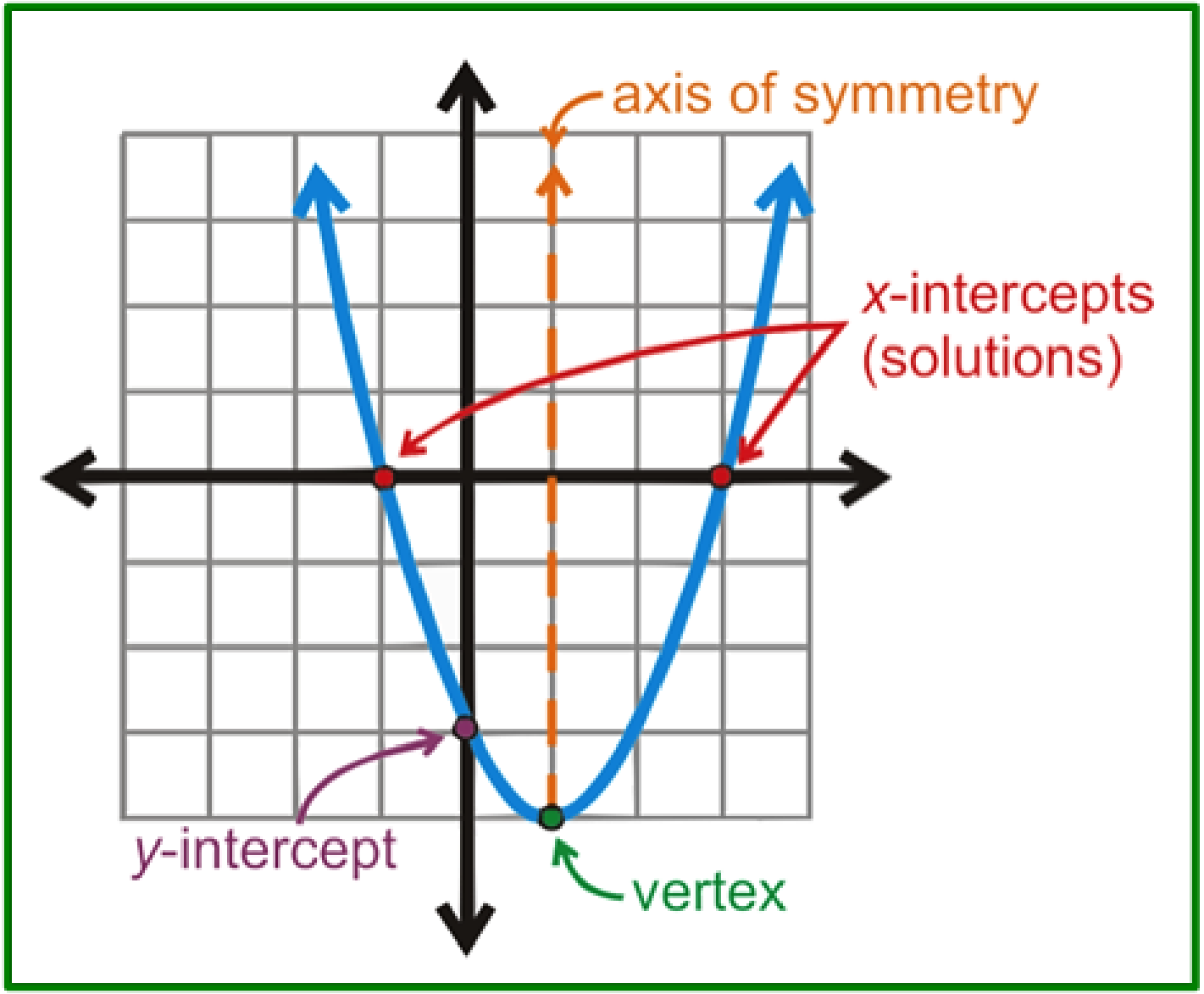
Quadratic Regression

Week 8 – Sept.11

Reference: Oxford Math AIHL, p.248

Recall


axis of symmetry
x-intercepts
(solutions)
y-intercept
vertex

A patient takes a specific drug in the form of a pill. Data is collected for the amount of the drug found in the bloodstream of the patient as soon as he has taken the medication. Time (t) is measured in hours and the concentration of the medication (C) is measured in milligrams of the medication found per litre of blood.

t	0	1	2	3	4	5	6	7	8	9	10
$C(t)$	0	4.87	7.17	10.27	12.81	13.05	15.03	13.3	12.22	11.29	8.26

a Plot a scatter plot of the given data.

b State a suitable type of function that would model this set of data points.

c Use your GDC to determine the model function for this set of data.

d Comment on the choice of model by determining the coefficient of determination.

e Sketch the model function over the scatter plot and comment on the closeness of fit to the original data.

f Hence, determine the time at which the medication is at its maximum effect.

g Using the model function, determine the time at which the medication will have been fully absorbed by the patient.

h Use the model to determine the concentration of the medication after 24 hours?

Coefficient of determination (r^2)

- This helps to determine whether the model fits the data well or not.
- It can be used for any curve.

r^2 value	Correlation
0	No correlation
$0 < \|r^2\| \leq 0.25$	Very weak
$0.25 < \|r^2\| \leq 0.5$	Weak
$0.5 < \|r^2\| \leq 0.75$	Moderate
$0.75 < \|r^2\| < 1$	Strong
1	Perfect

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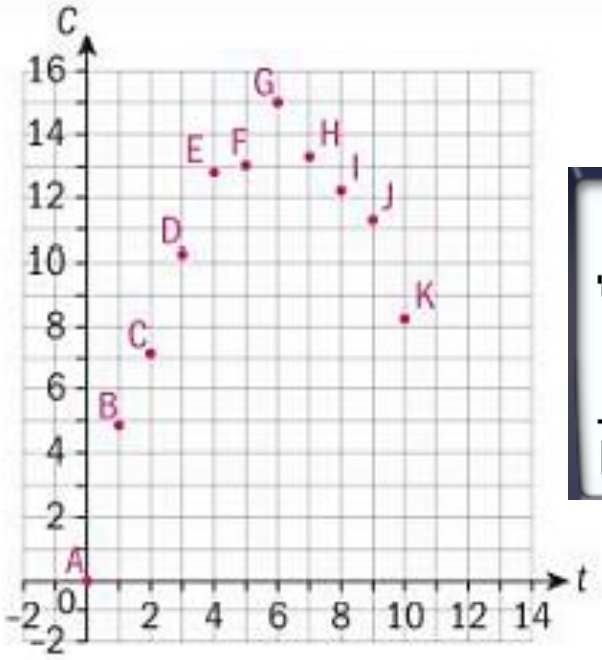
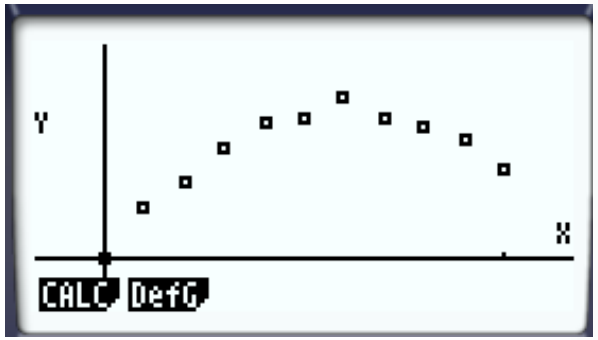
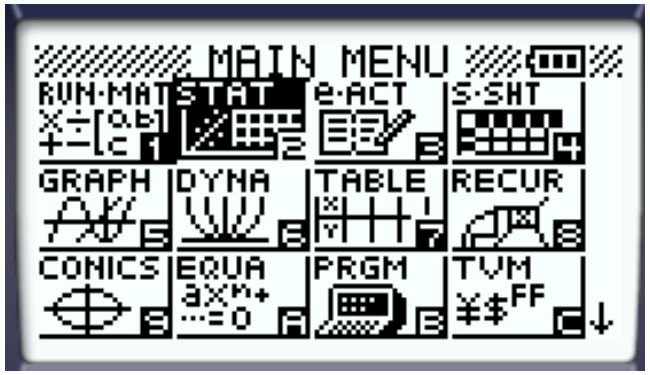
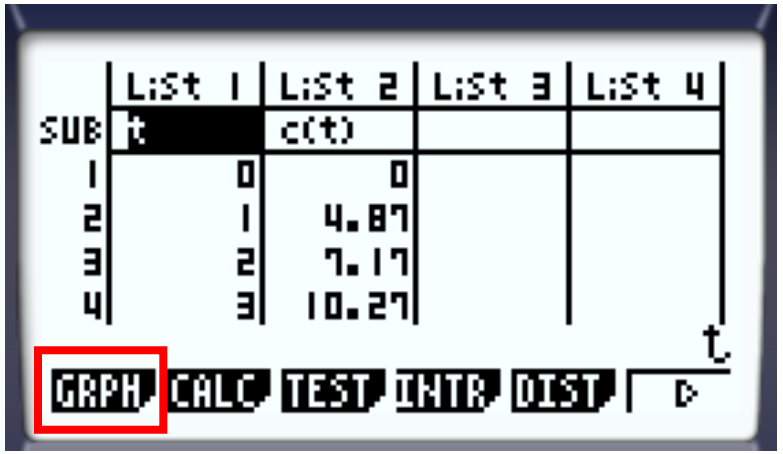
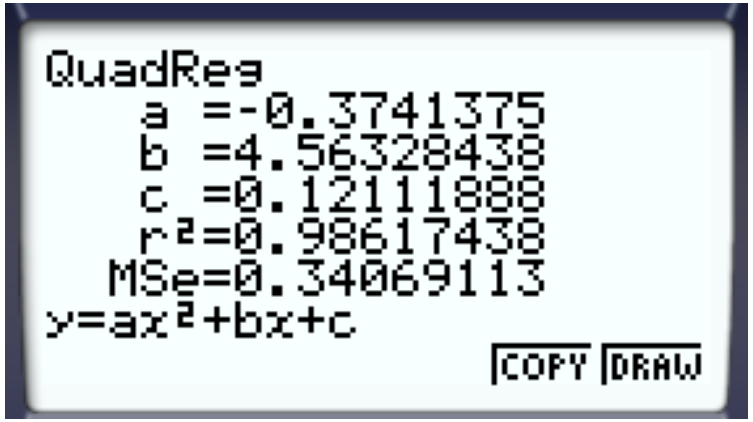
a









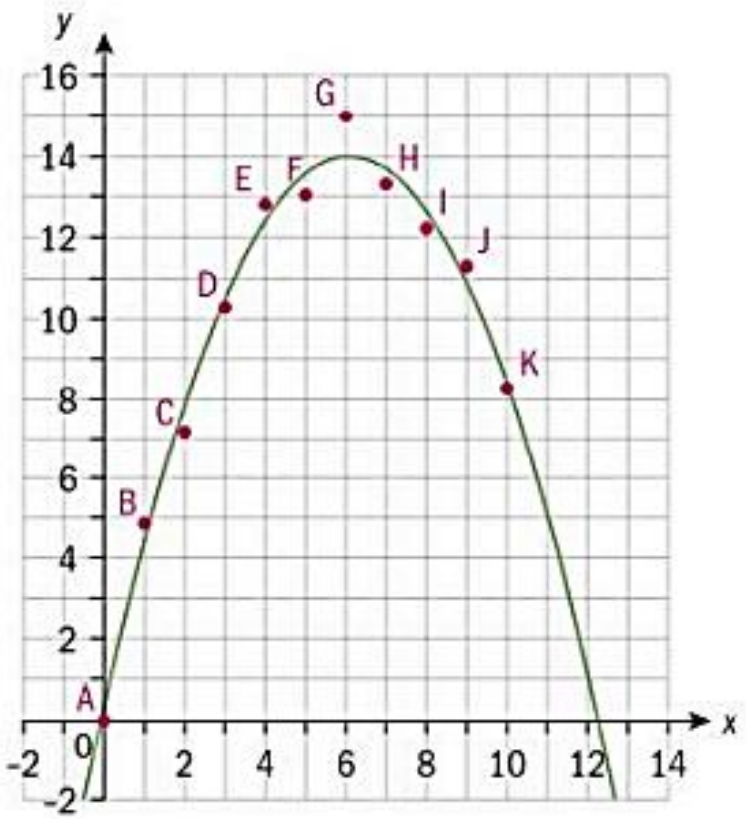


b There is strong evidence that the data depicts quadratic behaviour. There is a maximum point, there seems to be some kind of symmetry and the data follows a parabolic shape.

c $C(t) = -0.3741375t^2 + 4.56328438t + 0.12111888$

d The coefficient of determination is $R^2 = 0.98617438$, which shows very strong quadratic association.

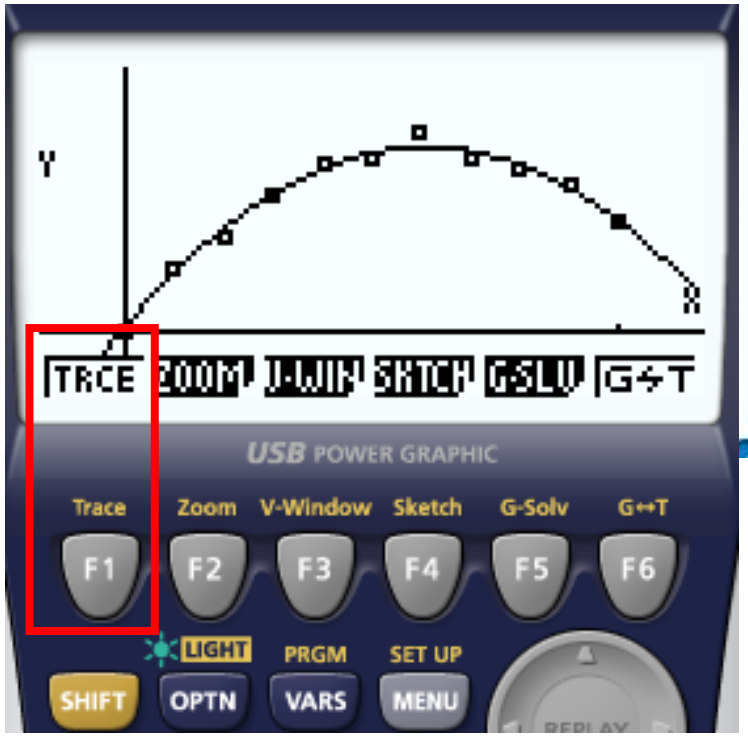
e



Use the '**trace**' function in your GDC to help you draw the parabola accurately.

Sketching the graph over the data points you get the graph shown.

It is now evident that the regression curve fits the data very closely.



f	The maximum amount of medication occurs around 6.1 hours after it was taken by the patient.	Using the graph of the model function on your GDC.
g	It will take about 12.2 hours for the whole of the medication to be fully absorbed and not be present any more in the bloodstream.	
h	No, as the value would be negative. $C(24) < 0$	After the time that the concentration of the medication becomes zero, the model cannot be used anymore as the value would become negative.

Examples
Quadratic Regression

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Exercise 6E

1 A company's weekly profit, in euros, in relation to the number of units sold each week, is given in the table below.

Number of units sold	200	300	400	500	600	700	800	900	1000	1100
Profit (euros)	4900	8000	10 000	12 000	12 600	13 200	13 000	10 000	8000	6000

a Plot these points on your GDC or other graphing software.
Put the number of units sold on the x-axis and the profit on the y-axis.

b Using your GDC or technology, find the best fit quadratic function through these points.

c State whether the function you found in part **b** is a good fit for this data. Justify your answer.

d Explain whether you could use this function to predict the value of the profit at a particular time during the year.

Extrapolation

- This means estimating a value at a point that is larger than (or smaller than) the data you have.

- Trends in data are only valid for the range of the study. We cannot extrapolate to draw conclusions outside of that range.

- Predictions are more valid when using **interpolation** to predict within the data set, rather than extrapolation that predicts values outside the data set.

- Predictions from a **linear** regression are only appropriate when the data displays a clear linear trend in its scatter plot (the correlation coefficient is **at least moderate**).

2 A company's weekly profit, in GBP in relation to the number of units sold each week, is given in the table below.

Number of units sold	0	5	10	15	20	25	30	35	40	45
Profit (GBP)	−350	−100	50	200	300	400	450	350	320	150

a Plot these points on your GDC or other graphing software.
Put the number of units sold on the x-axis and the profit on the y-axis.

b Using your GDC or technology, find the best fit quadratic function through these points.

c State whether the function you found in part **b** is a good fit for this data. Justify your answer.

d The company uses this function to predict what the profit would be in week 52. Explain whether this prediction would be reliable.