Direct Variation Function

Week 13 - October 25, 2023

Direct Variation

Two quantities that are related by a power law of the form $y = ax^n$, $a, n \in \mathbb{R}^+$ are said to **vary directly** with each other, or to be **directly proportional** to each other.

Example 1

The rate of the spread of fungus on a petri disk (x) varies directly with the square of the perimeter of the area covered by the fungus (p). If the rate is $2cm^2s^{-1}$ when the perimeter is 3.2 cm.

a Find the equation relating the rate of spread to the perimeter.

b Find the perimeter when the rate is $6\,cm^2\,s^{-1}$.

a $x = ap^2$	The general equation for the power function.
$2 = a \times 3.2^2$	Substituting the values.
$\Rightarrow a \approx 0.195 \Rightarrow x = 0.195p^2$	
b $6 = 0.195p^2$	
$p^2 = 30.72 \Rightarrow p = 5.54\,cm$	

Example 2

The distance, d metres, that a ball rolls down a slope varies directly with the square of the time, t seconds, it has been rolling. In 2 seconds the ball rolls 9 metres.

a Find an equation connecting d and t.

b Find how far the ball rolls in 5 seconds.

c Find the time it takes for the ball to roll 26.01 metres.

a $d = kt^2$, $t = 2$,

$$d = 9 \Rightarrow 9 = 4k \Rightarrow k = \frac{9}{4},$$

$$d = \frac{9}{4}t^2$$

b $d(5) = \frac{9}{4} \times 5^2 = 56.25 \text{ m}$

c $26.01 = \frac{9}{4}t^2 \Rightarrow t$

$$= \sqrt{\frac{4}{9} \times 26.01} = 3.4\text{s}$$

Inverse Variation Function

Week 13 - October 25, 2023

Investigation 6

1 Sketch the graphs of the following inverse variation functions:

a $f(x)=\dfrac{1}{x^2}$ b $f(x)=\dfrac{1}{x^3}$ c $f(x)=\dfrac{1}{x^4}$

d $f(x)=\dfrac{1}{x^5}$ e $f(x)=\dfrac{1}{x^6}$ f $f(x)=\dfrac{1}{x^7}$

2 How can you describe the shape of a inverse variation function?

3 Can you group inverse variation functions in smaller categories?

4 Are inverse variation functions symmetric? If yes, what type of symmetry do they possess?

5 **Factual** Describe what happens to the y-value of the function as

a x becomes very large

b as x approaches 0 from the positive side

c as x approaches 0 from the negative side.

6 What happens to the graph of inverse variation functions as the value of the power becomes more negative?

7 **Conceptual** What are the distinguishing geometrical features of inverse variation functions?

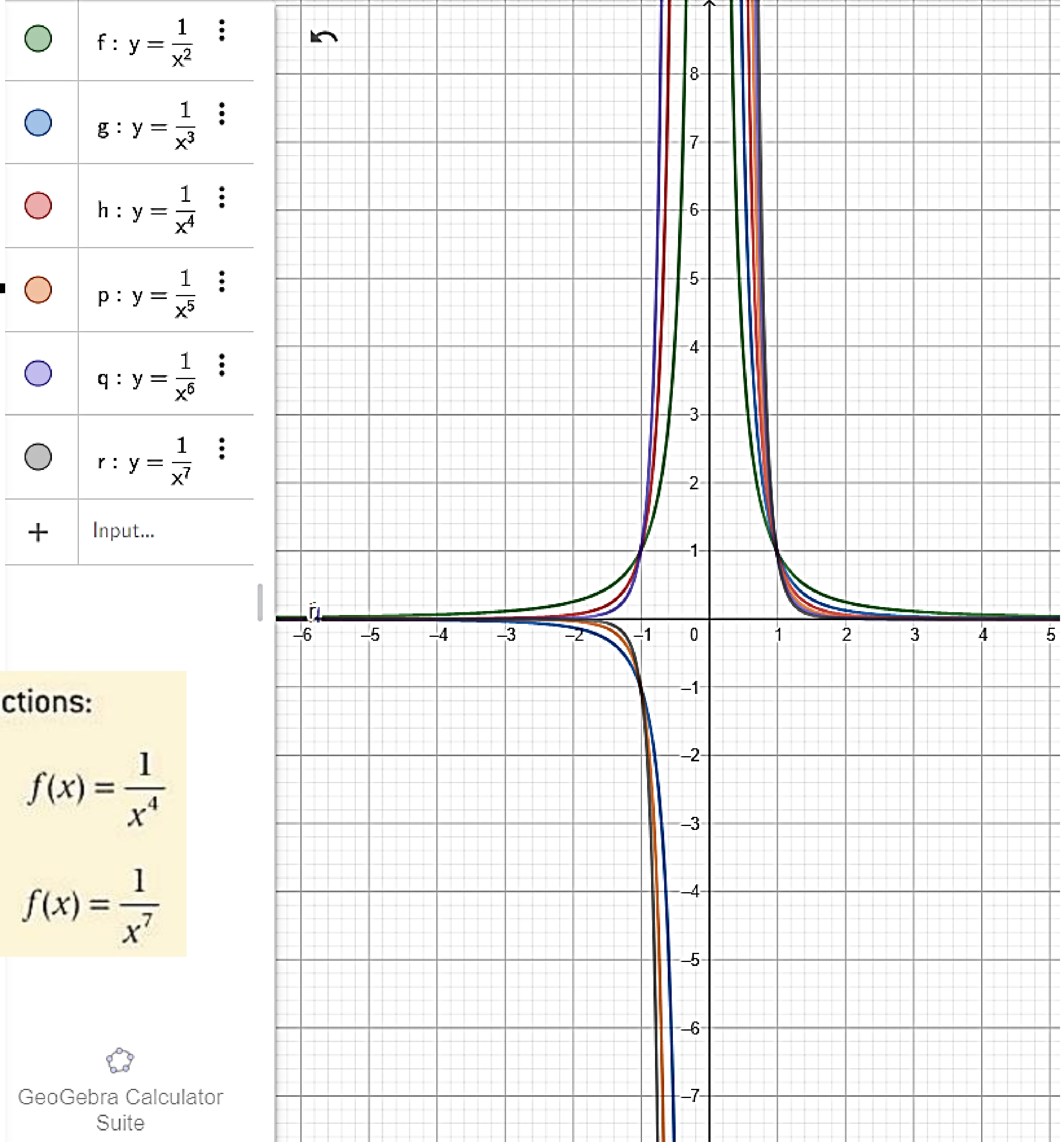
HINT

Use a view window of $-3 \le x \le 3$ and $-6 \le y \le 6$.

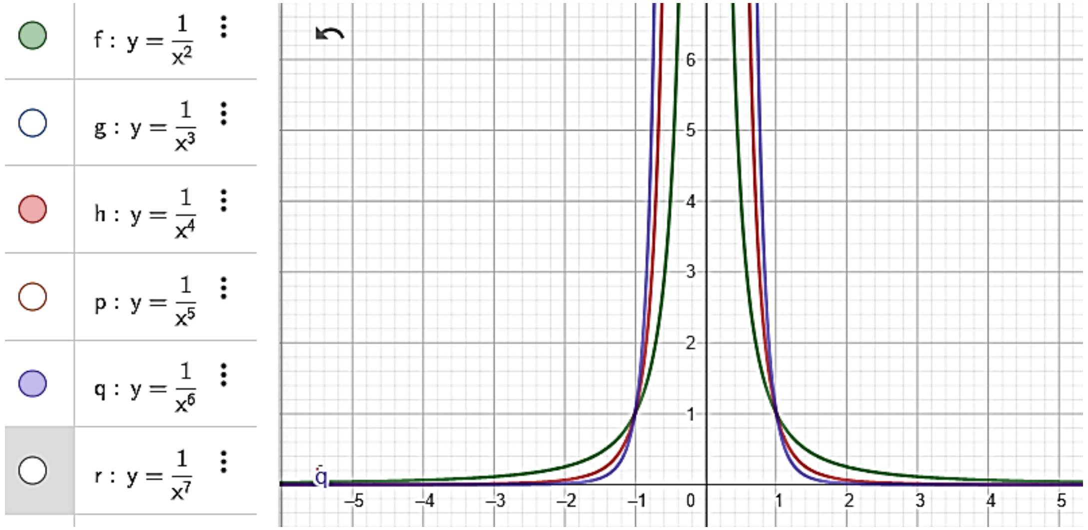
1 Sketch the graphs of the following inverse variation functions:

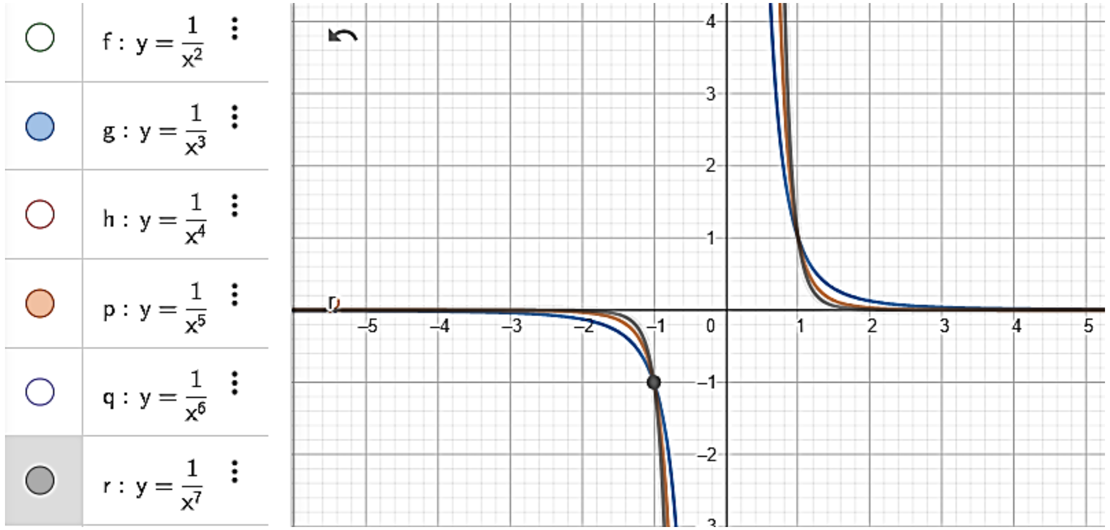
a $f(x)=\dfrac{1}{x^2}$ **b** $f(x)=\dfrac{1}{x^3}$ **c** $f(x)=\dfrac{1}{x^4}$

d $f(x)=\dfrac{1}{x^5}$ **e** $f(x)=\dfrac{1}{x^6}$ **f** $f(x)=\dfrac{1}{x^7}$



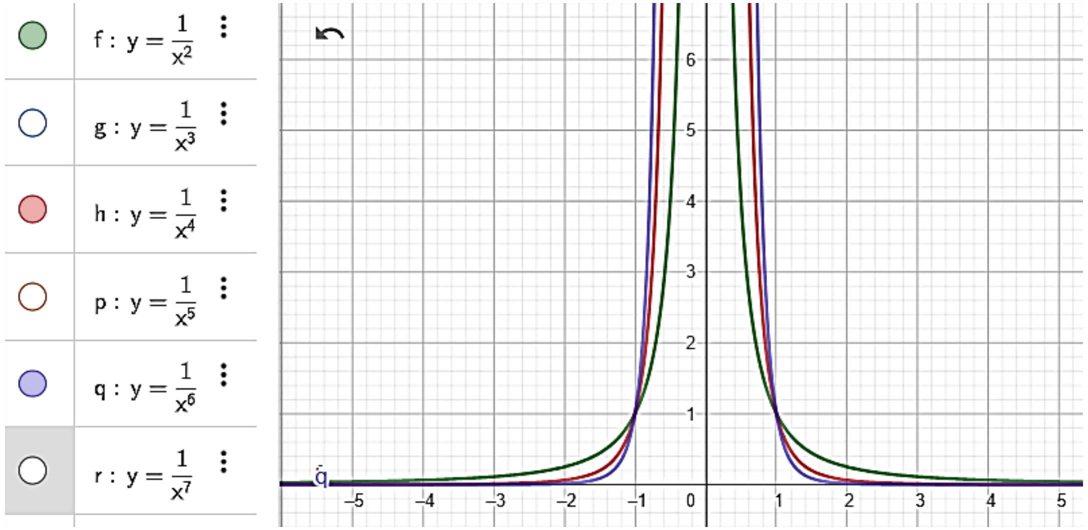
2 How can you describe the shape of an inverse variation function?

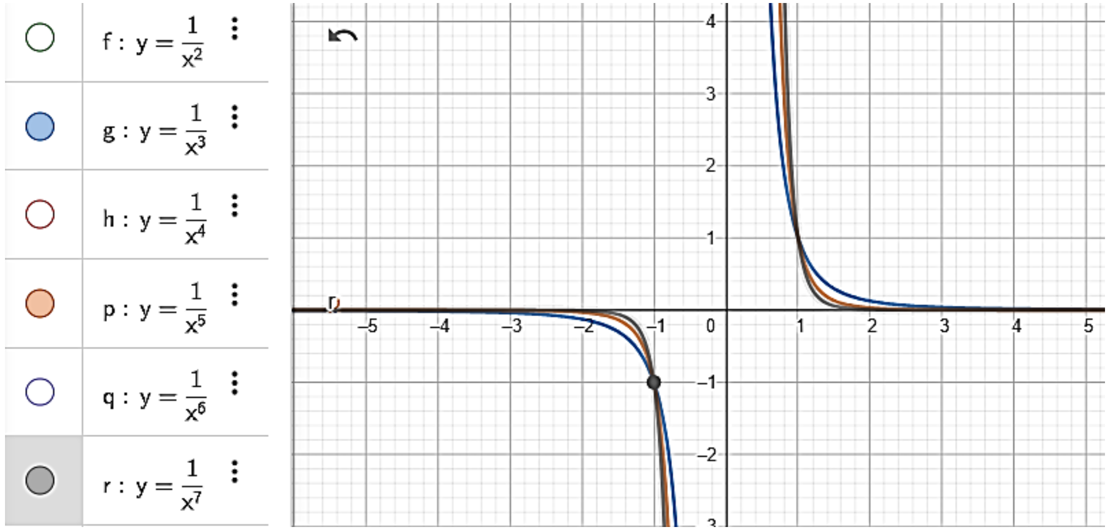




- The functions are asymptotic along the x– and y– axes.
  - Vertical asymptote: y–axis → x = 0
  - Horizontal asymptote: x–axis → y = 0

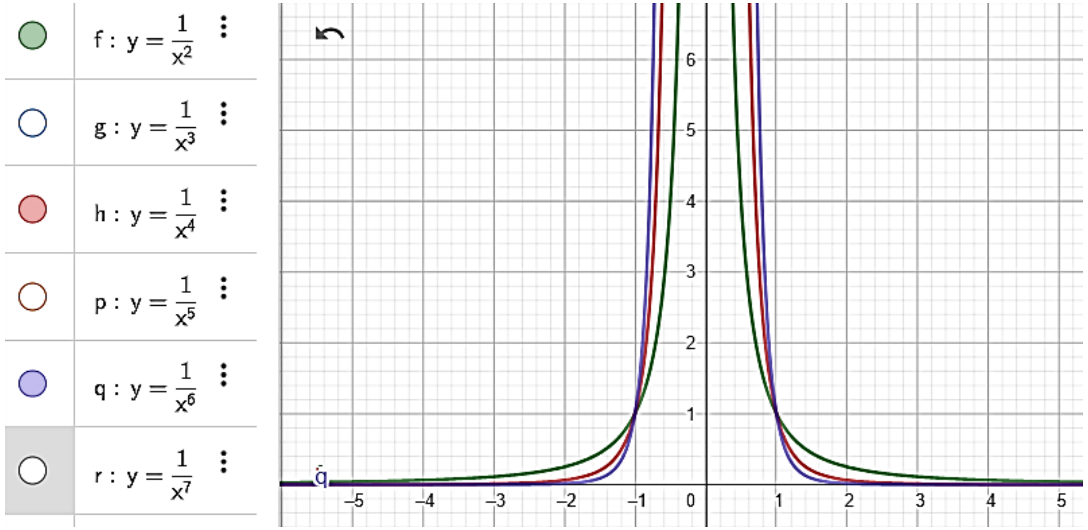
3 Can you group inverse variation functions in smaller categories?

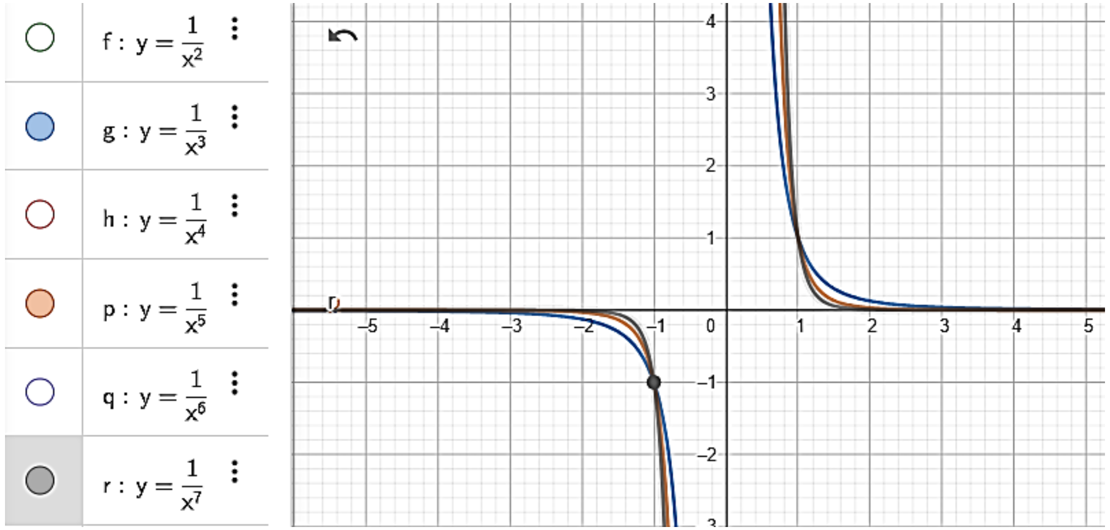




- Even negative power functions (Even power inverse functions)
- Odd negative power functions (Odd power inverse functions)

4 Are inverse variation functions symmetric? If yes, what type of symmetry do they possess?





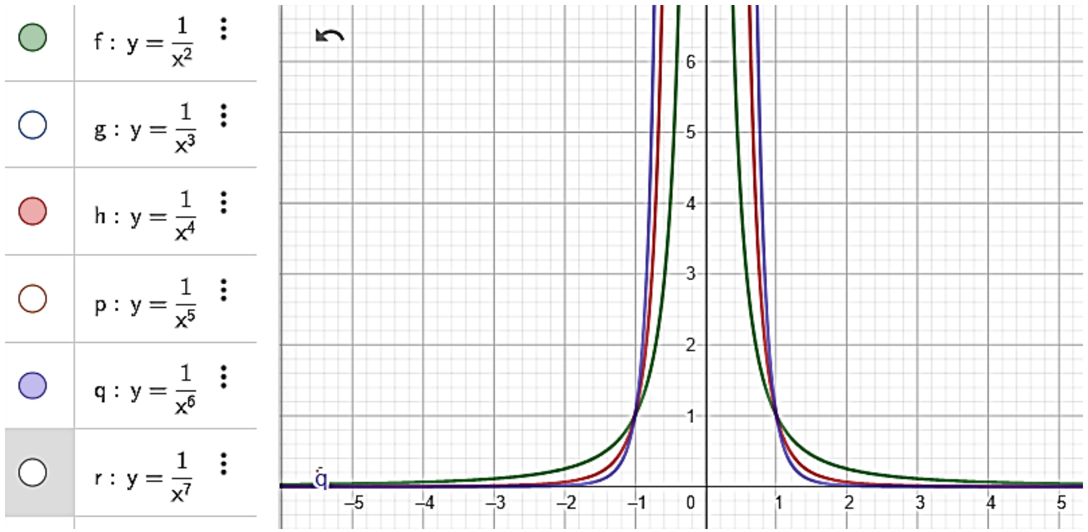
- Symmetrical about the y-axis
- Rotational symmetry about the origin

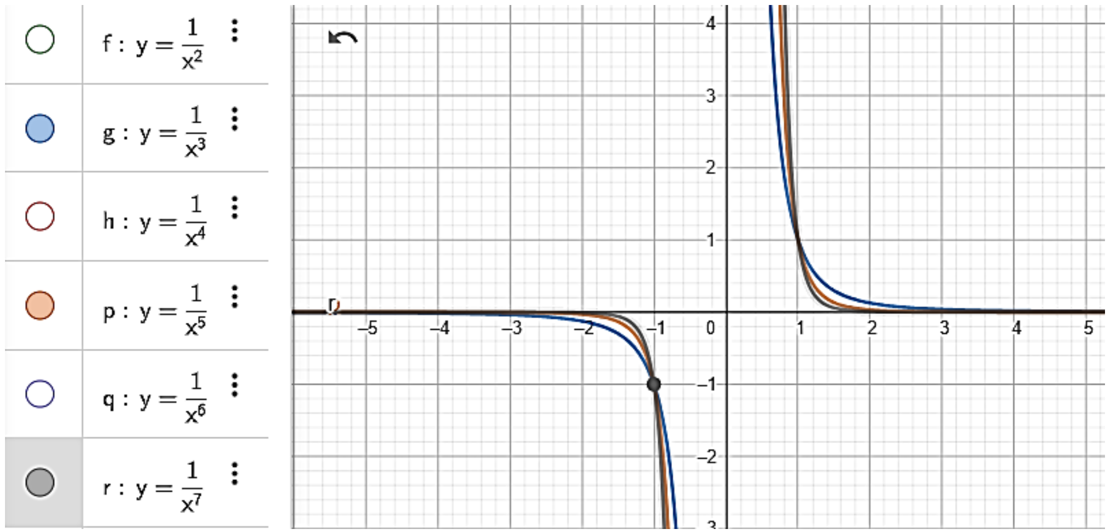
5 **Factual** Describe what happens to the y-value of the function as

a x becomes very large

b as x approaches 0 from the positive side

c as x approaches 0 from the negative side.





a. As x becomes larger, y approaches 0

b. As x approaches 0 from the positive side, y approaches positive infinity

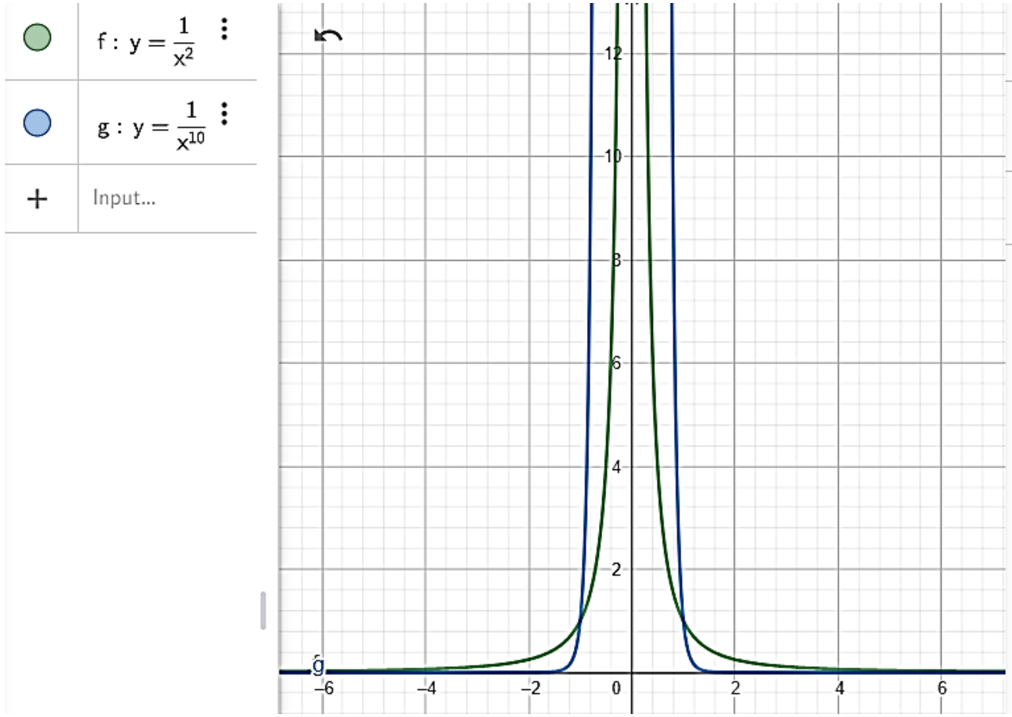
c. As x approaches 0 from the negative side, y approaches positive infinity

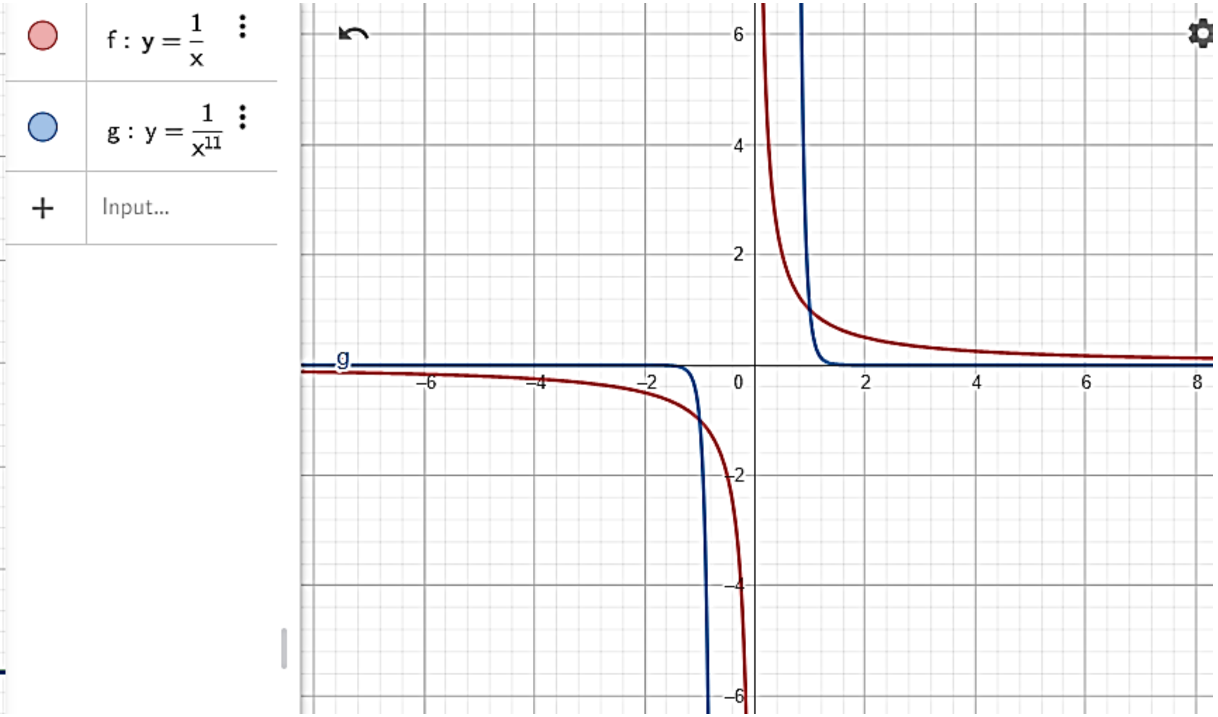
a. As x becomes larger, y approaches 0

b. As x approaches 0 from the positive side, y approaches positive infinity

c. As x approaches 0 from the negative side, y approaches negative infinity

6 What happens to the graph of inverse variation functions as the value of the power becomes more negative?





- As the value of the power becomes more negative, the graph tends to go farther away from the y-axis, but closer to the x-axis.

7 **Conceptual** What are the distinguishing geometrical features of inverse variation functions?

- Inverse variation functions have **vertical and horizontal asymptotes**.
- Odd power inverse variation functions (Odd negative power functions): Symmetrical about the origin
- Even power inverse variation functions (Even negative power functions): Symmetrical about the y-axis

Inverse Variation

When n is negative and $x > 0$, the function $f(x) = ax^n$ represents inverse variation.

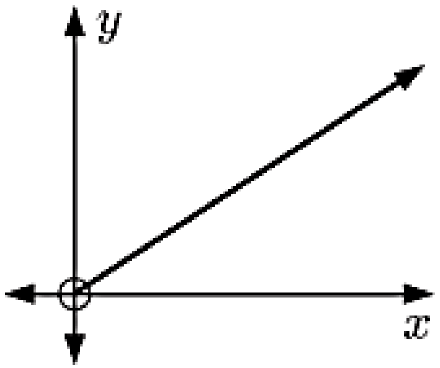
If a quantity y **varies inversely** with x^n for $x > 0$ then $y = kx^{-n}$ or $y = \frac{k}{x^n}$. In these situations y will decrease as x increases and vice versa.

As with direct variation a single point is needed to find the value of the parameter k.

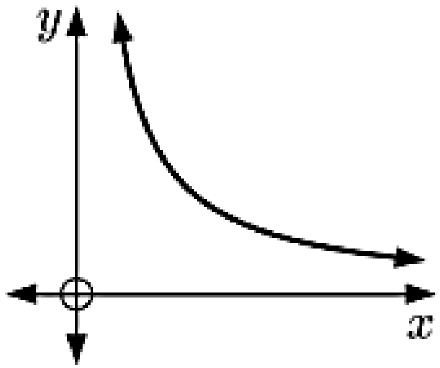
Example 1

Which graph could indicate that y is inversely proportional to x?

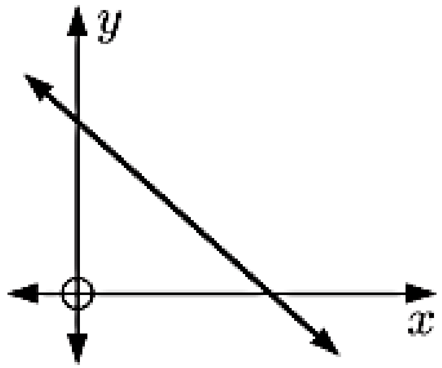
A



B



C



Example 2

The number of hours N taken to build a wall varies inversely with the number of people x who are available to work on it.

a When three people are available the wall takes two hours to build. Find the time it takes to build the wall when four people are available to work on it.

b Given it takes three hours to build the wall, state how many people worked on it.

a $N = \frac{k}{x}$	For inverse variation, the variable is written as $\frac{1}{x}$ or x^{-1} which is a power function.
$N = 2$ when $x = 3$ so $2 = \frac{k}{3}$ $k = 6$	Find the value of k using the given information.
$N = \frac{6}{x}$	Substitute the value of k you found into the equation for N.
$N = \frac{6}{4} = 1.5$ So, it takes 1.5 hours to build the wall when four people work on it.	Now find N when $x = 4$.
b $3 = \frac{6}{x}$ $3x = 6$ $x = \frac{6}{3} = 2$ So, two people were available to build the wall.	Substitute $N = 3$ into the equation $N = \frac{6}{x}$ which you found in part **a**.

Example 3

The volume V of a gas varies inversely with the pressure p of the gas. The pressure is 20 Pa when its volume is 180 m^3. Find the volume when the pressure is 90 Pa.

Example 4

A group of children attend a birthday party. The number of pieces of candy c that each child receives varies inversely with the number of children n.

When there are 16 children, each one receives 10 pieces of candy. Find the number of pieces of candy each child receives when there are 20 children.

Example 5

A company selling used cars wants to create mathematical models for the value of cars as a function of their age. Taking as reference one of its biggest selling models, it collected data relating the selling price to the number of years since the car was produced.

The data are shown in the table:

Age (years)	1	2	3	4	5	6	7	8	9	10
Price (€)	21641	17890	14709	12613	12239	10223	10501	9744	8257	6772

a Plot this data on your GDC.

b Explain why a linear function would not be suitable to model this set of data.

c Explain why an inverse variation function could be suitable to model this set of data.

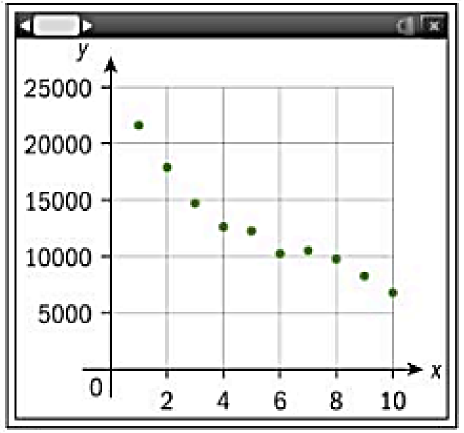
d Use your GDC to determine the power function model for this set of data.

e Find the coefficient of determination, and comment on your answer.

f Sketch the model function over the points and comment on the closeness of fit to the original data.

g According to this model, find the age of the car when its value falls below €4000?

6 **a**



x_1	1	2	3	4	5	6	7	8	9	10
y_1	21641	17890	14709	12613	12239	10223	10501	9744	8257	6772

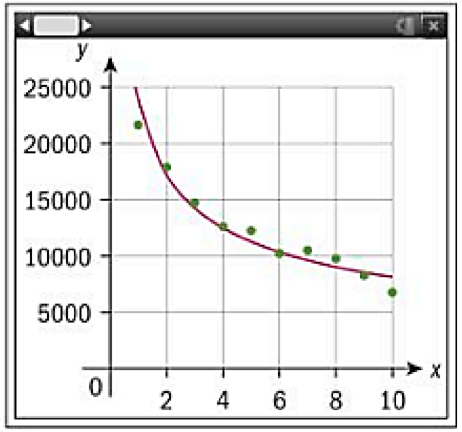
b The price of a car does not vary with age in a linear fashion. If it did, the value of the car would be worthless very quickly.

c The depreciation of the car is quite a lot in the early years, but levels off as the car gets older. An inverse variation function could model this.

d $P = 23688t^{-0.46253}$

e $R^2 = 0.939$ which is a very strong coefficient of determination, so the inverse variation function appears an appropriate model.

f



x_1	1	2	3	4	5	6	7	8	9	10
y_1	21641	17890	14709	12613	12239	10223	10501	9744	8257	6772

This model fits the general shape of the data well. There are a similar number of points above and below the curve.

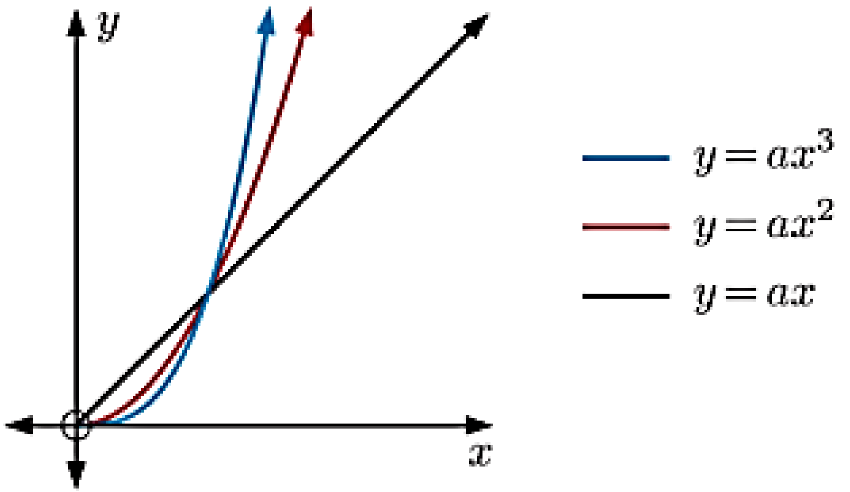
g $4000 = 23\,688t^{-0.46253}$

Solving using GDC or logs $\Rightarrow t = 46.8$ years

Determining the Variation Models

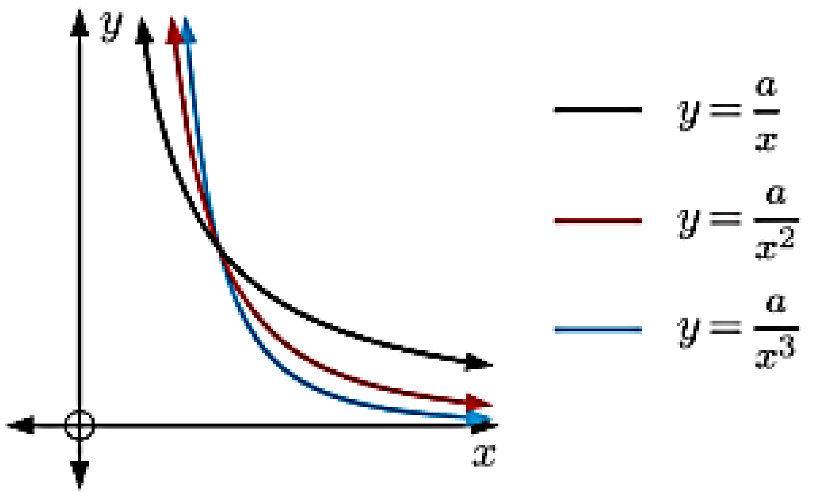
The direct and inverse variations we have studied have equations of the form $y = ax^n$ where $n \in \mathbb{Z}$, $n \neq 0$. These equations are called **variation models.**

- If $n > 0$ we have **direct variation.**



The graph passes through the origin $(0, 0)$.

- If $n < 0$ we have **inverse variation.**



The graph is asymptotic to both the x and y axes.