dfrac{1}{11}$	Gradient of the line segment connecting A and B.
$m_P = 11$	Gradient of the perpendicular line.
$\left(\dfrac{4+(-7)}{2}, \dfrac{-9+(-8)}{2}\right) = (-1.5,\ -8.5)$	Midpoint of the segment.
$y-(-8.5) = 11\,(x-(-1.5))$	Use the equation of a line with a given point and gradient.
$y = 11x + 8$	Rearrange.
So, the lightning strike was somewhere on the line $y = 11x + 8.$	

Your Turn

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