Euler's Method

Differential Equations

5.16	Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$, where h is a constant (step length)

Example 1.

Consider the differential equation $\frac{dy}{dx} = e^x + 1$ with $y(0) = 1$.

a. Estimate $y(0) = 0.5$ by applying Euler's method with:

   i. h = 0.25 for two steps

   ii. h = 0.1 for five steps.

b. Find $y(0) = 0.5$ exactly using the Fundamental Theorem of Calculus.

EULER'S METHOD

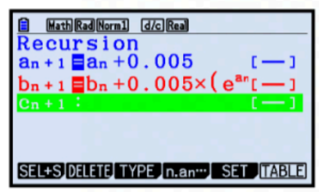
Casio fx-CG50

Consider the differential equation $\frac{dy}{dx} = e^x + 1$ with $y(0) = 1$.

To estimate $y(0.5)$ using Euler's method with step size 0.005, we have $x_0 = 0$, $y_0 = 1$, and

$$\begin{cases} x_i = x_{i-1} + 0.005 \\ y_i = y_{i-1} + 0.005(e^{x_{i-1}} + 1). \end{cases}$$

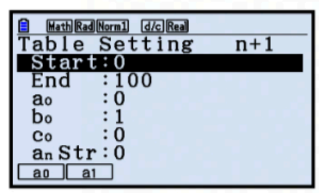
Select **Recursion** from the Main Menu, press F3 (**TYPE**), then F2 ($\mathbf{a_{n+1}}$).
Enter $a_n + 0.005$ into $\mathbf{a_{n+1}}$, and $b_n + 0.005(e^{a_n} + 1)$ into $\mathbf{b_{n+1}}$.



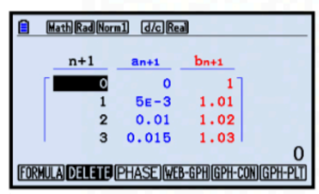
Note: a_n is entered by pressing F4 (**n.a$_n$···**), then F2 ($\mathbf{a_n}$).

b_n is entered by pressing F4 (**n.a$_n$···**), then F3 ($\mathbf{b_n}$).

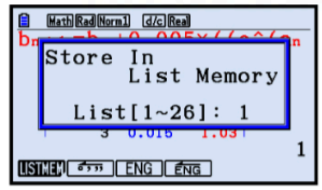
Press F5 (**SET**) and adjust the table settings.
Set **Start** $= 0$, and **End** $= 100$ since we are taking $\frac{0.5}{0.005} = 100$ steps.
Set $\mathbf{a_0} = 0$ since $x_0 = 0$, and $\mathbf{b_0} = 1$ since $y_0 = 1$.



Press **EXIT**, then F6 (**TABLE**) to view the table of values.



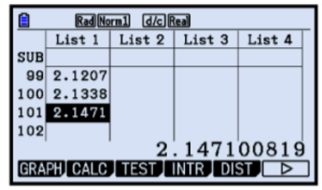
Press ▶ ▶ to highlight the first entry in the $\mathbf{b}_{n+1}$ column.



Press **OPTN** **F1** (**LISTMEM**), then enter 1 **EXE** to save the values of $\mathbf{b}_{n+1}$ into **List 1**.

To view **List 1** press **MENU** and select **Statistics**.

Press ▲ until the 101st entry is shown.



So, $y(0.5) \approx 2.1471$.

Example 2. 14 marks

Consider the differential equation

$\frac{dy}{dx} = \sqrt{x}y, x > 0,\ y > 0\ where\ y(1) = 4$

a. Use Euler's method with a step length of 0.25 to fill in the following table, rounding each value to two decimal places. [4]

x	1.25	1.50	1.75	2
y				

b. Solve the differential equation. [6]

c. Hence, find the exact value of y(2) to 2 decimal places [2]

d. Calculate the percentage error in the value of $y(2)$ found by using Euler's method. Give your answer to 2 significant figures. [2]

Example 3.

Consider the differential equation $\frac{dy}{dx} = x\cos(y)$ with initial point $(0, 0)$.

a. $h = 1$
b. $h = 0.1$
c. $h = 0.01$