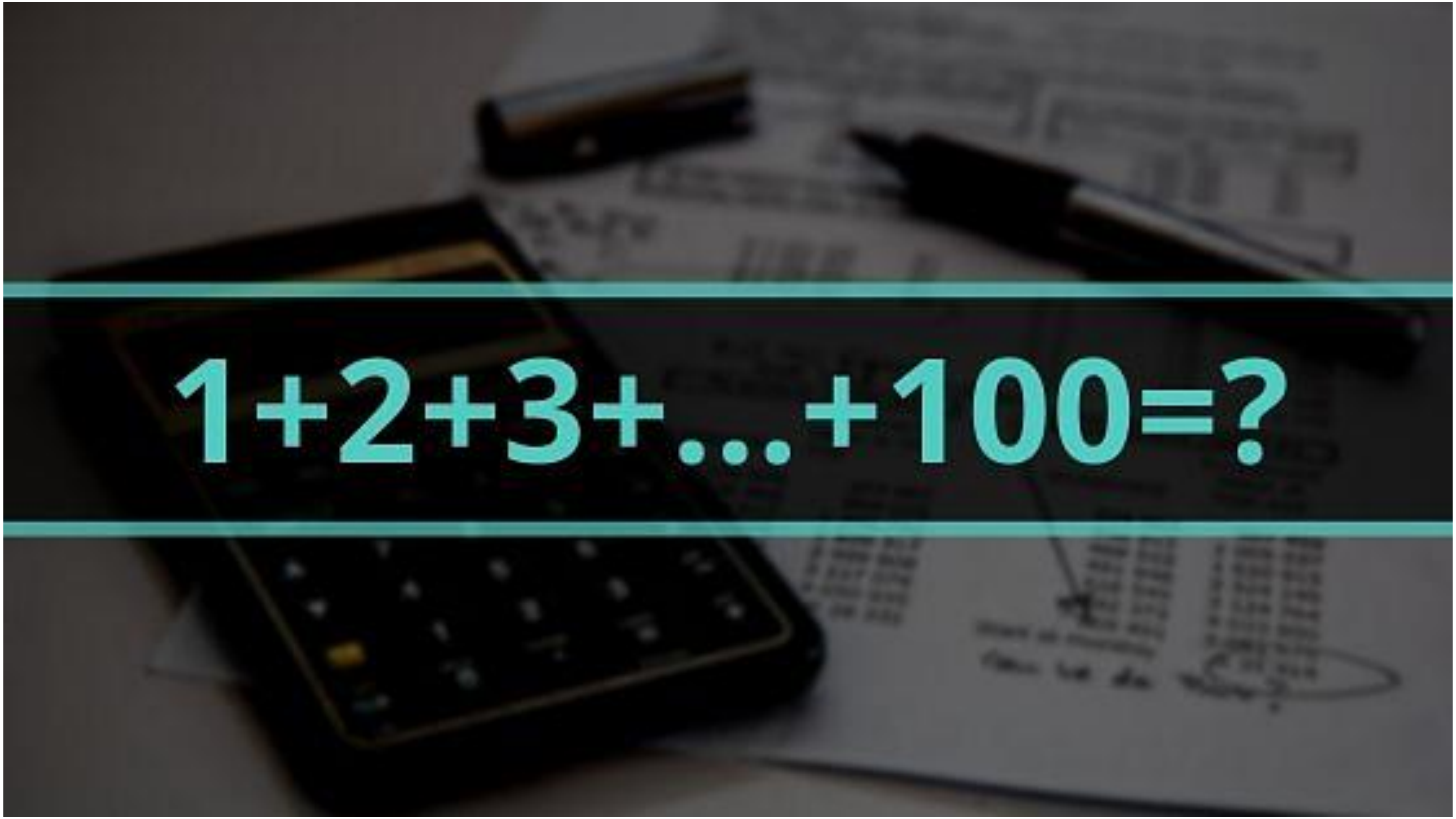
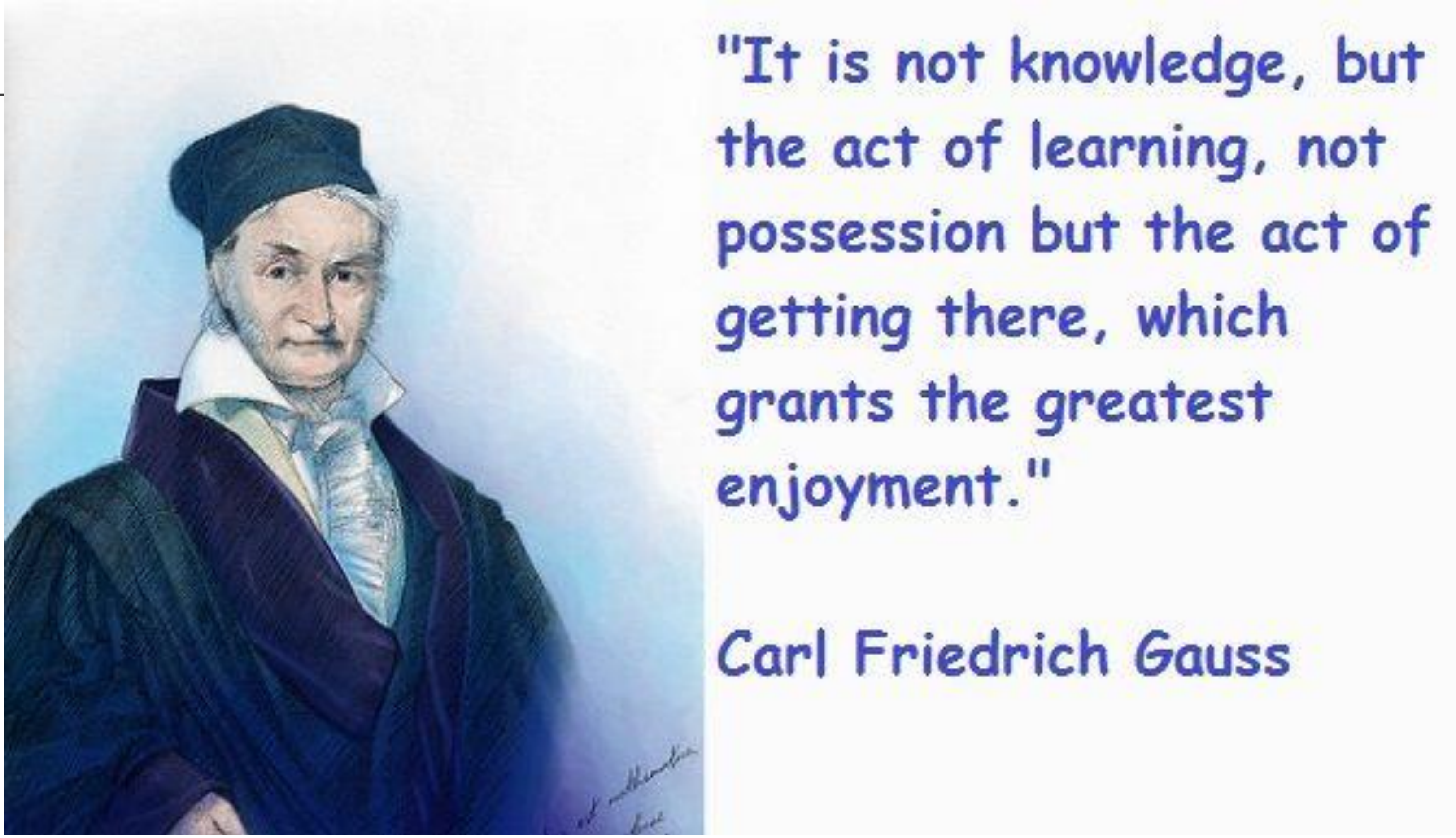
Welcome Back!

Arithmetic Sequences and Series

WEEK 20 – JANUARY 10, 2024


1+2+3+…+100=?

International-mindedness



https://i.pinimg.com/originals/56/8b/46/568b46f469ecc99fe0cb11c7cb647216.jpg

Read more about Gauss: https://www.britannica.com/biography/Carl-Friedrich-Gauss

A sequence of numbers is a list of numbers (of finite or infinite length) arranged in order that obeys a certain rule.

GENERAL SEQUENCES

Write down the next three terms in this sequence: 100, 75, 50, 25, …

The next three terms are 0, -25, and -50.

To get the nth term, subtract 25 to the previous term.

What is the general rule for this sequence? 2, 4, 6, 8, 10, …

Let n be the position of each term.

The general rule is **2n**.

What is the general rule for this sequence?1, 1/2, 1/3, 1/4, …

Let n be the position of each term.

The general rule is **1/n**.

Write down the first three terms of this sequence: $u_n = n + 1$, where n is a positive integer

$$u_1 = 1 + 1 = 2$$

$$u_2 = 2 + 1 = 3$$

$$u_3 = 3 + 1 = 4$$

Consider the sequence $u_n = 3 + [4(n-1)]$. Find the value of n for which $u_n = 111$.

$$111 = 3 + [4(n-1)]$$

$$111 - 3 = 4n - 4$$

$$108 + 4 = 4n$$

$$\boldsymbol{n = 28}$$

Arithmetic Sequences

A sequence in which the difference between each term and its previous one remains constant.

The constant difference is called the **common difference.**

Which of these sequences are arithmetic sequences? Choose all correct answers.

---

a. 2, 4, 6, 8, 10, ...

b. 1, 10, 100, 1000, ...

c. -1, -0.5, 0, 0.5, ...

d. $u_n = 5n + 2$

e. $u_n = n^2$

The general term of an arithmetic sequence

An arithmetic sequence with **first term *u_1*** and **common difference *d*** can be generated as:

u_1

$u_2 = u_1 + d$

$u_3 = u_1 + d + d = u_1 + 2d$

$u_4 = u_1 + d + d + d = u_1 + 3d$

$u_5 = u_1 + d + d + d + d = u_1 + 4d$

Following the pattern, $\boldsymbol{u_n = u_1 + (n-1)\,d}$, where n is a positive integer.

Consider this finite arithmetic sequence: -3, 5, … , 1189. Write down the common difference.

Answer: 8

$$n_2 - n_1 = 5 - (-3) = 8$$

Remember that the general term of an arithmetic sequence is $u_n = u_1 + (n\text{-}1)\ d.$
Find the number of terms in the sequence: -3, 5, ... , 1189

Answer: 150

$$u_n = u_1 + (n-1)d$$

$$1189 = -3 + (n-1)8$$

$$n = 150$$

Arithmetic Series

This is the **sum** of the terms of an arithmetic sequence.

The sum, S_n, of the first n terms of an arithmetic sequence u_1, u2, u_3, ... can be calculated using the formula $S_n = \frac{n}{2}(u_1 + u_n)$.

It can also be written as $S_n = \frac{n}{2}[2u_1 + (n-1)d]$.

Example

Calculate the sum of the first 20 terms of the series 60 + 57 + 54 + ...

common difference = 57 - 60 = -3

first term = 60

n = 20 terms

$$S_n = \frac{n}{2}\left[2u_1 + (n-1)\,d\right]$$

$$S_{20} = \frac{20}{2}\left[2\,(60) + (20-1)\,(-3)\right]$$

S_{20} = 630

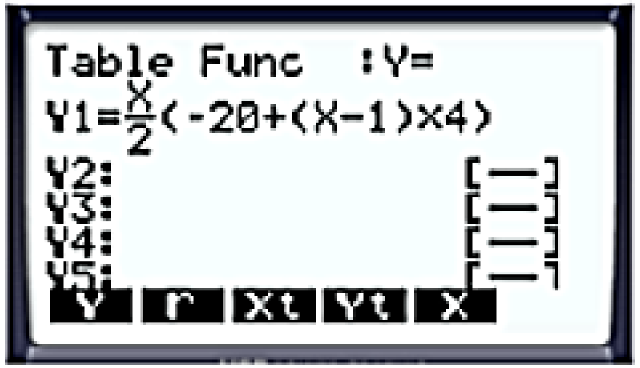
Find the sum of the arithmetic series (-10)+(-6)+(-2)+…+90.

Find the least number of terms that must be added to the series (-10)+(-6)+(-2)+... to obtain a sum greater than 100.

Given: d = 4 and sum > 100

Find: n (number of terms)

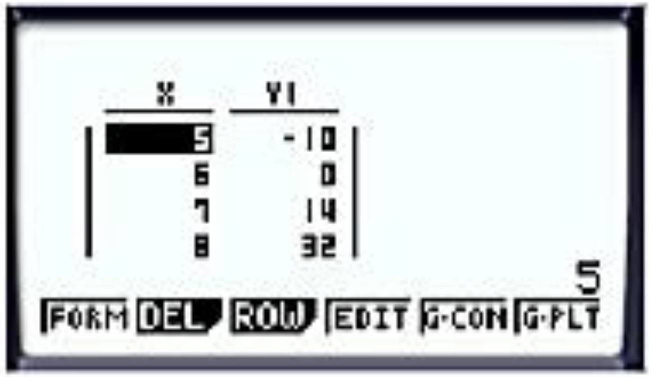
Step 1: Go to menu and select TABLE.

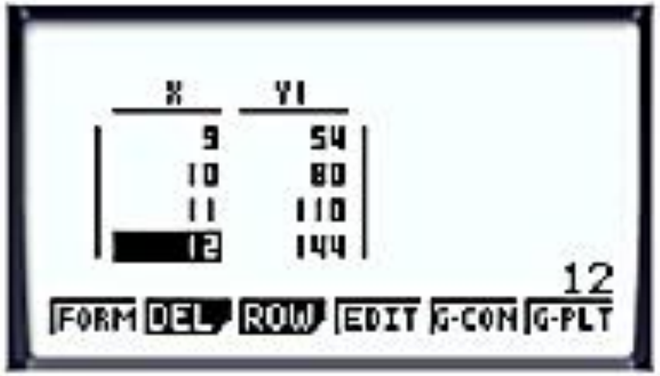


Step 2: Set (F5) your GDC as follows:

Continuation

Step 3: Press F6 (table function)





Adding 10 terms gives a sum of 80.

Adding 11 terms gives a sum of 110, so ***n* = 11**.

For each of the following arithmetic sequences:

i State its first term and common difference.

ii Find the 10th term of the sequence.

iii Determine, giving your reasons, whether 49 is an element of the sequence.

a $u_n = 3n + 1,\ n \in \mathbb{Z}^+$. Remember that $\mathbb{Z}^+$ is the set of positive integers: $\{1, 2, 3, \ldots\}$.

b 206, 199, 192, …

a	**i** Arithmetic. $u_1 = 4$, $d = 3$	The formula for calculating u_n is linear, so the sequence is arithmetic. The gradient or common difference is 3 and $u_1 = 3(1) + 1 = 4$.
	ii $u_{10} = 31$	Using the formula $u_n = u_1 + (n - 1)d$: $u_{10} = 4 + (10 - 1) \times 3 = 31$
	iii Yes, 49 is the 16th term.	You wish to determine whether any whole-number value of n results in $u_n = 49$. Use algebra or technology to solve the following: $49 = 4 + (n - 1) \times 3$ This gives $n = 16$.
b	**i** Arithmetic. $u_1 = 206$, $d = -7$	The sequence decreases by 7.
	ii $u_{10} = 143$	As d is negative, you subtract $7(n - 1)$: $u_{10} = 206 - 7(10 - 1) = 143$
	iii No, 49 lies between u_{23} and u_{24}.	You solve as in part **b**: $49 = 206 - 7(n - 1)$ Solving yields $n = 23.4$ (3 s.f.), a non-integer.

A piledriver is a machine used in construction to drive support poles into the ground by repeatedly striking them. Acme construction company uses a piledriver that drives support poles 0.12 m deeper into the ground with each strike. The current support pole has already been driven 13.6 m into the ground.

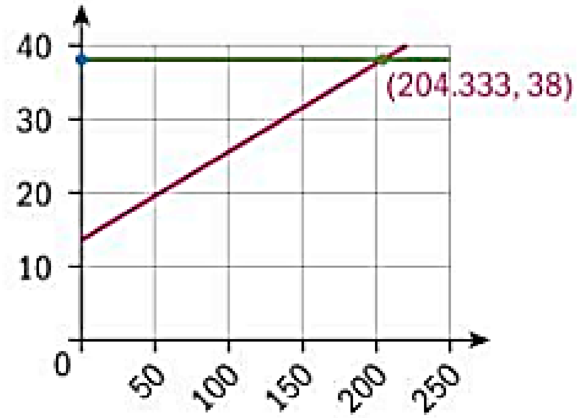
a If the sequence $\{u_n\}$ represents the depth of the support pole after n strikes, find the first three terms of the sequence.

b Write down an expression for the nth term of the sequence.

c The support poles must be driven to a depth of at least 38 m below ground. Determine

  i the number of strikes needed to reach this depth

  ii the exact depth it will then have reached.

a $u_1 = 13.6$, $u_2 = 13.72$, $u_3 = 13.84$	The pole is at an initial depth of 13.6 m, so $u_1 = 13.6$. The piledriver adds an additional depth of 0.12 m per strike, so $d = 0.12$.
b $u_n = 13.6 + 0.12(n - 1)$	Using $a_n = a_1 + (n - 1)d$.
c **i** 205 strikes	u_n represents the depth after n strikes, so you need to solve $u_n \geq 38$. Solve directly or using graphing technology, choosing an appropriate window:  This gives $n = 204.33$. As n must be a whole number and the depth must be 38 m or more, you choose the next largest whole number, so $n = 205$.
ii $u_{205} = 38.08$ m	Using the formula, $u_{205} = 13.6 + 0.12(205 - 1)$.

Julie swims each day in a 25 m pool. Today, she notes that she swims her first warm-up lap in 1 minute and 6 seconds. She then swims the remainder of her laps at a constant speed. After her fifth lap, she checks the clock and sees that 4 minutes 18 seconds have passed. Her entire swim takes 19 minutes 30 seconds.

a If the sequence $\{u_n\}$ represents Julie's total swim time after each lap in minutes, write down u_1 and u_5.

b Find the common difference, d, of this sequence. Explain its meaning in the context of the problem.

c Determine the number of laps that Julie swims.

a $u_1 = 1.1$, $u_5 = 4.3$	You convert minutes and seconds to minutes: 6 seconds = $\frac{6}{60}$ minutes = 0.1 minutes.
b $d = 0.8$ Julie takes 0.8 minutes, or 48 seconds, to swim one lap.	Four differences are added between the first and fifth terms, so $\frac{(4.3-1.1)}{4} = 0.8$. This can also be solved by substituting into the equation $u_5 = u_1 + 4d$: $4.3 = 1.1 + 4d$ $d = \frac{4.3-1.1}{4}$
c Julie swam 24 laps.	The last (nth) term of the sequence is 19.5: 1.1, u_2, u_3, u_4, 4.3, ..., 19.5 Substitute into the nth-term equation: $1.1 + (n - 1) \times 0.8 = 19.5$ Solving using algebra or technology gives $n = 24$.

Seatwork:

Exam.net: JYkTNN

Deadline: January 10

The examination will end at 5PM.

Next Topic

- Geometric Sequences and Series

Reference

OXFORD MAISL, p.235

OXFORD MAIHL, pp.178-182

Twinkl.com