Exponential Modelling

Week 11 – October 11, 2023

The function $M(t) = 85.7 \times 0.966^t$ models the amount (M) in grams of a radioactive material t years from its production.

a Determine the original mass of the radioactive material

b Determine the mass of the radioactive material after one decade?

c Calculate the complete number of years it would take for the radioactive material to reduce below 55 grams.

d Determine the half-life of the material

Answers

a	At $t = 0 \Rightarrow M(0) = 85.7$ grams	All these values should be found directly from the GDC.
b	At $t = 10 \Rightarrow M(10) = 60.6$ grams	
c	$M(t) < 55 \Rightarrow t > 12.8 \Rightarrow t = 13$ years	
d	$M(t) = \frac{85.7}{2} \Rightarrow t = 20$ years	

The function that determines the value of an investment in a fund is $V(t) = V_0 e^{kt}$, where t is measured in years.

a Given that the initial amount in the account was €5000, determine the value of V_0.

b The amount in the account increased by 25% after 6 years. Determine the value of k correct to 3 significant figures.

c Using the parameters found in parts **a** and **b**, determine the value of the investment after exactly eight and a half years.

d Calculate the number of complete years for the investment to double.

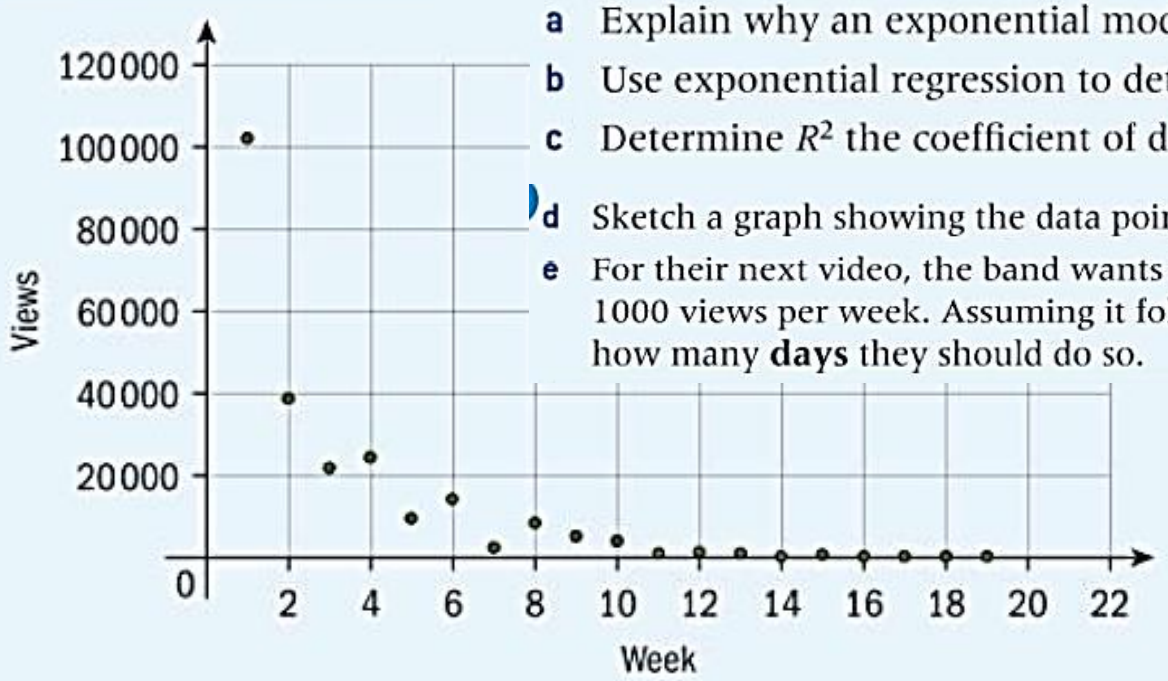
Answers

a	$5000 = V_0 e^{k0} \Rightarrow V_0 = €5000$	Using $t = 0$ and $V = €5000$
b	$6250 = 5000\ e^{k6} \Rightarrow k = 0.0372$	Using $t = 6$ and $V = 5000 \times 1.25 = €6250$
c	$V(8.5) = 5000e^{0.0372 \times 8} = €6859.52$	As a whole number of years is being looked for the value 19 can also be found directly from the table function of a GDC without the need to find 18.6.
d	$5000\ e^{0.0372t} > 10000 \Rightarrow t > 18.6$ $\Rightarrow$ the investment will double after 19 complete years.	

A rock band the VKs published a new video on their YouTube channel. The weekly views up to and including the 20th week of publication were as follows:

Week	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Views	102 365	38 716	21 617	24 305	9321	14 148	2103	8285	5098	3777
Week	11th	12th	13th	14th	15th	16th	17th	18th	19th	20th
Views	831	1007	834	34	378	204	6	42	54	31

A scatter plot of the data is shown below:



a Explain why an exponential model would be suitable to model this set of data.

b Use exponential regression to determine the best fit exponential model.

c Determine R^2 the coefficient of determination of the model function.

d Sketch a graph showing the data points with the model function.

e For their next video, the band wants to start advertising it as soon as it falls below 1000 views per week. Assuming it follows the same pattern as the first video, predict after how many **days** they should do so.

a The data shows the sales decaying at a diminishing rate continuous decay, which is a major characteristic of exponential decay functions.

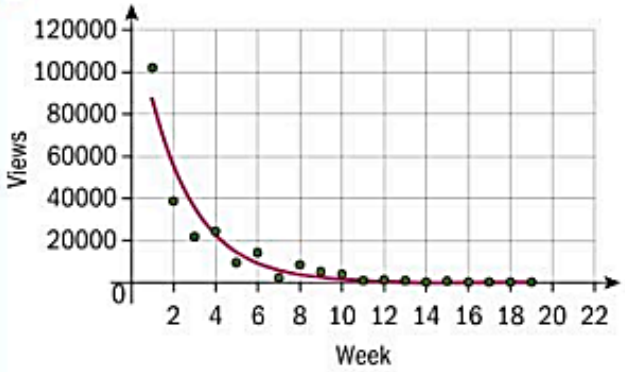
Also the data have a physical horizontal asymptote as the number of views can never fall below zero.

b The exponential function that models the data is $f(x) = 129\,000 \times 0.642^x$.

Exponential regression will give the function in the form *abx*. The next section will explain how to convert to base e if required.

c $R^2 = 0.888$

d



e $129\,000 \times 0.642^x = 1000$

$\Rightarrow x = 10.97$ weeks.

$\Rightarrow d = 7 \times 10.97 = 76.8$ days.

They have to start advertising their video again after 77 days.