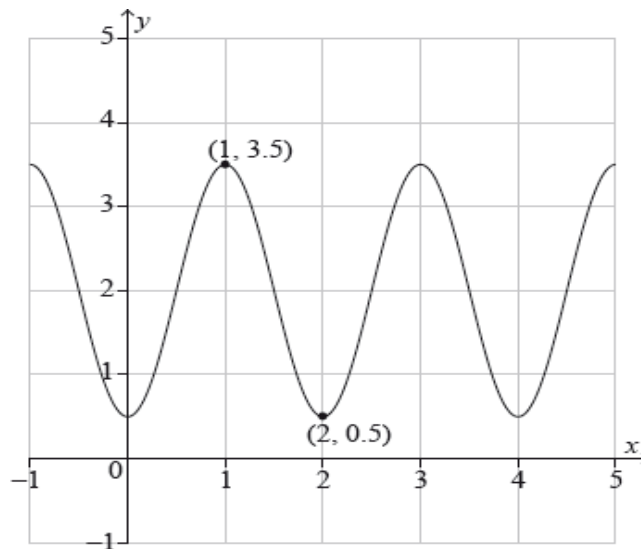


Answer the following problems involving periodic functions. Show your complete work.
Full marks are not necessarily awarded for incomplete working. [24 marks]

1a. [2] The following diagram shows the curve $y = a \sin(b(x - c)) + d$, where a, b, c and d are all positive constants. The curve has a maximum point at $(1, 3.5)$ and a minimum point at $(2, 0.5)$.

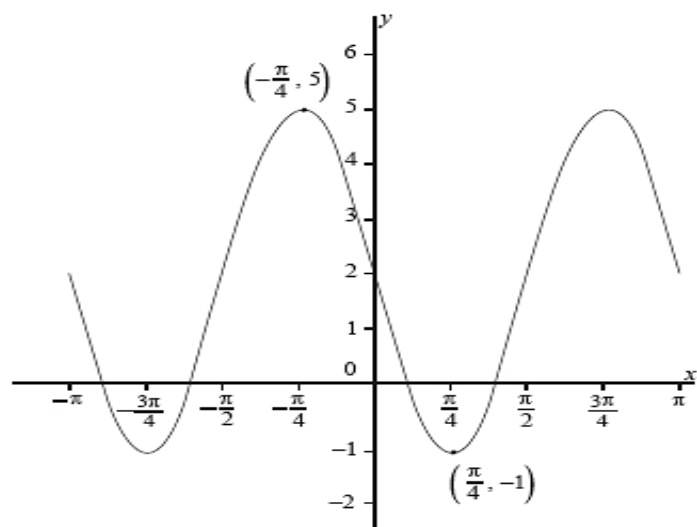


Write down the value of a and the value of d .

1b. [1] Find the value of b .

1c. [1] Find the smallest possible value of c , given $c > 0$.

2a. [3] A function is defined by $f(x) = A \sin(Bx) + C$, $-\pi \leq x \leq \pi$, where $A, B, C \in \mathbb{Z}$. The following diagram represents the graph of $y = f(x)$.

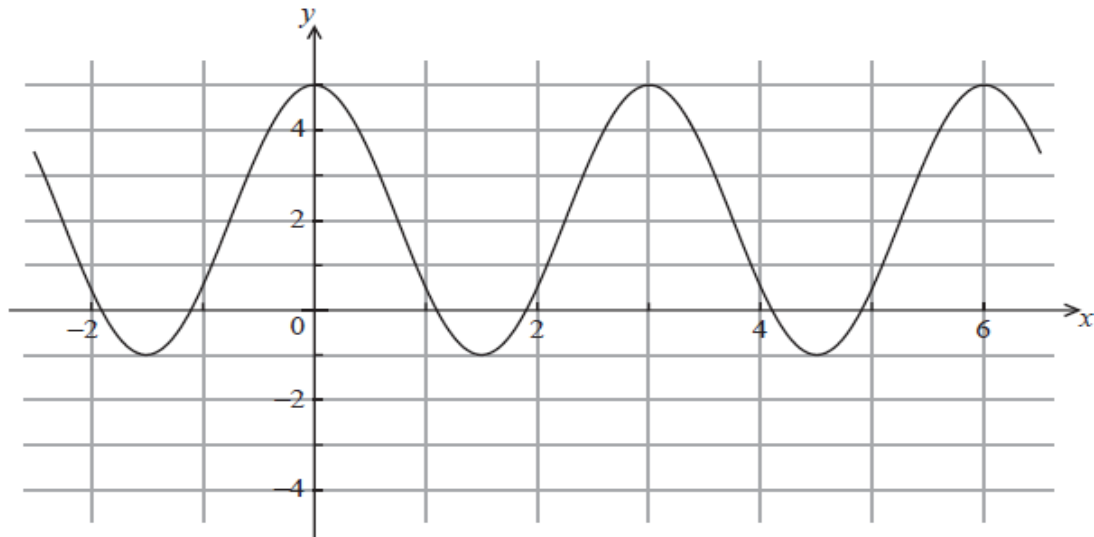


Find the value of

- (i) A ;
- (ii) B ;
- (iii) C .

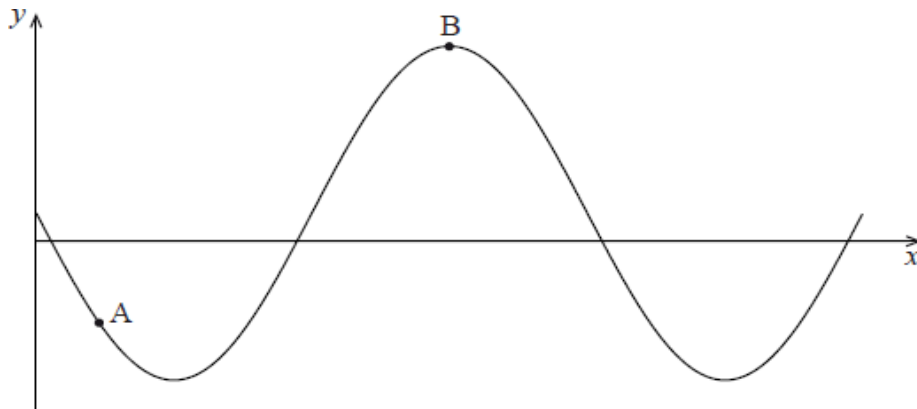
2b. [3] Solve $f(x) = 3$ for $0 \leq x \leq \pi$.

3. [3] The graph below shows $y = a \cos(bx) + c$.



Find the value of a , the value of b and the value of c .

4. The diagram below shows a curve with equation $y = 1 + k \sin x$, defined for $0 \leq x \leq 3\pi$.



The point $A\left(\frac{\pi}{6}, -2\right)$ lies on the curve and $B(a, b)$ is the maximum point.

(a) [2] Show that $k = -6$.

(b) [3] Hence, find the values of a and b .

5. [6] The depth, $h(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h(t) = 8 + 4 \sin\left(\frac{\pi t}{6}\right), \quad 0 \leq t \leq 24.$$

(a) Find the maximum depth and the minimum depth of the water.

(b) Find the values of t for which $h(t) \geq 8$.