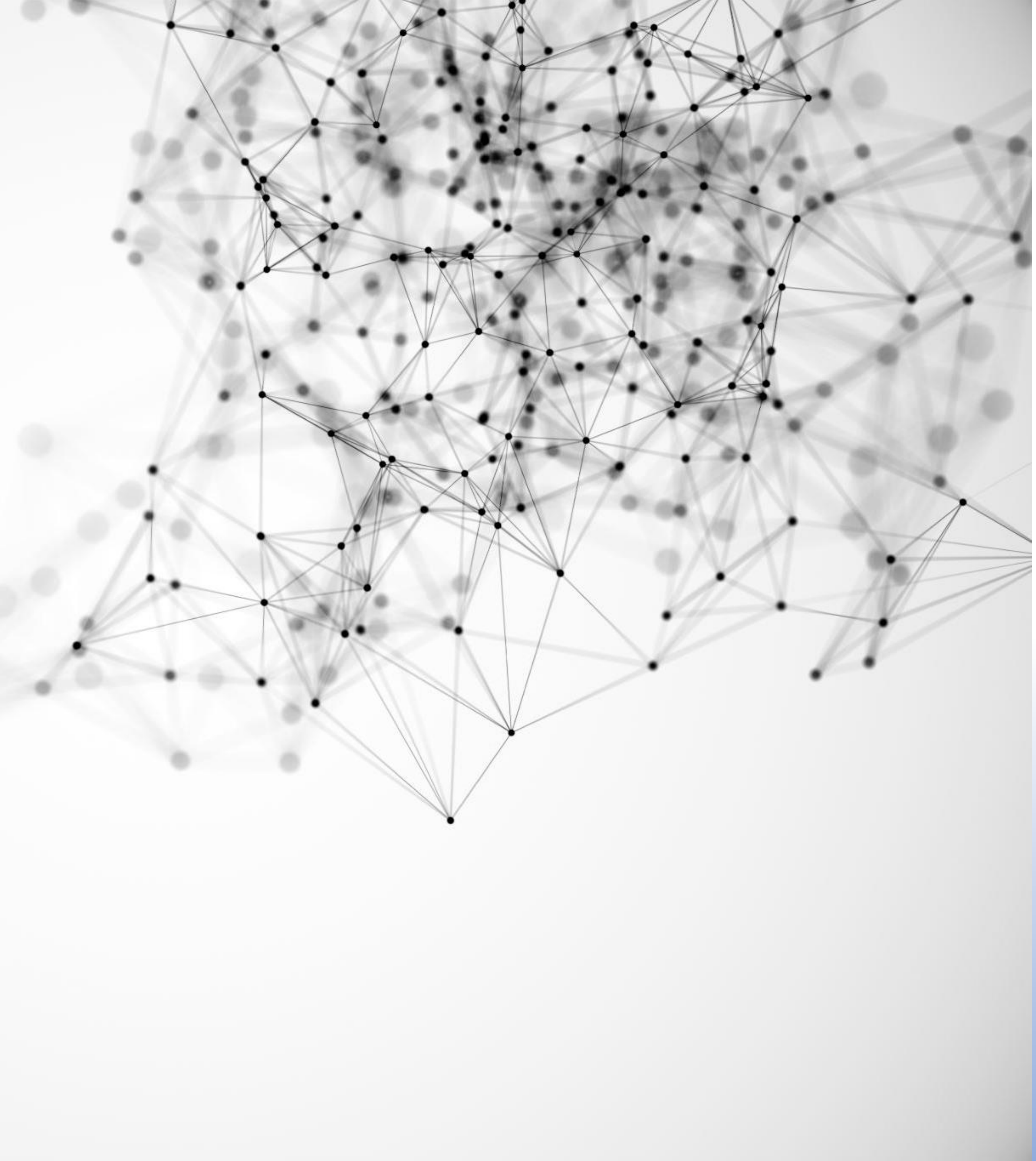


COMPLEX NUMBERS

Week 23

IMAGINARY NUMBER

Rafael Bombelli

It was **Rafael Bombelli** who defined the imaginary number $i = \sqrt{-1}$ in 1572.

The **imaginary number** is defined as $\boldsymbol{i = \sqrt{-1}}$ with the property that $i \times i = \sqrt{-1} \times \sqrt{-1} = -1$.

It is called "imaginary" because we cannot place it on the real number line.

RECALL AND SIMPLIFY:

- $\sqrt{81}$ $\boxed{=9}$
- $-\sqrt{3}$ $\boxed{=-\sqrt{3}}$
- $\sqrt{5}$ $\boxed{=\sqrt{5}}$
- $-\sqrt{36}$ $\boxed{=-6}$
- $\sqrt{\frac{1}{4}}$ $\boxed{=\frac{1}{2}}$

WRITE IN TERMS OF i:

- $\sqrt{-81}$ $\boxed{=9i}$
- $-\sqrt{-3}$ $\boxed{=-\sqrt{3}\,\boldsymbol{i}}$
- $\sqrt{-5}$ $\boxed{=\sqrt{5}\,\boldsymbol{i}}$
- $-\sqrt{-36}$ $\boxed{=-6\boldsymbol{i}}$
- $\sqrt{-\frac{1}{4}}$ $\boxed{=\frac{1}{2}\boldsymbol{i}}$

COMPLEX NUMBERS

The solutions to quadratic equations with $\Delta < 0$ have the form $x = a + bi$, where a and b are real.

Any number of the form $\boxed{a + bi}$ where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$, is called a **complex number**.

REAL AND IMAGINARY PARTS

If $z = a + bi$ where $a, b \in \mathbb{R}$, then:

- a is the **real part** of z, written $\mathcal{Re}(z)$
- b is the **imaginary part** of z, written $\mathcal{Im}(z)$.

For example:

- If $z = 2 + 3i$, then $\mathcal{Re}(z) = 2$ and $\mathcal{Im}(z) = 3$.
- If $z = -\sqrt{2}i$, then $\mathcal{Re}(z) = 0$ and $\mathcal{Im}(z) = -\sqrt{2}$.

COMPLEX CONJUGATES

Suppose $z = a + bi$ where $a, b \in \mathbb{R}$.

The **complex conjugate** of z is $z^* = a - bi$.

$\mathcal{Re}(z^*) = \mathcal{Re}(z)$ and $\mathcal{Im}(z^*) = -\mathcal{Im}(z)$

COMPLEX CONJUGATES

Suppose $z = a + bi$ where $a, b \in \mathbb{R}$.

The **complex conjugate** of z is $z^* = a - bi$.

$\mathfrak{Re}(z^*) = \mathfrak{Re}(z)$ and $\mathfrak{Im}(z^*) = -\mathfrak{Im}(z)$

z	$\mathfrak{Re}(z)$	$\mathfrak{Im}(z)$	z^*
$3 + 2i$	3	2	$3 - 2i$
$5 - i$	5	-1	$5 + i$
3	3	0	3
$4i$	0	4	$-4i$
0	0	0	0

z	$\mathfrak{Re}(z)$	$\mathfrak{Im}(z)$	z^*
$-3 + 4i$	-3	4	$-3 - 4i$
$-7 - 2i$	-7	-2	$-7 + 2i$
$-11i$	0	-11	$11i$
$i\sqrt{3}$	0	$\sqrt{3}$	$-i\sqrt{3}$
$1 - i\sqrt{2}$	1	$-\sqrt{2}$	$1 + i\sqrt{2}$

OPERATIONS WITH COMPLEX NUMBERS

$$(a+bi)+(c+di)=(a+c)+(b+d)i \qquad \textbf{addition}$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i \qquad \textbf{subtraction}$$

$$(a+bi)(c+di)=ac+adi+bci+bdi^2 \qquad \textbf{multiplication}$$

$$\frac{a+bi}{c+di}=\left(\frac{a+bi}{c+di}\right)\left(\frac{c-di}{c-di}\right)=\frac{ac-adi+bci-bdi^2}{c^2+d^2} \qquad \textbf{division}$$

The complex conjugate of the denominator

GDC

- Run Math Mode
- Press Shift + Menu (Do this to Set Up your GDC for the use of complex numbers)
- Browse down (Until you see the "complex Mode" on the screen)
- Press F2 ($a + bi$ button)
- Exit and try

**Note:* the imaginary number symbol, i, is found by pressing shift + 0

EXAMPLES (ANSWER IN EXACT (FRACTION) FORM)

Use technology to calculate:

a $\dfrac{1}{(1+2i)^3}$

b $\dfrac{(2+5i)^4}{1-i}$

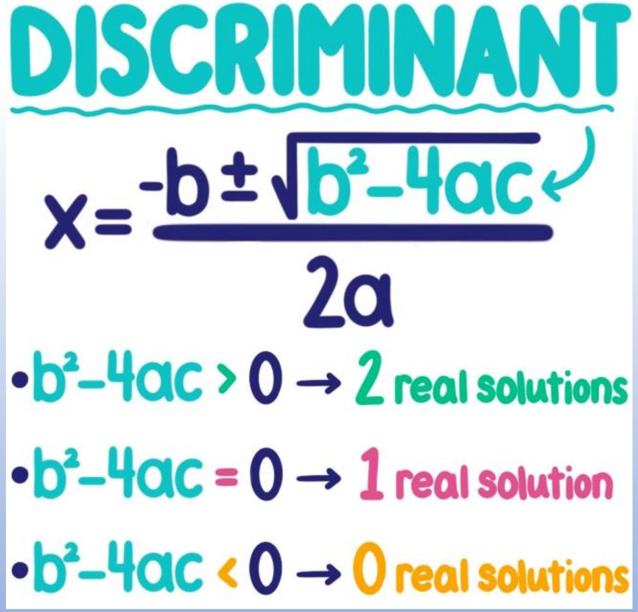
a $\dfrac{1}{(1+2i)^3} = -\dfrac{11}{125} + \dfrac{2}{125}i$

b $\dfrac{(2+5i)^4}{1-i} = \dfrac{881}{2} - \dfrac{799}{2}i$

REAL QUADRATICS WITH DISCRIMINANT < 0

If $ax^2 + bx + c = 0$, $a \neq 0$ and $a, b, c \in \mathbb{R}$, then the solutions or roots are found using the formula

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$ where $\Delta = b^2 - 4ac$ is known as the **discriminant**.



EXAMPLE: SOLVE FOR X: $\boldsymbol{x^2 - 10x + 29 = 0}$

$The\ roots\ are$: $\boldsymbol{5 + 2i\ and\ 5 - 2i}$

What does this mean?

If the discriminant is < 0, the equation has no real solutions.

MORE EXAMPLES

Solve for x using the quadratic formula:

a $x^2 - 10x + 29 = 0$ **b** $x^2 + 6x + 25 = 0$ **c** $x^2 + 14x + 50 = 0$

d $x^2 - 3x + 5 = 0$ **e** $x^2 + 4 = x$ **f** $3x^2 + 6x + 5 = 0$

SEATWORK

- Exam.net key code: NAB562

Use your GDC to answer the questions correctly.