Relations and Functions

At the end of the lesson, the students will be able to:

1. differentiate between relations and functions.
2. represent functions and non-functions as equations, tables, or mapping diagrams.
3. determine, with a reason, whether y is a function of x.
4. identify the domain and range of a relation, as well as the dependent and independent variables.

Essential Questions:

- What is a function?

- What are the characteristics of a function?
- How do you determine if a relation is a function?
- How is a function different from a relation?
- Why is it important to know which variable is the independent variable?

Definition of terms

A **relation** is a set of ordered pairs that specifies corresponding values of the independent (x) and dependent (y) variables.

An **ordered pair** consists of an *x* and *y*-coordinate.

- **A relation** may be viewed as *ordered pairs, mapping diagram, graph, table, equation, or written in sentences.*

x-values are **input, independent variable, domain.**

y-values are **output**, **dependent variable, range**

Example 1:
What makes this a relation?

$$\{(0,-5),(1,-4),(2,-3),(3,-2),(4,-1),(5,0)\}$$

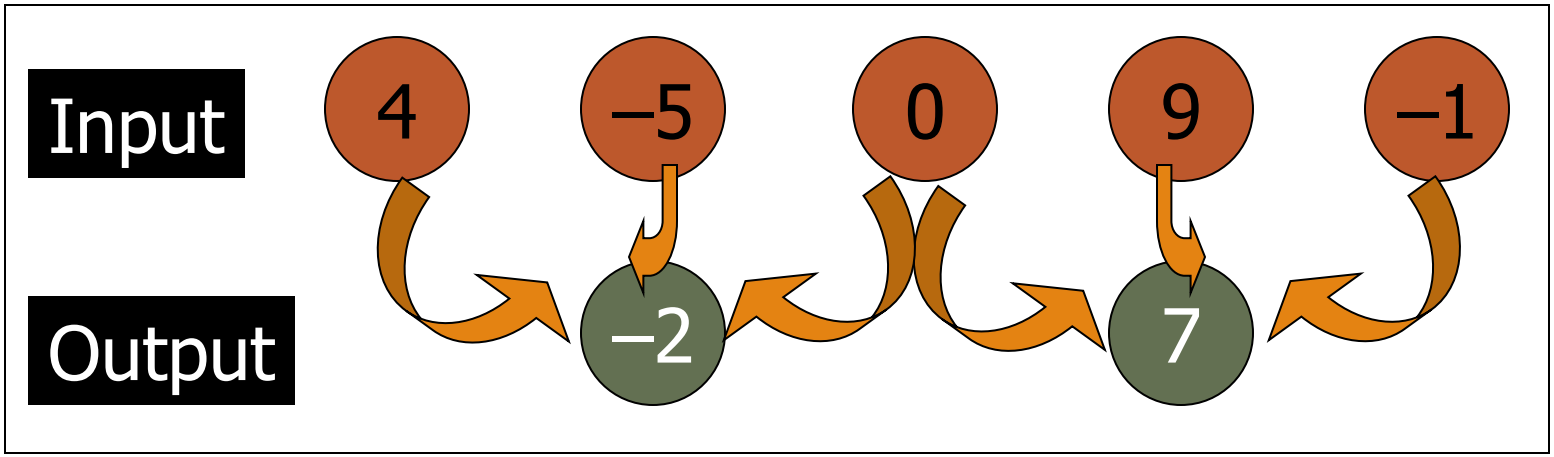
- What is the **domain**?

{0, 1, 2, 3, 4, 5}

What is the **range**?

{-5, -4, -3, -2, -1, 0}

Example 2 – Is this a relation?



- What is the **domain**?

{4, -5, 0, 9, -1}

- What is the **range**?

{-2, 7}

Is a relation a function?

What is a **function**?

It is a relation between two sets in which every element of the first set (input, independent variable) is mapped onto **one and only one** element of the second set (output, dependent).

That is, y is a function of x if, for each x-value, there is exactly one corresponding value of y.

Is a relation a function?

- Focus on the **x-coordinates**, when given a relation.

If the set of ordered pairs has **different x-coordinates**,

it **IS A** function.

If the set of ordered pairs has **same x-coordinates**,

it is **NOT** a function.

- **Y-coordinates** have no bearings in determining functions.

Functions and mere relations

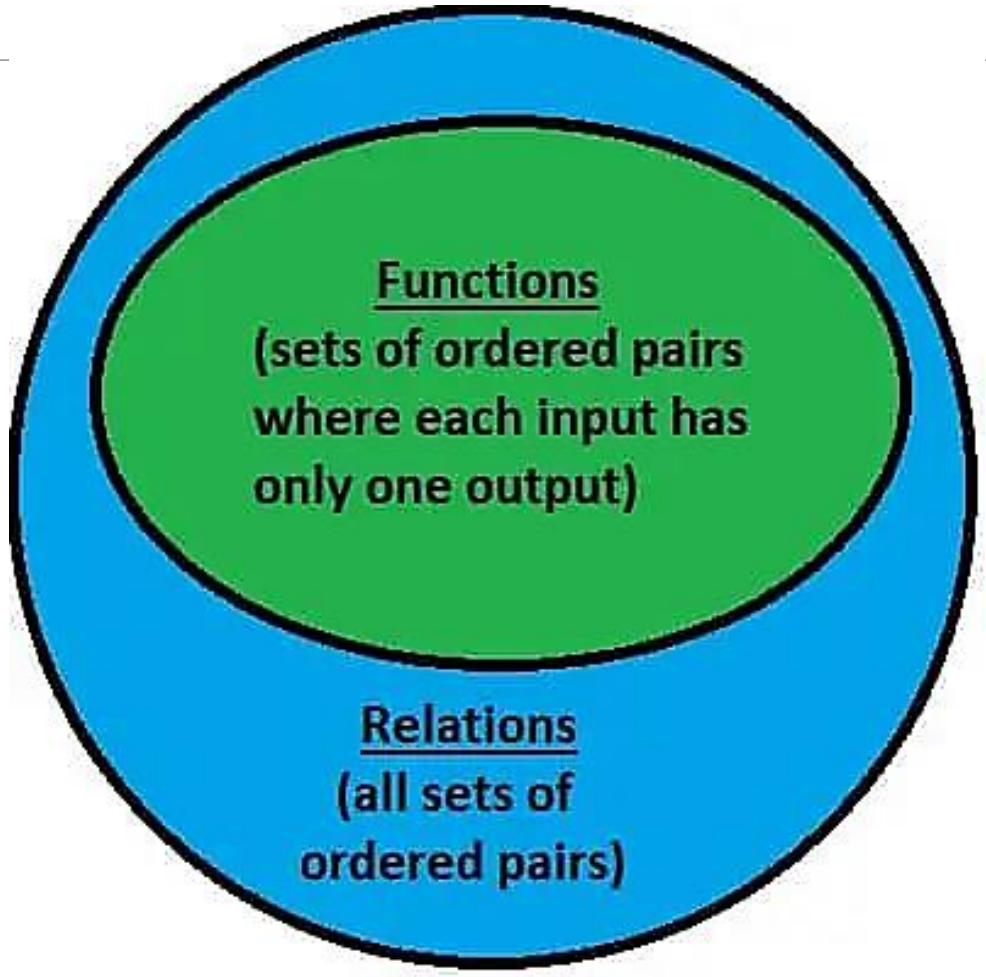


Image Source: https://jdmeducational.com/wp-content/uploads/2021/05/Functions-vs-Relations-diagram.jpg

Example 3

$$\{(0,-5),(1,-4),(2,-3),(3,-2),(4,-1),(5,0)\}$$

- ***Is this a relation?*** **YES**

- ***Is this a function?***
  - ***Hint:*** *Look only at the* ***x-coordinates***

YES, a one-to-one relation

Example 4

$$\{(-1,-7),(1,0),(2,-3),(0,-8),(0,5),(-2,-1)\}$$

- ***Is this a function?***
  - ***Hint:*** *Look only at the* ***x-coordinates***

NO

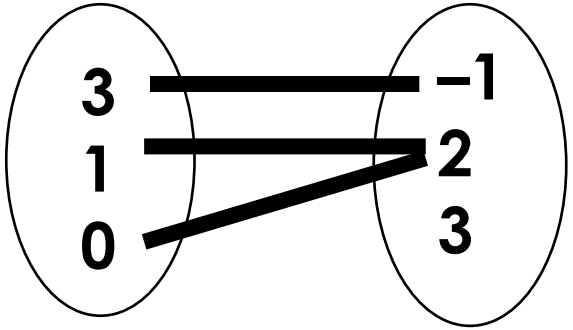
- ***Is this still a relation?***

YES, a one-to-many relation

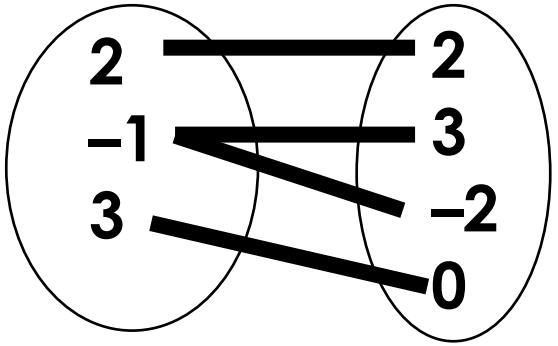
Example 5

Which relation mapping diagram represents a function?

Choice One



Choice Two



Choice 1

a many-to-one relation

Functions and mere relations

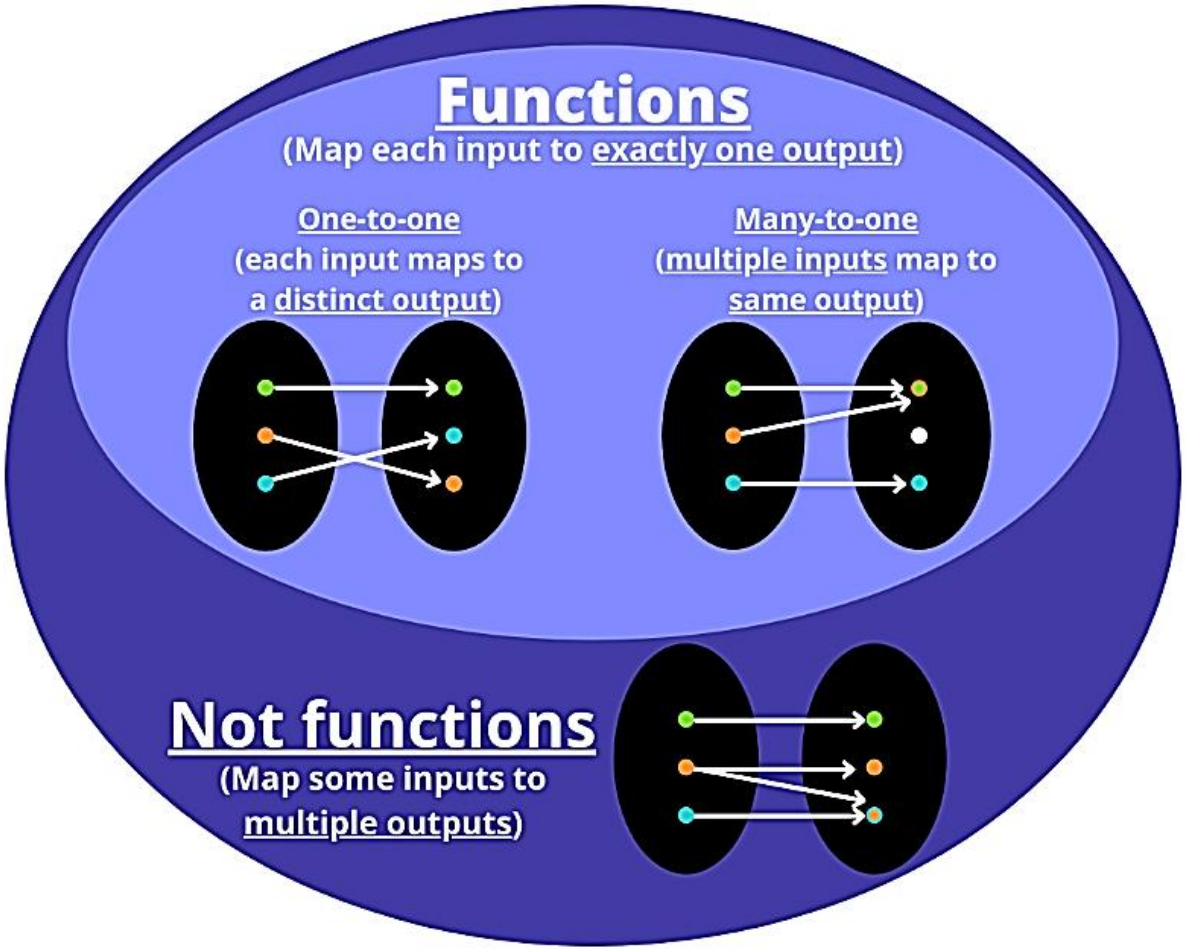


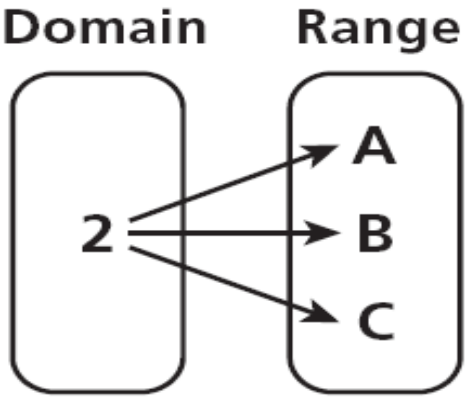
Image source: By Caroline Kulczycky

Source: https://www.expii.com/t/when-is-a-relation-a-function-4325

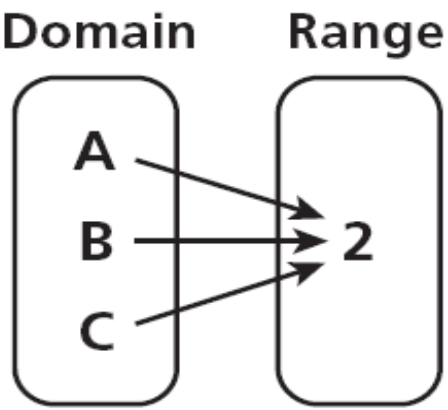
Example 6

Which mapping diagram represents a function?

A.



B.



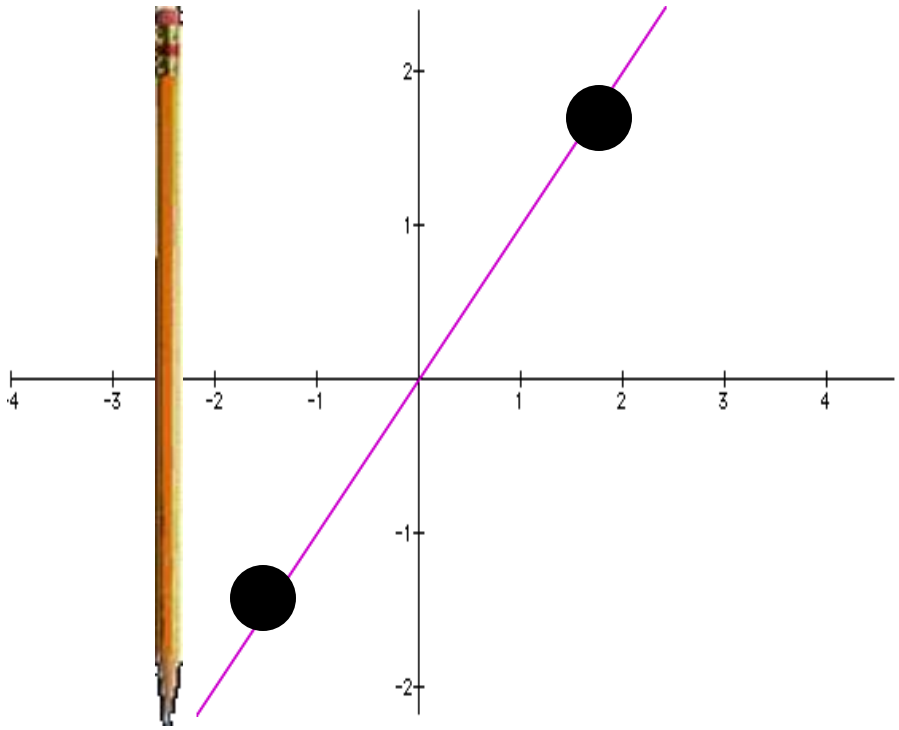
B

a many-to-one relation

Vertical Line Test

(Also known as the pencil test) If you can draw any vertical line that intersects more than one point on the relationship, then it is not a function.

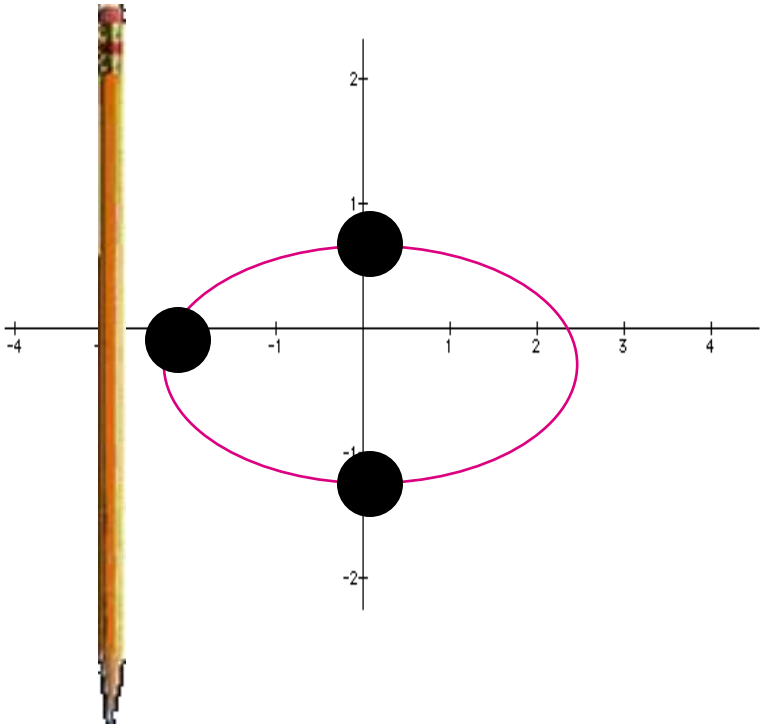
Vertical Line Test



Would this graph be a function?

YES

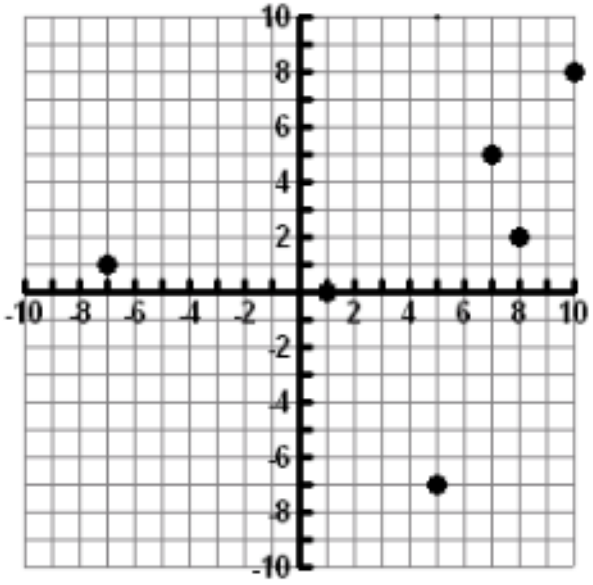
Vertical Line Test



Would this graph be a function?

NO

Is the following function discrete or continuous?
What is the Domain?
What is the Range?



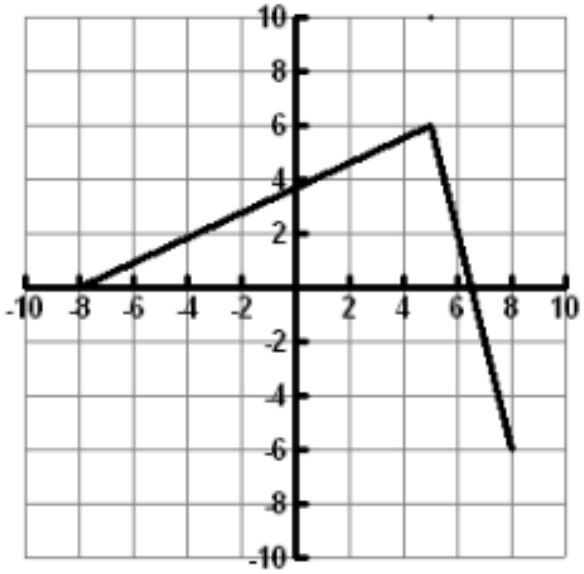
Type: Discrete

Domain: $\{-7, 1, 5, 7, 8, 10\}$

Range: $\{1, 0, -7, 5, 2, 8\}$

A diagram of the discrete function shows distinct points that remains unconnected.

Is the following function discrete or continuous?
What is the Domain?
What is the Range?



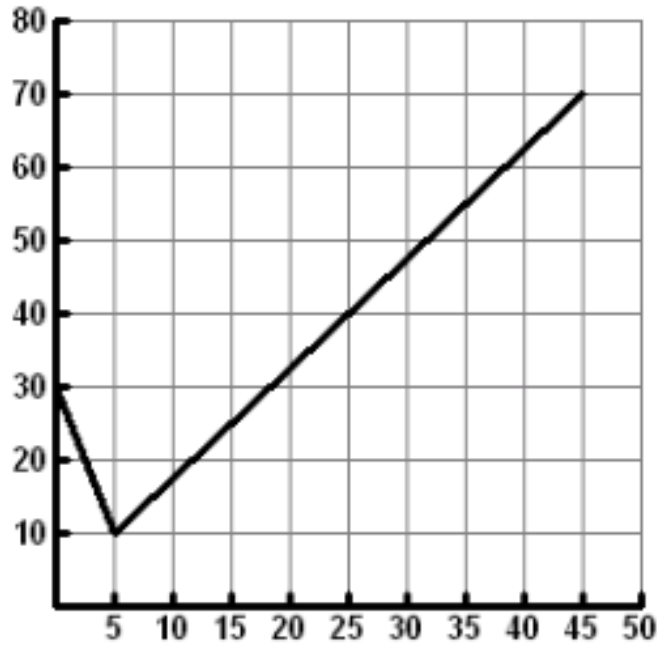
Type: continuous

Domain: $[-8, 8]$

Range: $[-6, 6]$

In a continuous function graph, the points are connected with an unbroken line.

Is the following function discrete or continuous?
What is the Domain?
What is the Range?

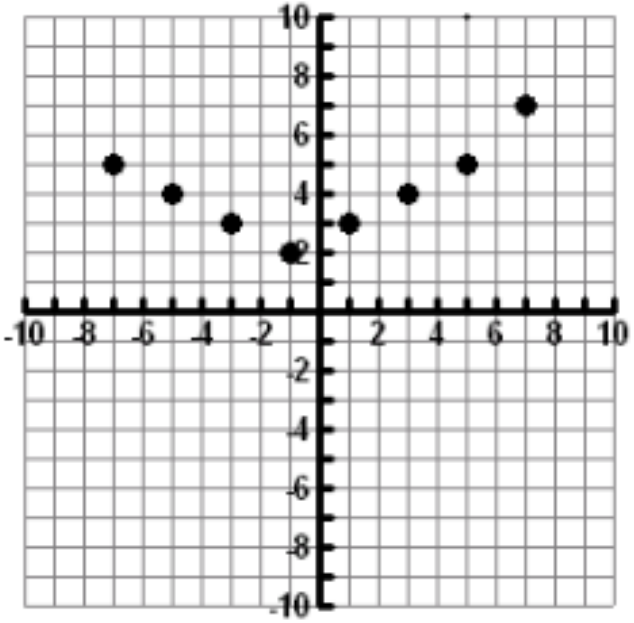


Type: continuous

Domain: $[0,45]$

Range: $[10,70]$

Is the following function discrete or continuous?
What is the Domain?
What is the Range?



Type: discrete

Domain: $\{-7, -5, -3, -1, 1, 3, 5, 7\}$

Range: $\{2, 3, 4, 5, 7\}$

Example 7

Which situation represents a function?

a. **The items in a store to their prices on a certain date**

b. **Types of fruits to their colors**

There is only one price for each different item on a certain date. The relation from items to price makes it a function.

A fruit, such as an apple, from the domain would be associated with more than one color, such as red and green. The relation from types of fruits to their colors is not a function.

Domain and Range in Real Life

The total number of shoes in x **pairs** of shoes can be expressed by the equation $y = 2x$.

What is the independent variable?

The **number of pairs** of shoes.

What is the dependent variable?

The **total number** of shoes.

Domain and Range in Real Life

Mr. Landry is driving to his hometown. It takes four hours to get there. The distance he travels at any time, t, is represented by the function $d = 55t$ (his average speed is 55mph.

What is the independent variable?

The time that he drives.

What is the dependent variable?

The total distance traveled.

Domain and Range in Real Life

Johnny bought at most 10 tickets to a concert for him and his friends. The cost of each ticket was $12.50.
Complete the table below to list the possible domain and range.

1	2	3	4	5	6	7	8	9	10
12.50	25.00	37.50	50	62.50	75	87.50	100	112.50	125

What is the independent variable?
The number of tickets bought.

What is the dependent variable?
The total cost of the tickets.

Domain and Range in Real Life

Pete's Pizza Parlor charges $5 for a large pizza with no toppings. They charge an additional $1.50 for each of their 5 specialty toppings (tax is included in the price).

What is the independent variable?

The number of toppings

What is the dependent variable?

The cost of the pizza

For each relation, determine, with a reason, whether y is a function of x.

a. In an annual survey of panda populations, x = time (in year since 1977), y = number of pandas.

b. The relation $x \rightarrow 2x^2 - 1$ with the set of inputs A = {-3, -2, 0, 2}. ***Note*** *that the mapping notation* $x \rightarrow 2x^2 - 1$ represents the same relation as $y = 2x^2 - 1$.

c. The relation $x = y^2$.

a Yes, because each year will have exactly one measurement of the panda population.

b Yes, because each input has exactly one corresponding output:

x	$y = 2x^2 - 1$
−3	17
−2	7
0	−1
2	7

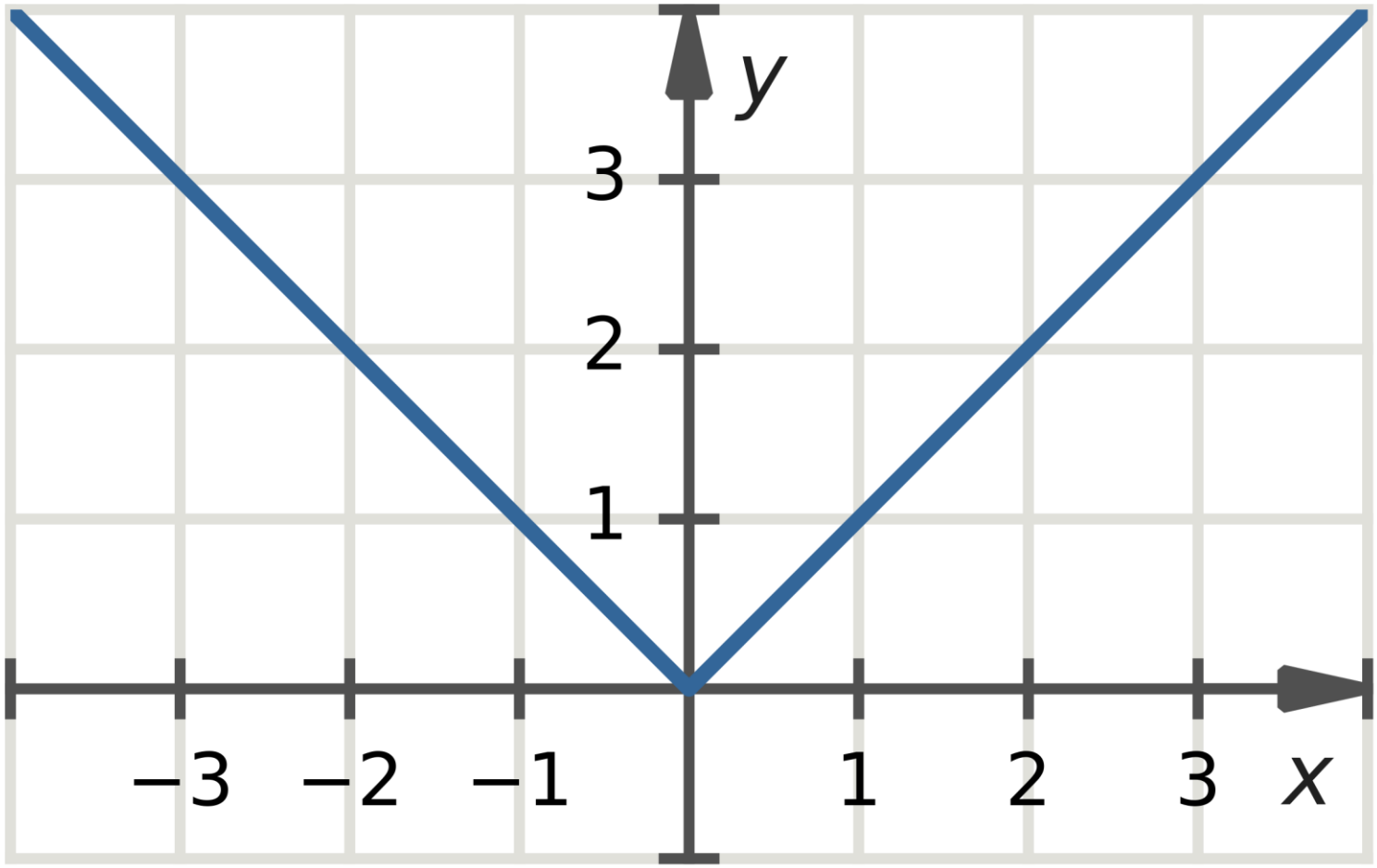
c No, because an x-value can have two corresponding y-values. For example, $x = 9$ corresponds to $y = 3$ and $y = -3$.

Substitute each x-value into the formula to find the corresponding y-value. For example, when $x = -3$:

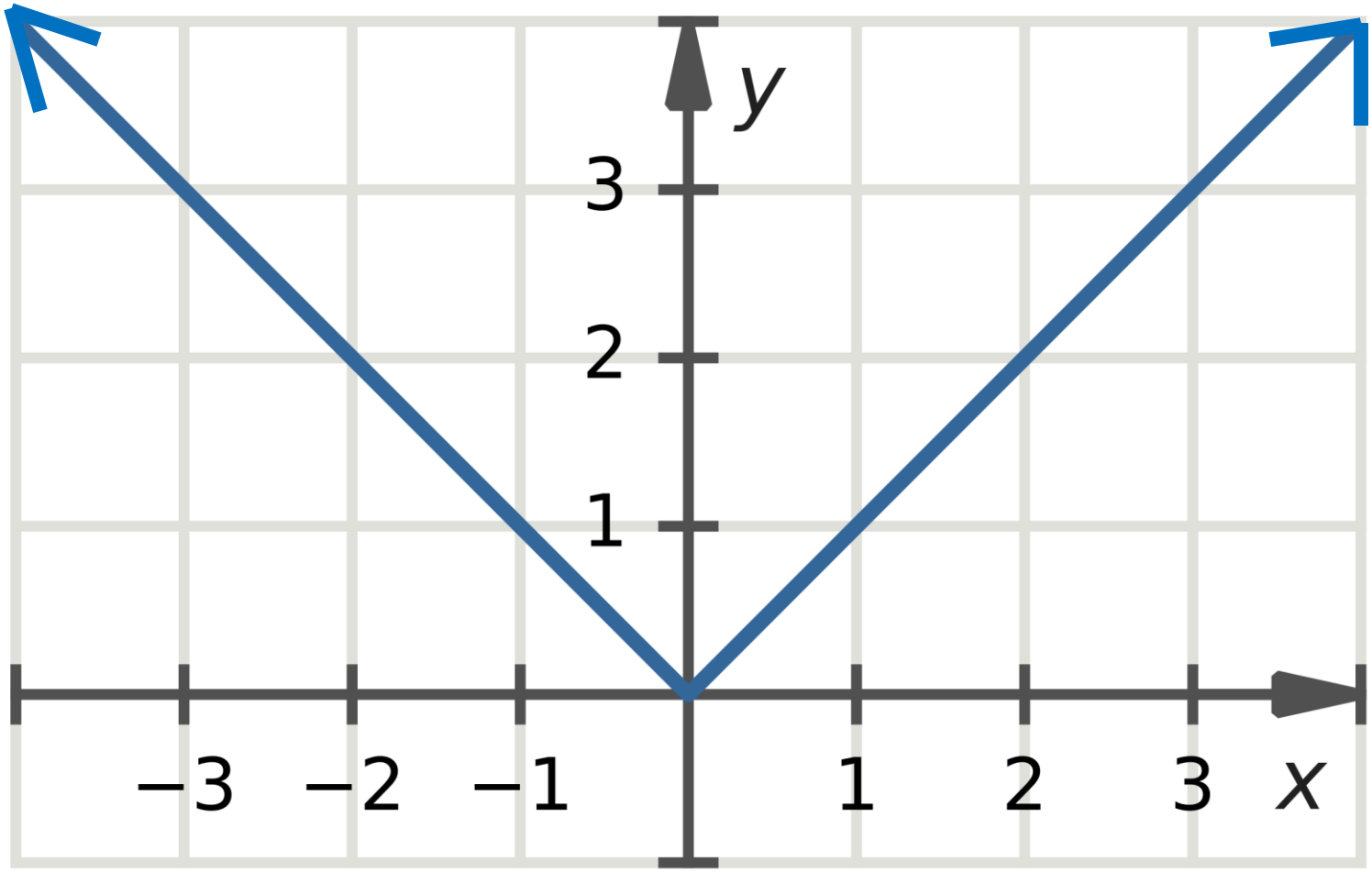
$$\begin{aligned} y &= 2(-3)^2 - 1 \\ &= 2(9) - 1 \\ &= 18 - 1 \\ &= 17 \end{aligned}$$

Note that two inputs can have the same output.

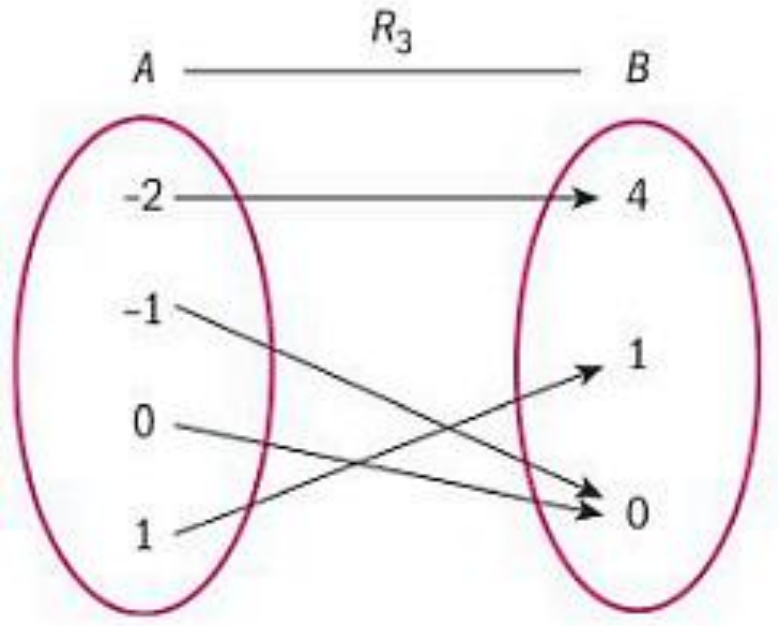
More Examples



Write the domain and range.

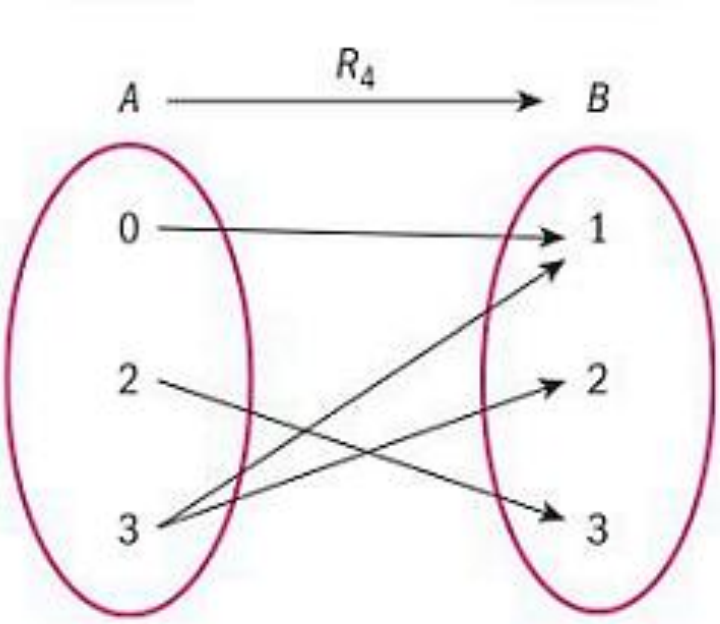


Write the domain and range.



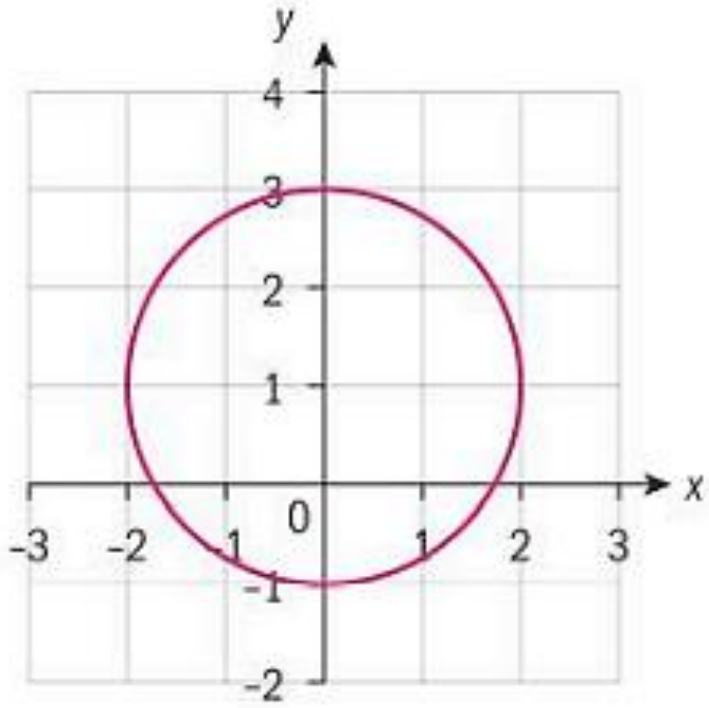
Function, every element from first set maps onto one element from the second set

Function or not? Explain.



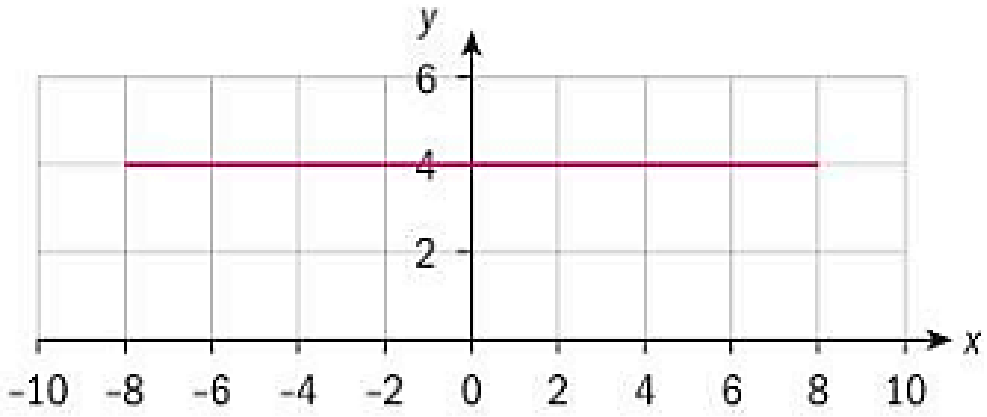
Not a function, input 3 has two outputs

Function or not? Explain.



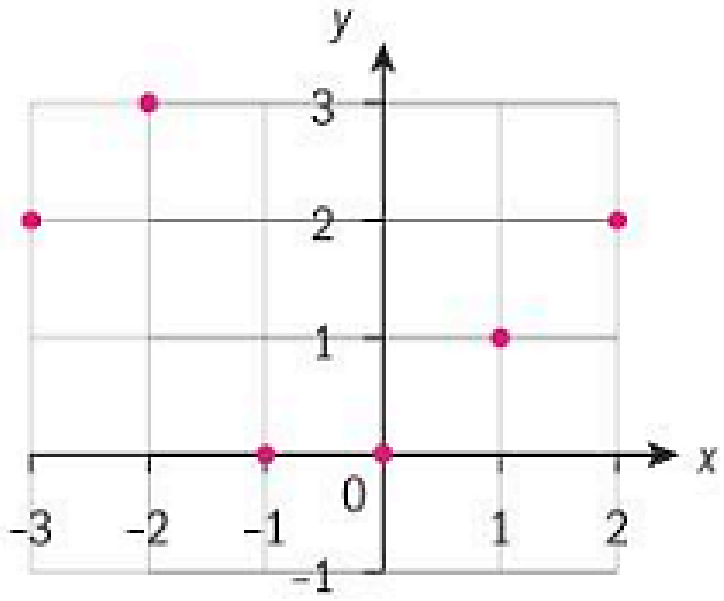
Not a function, fails the vertical line test

Function or not? Explain.



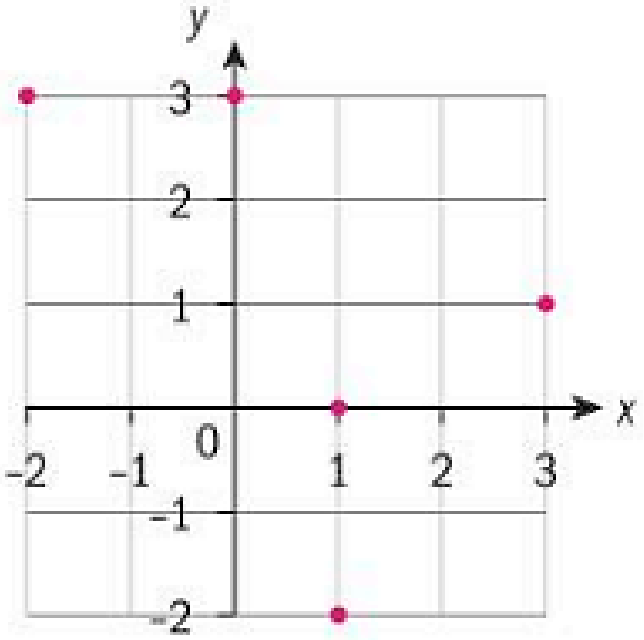
Function, passes the vertical line test

Function or not? Explain.



Function, no two first elements are the same

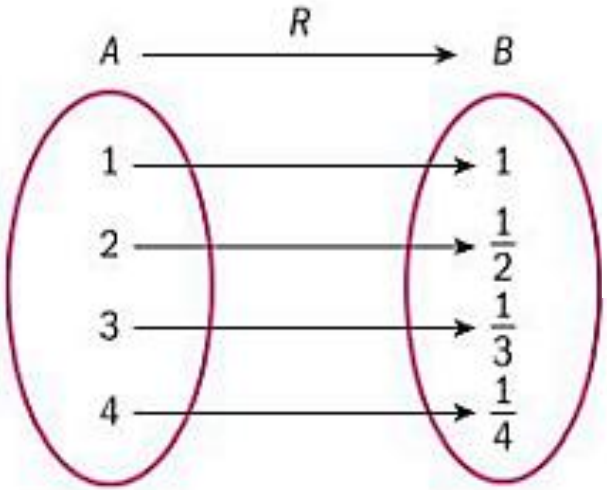
Function or not? Explain.



Not a function, fails the vertical line test

Function or not? Explain.

a Write down the mapping R in the form $x \rightarrow \ldots$



a. $x \rightarrow \frac{1}{x}$

b State whether or not R is a function.

b. function

c Write down the output of $x = 2$ under R.

c. $\frac{1}{2}$

d The set A is extended to $\{1, 2, 3, 4, a\}$, and a is mapped onto 3. State the value of a.

d. $a = \frac{1}{3}$

Consider the mapping $y = x^3$.

a State the output when the input is $x = -1$.

-1

b State the input when $y = -64$.

-4

c Let the set of inputs be $A = \{-1, 0, 1, 2, 3, 4\}$. Find B, the set of outputs.

$B = \{-1,0,1,8,27,64\}$

d State, with a reason, whether or not the mapping $y = x^3$ from A to B is a function.

It is a (one-to-one) function. Every input has a corresponding output.

Essential Questions:

- What is a function?

- What are the characteristics of a function?
- How do you determine if a relation is a function?
- How is a function different from a relation?
- Why is it important to know which variable is the independent variable?

Sources

(No date) *Relations and functions - paulding county school district*. Available at: https://www.paulding.k12.ga.us/cms/lib/GA01903603/Centricity/Domain/3809/Relations%20and%20Functions%20Power%20Point.ppt (Accessed: 06 August 2023).

(No date) *Google search*. Available at: https://www.google.com/search?q=discrete+and+continuus+graph (Accessed: 07 August 2023).