Equations of a Straight Line: Recall

Week 2 - July 31 to August 4, 2023

Skills Check:

1. For the line $y = 4x + 5$, what is the gradient?

   A. 4 ☐

   B. 5 ☐

   C. -4 ☐

   D. 1 ☐

Skills Check:

2. For the line $2y - 6x - 5 = 0$, what is the gradient?

A. 5 ☐

B. -6 ☐

C. 6 ☐

D. 3 ✓

Skills Check:

3. What is the gradient of the line between (-2, 5) and (3, -10)?

   A. 3 ☐

   B. $\frac{1}{3}$ ☐

   C. -3 ✓

   D. $-\frac{1}{3}$ ☐

Skills Check:

4. For the line $y = 2x - 4$, what is the intercept?

   A. 2 ☐

   B. -2 ☐

   C. 4 ☐

   D. -4 ✓

Skills Check:

5. For the line $5y + 2x + 12 = 0$, what is the intercept?

   A. $\frac{2}{5}$

   B. $\frac{12}{5}$

   C. $-\frac{2}{5}$

   D. $-\frac{12}{5}$

Skills Check:

6. Find the coordinate where $y = -2x + 1$ intercepts the line $y = x - 8$.

   A. (3, -5) ✓

   B. (-2, 1) ☐

   C. (1, 8) ☐

   D. (0, -7) ☐

Skills Check:

7. What is the gradient of a line that is parallel to $y = 5x - 3$?

A. $-\frac{1}{5}$ ☐

B. -5 ☐

C. $\frac{1}{5}$ ☐

D. 5 ✓

Skills Check:

8. What is the gradient of a line that is perpendicular to $y = -2x + 4$?

A. $-\frac{1}{2}$ ☐

B. -2 ☐

C. $\frac{1}{2}$ ✓

D. 2 ☐

Skills Check:

9. Find the equation of a line with a gradient of 7 that goes through the point (9, -11).

   A. $y = 7x + 86$ ☐

   B. $y = 7x - 74$ ✓

   C. $y = \frac{4}{9}x + 7$ ☐

   D. $y = -\frac{2}{11}x + 7$ ☐

Skills Check:

10. Find the equation of the line which passes through the points (-15, -5) and (3, 7).

A. $y = \frac{3}{2}x + \frac{5}{2}$ ☐

B. $y = \frac{2}{3}x + 5$ ✓

C. $y = -\frac{3}{2}x + \frac{21}{2}$ ☐

D. $y = -\frac{2}{3}x + 9$ ☐

Different Forms of the Equation of a Straight Line

Week 2 - July 31 to August 4, 2023

Equations of a Straight Line

1. Standard (General Form): $ax + by + c = 0$ ($a, b, and\ c$ are integers)

2. Point-Gradient (Point-Slope) Form: $y - y_1 = m(x - x_1)$ (m is the gradient of the line)

3. Two-Point Form $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

4. Gradient-Intercept Form: $y = mx + c$ (m is the gradient of the line and c is the y-intercept)

5. Equation of a Horizontal Line: $y = c$ (c is a constant)

6. Equation of a Vertical Line: $x = c$ (c is a constant)

7. Intercept Form: $\frac{x}{a} + \frac{y}{b} = 1$ (a is the x-intercept and b is the y-intercept)

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Example 1

For the two points A(2, 2) and B(6, 1)

a Find the gradient m of (AB) (the line passing through A and B).

b Find the equation of (AB) in the form $y = mx + c$.

c Sketch the line for $-2 \leq x \leq 12$.

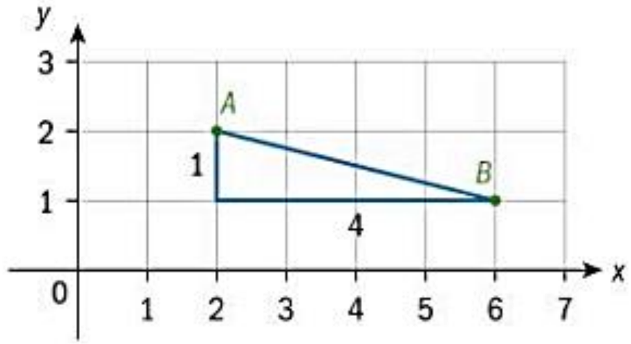
d Find:

i the value of y when x is 4.7 **ii** the y-intercept.

Answer for Example 1

a $m = \dfrac{1-2}{6-2} = \dfrac{-1}{4}$

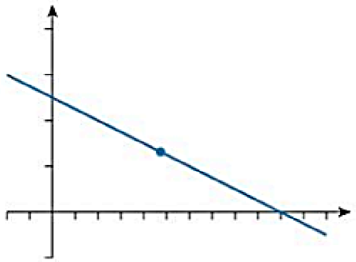
a It is important you are careful to subtract both coordinates in the same order. The gradient can also be found from a sketch:



From the diagram $m = \dfrac{-1}{4}$

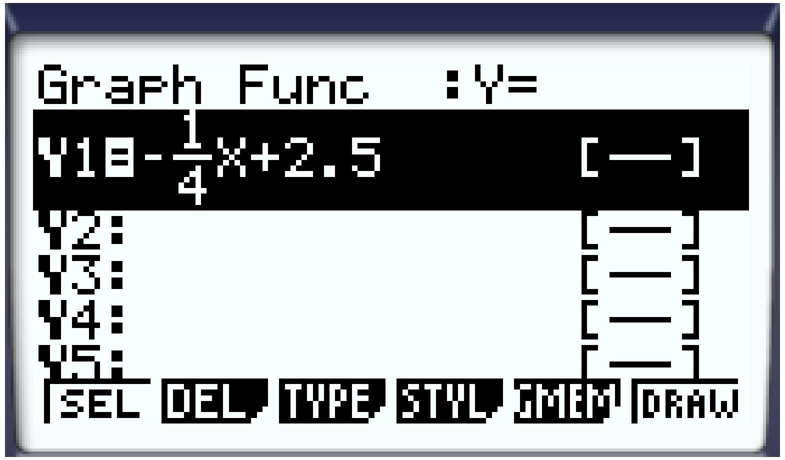
Because the line goes down as you look from left to right the gradient is negative.

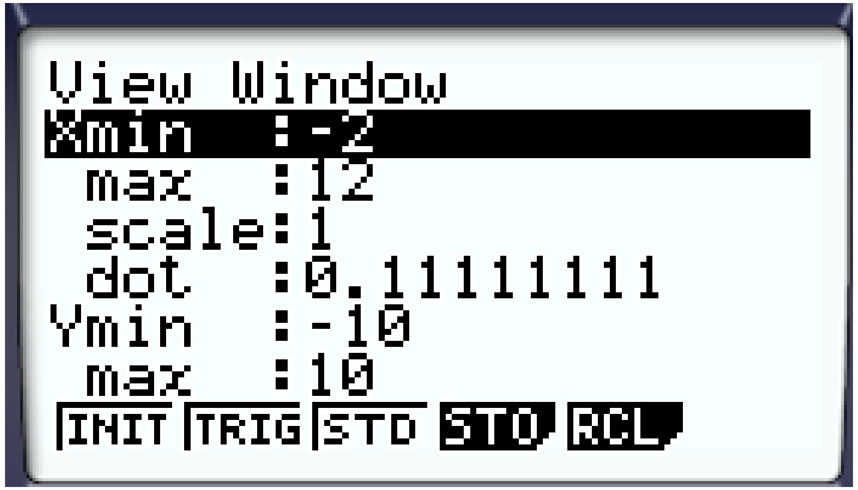
Answer for Example 1

b $y = -\frac{1}{4}x + c$ Substitute one of the points on the line; for example, (2, 2). $2 = -\frac{1}{4} \times 2 + c$ $c = 2.5$ Equation is $y = -\frac{1}{4}x + 2.5$	**b** The same result would have been obtained if (6, 1) had been substituted.
c 	**c** The question will often specify the domain and hence the required range on the *x*-axis. The required range for the *y*-axis can, if necessary, be found from the "table" function on the GDC.
d **i** When $x = 4.7$ $y = 1.325$ **ii** 2.5	**d** These are easy to calculate without the graph but make sure you know how to get these values from your GDC.

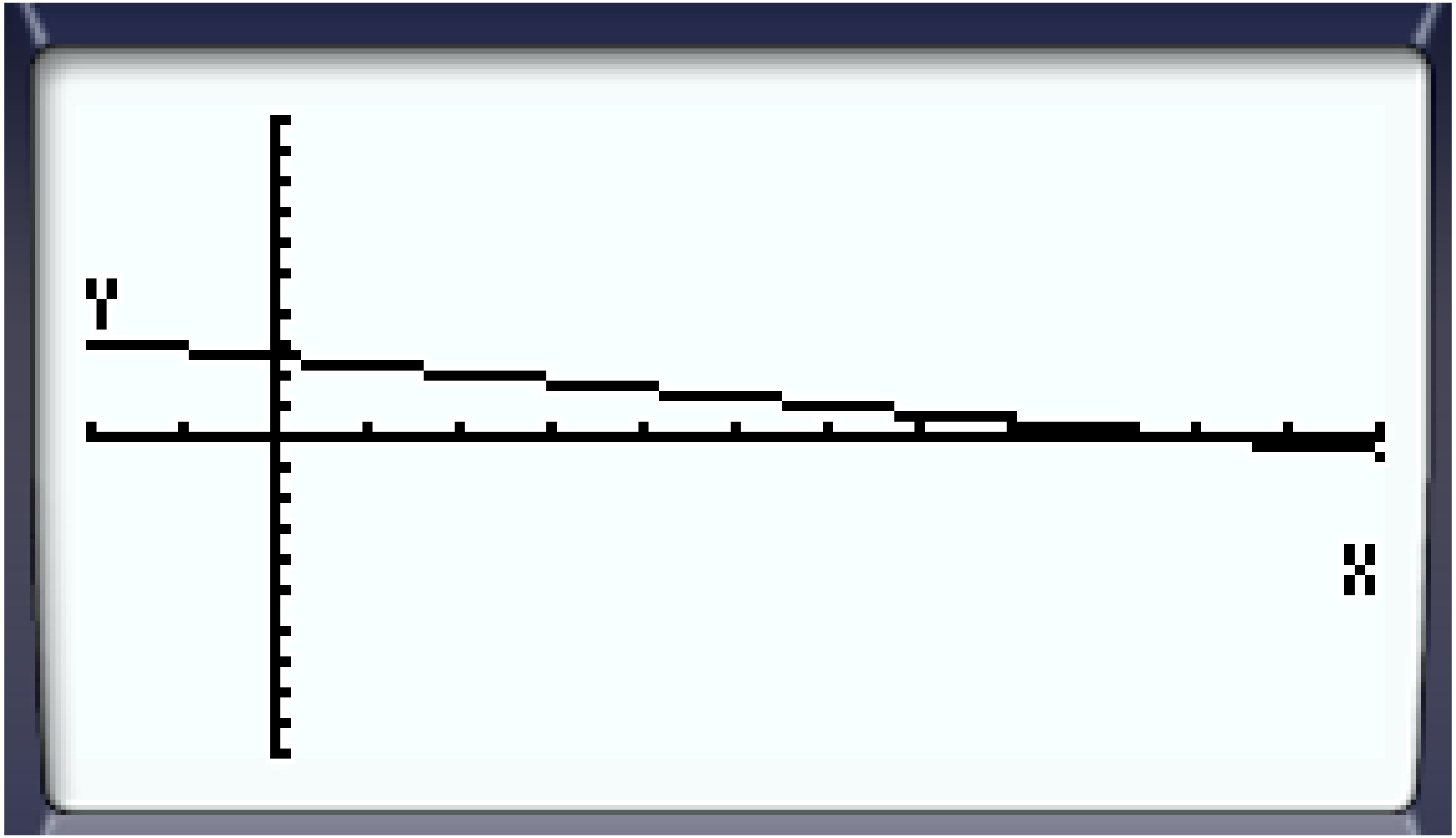
GDC use: $y = -\frac{1}{4}x + 2.5$

- Menu
- Graph
- Type in the function: $\boldsymbol{y = -\frac{1}{4}x + 2.5}$
- F6 (Draw)
- F3 (V-Window - to adjust the min and max x and y values)
- EXE (Enter)





$$y = -\frac{1}{4}x + 2.5$$



Example 2

a Find the coordinates of the x- and y-intercepts for the graph of $2x + 3y - 6 = 0$.

b Write the following equation in general form, $y = x - \frac{1}{2}$

c Find the point of intersection of the two lines **i** analytically **ii** using an appropriate application on your technology.

a When $x = 0$, $3y = 6$ so $y = 2$ When $y = 0$, $2x = 6$ so $x = 3$ Coordinates are $(0, 2)$ and $(3, 0)$	**a** x-intercepts occur when $y = 0$ and y-intercepts occur when $x = 0$. The intercepts are the values of x or y so you need to check whether the question is asking for the intercepts or the coordinates of the intercepts.
b $2x - 2y - 1 = 0$	**b** It is usual to avoid beginning the equation with a negative coefficient but $-2x + 2y + 1 = 0$ is an equally valid answer.
c **i** $2x + 3y = 6$ $2x - 2y = 1$ subtract the two equations to get $5y = 5 \Rightarrow y = 1$ substitute this value into either equation to get $x = 1.5$	**c** **i** The solution can be found using elimination or substitution, for example by replacing y in $2x + 3y - 6 = 0$ with $y = x - \frac{1}{2}$ to get $2x + 3\left(x - \frac{1}{2}\right) - 6 = 0$ $\Rightarrow 5x - 1.5 - 6 = 0 \Rightarrow x = 1.5$
ii $2x + 3y = 6$ $2x - 2y = 1$ $x = 1.5$, $y = 1$	**ii** When solving a system of equations using your technology, there is no need to write down details of the method. Some calculators have an inbuilt simultaneous equation solver. If not the solution can be found by drawing both lines and finding the point of intersection, $(1.5, 1)$

More Examples

1. Find the solutions to:

$$a.)\ 2x + y = 8$$
$$3x - 2y = 33$$

$$b.)\ y - 2 = 3(x - 4)$$
$$y = 2x - 9$$

$$c.)\ x + 3y = 1$$
$$5x + 16y = 8$$

2.

2 Two friends, Alison and Bernard are walking along two different roads. The roads can be represented in the Cartesian plane by the lines with equations

$y = -x + 410$ and $y = \frac{1}{2}x - 100.$

At 2:00 pm Alison is on the first road at the point with coordinates $(0,\ 410)$ and Bernard is at the point with coordinates $(50, -75)$, where the units are in meters.

a Verify that Bernard is on the road with equation $y = \frac{1}{2}x - 100$ at 2:00 pm.

b Find the coordinates of the point of intersection of the two roads.

At 2:00 pm the two friends begin walking at $4\,\text{kmh}^{-1}$ towards the intersection.

c **i** Show that Bernard arrives at the intersection first.

ii Find the length of time he needs to wait before Alison arrives.

3 Road signs showing the steepness of hills are often given as percentages where the figure is derived using the following formula

$$\frac{\text{vertical height gained or lost}}{\text{horizontal distance covered}} \times 100$$

a A road gains 5 m while covering 20 m horizontally. State the percentage that would be written on the road sign.

There is a triangular hill directly outside my house. On the way up the hill from my house I pass a sign indicating the slope is 10%. On the way down the other side of the hill, I pass one indicating the slope is 15%.

b State which road is steeper.

I decide to take my house to be the origin for a coordinate system and one day I go over the hill to the other side and reach my local shop. My GPS tells me the horizontal distance of the shop from my home is 2.45 km and I am at the same level as my house.

c Assuming the roads up and down the hill are straight lines and lie in the plane of the coordinate system find

  i the equation of the road going up from my house

  ii the equation of the road going down from the top of the hill to the shop.

d Find the height of the hill

e Find the total distance of my journey from my house to the shop.

Exercises

Answer the following on your math (graph) notebook. Show all necessary workings.

Exercise 3B

1 Write down:

a the equation of the vertical line passing through (2, 7)

b the equation of the horizontal line passing through (4, 6)

c the point of intersection of the two lines.

2 Find the equation of the following lines.

a gradient = 3, y-intercept 5

b gradient = –2, y-intercept 0.4

c gradient = 4.5, passing through (0, 5)

d gradient = 2, passing through (3, 5).

3 Find the equation of the line passing through the two points given.

a (3, 5) and (5, –3)

b (2, –1) and (3, 4)

c (–3, 2)and (3, 4)

4 A Cartesian grid is superimposed on a map of a city. The y-axis lies in the direction of North and the x-axis due East. The North-South boundaries of the city lie along the lines $x = 0$, $x = 22$ and the distances are in kilometres.

A straight road passes through the city. The road passes through the points with coordinates (6, 8) and (12, –4) and stays within the city boundaries for $0 \le x \le 22$.

a Find the equation of the road.

b Find the coordinates of the road as it crosses:

i the western boundary of the city

ii the eastern boundary of the city.

c Find the length of the road within the city boundary.