TEST OF HYPOTHESIS

AGENDA

Recall Binomial and Poisson Distributions

Use Binomial and Poisson Distributions to test hypotheses

Test hypotheses using Goodness of Fit

USING BINOMIAL AND POISSON DISTRIBUTIONS

EXAMPLE 1: RECALL AND USE OF BINOMIAL DISTRIBUTION

A six-sided dice is rolled 30 times, obtaining 8 sixes. Is there evidence, at the 10% significance level, that the probability of rolling a six is more than 1/6

Null hypothesis (Ho): $p = 1/6$ (The probability of rolling a six is 1/6)
Alternative (Ha): $p > 1/6$ (The probability of rolling a six is greater than 1/6)

Let X = number of sixes from 30 rolls
Assuming that Ho is true, $X \sim B(30, 1/6)$

$$P(X \geq 8) = 1 - P(X \leq 7)$$
$$= 0.114$$

$0.114 > 0.10$
There is insufficient evidence, at the 10% significance level, that the probability of rolling a six is more than 1/6

EXAMPLE 2:

It is known that 77% of UK households own at least one car. Ganesh lives in an urban area and suspects that car ownership in his neighbourhood is lower than this. To test his belief, he intends to take a random sample of 50 households and find out how many own a car. Write down suitable hypotheses and find the critical region for the test at the 5% significance level.

Let X = the number of households out of 50 which own a car.
Then $X \sim B(50, p)$

Ho : $p = 0.77$, Car ownership is 77% or The proportion of car ownership is the same as the national average
Ha: $p < 0.77$, Car ownership is less than 77% or The proportion of car ownership in Ganesh's neighborhood is lower than the national average

Assuming $X \sim B(50, 0.77)$

$P(X \leq 34) = 0.0925 > 0.05$
$P(X \leq 33) = 0.0508 > 0.05$
$P(X \leq 32) = 0.0258 < 0.05$,

The critical value is 32, the critical region is $X \leq 32$

EXAMPLE 3: RECALL AND USE OF POISSON DISTRIBUTION

Ingrid runs an IB revision website which gets an average of 18 visits per day. Following the launch of a rival website, she wants to test whether this average rate has decreased. On a randomly selected day, there were 10 visits to her website. Test, using a 5% significance level, whether the average rate of visiting has decreased.

Null hypothesis (Ho): $\lambda = 18$
Alternative hypothesis (Ha): $\lambda < 18$

X = the number of visits in one day Assuming that Ho is true, $X \sim Po(18)$

$P(X \leq 10) = 0.0304 < 0.05$

There is sufficient evidence, at the 5% significance level, that the average rate of visits has decreased.

EXAMPLE 4:

The number of outbreaks of a certain disease in a country is modelled by a Poisson distribution. Over a long period, the average rate of outbreaks has been constant at 2.4 per year. A doctor suspects that the average rate has increased, and wants to test his suspicion by looking at the number of outbreaks in the past year. Find the critical region for this test at the 2% significance level.

Let X = the number of outbreaks in a year.
Then $X \sim Po(\lambda)$

Null hypothesis	(Ho):	$\lambda = 2.4$
Alternative hypothesis	(Ha):	$\lambda > 2.4$

Assuming $X \sim Po(2.4)$

$P(X \geq 5) = 1 - P(X \leq 4) = 0.096 > 0.02$
$P(X \geq 6) = 1 - P(X \leq 4) = 0.036 > 0.02$
$P(X \geq 7) = 1 - P(X \leq 4) = 0.012 < 0.02$

GOODNESS OF FIT

The **Goodness of Fit Test** is a statistical hypothesis test used to determine how well a sample distribution matches an expected (theoretical) distribution. It is often applied to categorical data and tests whether observed frequencies differ significantly from expected frequencies under a specific hypothesis.

When to Use It

You would use a goodness of fit test when you want to evaluate whether:

1. A sample of data fits a theoretical probability distribution (e.g., uniform, binomial, or normal distribution).

2. The observed frequencies in categories differ significantly from the expected frequencies based on a known distribution.

1. State the null hypothesis and the alternative hypothesis.

2. State the significance level α.

3. Calculate the value of the test statistic: $X^2 = \sum \frac{(observed - expected)^2}{expected}$.

4. Calculate the p-value, using df = number of categories — 1.

5. Come up with the decision based on the computed values.

6. Interpret your decision in the context of the problem. Write your conclusion in a sentence.

EXAMPLE 5:

After losing a board game, your friend believes she might have lost because of a problem with your dice. To find out, she rolls your dice 60 times and obtains the following frequencies:

Number	Frequency
1	8
2	11
3	6
4	9
5	12
6	14

EXAMPLE 5:

After losing a board game, your friend believes she might have lost because of a problem with your dice. To find out, she rolls your dice 60 times and obtains the following frequencies:

Number	Frequency (Observed)	Expected
1	8	10
2	11	10
3	6	10
4	9	10
5	12	10
6	14	10

O – E	(O – E)^2	(O – E)^2/E
-2	4	0.4
1	1	0.1
-4	16	1.6
-1	1	0.1
2	4	0.4
4	16	1.6

$$X^2 = 0.4 + 0.1 + 1.6 + 0.1 + 0.4 + 1.6 = 4.2$$

EXAMPLE 5:

After losing a board game, your friend believes she might have lost because of a problem with your dice. To find out, she rolls your dice 60 times and obtains the following frequencies:

Chi calculated value : $X^2 = 0.4 + 0.1 + 1.6 + 0.1 + 0.4 + 1.6 = 4.2$

Critical Value:
Stat > Dist (F5) > Chi (F3) > InvC (F3)

Area : 0.05
df : $n - 1 = 6 - 1 = 5$

$X^2 = 11.07$

Calculated Value < Critical Value
Fail to reject the null hypothesis

EXAMPLE 6:

You work at a nut factory and you're in charge of quality control. The nut factory produces a nut mix that's supposed to be 50% peanuts, 30% cashews, and 20% almonds.

To check that the nut mix proportions are acceptable, you randomly sample 1000 nuts and find the following frequencies:

Nut	Frequency
Peanuts	621
Cashew	189
Almonds	190

Should you reject the null hypothesis that the nut mix has the desired proportions of nuts?

THANK YOU

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