Exponential Functions

Investigation #1

a Copy and complete the table below showing the number of bacteria at the specified number of hours after the beginning of the experiment

Hours after	0	1	2	3	4
Number of bacteria	200				

b What sequence does the trend of the number of bacteria follow?

c What is the formula of the nth term of the above sequence of numbers?

Assume that we now want to determine the number of bacteria 1.5 hours after the experiment begun.

d Why is a sequence insufficient to determine this value?

If the same relation is used with the independent variable being any positive real number it now becomes a function.

e Use this function in order to determine the number of bacteria present after 1.5 hours.

f Sketch the graph of the function representing the number of bacteria.

g What do you notice? What is the domain and range of this function?

Exponential Function

- This is a function in which the input or independent variable is the exponent of a number called the base.

Exponential Functions Definition

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An exponential function can be in one of the following forms:

→ $f(x) = b^x$

→ $f(x) = ab^x$

→ $f(x) = ab^{cx}$

→ $f(x) = e^x$

→ $f(x) = e^{kx}$

→ $f(x) = p\,e^{kx}$

Here $b>0$ and $b \neq 1$

Investigation #2

Investigation 9

1 Use technology to sketch the graphs of these functions.

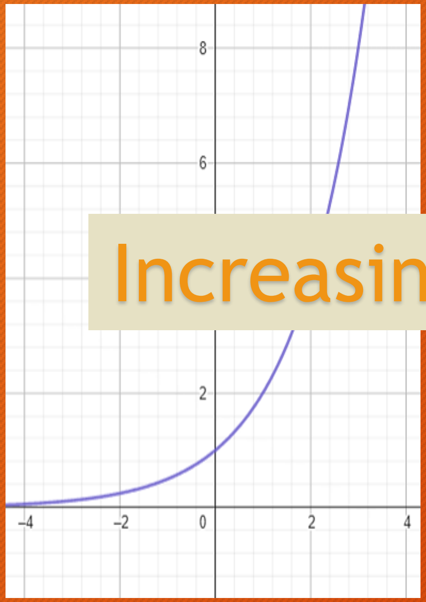
a $f(x)=2^x$ **b** $f(x)=3^x$ **c** $f(x)=0.5^x$

d $f(x)=0.2^x$ **e** $f(x)=2^{-x}$

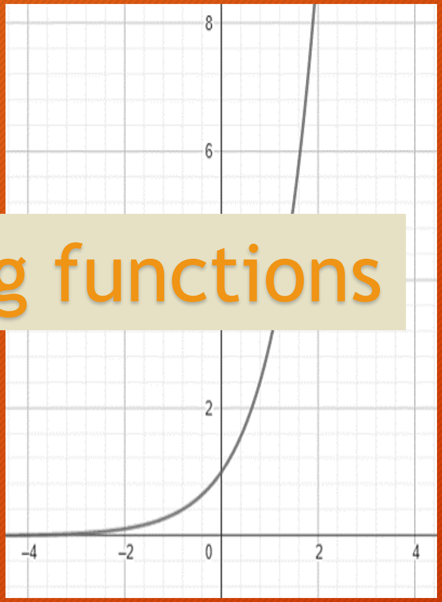
2 Explain why the exponential equation $f(x)=\left(\frac{1}{2}\right)^x$ can be written as $f(x)=2^{-x}$.

3 Give two different conditions on the base and the exponent for an exponential function to be a decreasing function.

Increasing functions

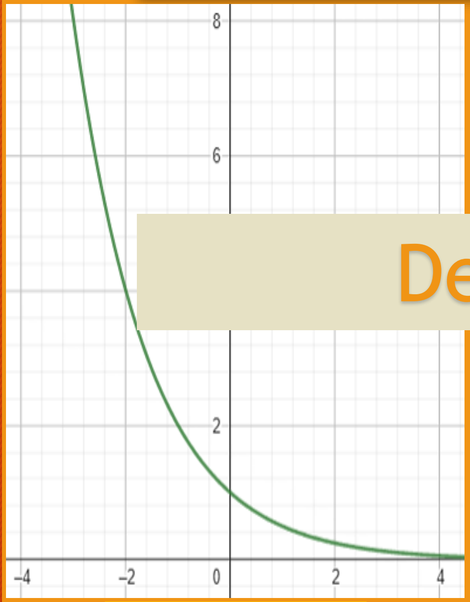


$f(x)=2^x$

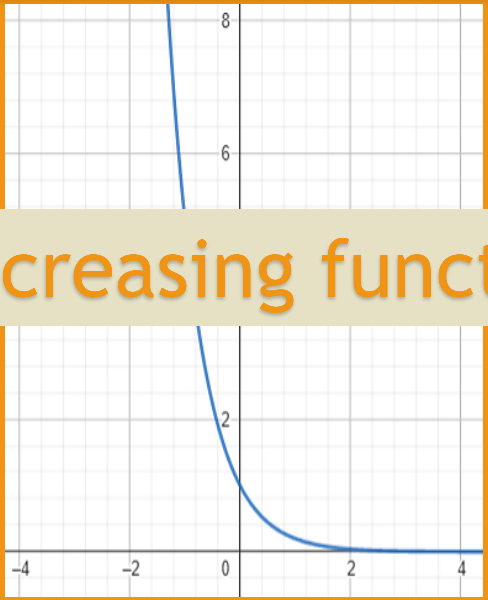


$f(x)=3^x$

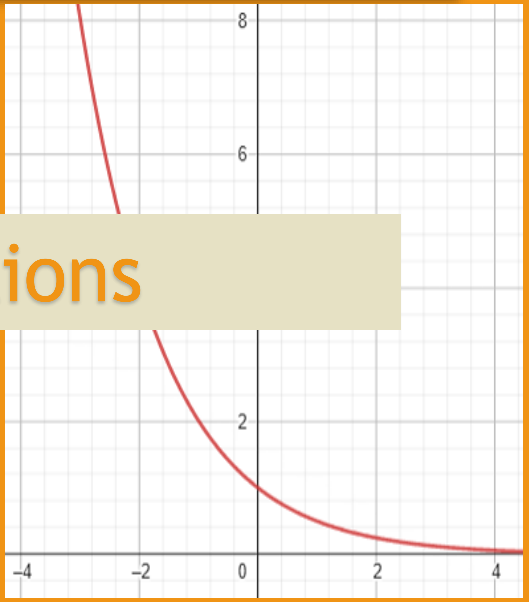
Decreasing functions



$f(x)=0.5^x$



$f(x)=0.2^x$



$f(x)=2^{-x}$

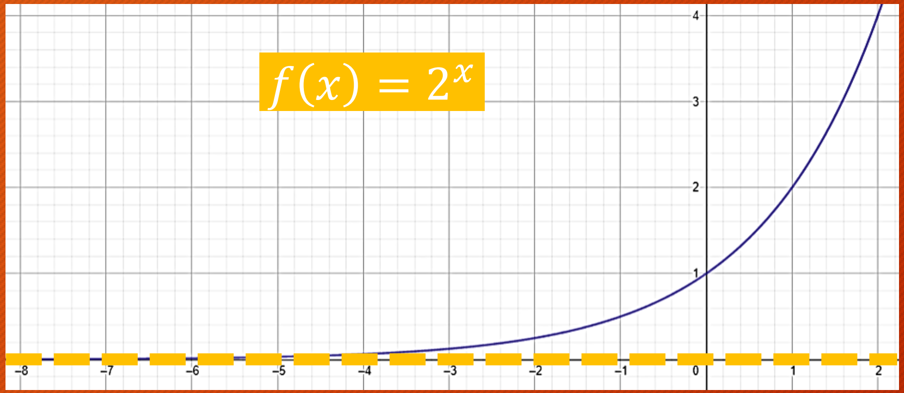
Exponential Function

The independent or input variable is the exponent.

Exponential Function

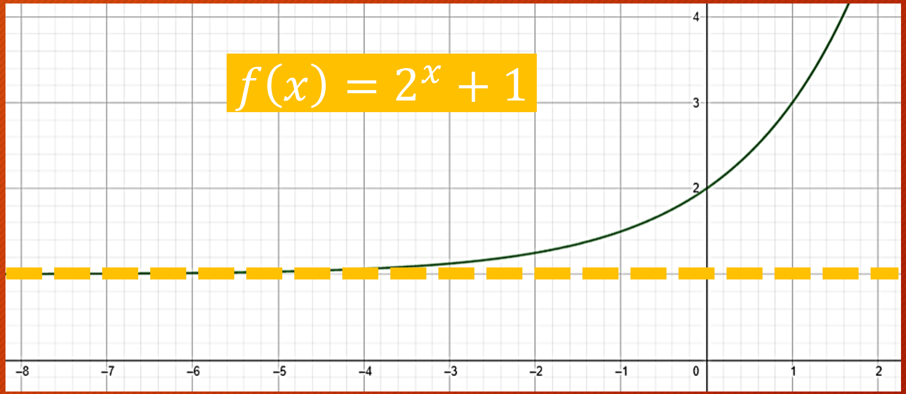
- All exponential functions have a **horizontal asymptote.**
- Exponential functions of the form $f(x) = b^x$ have the line $y = 0$ as the horizontal asymptote.

A horizontal asymptote is **a line in which a graph approaches but never intersects.**



$Horizontal\ Asymptote: y = 0$

- The general form $f(x) = ab^x + c$ has a horizontal asymptote at $y = c$.



$Horizontal\ Asymptote: y = 1$

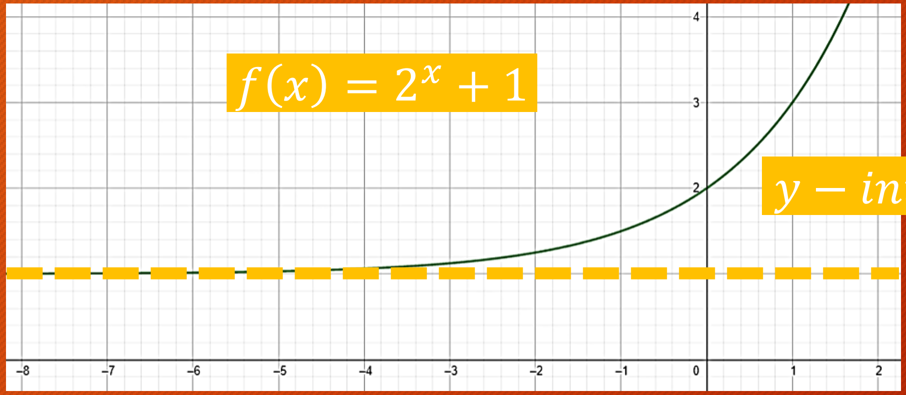
Exponential Growth and Decay

The exponential function $f(x) = ka^x + c$ has the following properties:

The straight line $y = c$ is a horizontal asymptote.

It crosses the y-axis at the point $(0, k + c)$.

Is increasing for $a > 1$ and decreasing for $0 < a < 1$.



$f(x) = 2^x + 1$

$y - intercept: (0, k + c) \rightarrow (0, 1 + 1) \rightarrow (0, 2)$

$Horizontal\ Asymptote: y = 1$

$Since\ a > 1, the\ function\ is\ increasing.$

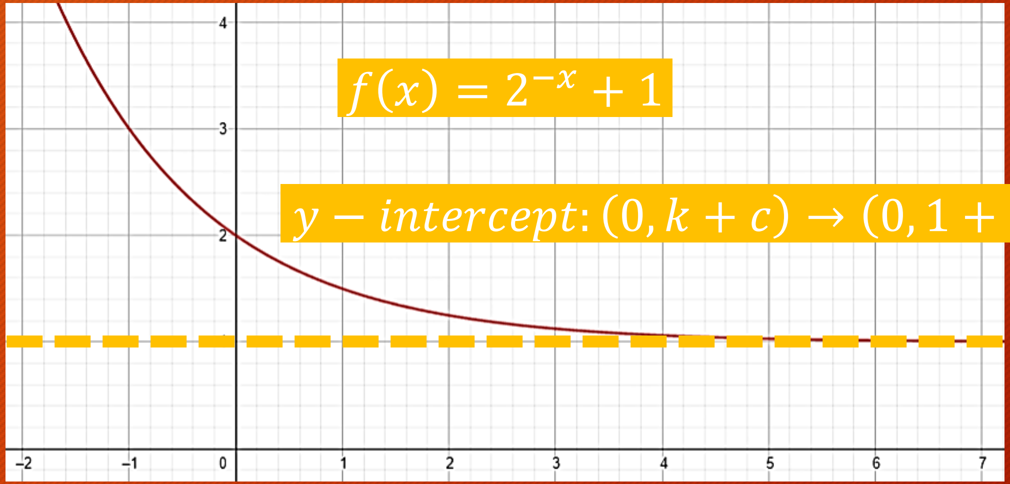
Exponential Growth and Decay

The exponential function $f(x) = ka^{-x} + c$ has the following properties:

The straight line $y = c$ is a horizontal asymptote as $x \to \infty$.

It crosses the y-axis at the point $(0,\ k + c)$

Is decreasing for $a > 1$ (exponential decay)



$f(x) = 2^{-x} + 1$

$y - intercept: (0, k + c) \to (0, 1 + 1) \to (0, 2)$

$Horizontal\ Asymptote: y = 1$

$Since\ a > 1, the\ function\ is\ decreasing.$

Examples

For each of the following functions:

- i find the equation of the horizontal asymptote
- ii find the coordinates of the point where the curve cuts the y-axis.
- iii state if the function is increasing or decreasing.

- a $f(x) = 3^x + 2$
- b $f(x) = 5 \times 0.2^x - 3$

Answer

a i The horizontal asymptote has equation $y = 2$.	2 is the value for c.
ii The curve cuts the y-axis at the point (0, 3).	$k = 1$ and $c = 2$, so, $1 + 2 = 3$
iii The function is increasing because 3 is greater than 1.	
b i The horizontal asymptote has equation $y = -3$.	-3 is the value for c.
ii The curve cuts the y-axis at the point (0, 2).	$k = 5$ and $c = -3$, so $5 - 3 = 2$
iii The function is decreasing because 0.2 lies between 0 and 1.	

Your Turn

1 For the graphs of the following equations:

i find the y-intercept

ii find the equation of the horizontal asymptote

iii state whether the function shows growth (increasing) or decay (decreasing).

a $f(x) = 4^x + 1$

b $f(x) = 0.2^x - 3$

c $f(x) = 5^{2x}$

d $f(x) = 3^{0.1x} + 2$

e $f(x) = 3(2)^x - 5$

f $f(x) = 4(0.3)^{2x} + 3$

g $f(x) = 5(2)^{0.5x} - 1$

h $f(x) = 2(2.5)^{-x} - 1$

2 Write the following functions in the form $f(x) = ka^x + c$.

a $f(x) = 2(4)^{2x} + 5$

b $f(x) = 7(0.5)^{-3x} + 2$