Quadratic Models

Week 7 – Sept.7

Reference: Oxford Math AIHL, p.234

Objectives

- To identify the key features of the graph of a quadratic function
- To learn how to transfer graphs from “screen” to paper

How can you find out how far the dolphin jumped and how high it jumped?

- A dolphin jumps above the surface of the ocean. What does the path of its jump look like?
- The path of the jump can be modelled by the equation

$$f(x) = -0.09375x^2 + 1.875x - 3.375$$

where:

x = distance (in metres) from the point where it left the water

f(x) = vertical height in metres, above the surface of the water

Quadratic Functions

- These are polynomial functions where the highest power of *x* is two.
- $f(x) = ax^2 + bx + c$; $a \neq 0$ and *a, b,* and *c* are real numbers.
- *a, b,* and *c* are also called the parameters of the function. They help determine the shape and behavior of the graph.

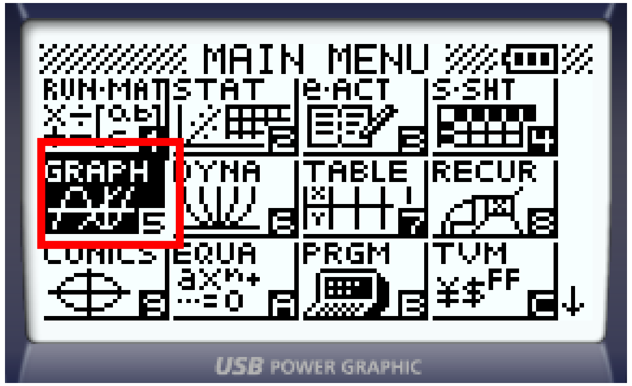
Function	Value of *a*	Value of *b*	Value of *c*
$f(x) = x^2 + 4x + 4$			
$f(x) = -0.5x^2 + 7.5x - 18$			
$f(x) = 2x^2 + 5x + 2$			

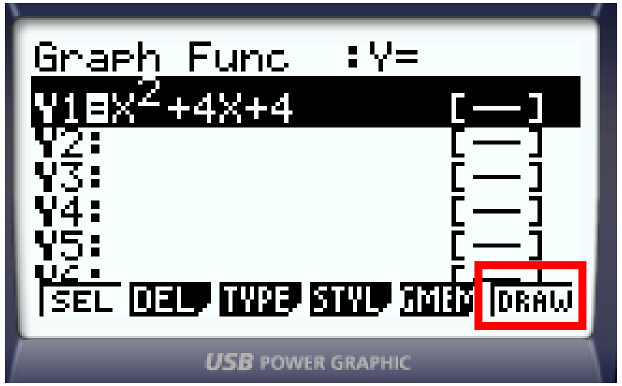
- The graph is called a parabola.
- Zeros of the function → the x-intercepts
- Axis of symmetry – the line of symmetry → $x = \frac{-b}{2a}$
- Vertex – maximum or minimum point on the graph

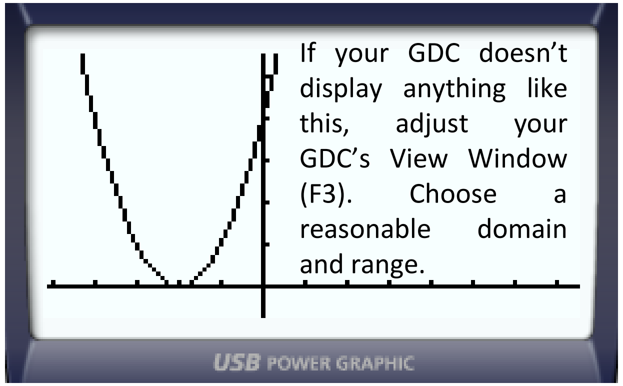
Use of GDC

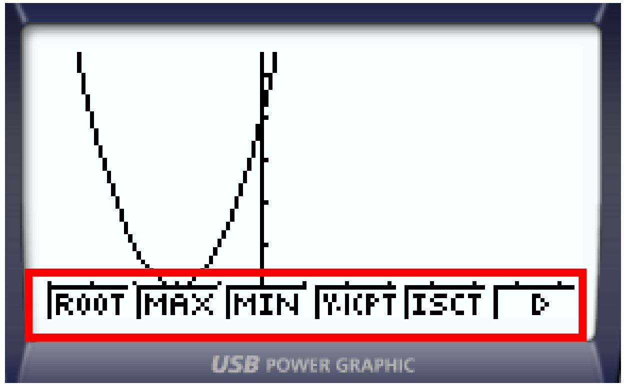
Graph the quadratic function $y = x^2 + 4x + 4$ and identify the:

- y – intercept
- zeros (or the roots) of the function
- vertex (min or max?)
- axis of symmetry
- domain and range



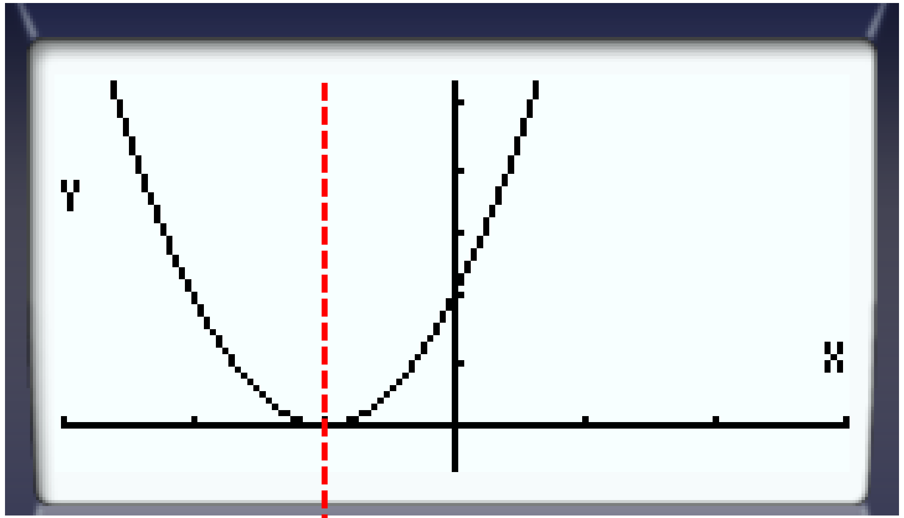






Use of GDC

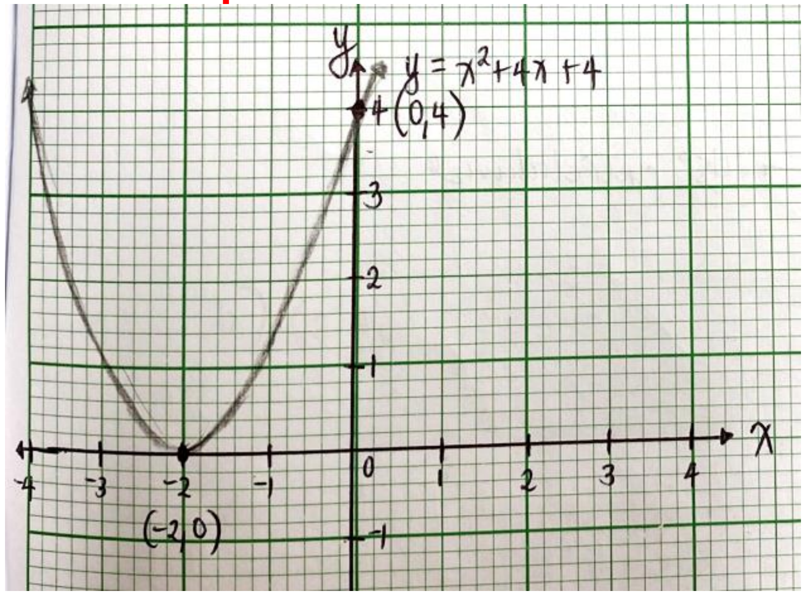
Graph the quadratic function $y = x^2 + 4x + 4$ and identify the:



- y – intercept (0, 4)
- zeros (or the roots) (-2, 0)

of the function

- vertex (min or max?) (-2, 0) Minimum point
- axis of symmetry $x = \frac{-b}{2a} = \frac{-4}{2(1)} \rightarrow x = -2$
- domain and range D = (-∞, ∞)
  R = [0, ∞)



$$f(x) = -0.5x^2 + 7.5x - 18$$

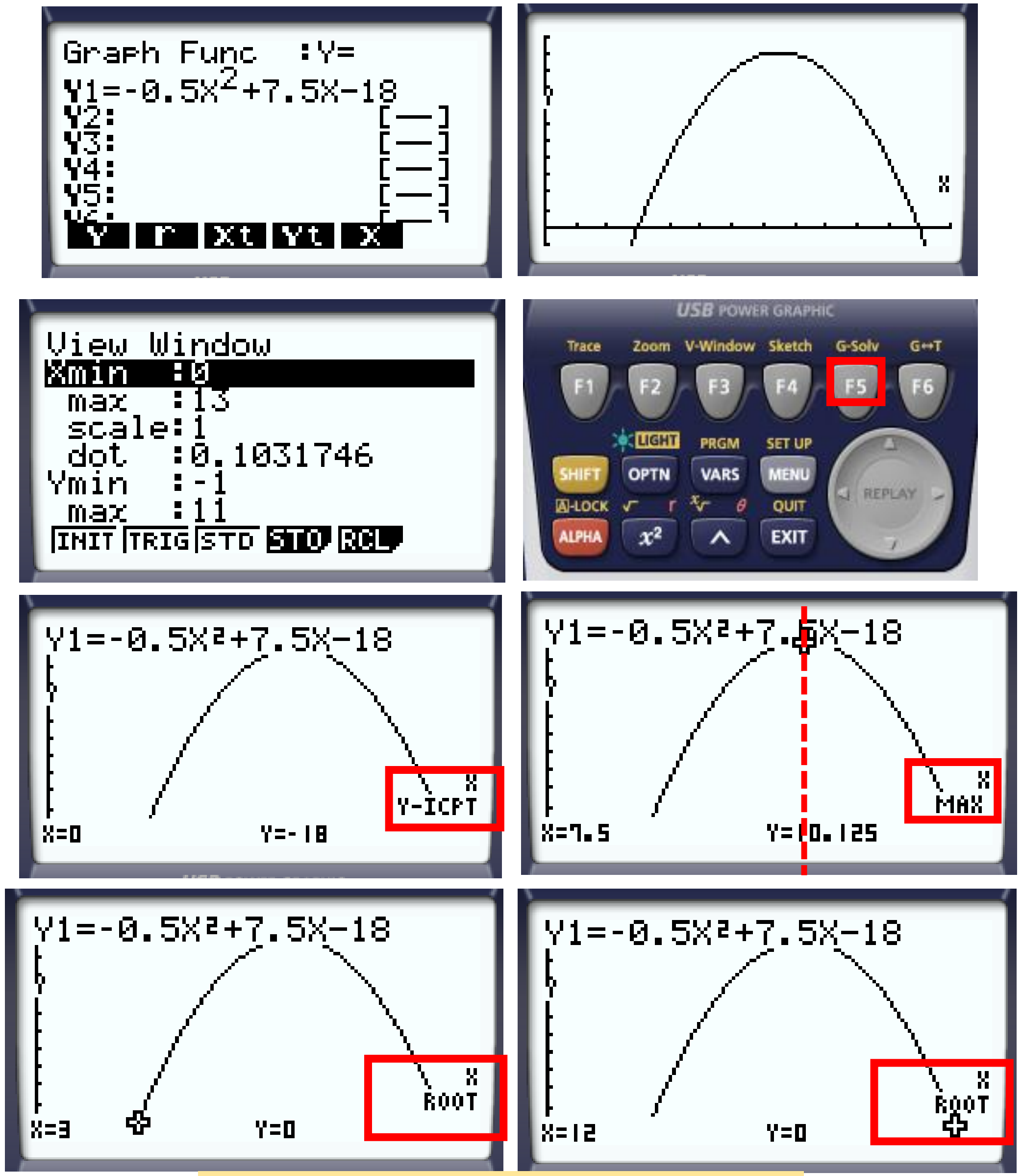
- y – intercept (0, -18)
- zeros (or the roots) (3, 0) and (12, 0)

of the function

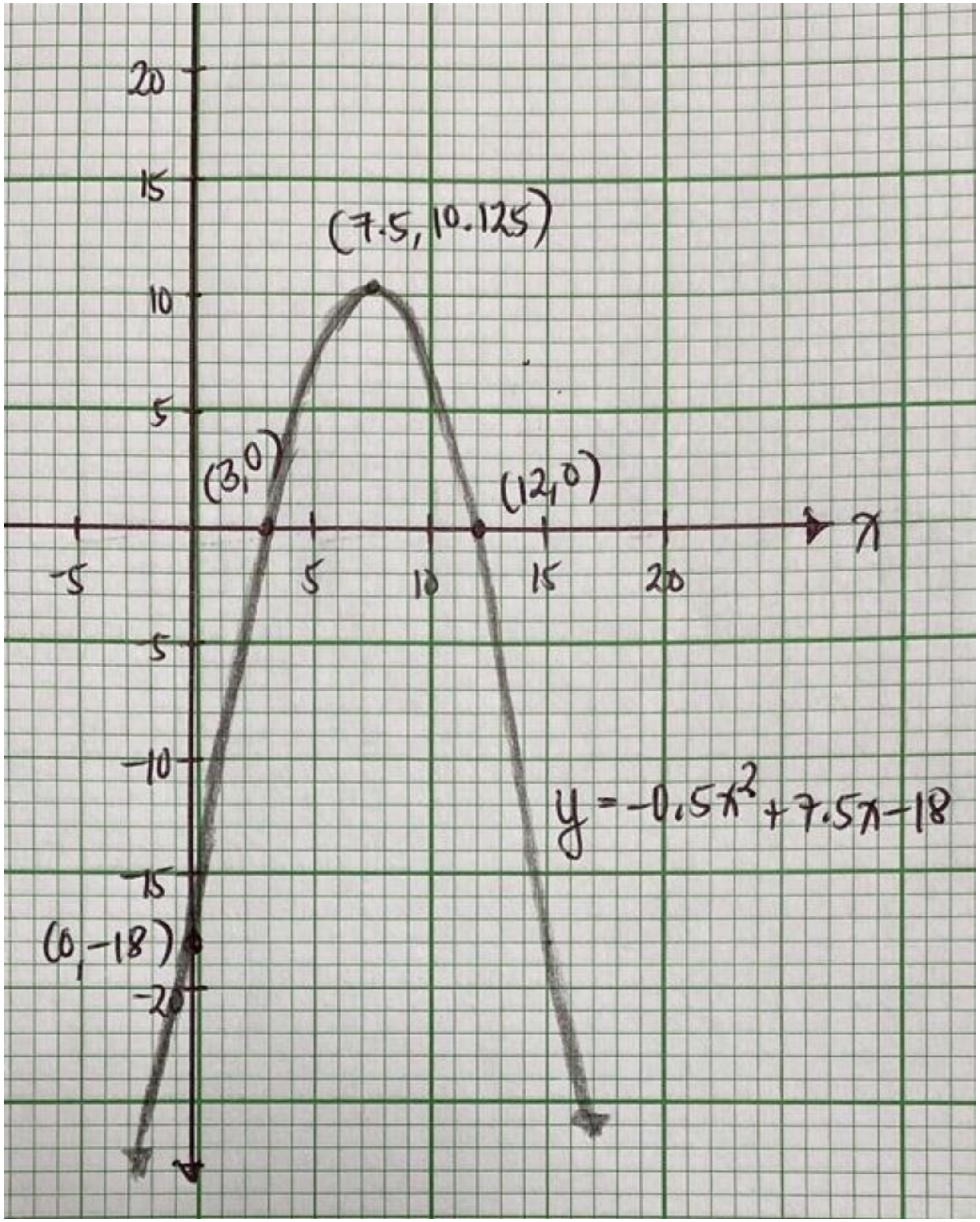
- vertex (min or max?) (7.5, 10.125) Max. point
- axis of symmetry $x = \frac{-b}{2a} = \frac{-7.5}{2(-0.5)} \rightarrow x = 7.5$
- domain and range

D = (-∞, ∞)

R = (-∞,10.125]



Press the left and right arrow keys of your GDC to switch from one root to another.



20
15
10
5
-5
-10
-15
-20
-5
5
10
15
20
x
(7.5, 10.125)
(3, 0)
(12, 0)
(0, -18)
$y = -0.5x^2 + 7.5x - 18$

$$f(x) = 2x^2 + 5x + 2$$

- y – intercept (0, 2)
- zeros (or the roots)

of the function (-2, 0) and (-0.5, 0)

- vertex (min or max?) (-1.25, -1.125)

Min. point

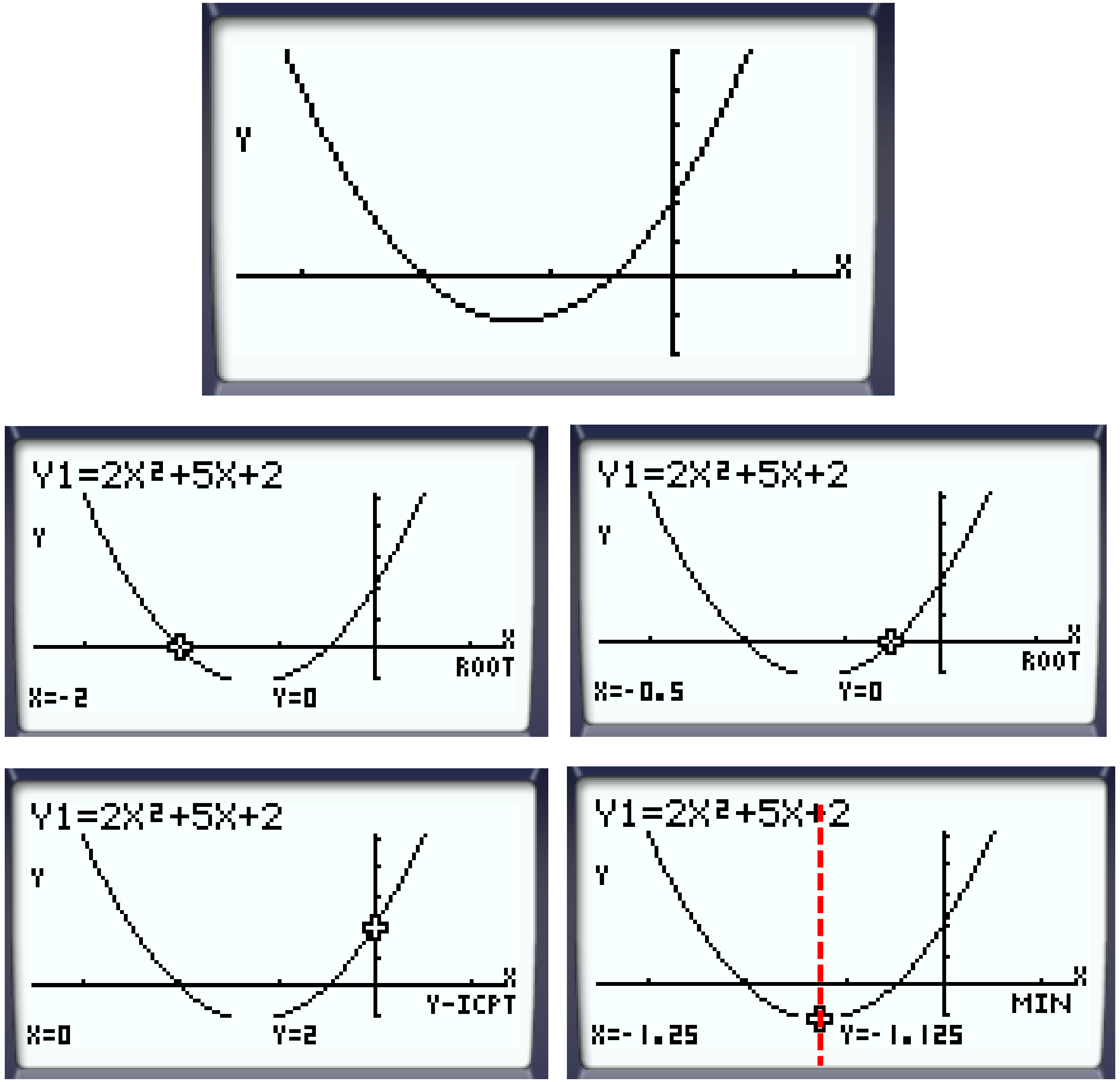
- axis of symmetry $x = \frac{-b}{2a} = \frac{-5}{2(2)}$

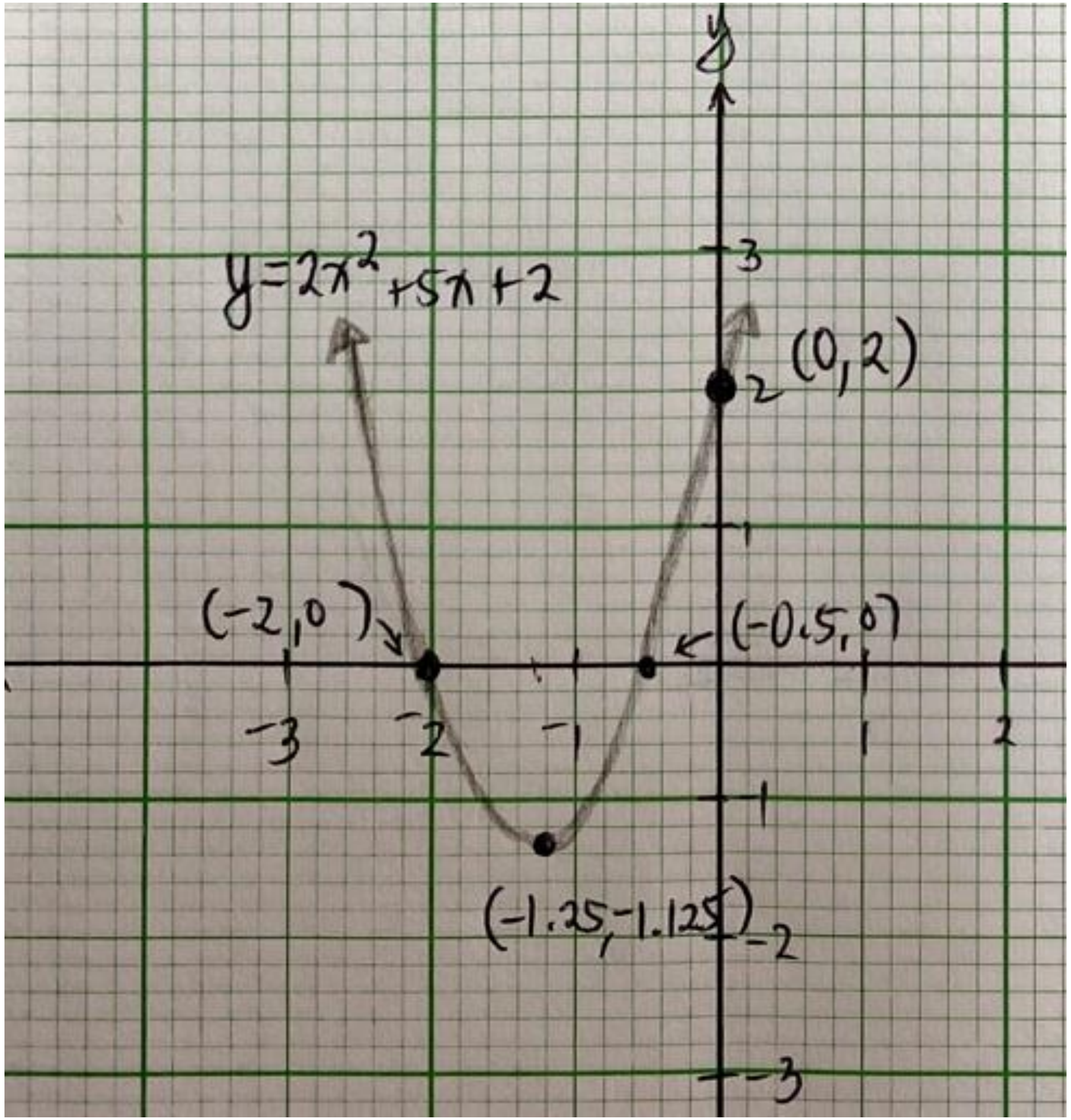
$\rightarrow x = -1.25$

- domain and range

D = (-∞, ∞)

R = [-1.125, ∞)




y
$y=2x^2+5x+2$
(0,2)
(-2,0)
(-0.5,0)
(-1.25,-1.125)
3
2
1
-1
-2
-3
-3
-2
-1
1
2

Let $\boldsymbol{f(x) = ax^2 + bx + c}$; $\boldsymbol{a \neq 0}$ and *a, b,* and *c* are real numbers

- Now, complete the table and answer the questions that follow.

Function	Value of *a*	Value of *b*	Value of *c*	y-int.	Is the vertex a minimum or maximum point?
$f(x) = x^2 + 4x + 4$	1	4	4	4	minimum
$f(x) = -0.5x^2 + 7.5x - 18$	-0.5	7.5	-18	-18	maximum
$f(x) = 2x^2 + 5x + 2$	2	5	2	2	minimum

- How does the sign of *a* in a particular quadratic function determine whether a vertex is a maximum or minimum?

If *a* is positive, the parabola opens upward and the vertex is a minimum point.
If *a* is negative, the parabola opens downward, and the vertex is a maximum point.

- What is the connection between the value of *c* and the y-intercept?

The value of *c* is equal to the value of the *y*-intercept.

Going back to the dolphin problem

- Use what you have learned to find out how high the dolphin jumped above the surface of the ocean and how far it jumped.

- The path of the jump was modelled by the equation

$$f(x) = -0.09375x^2 + 1.875x - 3.375$$

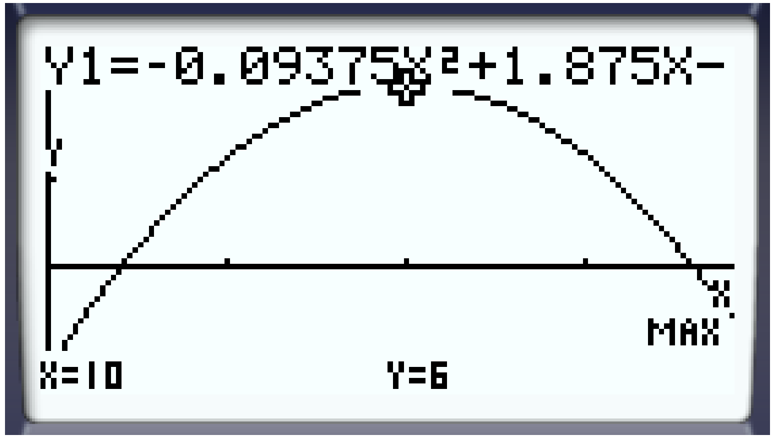
where:

x = distance (in metres) from the point where it left the water

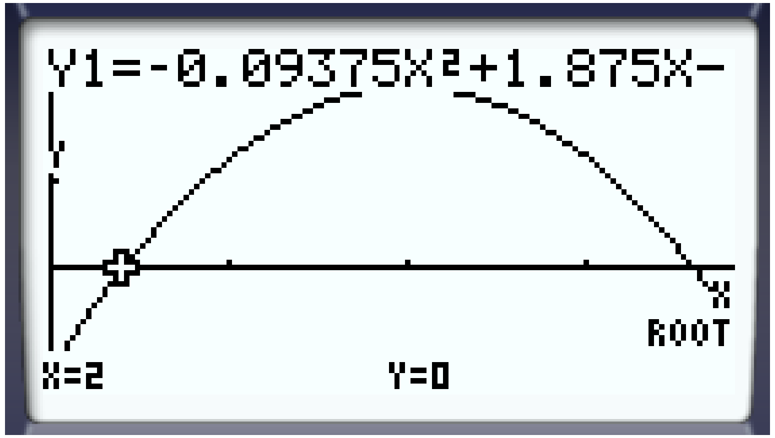
f(x) = vertical height in metres, above the surface of the water

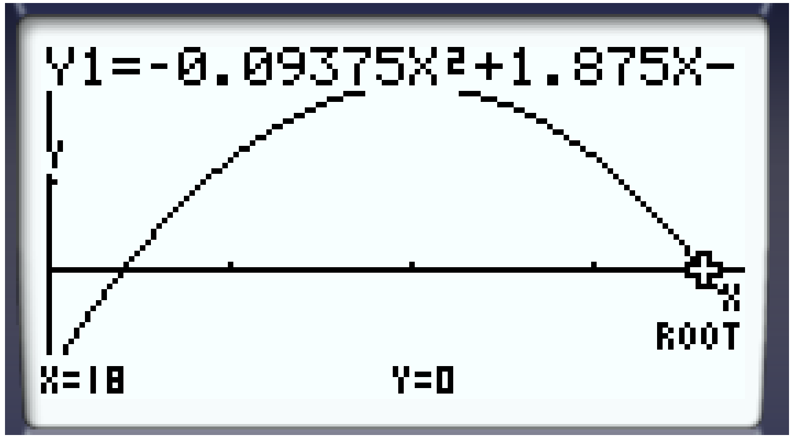
$$f(x) = -0.09375x^2 + 1.875x - 3.375$$

- How high did the dolphin jump above the surface of the ocean?
- *6 metres (the y-coordinate of the vertex).*



- How far did it jump?
- *16 metres (the x-intercepts are 2 and 18).*

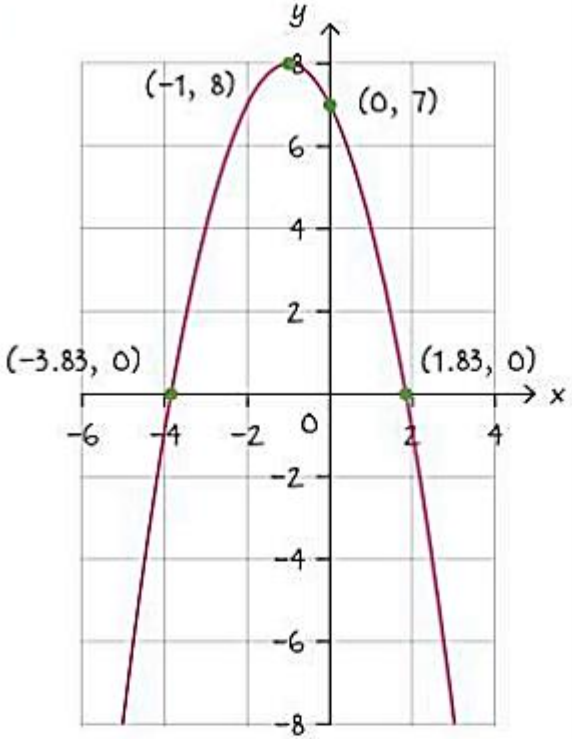




a Sketch the graph of the function $f(x) = 7 - 2x - x^2$, for $-5 \le x \le 3$, and hence determine the range.

b State the range if the domain were unrestricted.

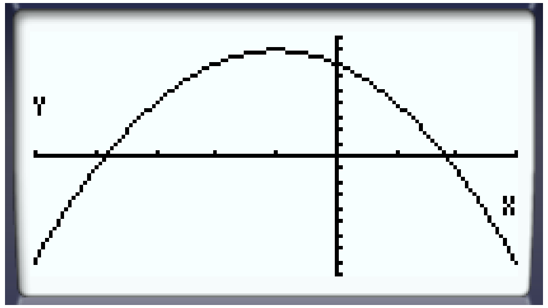
a



The range of the function is $-8 \le y \le 8$.

Use your GDC to sketch a graph within a given domain.

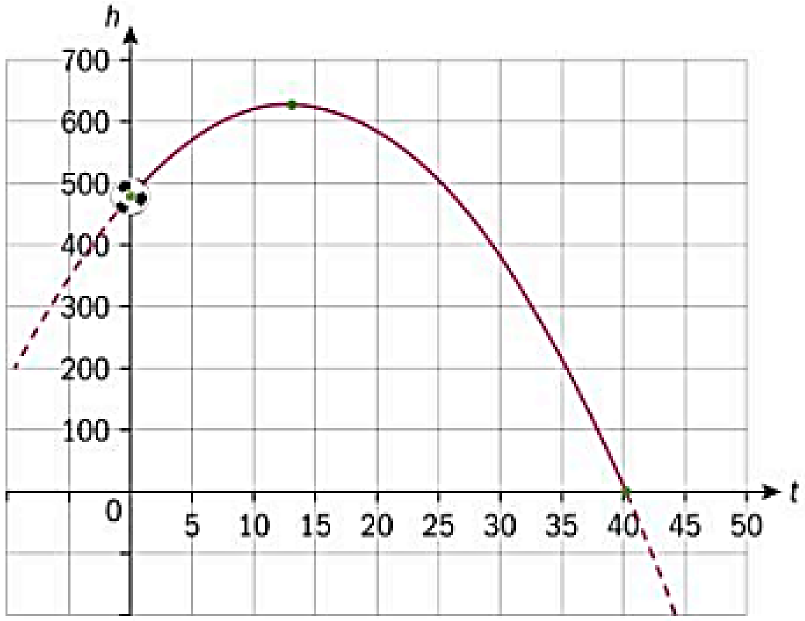
The end points, the y-intercept, the x-intercepts and the vertex (maximum) can all be obtained from the graph on your GDC. Make sure you clearly show these points on your diagram.



The x- and y-intercepts, vertex, and roots of the graph must be shown or labelled when you sketch it on a paper.

Pay attention to the given domain and range, too.

Sketch the graph of the function $h(t) = -0.846t^2 + 22.2t + 478$, representing the height h, in metres above the ground, of a projectile at time t minutes after it was launched. Find the maximum height that the projectile will reach.



The maximum height above the ground of the projectile is $h = 624$.

You first need to decide on a reasonable domain and range in the context of the question.

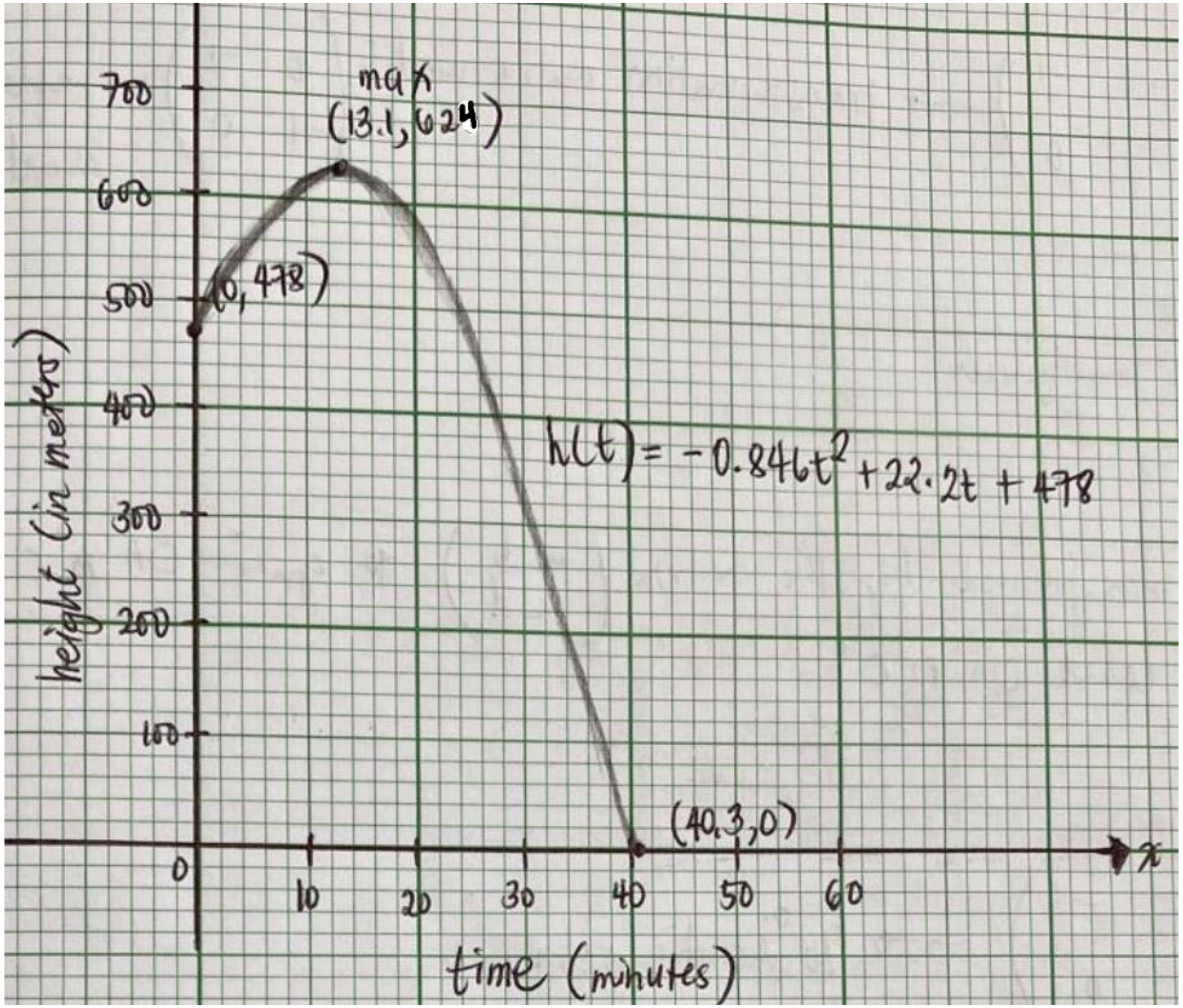
Here the input variable is time, so the domain cannot contain negative values, meaning that $t \geq 0$.

Similarly, since the output variable is the height in metres above the ground, the range cannot contain negative values.

The starting point should be at (0, 478) and the ending point at approximately (40.3, 0). The vertex should be placed at approximately (13.1, 624).

This will further restrict the domain. The resulting graph should only appear within the first quadrant.

In a model of a real-life situation you might need to consider the domain and the range without them being explicitly given.


max
(13.1, 624)
(0, 478)
(40.3, 0)
h(t) = -0.846t² + 22.2t + 478
700
600
500
400
300
200
100
0
10
20
30
40
50
60
x
height (in meters)
time (minutes)

HINT

When the **domain** of the function is **not explicitly given**, you might need to do some working in order to choose an appropriate view window in your GDC. The most important point of a parabola that should be in your view window is the vertex. This means that finding the vertex analytically or by using the table function in the GDC might prove to be very helpful.