RULES FOR DIFFERENTIATION

AHL 5.9	Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \dfrac{1}{\cos^2 x}$
	Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \dfrac{1}{x}$
	Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$ $y = g(f(x))$
	Product rule	$y = uv \Rightarrow \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
	Quotient rule	$y = \dfrac{u}{v} \Rightarrow \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Recall:

Find the derivative of:

a.
$y = 3x^2 + 2x - 7$ $\quad y' = 6x + 2$

b.
$f(x) = 3\sqrt{x}$ $\quad f(x) = 3x^{\frac{1}{2}}$ $\quad f'(x) = \frac{3}{2}x^{-\frac{1}{2}} = \frac{3}{2\sqrt{x}}$

c.
$y = (x^2 - 3)^2$ $\quad y = x^4 - 6x^2 + 9$
$y' = 4x^3 - 12x$

Now, try this:

d.
$y = (x^3 + 1)^3$ $\quad y' = 3(x^3+1)^2(3x^2)$ $\quad y = (x^3+1)^3$
$y' = 3(x^3+1)^2 \cdot 3x^2$

x^3+1

$u = x^3+1$ $\quad y = u^3$
$u' = 3x^2$ $\quad y' = 3u^2$

CHAIN RULE:

Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

e.

$y = (3x^2 + 2x - 3)^2$

$y' = 2(3x^2+2x-3)(6x+2)$ $\quad y' = (12x+4)(3x^2+2x-3)$

f.

$y = 3\sqrt{2x^2 - x}$

$y = 3(2x^2 - x)^{\frac{1}{2}}$

$y' = \frac{3}{2}(2x^2-x)^{-\frac{1}{2}}(4x-1)$

$y' = \frac{12x-3}{2\sqrt{2x^2-x}}$

g.

$y = \left(3x^3 + \frac{2}{x}\right)^4$

$y = (3x^3 + 2x^{-1})^4$

$\frac{dy}{dx} = 4(3x^3+2x^{-1})^3(9x^2-2x^{-2})$

$= \left(3x^3+\frac{2}{x}\right)^3\left(36x^2-\frac{8}{x^2}\right)$

h.

$y = \frac{2}{3(x^2+2x)^4}$

$y = \frac{2}{3}(x^2+2x)^{-4}$

$y' = -\frac{8}{3}(x^2+2x)^{-5}(2x+2)$

$= \frac{-(16x+16)}{3(x^2+2x)^5}$

i.

$y = \frac{5}{2x^2 - 3x}$

$y = 5(2x^2-3x)^{-1}$

$\frac{dy}{dx} = -5(2x^2-3x)^{-2}(4x-3)$

$\frac{dy}{dx} = \frac{-5(4x-3)}{(2x^2-3x)^2}$ ✓

j. $\quad y = \cos x \quad y' = -\sin x \leftarrow$

$g(x) = 2\cos 4x$

$g'(x) = 2(-\sin 4x)(4)$

$g'(x) = -8\sin 4x$

$f(x) = \dfrac{2x+4}{3x-1}$

$y = \dfrac{u}{v}$ $\quad \dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$u = 2x+4 \quad u' = 2$

$v = 3x-1 \quad v' = 3$

$$f'(x) = \frac{2(3x-1) - 3(2x+4)}{(3x-1)^2}$$

$$= \frac{\cancel{6x} - 2 - \cancel{6x} - 12}{(3x-1)^2}$$

$$= \frac{-14}{(3x-1)^2}$$

$f(x) = (2x+4)(3x-1)^{-1}$

$u = 2x+4 \qquad v = (3x-1)^{-1}$

$y = uv \quad y' = uv' + u'v$

$u' = 2 \qquad v' = -1(3x-1)^{-2}(3)$

$= -3(3x-1)^{-2}$

$$f'(x) = (2x+4)(-3(3x-1)^{-2}) + 2(3x-1)^{-1}$$

$$f'(x) = \frac{-3(2x+4)}{(3x-1)^2} + \frac{2}{3x-1} \checkmark$$

$$= \frac{-6x - 12 + 2(3x-1)}{(3x-1)^2}$$

$$= \frac{\cancel{-6x} - 12 \cancel{+6x} - 2}{(3x-1)^2}$$

$$= \frac{-14}{(3x-1)^2}$$

k. $y = \ln x \quad y' = \frac{1}{x} \quad y = \sin x \quad y' = \cos x$

$y = \ln(\sin 2x)$

$y' = \frac{1}{\sin 2x} \cdot \cos 2x \cdot 2$

$y' = \frac{2\cos 2x}{\sin 2x}$

l. $y = e^x \quad y' = e^x$

$y = e^{\cos 2x}$

$y' = e^{\cos 2x} \cdot -\sin 2x \cdot 2$

$y' = -2\sin(2x)e^{\cos 2x}$

$f'(x)$

EQUATIONS OF TANGENTS AND NORMALS $\frac{1}{f'(x)}$

m. $f(x) = 2 + x - 3x^{-1}$

Let $f(x) = 2 + x - \frac{3}{x}$. The line L is the tangent to the curve of f at $(1, 0)$.

(a) Find the expression of $f'(x)$. [2]

(b) Find the gradient of L. [2]

(c) Find the equation of L in the form $y = ax + b$. [2]

gradient Function

a. $f'(x) = 1 + \frac{3}{x^2}$

b. $f'(1) = 1 + \frac{3}{1}$

$= 4$

c. $y - 0 = 4(x - 1)$

$y = 4x - 4$

n. $3x - 4x^{-2}$

Let $f(x)=3x-\frac{4}{x^2}$. The line L is the normal to the curve of f at $(1,-1)$.

(a) Find the expression of $f'(x)$.

[2]

(b) Find the gradient of L.

[2]

(c) Find the equation of L in the form $y=ax+b$.

[2]

a. $f'(x) = 3 + \frac{8}{x^3}$

b. $f'(1) = 3+8$

$= 11$

c. $y+1 = \frac{-1}{11}(x-1)$

$y = -\frac{1}{11}x + \frac{1}{11} - \frac{11}{11}$

$y = -\frac{1}{11}x - \frac{10}{11}$

o.

Let $f(x)=ax^3-2x^2+1$. The line L is the tangent to the curve of f at $(3, 27a-17)$.

(a) Find the expression of $f'(x)$ in terms of a.

[2]

(b) The gradient of L is 96. Find the value of a.

[2]

(c) Find the equation of L in the form $y=mx+c$.

[2]

Practice Questions

1.

Let $f(x) = e^x \sin x$.

(a) Find $f'(x)$.

[2]

(b) (i) Find the gradient of the tangent to the curve of f at $x = \frac{\pi}{2}$.

(ii) Hence, find the gradient of the normal to the curve of f at $x = \frac{\pi}{2}$.

[4]

2.

Let $f(x) = 3x\cos x$.

(a) Find $f'(x)$.

[2]

(b) (i) Find the gradient of the tangent to the curve of f at $x = \frac{3\pi}{2}$.

(ii) Hence, find the gradient of the normal to the curve of f at $x = \frac{3\pi}{2}$.

[4]

3.

Let $f(x) = e^{-3x}$.

(a) Find $f'(x)$.

[2]

(b) (i) Find the gradient of the tangent to the curve of f at $x = 0.1$.

(ii) Hence, find the gradient of the normal to the curve of f at $x = 0.1$.

[4]

4.

Let $f(x) = 2e^{-x}$.

(a) Find $f'(x)$.

[2]

(b) (i) Find the gradient of the tangent to the curve of f at $(0, 2)$.

(ii) Hence, find the equation of the tangent to the curve of f at $(0, 2)$, giving the answer in the form $y = ax + b$.

[5]

5.

Let $f(x) = \ln\sqrt{x}$.

(a) Find $f'(x)$.

[2]

(b) Hence, write down the gradient of the tangent to the curve of f at $(2, \ln\sqrt{2})$.

[1]

It is given that the equation of the normal to the curve of f at $(2, \ln\sqrt{2})$ is $y = mx + (\ln\sqrt{2} - 2m)$, where m is a constant.

(c) Find the exact value of the y-intercept of the normal to the curve of f at $(2, \ln\sqrt{2})$.

[3]