$X \sim B(n, p)$

BINOMIAL MODELS

- Interested in the number of successes in a set number of trials
- 4 conditions that must apply:
  - Only 2 possible outcomes (success/failure)
  - Probability of success remains constant (called ***p***)
  - Number of trials is set/known (called ***n***)
  - Independent trials
    - **10% Condition:** If we cannot assume independence, we can proceed as long as the sample is smaller than 10% of the population
- If these 4 conditions apply, we have a **Bernoulli trial [another name for Binomial trial]**

$q = 1 - p$

Notation:

$\mu_X = np$

$\sigma_X = \sqrt{npq}$

$\sigma_X^2 = npq$

01

Example: Computer chips have a 25% chance of being defective. Create the probability distribution for X, if X is the # of defective chips in a sample of 3. What is the probability of having 2 or more defective chips?

p

n

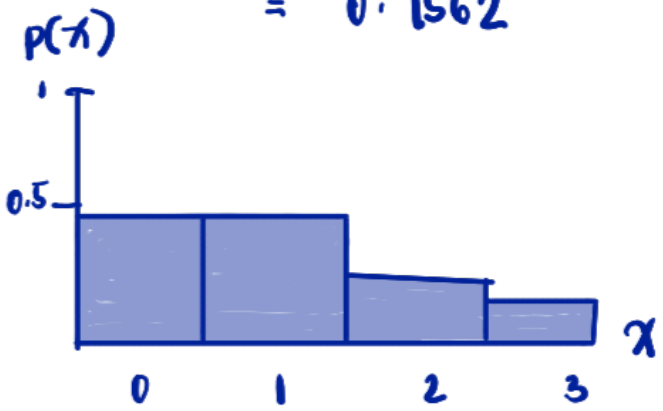
X	0	1	2	3
P(X)	${}_3C_0 (0.25)^0 (0.75)^3$ 0.4219	${}_3C_1 (0.25)^1 (0.75)^2$ 0.4219	${}_3C_2 (0.25)^2 (0.75)^1$ 0.1406	${}_3C_3 (0.25)^3 (0.75)^0$ 0.0156

$P(x \geq 2)$

$P(x \geq 2) = P(x = 2) + P(x = 3)$ OR $P(x \geq 2) = 1 - P(x < 1)$

$= 0.1406 + 0.0156$ $= 1 - 0.8438$

$= 0.1562$ $= 0.1562$



02

Example: I am playing a game in which I have only a 39% chance of winning. I am playing 4 times. Create the probability distribution below:

P

n

X	P(X)
0	0.1385
1	0.3541
2	0.3396
3	0.1447
4	0.0231

Easy questions: Do twice, once using the table, the second time using binomialpdf or cdf.

P(X=2) =

P(X<2) =

P(X≥3) =

P(2≤X≤4) =

Now let's look at changing the sample size to **10**, and answering similar questions:

Example: John is taking archery. He has a 30% chance of hitting the target each time he shoots. He shoots 8 times

1) What is the probability that he hits the target 4 times?

Binomial pdf (8, 0.3, 4)

2) What is the probability that he hits the target 2 times or less?

Binomial cdf (8, 0.3, 2)

3) What is the probability that he hits the target at least 3 times?

1 − Binomialcdf (8, 0.3, 2)

4) What is the probability that he hits the target less than 5 times?

Binomialcdf (8, 0.3, 4)

5) What is the probability that he hits the target more than 6 times?

1 − Binomialcdf (8, 0.3, 6)

6) How many times do we expect him to hit the target? (average!)

$E(x) = np = 8 \times 0.3 = 2.4$ hits per 8 shots

7) What is the standard deviation of the number of times he hits the target?

$\sigma_x = \sqrt{npq} = \sqrt{8 \times 0.3 \times 0.7} = 1.3$