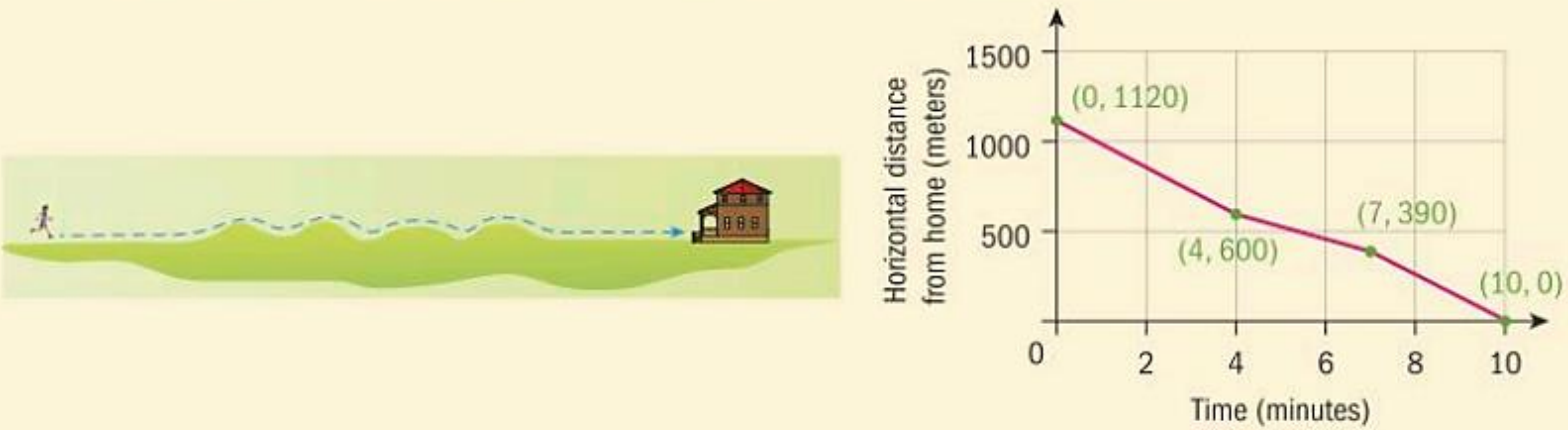
PIECEWISE FUNCTIONS


INVESTIGATE

1 What does the graph tell you about the rate of change of each section of Amir's run? Explain how this relates to the context.



2 Why can this particular context not be modelled accurately by a single linear function?

3 Describe how you could model sections of Amir's run accurately with linear functions.

4 Find a linear equation in point-gradient form to model each section separately, filling in the table.

	Starting point	Ending point	Gradient	Point-gradient equation
Section 1				
Section 2				
Section 3				

5 Convert each equation to gradient-intercept form. In the "domain" column, write the domain of x-values to which the equation applies.

	Equation	Domain
Section 1		
Section 2		
Section 3		

$$f(x)=\begin{cases}-130x+1120 & 0\le x<4\\ -70x+880 & 4\le x<7\\ -130x+1300 & 7\le x\le 10\end{cases}$$

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6 How fast is Amir running on each piece of his run, and for how long does he run at that speed?

$$f(x)=\begin{cases}-130x+1120 & 0\le x<4\\ -70x+880 & 4\le x<7\\ -130x+1300 & 7\le x\le 10\end{cases}$$

7 **a** If $x=7$, would you use the second or third piece to evaluate $f(7)$? Justify your answer.

b Based on this, why is it important that the pieces of the function do not overlap on their domains?

$$f(x)=\begin{cases}-130x+1120 & 0\le x<4\\ -70x+880 & 4\le x<7\\ -130x+1300 & 7\le x\le 10\end{cases}$$

8 Use your piecewise model to find each of the following and interpret it in the context of the problem:

a $f(3)$ **b** $f(8)$ **c** x if $f(x)=500$

A piecewise function is **continuous** if one piece connects to the next with no breaks or jumps.

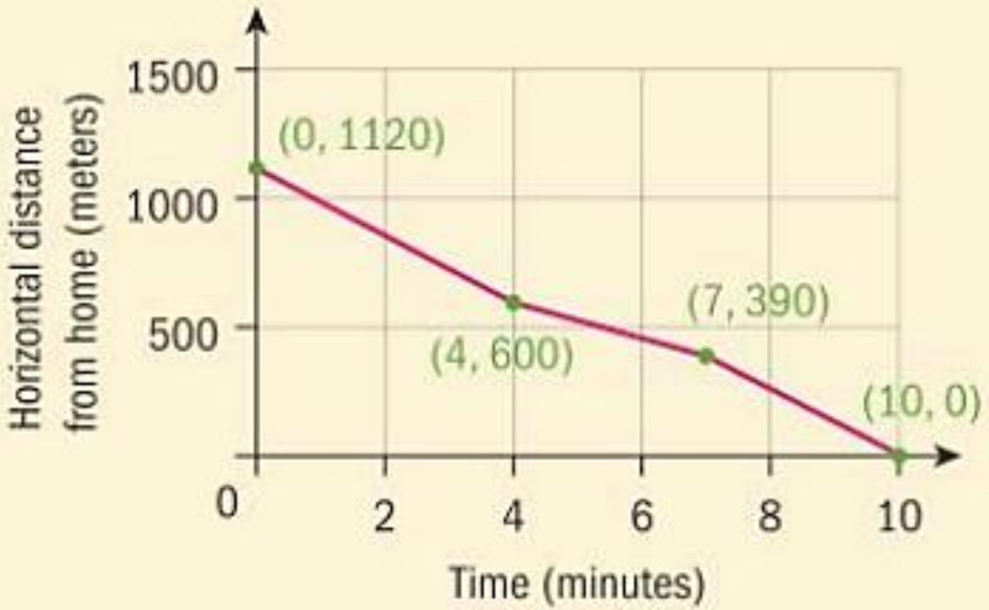
9 Check that your piecewise function is continuous with the following steps:

a Verify using the equations that the endpoint of the first section matches the beginning point of the second section.

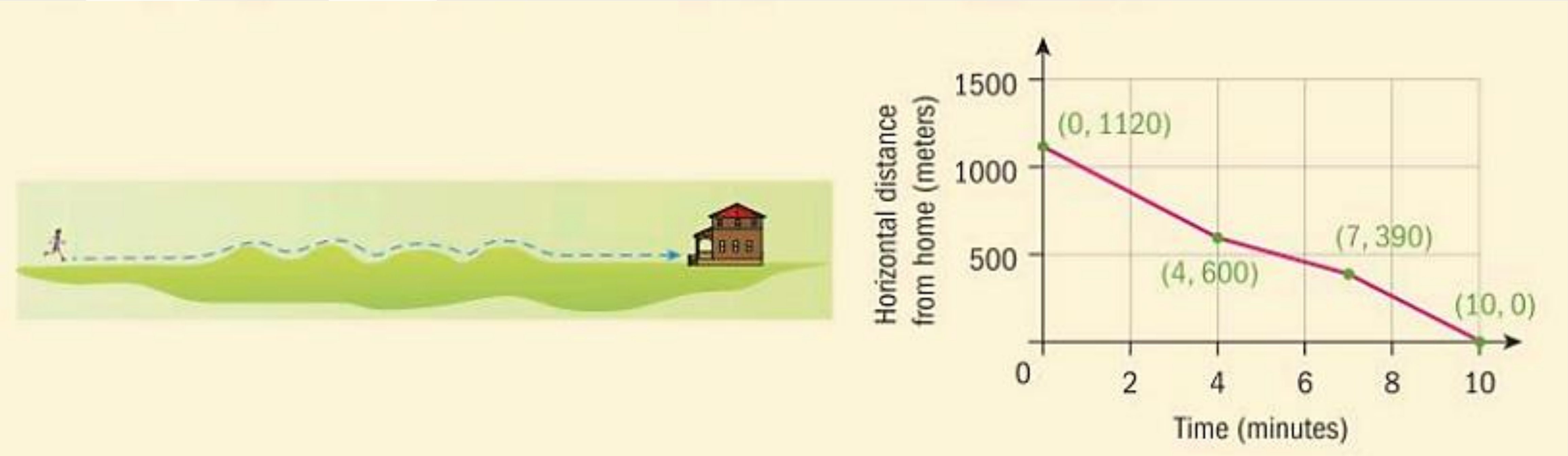
b Repeat for the endpoint of the second section and the beginning point of the third section.

c Why can there not be any other breaks or jumps elsewhere in the function?

10 **Factual** What is a piecewise function and how do you evaluate a piecewise function for a specific value?



11 **Factual** How do you check that a piecewise function is continuous?

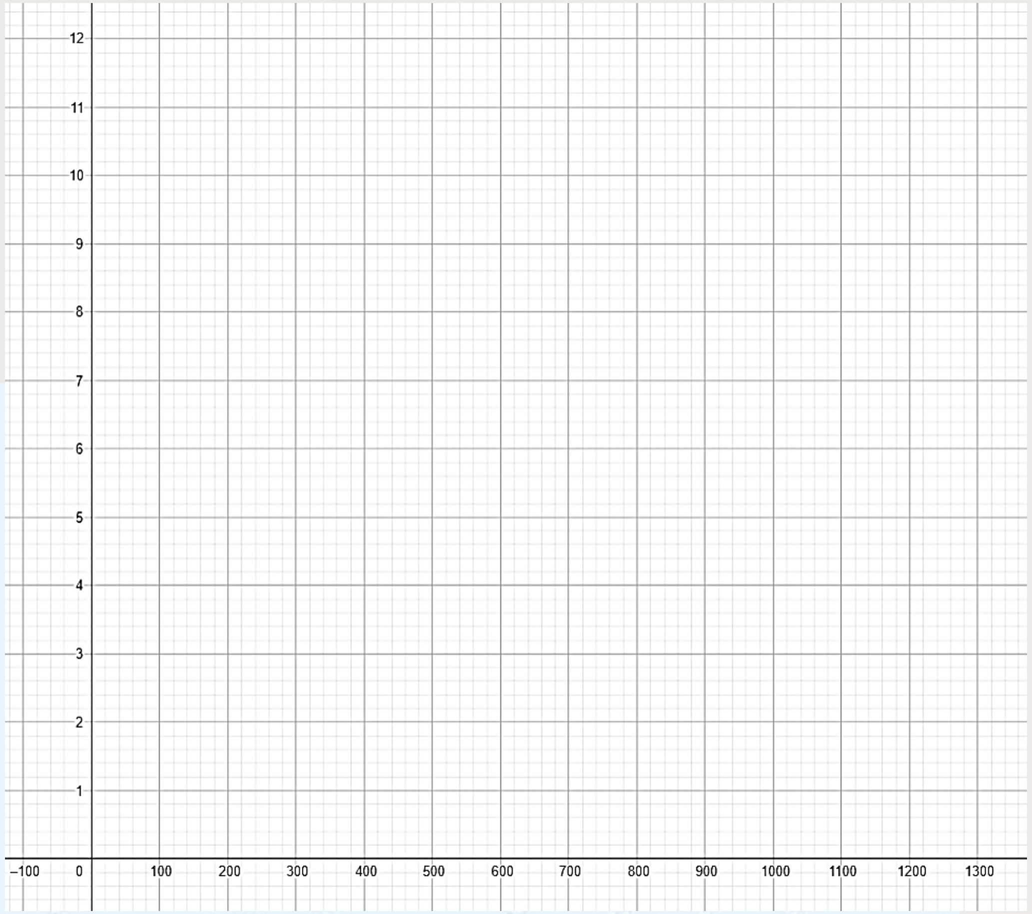


12 **Conceptual** When is a piecewise linear function a useful model?

A piecewise linear function is a useful model when the data follows different linear trends over different regions of the data.

A **piecewise function** is a function where more than one function is used to define the output over different pieces of the domain.

Example



Consider the piecewise function

$$h(t) = \begin{cases} \dfrac{1}{30}t & 0 \le t < 315 \\ 15 - \dfrac{1}{70}t & t \ge 315 \end{cases}$$

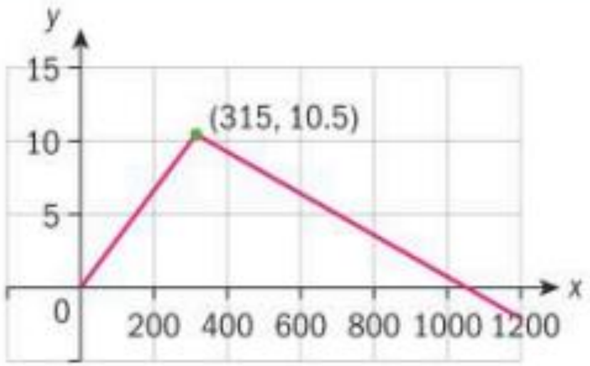
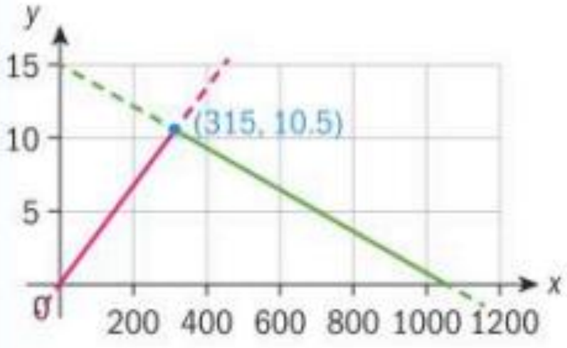
a Sketch the graph of the function.

Suppose that $h(t)$ is modelling the height h (in centimetres) of water in a bathtub as a function of time t (in seconds).

b Give a possible explanation for what happens at $t = 315$.

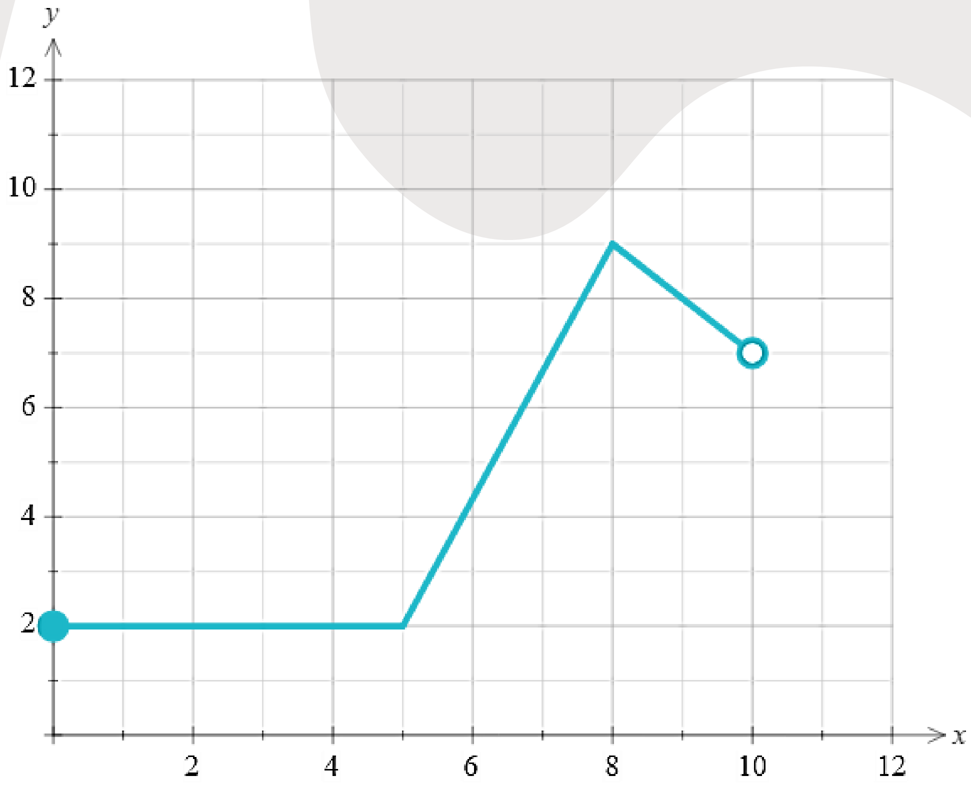
c Find the number of minutes until the bathtub is empty.

d Hence, write down a practical domain for $h(t)$.

a 	Graph both lines completely using technology or by hand. Then remove the portions of the lines that do not fall within the domain of each piece. 
b The bathtub begins to drain at this time, as the height of the water changes from increasing to decreasing.	Use technology or substitution to find the point where the pieces connect, at $t = 315$.
c $t = 1050$ seconds or 17.5 minutes **d** $0 \le t \le 17.5$	The bathtub will empty when $h = 0$. Find the t-intercept with technology or by substitution.

Write a piecewise function for each graph.

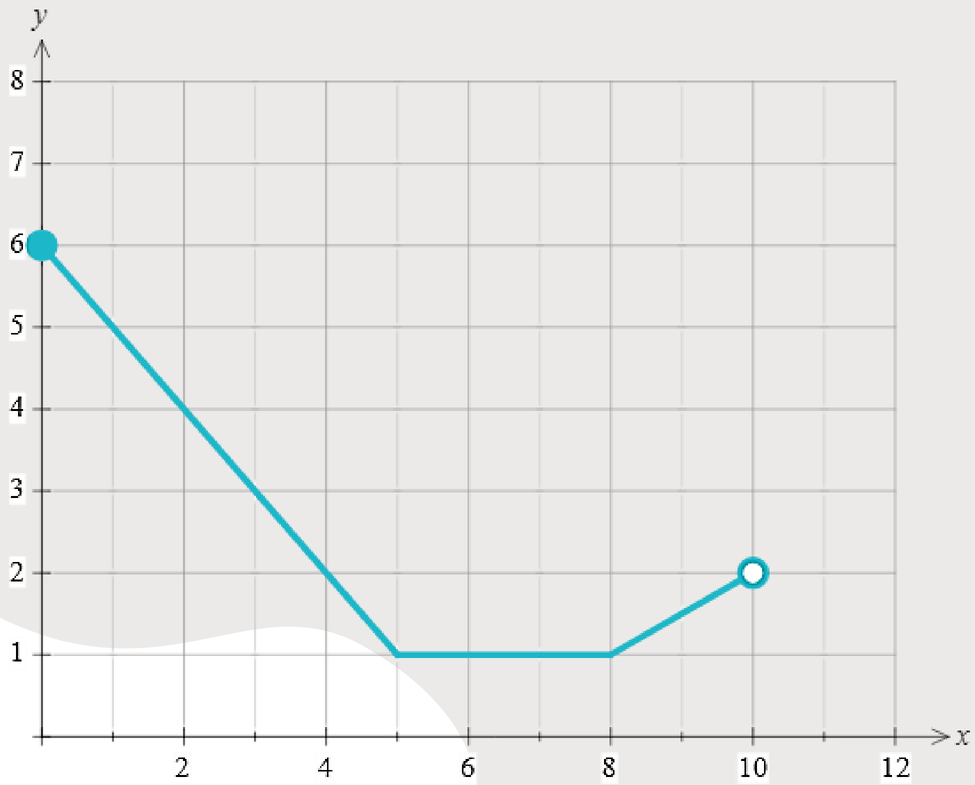
State the domain and range of each function.



$$f(x) = \begin{cases} 2 & 0 \leqslant x < 5 \\ \frac{7}{3}x - \frac{29}{3} & 5 \leqslant x < 8 \\ -x + 17 & 8 \leqslant x < 10 \end{cases}$$

Domain: $x \in [0, 10)$ *or* $0 \leqslant x < 10$

Range: $y \in [2, 9]$ *or* $2 \leqslant y \leqslant 9$



$$f(x) = \begin{cases} -x + 6 & 0 \leqslant x < 5 \\ 1 & 5 \leqslant x < 8 \\ -\frac{1}{2}x - 3 & 8 \leqslant x < 10 \end{cases}$$

Domain: $x \in [0, 10)$ *or* $0 \leqslant x < 10$

Range: $y \in [1, 6]$ *or* $1 \leqslant y \leqslant 6$

More Practice

Exercise 4G

1 Consider $f(x) = \begin{cases} 2x-1 & -5 \le x < 2 \\ 4-\dfrac{1}{2}x & 2 \le x < 7 \end{cases}$

a Sketch a graph of $f(x)$.

b Find

i $f(5.7)$ ii $f(-3.2)$

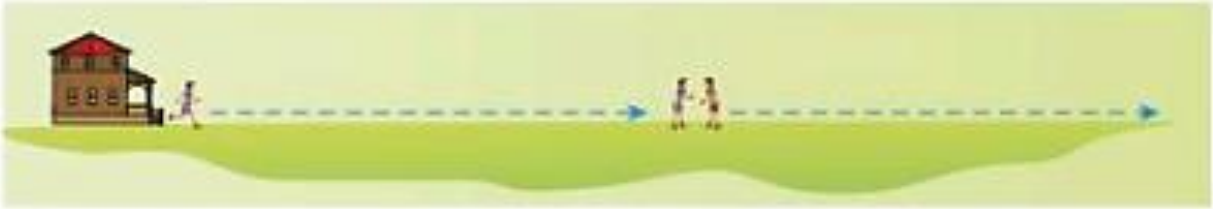
c Find the value(s) of x for which $f(x) = 2$.

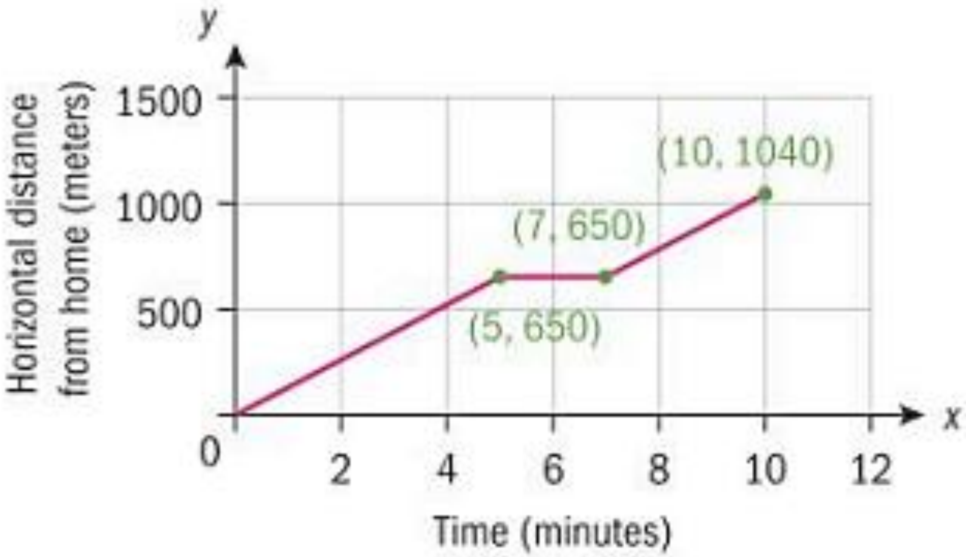
d Verify that $f(x)$ is continuous.

e State the domain and range of $f(x)$.

More Practice

2 a Construct a piecewise function for Amir's Run





b Verify from the formulas that the function is continuous.

c Determine how long it took Amir to run from 300 m to 800 m away from home.

More Practice

3 A cell phone company offers a plan with a flat rate of \$35 per month for up to one gigabyte (GB) of data. Any data used beyond 1 GB costs 6 cents per megabyte (MB), and 1000 MB = 1 GB.

a Find a piecewise linear model for the monthly cost C (in \$) of the phone as a function of the amount of data d (in GB) used.

b Determine the monthly cost if

i 500 MB is used **ii** 2 GB is used.

c The cell phone company offers an alternative plan that costs \$59/month for unlimited data. Determine when this plan is the better deal.

d It is January 3rd, and Yaqeen has used 172 MB of data. She is currently on the original \$35/month plan but is considering switching. Assume that her data usage will continue at the same constant rate.

i Estimate her data usage for the month of January.

ii Determine which plan is cheaper for Yaqeen, and how much cheaper it will be than the other plan.