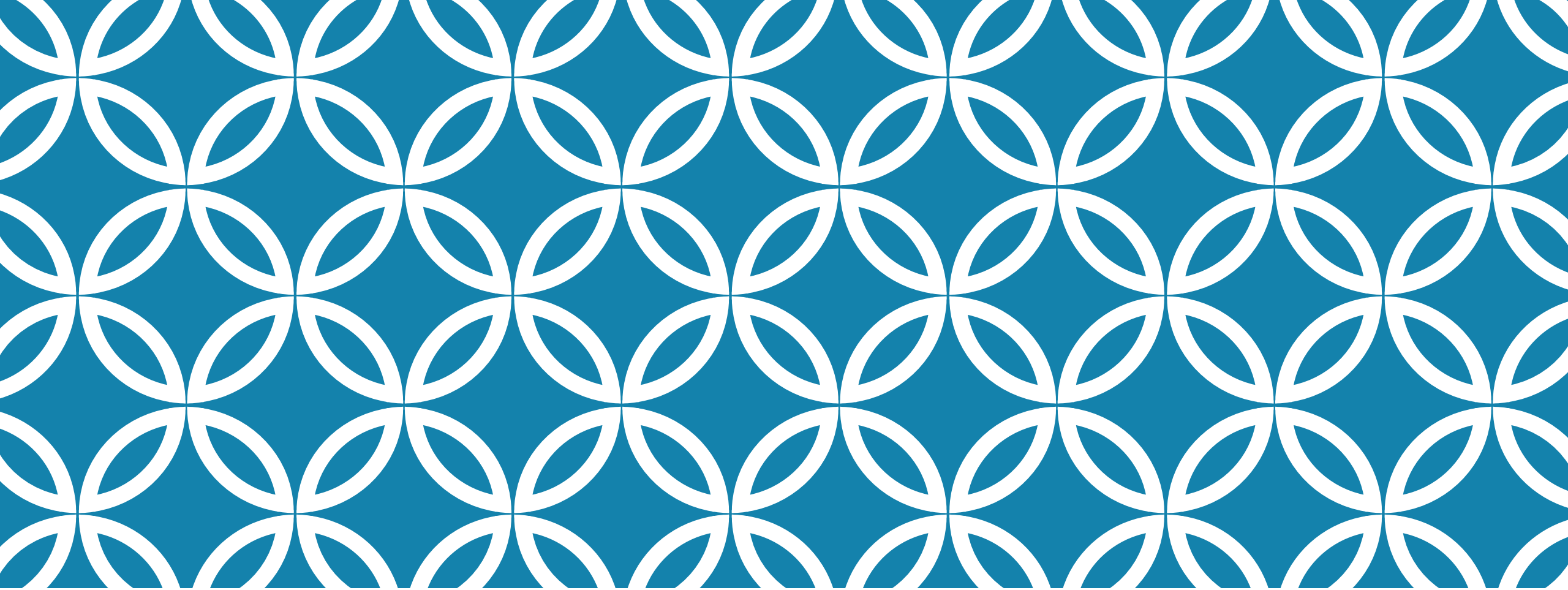
- What is the difference between a line segment and a directed line segment?
- Which between the two above is used to represent a vector?
- What is a vector?
- What are opposite vectors?
- What do we call the vector that resulted from adding two or more vectors?
- What are the two ways to add vectors geometrically?
- How are these ways different from each other?
- Rewrite (3, 5) as a column vector and using base vectors (also called component forms)
- How do we add vectors algebraically?

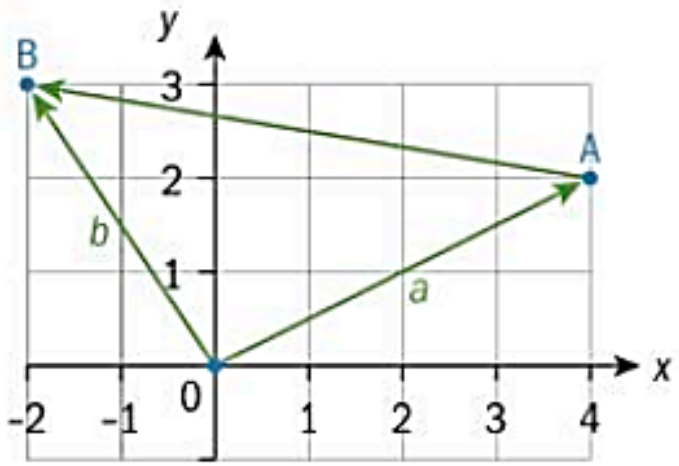


POSITION VECTORS

Week 34

Position vectors

A position vector gives a point's displacement from the origin.



From the illustration above, how can $\overrightarrow{AB}$ be written as the result of adding $\overrightarrow{OA}$ and $\overrightarrow{OB}$ using the triangle law of vector addition?

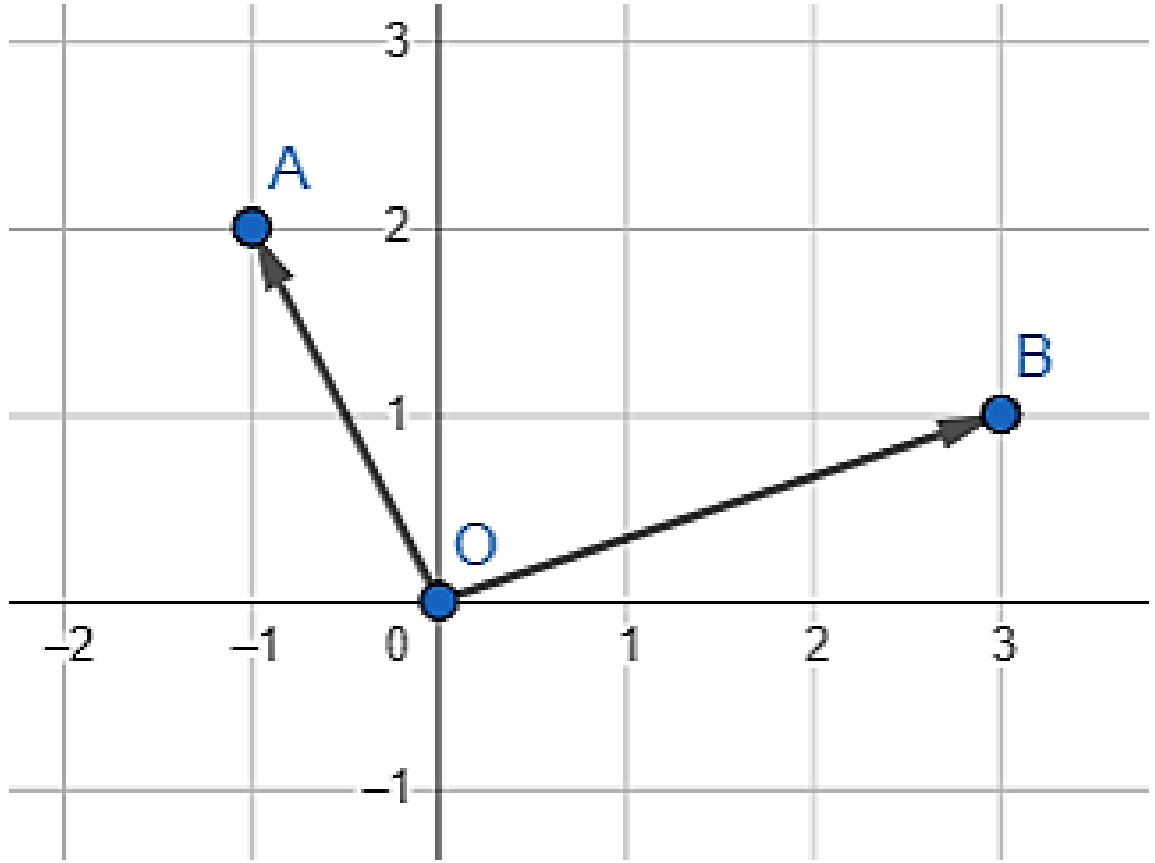
It can be seen from the diagram and using the triangle law of vector addition that $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$

This is normally written as $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or $\overrightarrow{AB} = b - a$.

Given A(−1, 2) and B(3, 1),

a write down the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ as column vectors

b find $\overrightarrow{AB}$.



Answers

a $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

b $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

Subtract the components.

A is the point with coordinates $(6, 1)$ and B is the point with coordinates $(-2, 5)$.

a Find the vector $\overrightarrow{AB}$.

A ship moves from the point A to a point C which has coordinates $(7, 4)$ and then onto point B.

b Find the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$.

c Write down an equation linking $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{CB}$.

d Verify your answer is true for the values obtained in parts **a** and **b**.

From point B the ship moves on to the point D where $\overrightarrow{BD} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

e Find the vector $\overrightarrow{AD}$.

f Find the coordinates of D.

a $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$

Using $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$

b $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

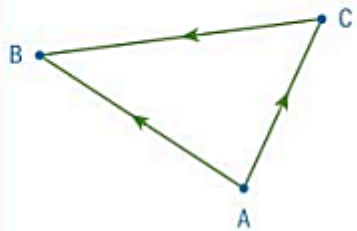
$\overrightarrow{CB} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$

Using $\overrightarrow{AC} = \boldsymbol{c} - \boldsymbol{a}$ and $\overrightarrow{BC} = \boldsymbol{b} - \boldsymbol{c}$

c $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

Care needs to be taken as the triangle law for vector addition requires the vectors to follow on from each other. The endpoint of the first vector must be the first point of the second.

The sketch below indicates the order required.



d $\begin{pmatrix} -8 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -9 \\ 1 \end{pmatrix}$

e $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$\begin{pmatrix} -8 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

f $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$\overrightarrow{OD} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

Hence coordinates are (3, 7).

Remember the coordinates of D are obtained from its position vector $\overrightarrow{OD}$.

Give the answer as coordinates rather than a position vector if this is what is asked for.