Continuous Probability Distributions

Probability Distributions

Know this

Continuous Probability Distributions

A continuous probability density function (or continuous probability distribution), f(x), is a function satisfying the following properties:

1. The variable is continuous and can assume all real-valued numbers.

2. The function is non-negative, that is $f(x) \geq 0$.

3. The total area contained between the graph and the horizontal axis is 1, that is $\int_{-\infty}^{\infty} f(x)dx = 1$

Continuous Probability Distributions

Example: The continuous random variable X has a probability density function defined by $f(x)=\begin{cases} k(2-x)^2, & 0\le x\le 2 \\ 0, & otherwise \end{cases}$

a) Find k.

b) Find the probability that i) $0.5 < X \le 1$ ii) $X > 1 | X > 0.5$

Know this

Mode is the most commonly occurring value. Graphically this is the maximum. Mode of a continuous probability distribution occurs when $f'(x) = 0$.

Mean, μ, for a continuous probability distribution $E(X) = \int_{-\infty}^{\infty} xf(x)dx$

Median, m, is the value of the variable such that half the probability is below that value (and half is above), thus m can be found using $\int_{-\infty}^{m} f(x)dx = \frac{1}{2}$

Variance, $Var(X) = E\left(X^2\right) - \mu^2$

$$Var(X) = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

Continuous Probability Distributions

Example: Find the mode, mean, median, variance and standard deviation of the probability distribution function $f(x)=\begin{cases} 12x^2(1-x) & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$

Continuous Probability Distributions

The probability density function of a continuous random variable X is given by

$$f(x) = \frac{1}{1+x^4}, \quad 0 \leq x \leq a$$

(a) Find the value of a. (3 marks)

(b) Find the mean of X. (2 marks)

Transformations on Random Variables

Probability Distributions

Transforming Random Variables

4.14	Linear transformation of a single random variable	$\mathrm{E}(aX+b)=a\mathrm{E}(X)+b$ $\mathrm{Var}(aX+b)=a^2\,\mathrm{Var}(X)$
	Linear combinations of n independent random variables, $X_1, X_2, \ldots, X_n$	$\mathrm{E}(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)=a_1\mathrm{E}(X_1)\pm a_2\mathrm{E}(X_2)\pm \ldots \pm a_n\mathrm{E}(X_n)$ $\mathrm{Var}(a_1X_1 \pm a_2X_2 \pm \ldots \pm a_nX_n)$ $=a_1^2\,\mathrm{Var}(X_1)+a_2^2\,\mathrm{Var}(X_2)+\ldots+a_n^2\,\mathrm{Var}(X_n)$

Transforming Random Variables

Suppose the equation Y = 20 + 10X converts a PSAT math score, X, into an SAT math score, Y. Suppose the average PSAT math score is 48. What is the average SAT math score?

Let $\mu_X = 625$ represent the average SAT math score.

Let $\mu_Y = 590$ represent the average SAT verbal score.

$\mu_{X+Y} = \mu_X + \mu_Y$ represents the average combined SAT score. Then

Transforming Random Variables

Suppose the equation $Y = 20 + 10X$ converts a PSAT math score, X, into an SAT math score, Y. Suppose the standard deviation for the PSAT math score is 1.5 points. What is the standard deviation for the SAT math score?

Suppose the standard deviation for the SAT math score is 150 points, and the standard deviation for the SAT verbal score is 165 points. What is the standard deviation for the combined SAT score?

Transforming Random Variables

Random variable X has a mean of 6.2 and a standard deviation of 3.1.

a. Find the new mean and standard deviation if we multiply by 3

b. Find the new mean and standard deviation if we subtract 10

c. Find the new mean and standard deviation if we multiply by 5 and add 10.

Transforming Random Variables

Random variable Y has a mean of 3.4 and a standard deviation of 1.4.

a. Find the mean and standard deviation of $X + Y$

b. Find the mean and standard deviation of $X - Y$

c. Find the mean and standard deviation of $2X + 3Y$

d. Find the mean and standard deviation of $3X + Y - 4$

Thanks

Do you have any question?

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