Geometric Sequences

Week 21 – January 17, 2024

Objectives

01 Define a geometric sequence

02 Describe the rule in finding the next term

03 Find the nth term of a geometric sequence

04 Find the sum of a geometric series

International-mindedness

According to an old legend, the game of chess was invented by a skilled mathematician named Sissa Ben Dahir, who served as a Grand Vizier in an ancient Mughal kingdom in India.

The King, Shirham, was so impressed with the game that he offered Sissa a reward of gold and silver.

https://muaw1yah.medium.com/the-legend-of-sissa-ben-dahir-a-cautionary-tale-on-the-power-of-exponential-growth-32120f01dbf3

International-mindedness

Do you think Sissa accepted the offer of the king? Why or why not?

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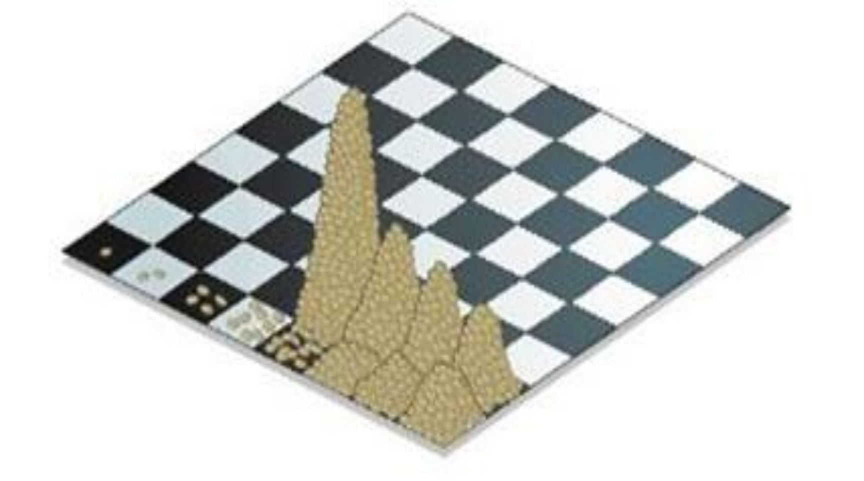
International-mindedness



At first, the King thought the reward would not cost him much, but as the counting began, the number of grains required for each square increased rapidly.

Despite bringing in more bags of wheat, it soon became clear that the King could not fulfill Sissa's request.

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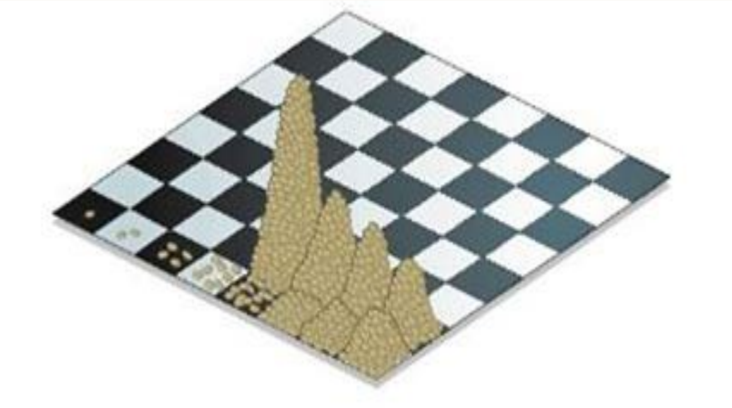
International-mindedness

- How would you be able to find out the number of grains of wheat on the **last square** of the chessboard?
- How would you be able to find the **total** number of grains of wheat on the chessboard?



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Geometric Sequence

This is a sequence of numbers in which each term can be found by multiplying the preceding term by a **common ratio.**

For a sequence to be geometric, $r \neq 1$.

Investigate

For each of the following sequences, find the next term and describe the rule for finding the next term.

a 2, 4, 6, 8, …

b 1, 3, 9, 27, …

c 1, 1, 2, 3, 5, …

d −3, −1.5, 0, 1.5, …

e 1, 4, 9, 16, …

f 5, 15, 45, 135, …

To check whether a sequence is geometric, find the ratio of pairs of consecutive terms. If this ratio is constant, it is the common ratio and the sequence is geometric.

The nth term in a geometric sequence is given by the formula $u_n = u_1 r^{n-1}$.

Example

1 For each of these geometric sequences, write down:

i the first term, u_1 **ii** the common ratio, r **iii** u_{10}.

a 2, 6, 18, 54, … **b** –3, 6, –12, 24, … **c** 16, 8, 4, 2, …

Geometric Series (Finite Sequence)

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The sum to n terms of a geometric series is written as S_n. The formula for S_n is

$$S_n = \sum_{i=1}^{n} u_i = \boxed{\frac{u_1(r^n - 1)}{r - 1}} = \boxed{\frac{u_1(1 - r^n)}{1 - r}}, r \neq 1$$

You can use either form.

A **geometric series** is the sum of the terms of a geometric sequence.

Find the sum to eight terms of the geometric series

7, 28, 112, 448, ...

$$u_1 = 7$$

$$r = \frac{28}{7} = 4$$

$$\text{So, } S_8 = \frac{7(4^8 - 1)}{4 - 1} = 152\,915$$

When Kenzo starts Bright Academy, the school fees are JYN 2 500 000 (Japanese Yen). The fees increase by 2% each year. Kenzo attends the school for a total of six years.

a. Write down the common ratio.
b. Calculate the school fees in year six.
c. Calculate the total fees paid for the six years.

a $r = 1.02$

$1 + 2\% = 1 + 0.02 = 1.02$

b $u_6 = u_1 r^5 = 2\,500\,000 \times 1.02^5$

$= \text{JYN } 2\,760\,202$

Remember, $u_n = u_1 r^{n-1}$

c $S_6 = \dfrac{u_1(r^6 - 1)}{(r-1)} = \dfrac{2\,500\,000(1.02^6 - 1)}{(1.02 - 1)}$

$= \text{JYN } 15\,770\,302$

$$S_n = \sum_{i=1}^{n} u_i = \frac{u_1(r^n - 1)}{r - 1}$$

The second term of a geometric sequence is 6 and the fourth term is 24. All the terms are positive.

a. Find the first term and the common ratio.
b. Find the sum of the first 10 terms.

a $u_2 = u_1 r = 6$

$u_4 = u_1 r^3 = 24$

$$\frac{u_4}{u_2} = \frac{u_1 r^3}{u_1 r} = \frac{24}{6} = 4$$

$r^2 = 4$

$\boxed{r = 2}$

$u_1 r = 6$

$u_1(2) = 6$

$\boxed{u_1 = 3}$

b $S_{10} = \dfrac{3(2^{10} - 1)}{(2 - 1)} = 3069$

You do not need $r = -2$ because all the terms are positive.

Geometric Series (Infinite Sequence)

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Sum of Infinite Geometric Series

$$S_\infty = \frac{u_1}{1-r}, \ |r| < 1$$

In the case of an infinite geometric series when the common ratio *is greater than one, the terms in the sequence will get larger and larger and if you add the larger numbers, you won't get a final answer. The only possible answer would be infinity.*

Find the sum of the geometric series

$125 + 25 + 5 + 1 + \ldots$

The nth term of a geometric sequence The sum of n terms of a finite geometric sequence	$u_n = u_1 r^{n-1}$ $S_n = \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
The sum of an infinite geometric sequence	$S_\infty = \dfrac{u_1}{1 - r}$, $\lvert r \rvert < 1$

Task: Geometric Sequence Puzzler

Thank you!

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