Week 32

# Arc Length and Area of Sector

**Prior Knowledge:**

- Finding the area of a circle.
- Finding the circumference of a circle.
- Expressing answers in terms of $\pi$.
- Rearranging formulae.

**Length of arc formula**

The length of the arc of a circle with radius $r$ and with central angle $\theta$ (in degrees) is:

$$\frac{\theta}{360°} \times 2\pi r$$

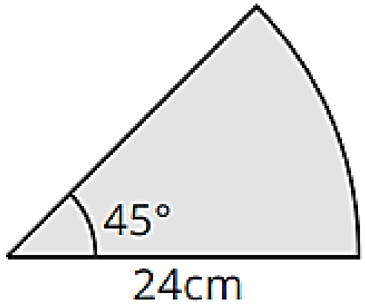
**Area of sector formula**

The area of a sector with central angle $\theta$ is:

$$\frac{\theta}{360°} \times \pi r^2$$

**Example 1:**

Find the area and perimeter of the following sector. Give your answers correct to 2 d.p.




We'll start by finding the area. This sector is $\frac{45}{360}$ (or $\frac{1}{8}$) of a full circle so the area is:

Area of sector = $\frac{1}{8} \times \pi \times 242$ = $\mathbf{226.19cm^2}$

We can find the length of the arc in a similar way:

Length of arc = $\frac{1}{8} \times \pi \times 48 = 18.85$cm

Be careful here. The question doesn't ask for the length of the arc, but the perimeter of the sector, and the arc is just one of its three sides:

Perimeter = 24 + 24 + 18.85 = **66.85cm**

**Example 2:**

A sector of a circle of radius 15m has an area of $135\pi$ m$^2$. Find the angle of the sector.

Here, we know the area and need to work backwards to find the angle. We will substitute the values we know into the formula for the area of a sector, then rearrange:

$$\text{Area of sector} = \frac{\text{angle}}{360} \times \pi \times r^2$$

$$135\pi = \frac{\text{angle}}{360} \times \pi \times 15^2$$

We'll start by dividing both sides by $\pi$:

$$135 = \frac{\text{angle}}{360} \times 225$$

We can divide both sides by 225, then simplify:

$$\frac{135}{225} = \frac{\text{angle}}{360}$$

$$\frac{3}{5} = \frac{\text{angle}}{360}$$

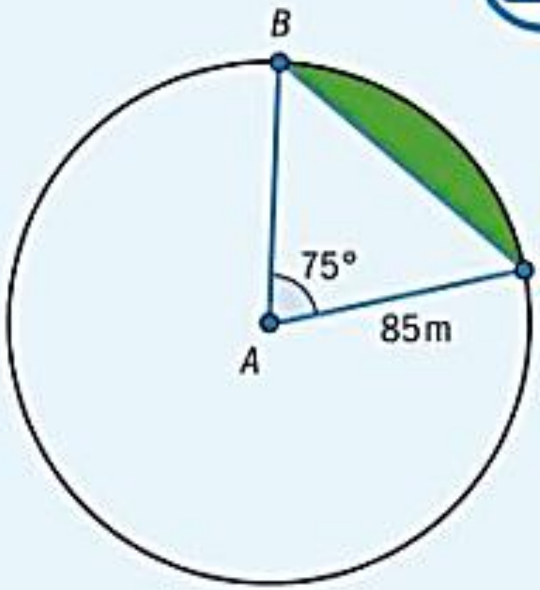
Finally, multiply both sides by 360:

$$\text{angle} = \frac{3}{5} \times 360 = \mathbf{216°}$$

A city park with a circular perimeter contains several sidewalks represented by the sides of triangle ABC. The length of sidewalk [AC] = 85 m and $B\hat{A}C = 75°$.

**a** Find how much longer the circular arc $\overset{\frown}{BC}$ is than the straight path [BC].

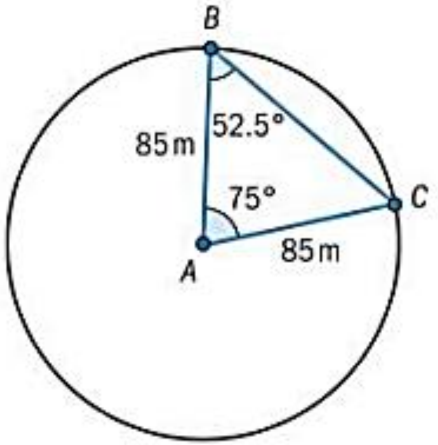
**b** Find the area of the segment that is between the chord [BC] and the arc $\overset{\frown}{BC}$



**a** $\widehat{BC} = \dfrac{75°}{360°} \times 2\pi(85) = 111.265 \text{ m}$

Using length of arc formula.

Since AB = AC = 85 m, $\Delta ABC$ is isosceles, so $\hat{B} = \hat{C} = \dfrac{180 - 75}{2} = 52.5°$.




$$\frac{BC}{\sin 75°} = \frac{85}{\sin 52.5°}$$

$BC = 103.489 \text{ m}$

Now there is sufficient information to use the Sine Rule.

$111.265 - 103.489 = 7.78 \text{ m}$ longer

Find the difference between the length of the arc and the length of the chord.

**b** $A_{\text{sector}} - A_{\text{triangle}}$

$= \dfrac{75}{360} \times \pi \times 85^2 - \dfrac{1}{2}(85)(85)\sin 75°$

$= 1240 \text{ m}^2$

Use formula $A = \dfrac{1}{2}ab\sin C$ for the area of the triangle.

## Exercise 1G

1 Determine **i** the length of the each arc **ii** the area of each sector for the radius $r$ and central angle $\alpha$ given below. Give your answer correct to 2 d.p.

**a** $r = 5\,\text{cm};\ \alpha = 70°$

**b** $r = 4\,\text{cm};\ \alpha = 45°$

**c** $r = 10.5\,\text{cm},\ \alpha = 130°$

2 A clock is circular in shape with diameter 25 cm. Find the length of the arc between the markings 12 and 5 rounded to the nearest tenth of cm.

# SEAT WORK

- Exam.net key code: wRvCpi

# References

- Oxford MAIHL, pp.25-26
- Twinkl.com