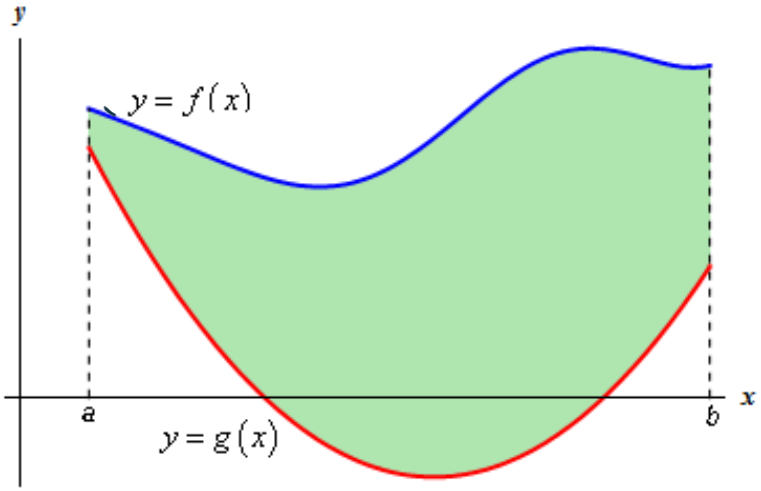
Area Between Curves

5.12	Area of region enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y \rvert \, dx$ or $A = \int_a^b \lvert x \rvert \, dy$
	Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2 \, dx$ or $V = \int_a^b \pi x^2 \, dy$

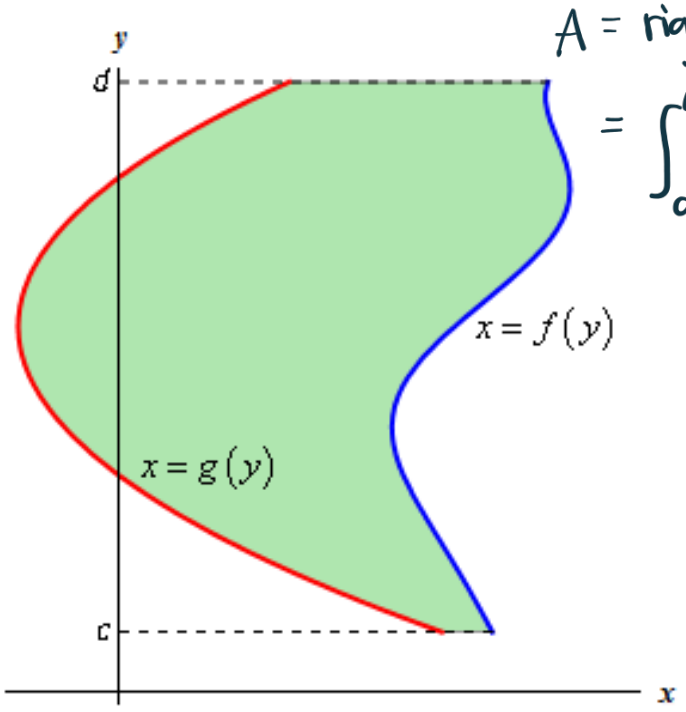
$A = \int f(x)\,dx - \int g(x)\,dx$

graph

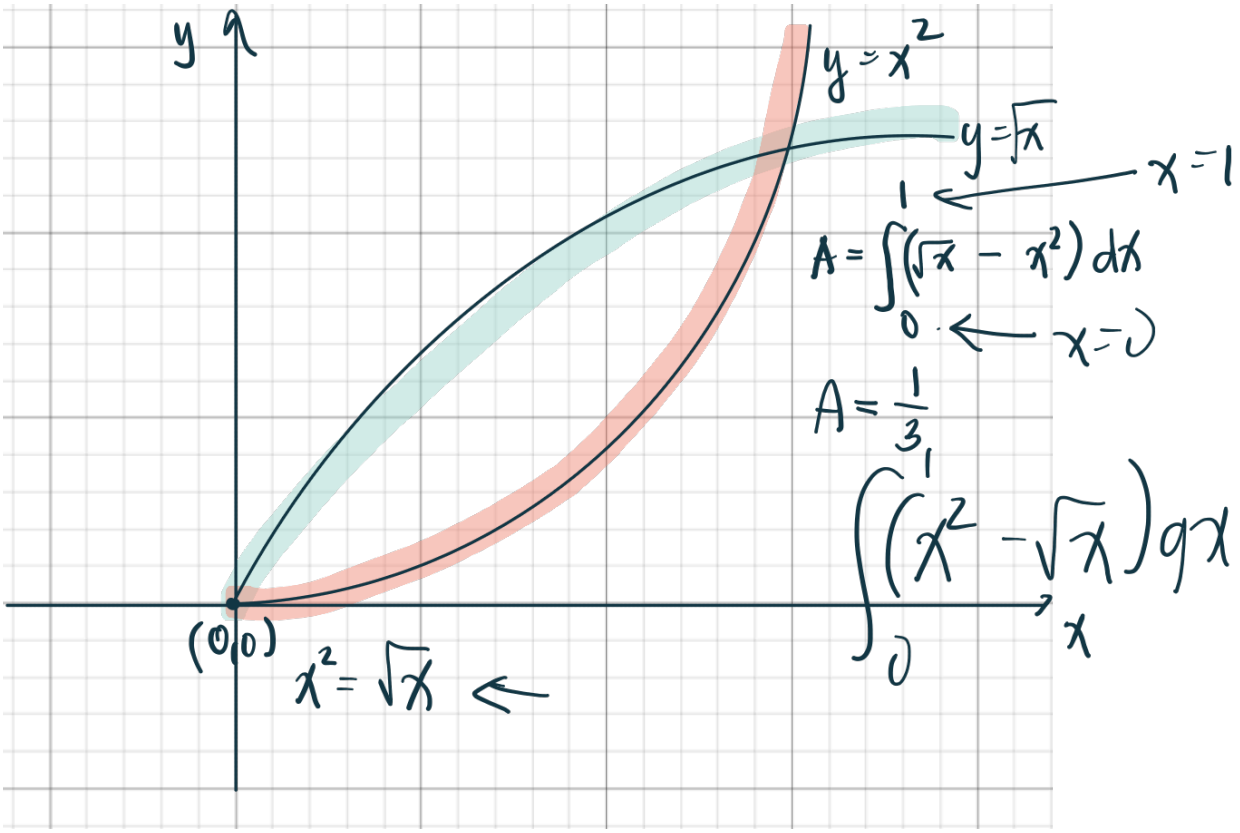


A = right − left

$= \int_c^d f(y) - g(y)\,dy$

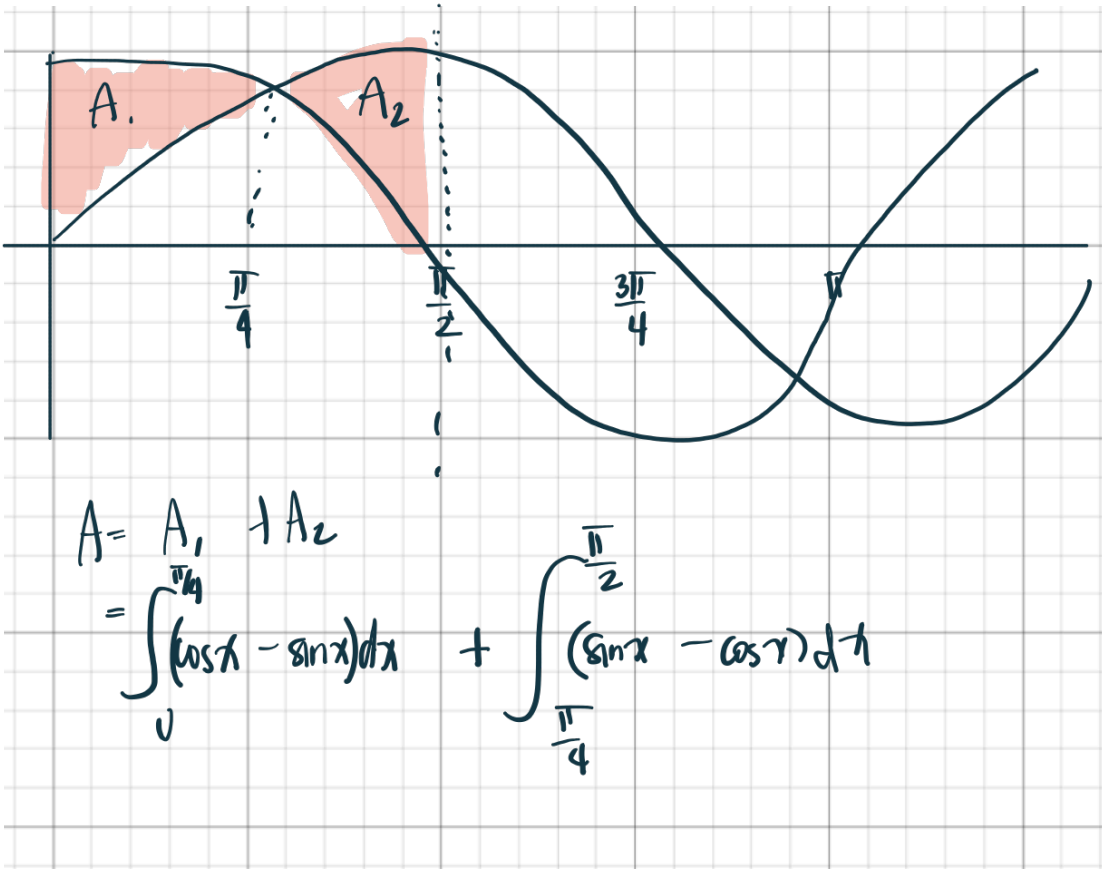


Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$.



Determine the area of the region bounded by $y = xe^{-x^2}$, $y = x + 1$, $x = 2$, and the y-axis.

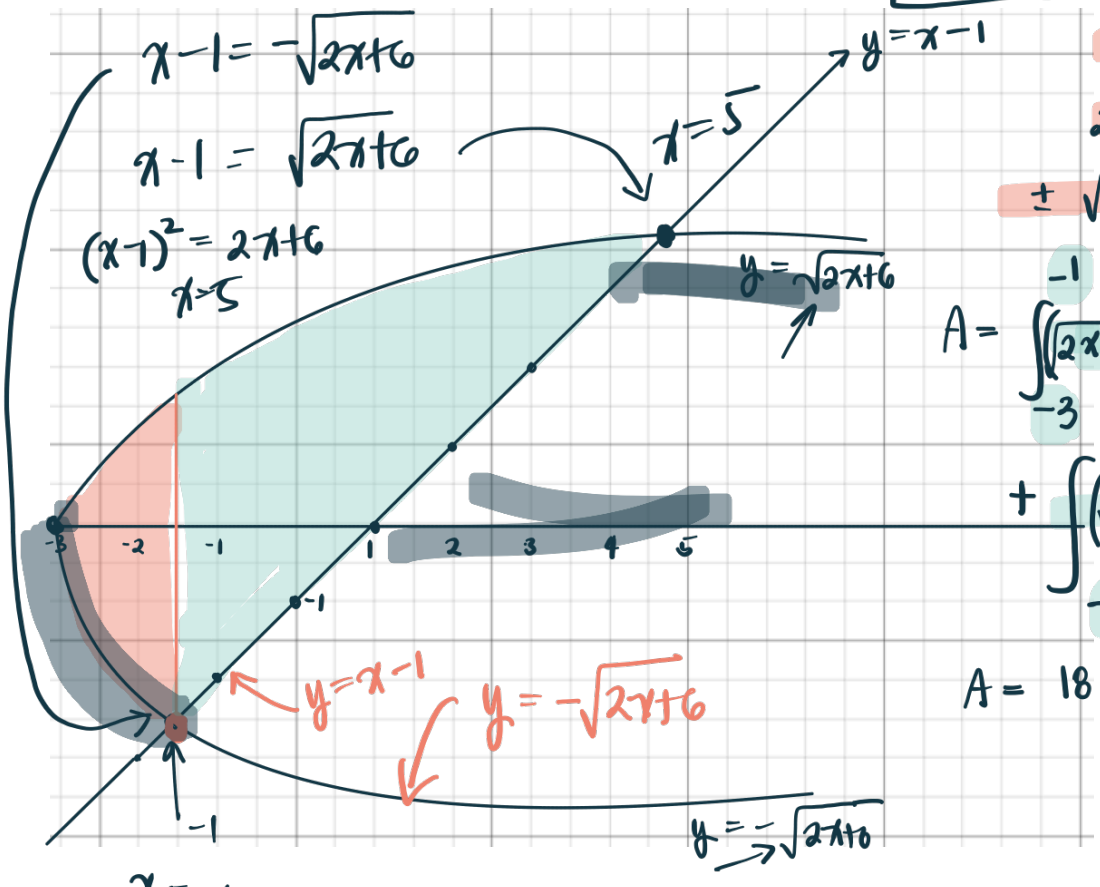
Determine the area of the region enclosed by $y = \sin x$, $y = \cos x$, $x = \frac{\pi}{2}$, and the y-axis.



$$A = A_1 + A_2$$

$$= \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.



$x = \frac{1}{2}y^2 - 3$

$x + 3 = \frac{1}{2}y^2$

$2x + 6 = y^2$

$\pm\sqrt{2x+6} = y$

$$A = \int_{-3}^{-1} \left(\sqrt{2x+6} + \sqrt{2x+6}\right) dx + \int_{-1}^{5} \left(\sqrt{2x+6} - (x-1)\right) dx$$

$A = 18$

horizontal

$y = x - 1$
$x = y + 1$

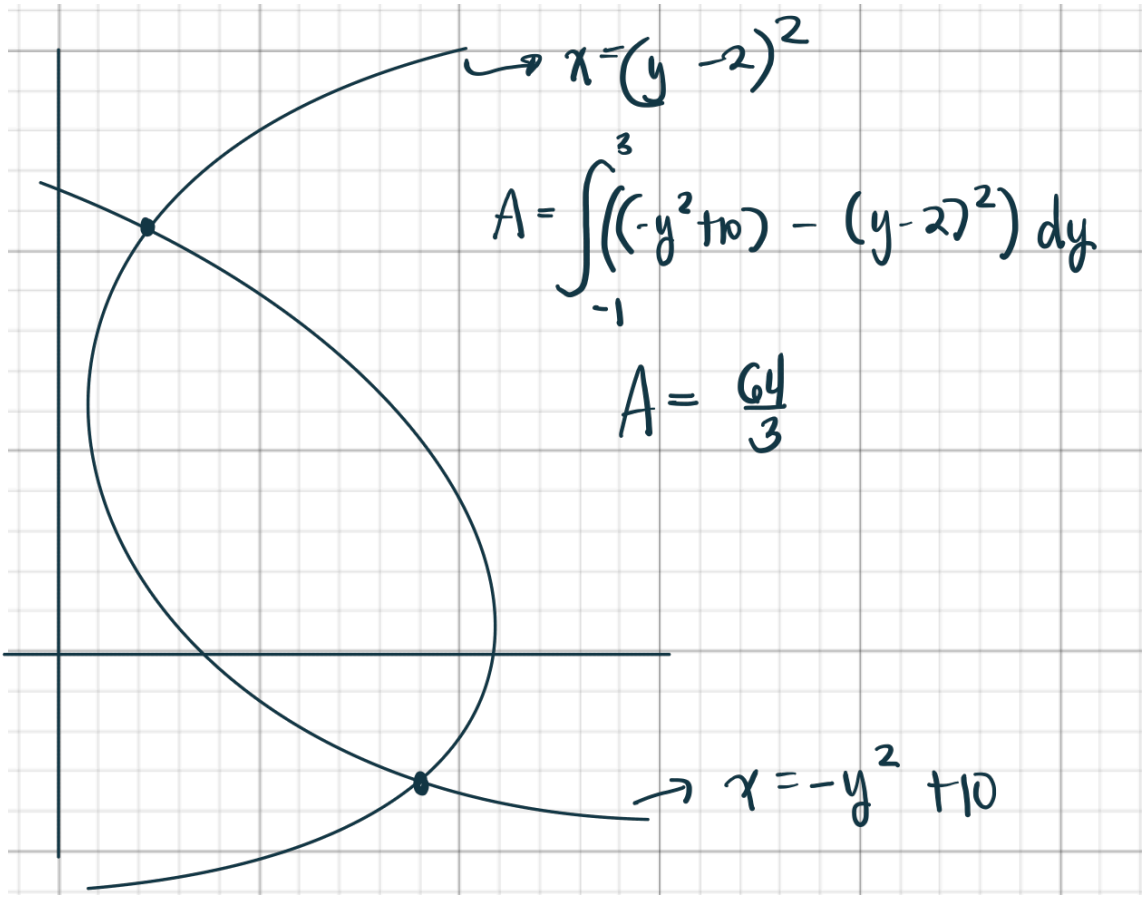
$x = \frac{1}{2}y^2 - 3$

$$A = \int_{-2}^{4} \left((y+1) - \left(\frac{1}{2}y^2 - 3\right)\right) dy$$ PAPER

$$\int_{-2}^{4} \left((x+1) - \left(\frac{1}{2}x^2 - 3\right)\right) dx$$ GDC

$-y^2+10=(y-2)^2$

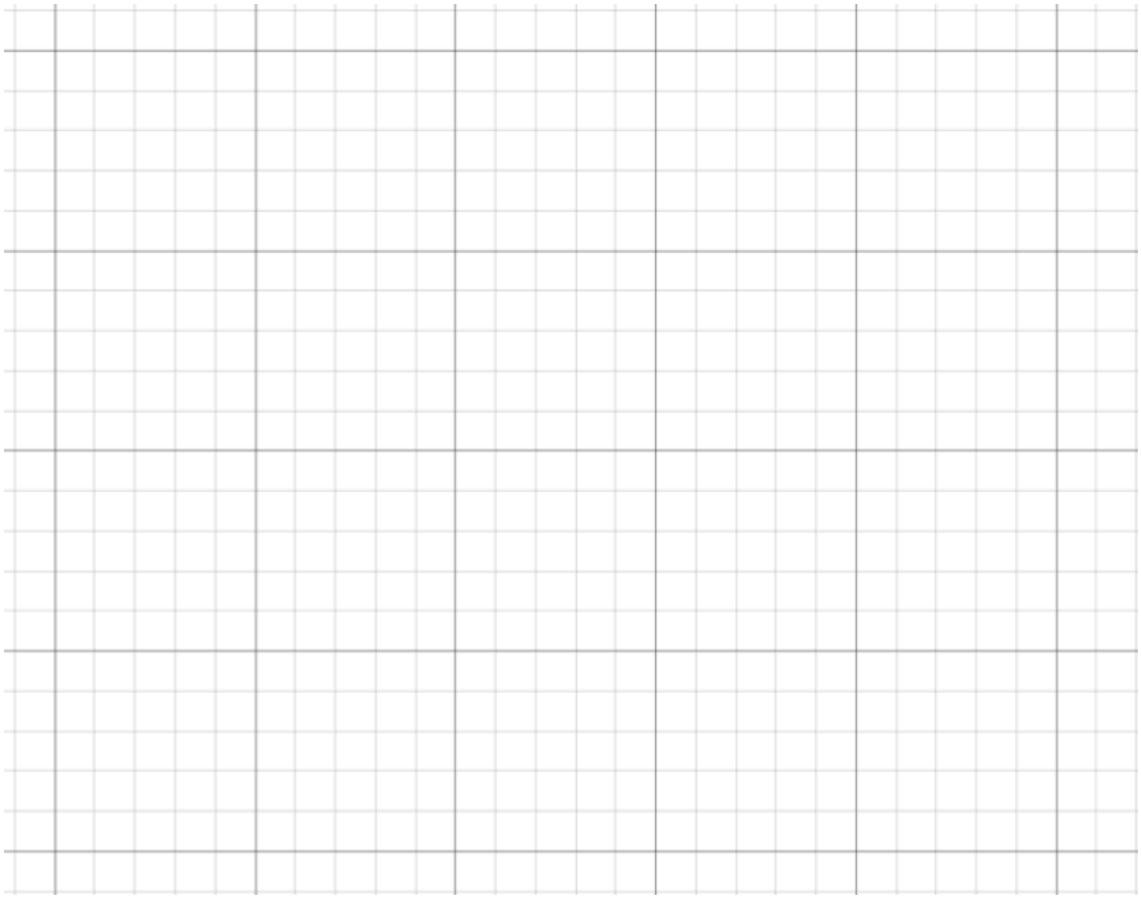
Determine the area of the region bounded by $x=-y^2+10$ and $x=(y-2)^2$.



Volumes of Revolution

5.12	Area of region enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y \rvert \mathrm{d}x$ or $A = \int_a^b \lvert x \rvert \mathrm{d}y$
	Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2 \mathrm{d}x$ or $V = \int_a^b \pi x^2 \mathrm{d}y$

Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.



Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.

Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$.

Determine the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$.