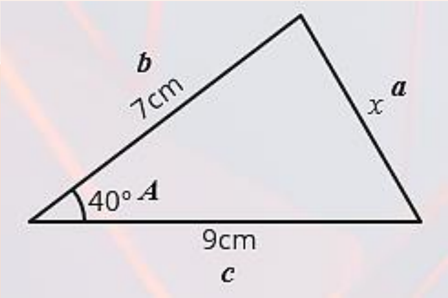
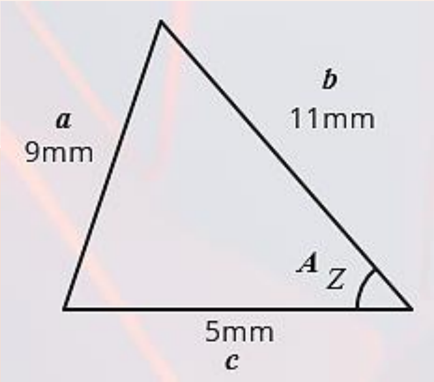
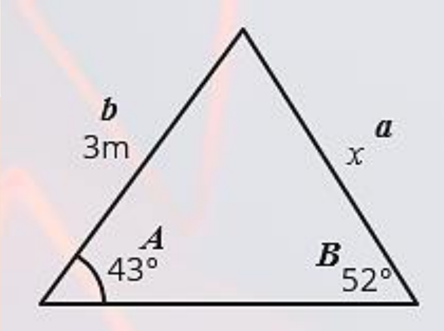
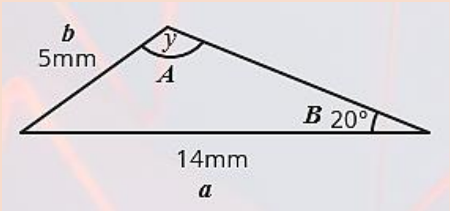
Recall

Cosine Rule		Sine Rule	
Missing Side	**Missing Angle**	**Missing Side**	**Missing Angle**
the size of the angle opposite the missing side and the length of the other two sides are known	the length of all three sides of a triangle are known	the size of the opposite angle and a separate side-angle pair are known	the length of the opposite side and a separate side-angle pair are known Ambiguous Case: When two sides and one angle are known, and the unknown angle is opposite the longer of the two sides
			

TRIANGULATION & AREA OF A TRIANGLE

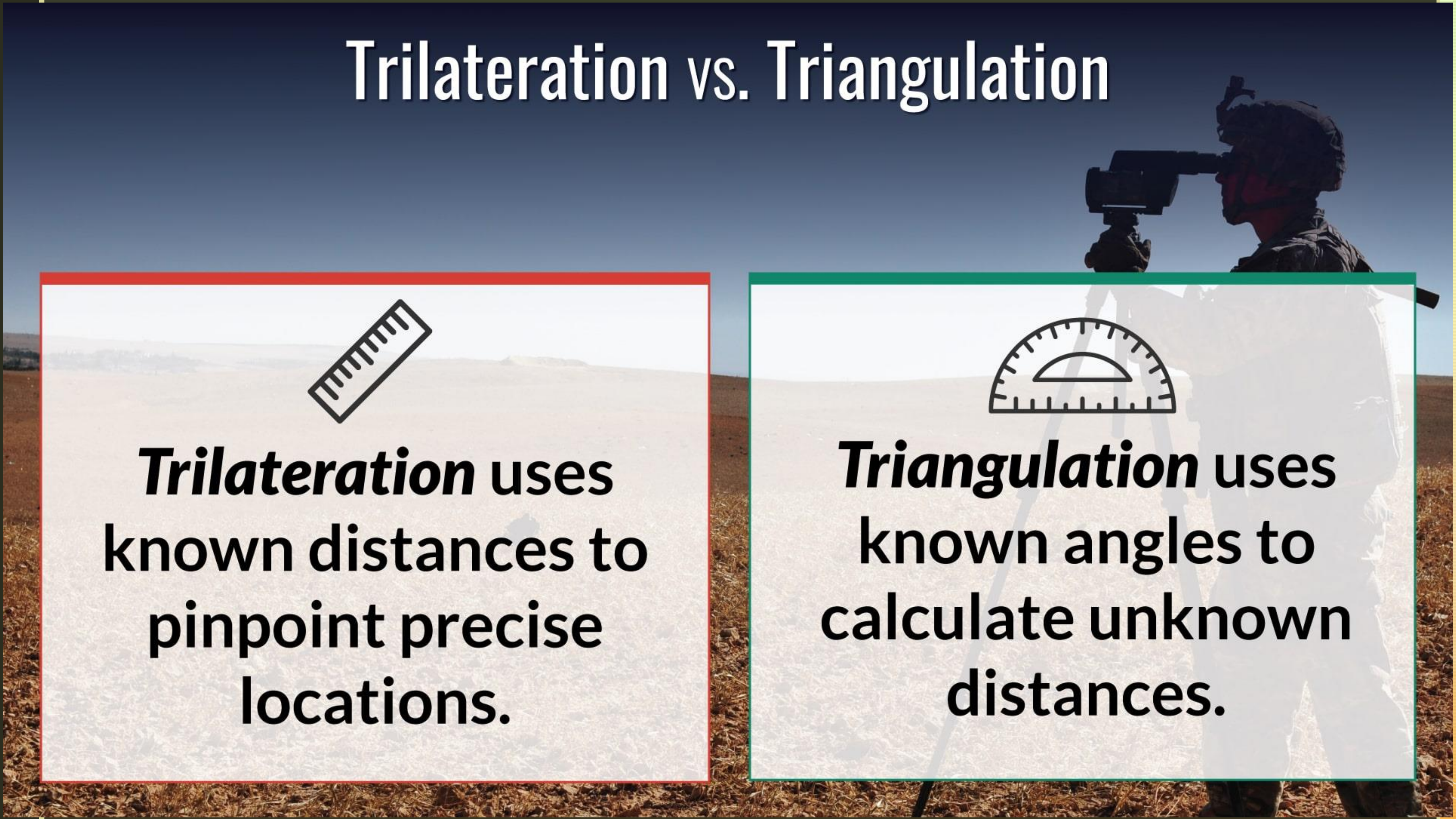
Objective

- To solve problems using triangulation and area of a triangle

Triangulation

- Surveyors extensively use **triangulation** to indirectly calculate large distances.
- By measuring the distance between two landmarks and the angles between those landmarks and a third point, the surveyor can calculate the other two distances in the triangle formed by those points.
- This process can then be repeated to form a chain of triangles.

https://static.vecteezy.com/system/resources/previews/000/133/120/non_2x/engineer-surveyor-vector.jpg



https://spaceplace.nasa.gov/gps/en/

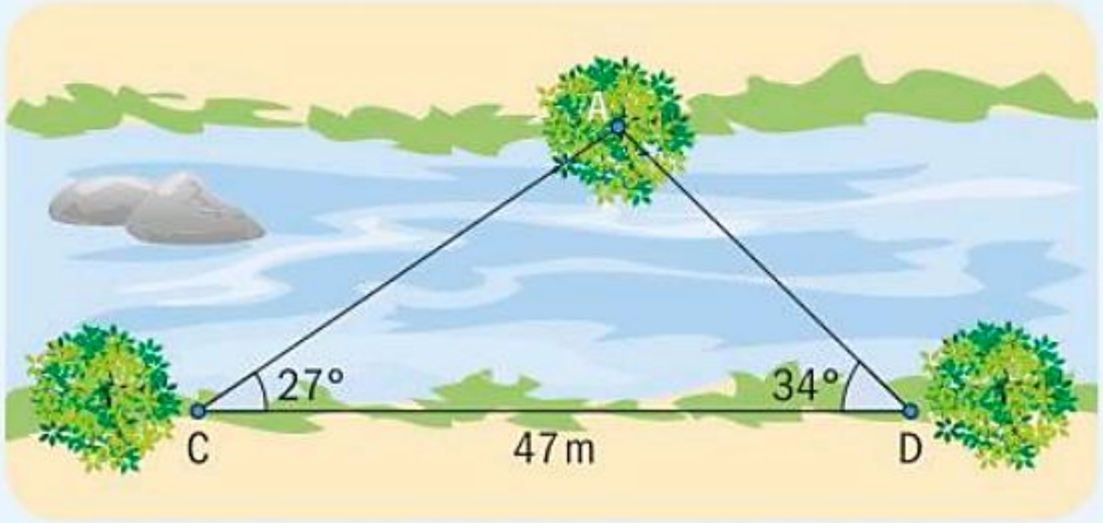
The area of any triangle ABC can be calculated given two sides and the angle between them:

$$\text{area} = \frac{1}{2}ac\sin B$$

1

A surveyor triangulating a region uses a tree at the opposite side of the river as reference point. He measures $A\hat{D}C = 34°$. He walks along the bank of the river on a straight line and measures the distance he walked as DC = 47 m, to the nearest metre. From point C, he measures angle $\hat{C} = 27°$.

Calculate the area covered.



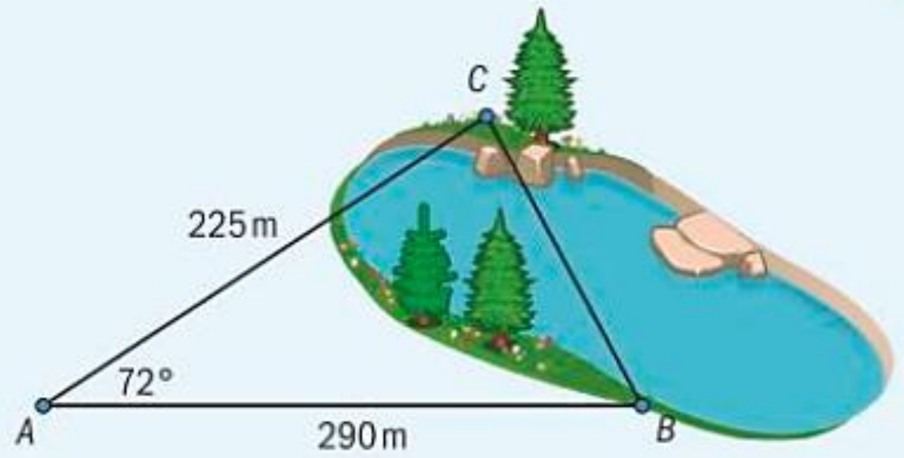
	To find the area we need to find one additional side length; choosing AD.
$\hat{A} = 180° - 34° - 27°$ $\hat{A} = 119°$	Determine the angle opposite the known side.
$\dfrac{AD}{\sin 27°} = \dfrac{47}{\sin 119°}$ $AD = \dfrac{47\sin 27°}{\sin 119°}$ .....(1)	When applying the Sine Rule note the ambiguous case does not apply as a length (not an angle) is being found.
$\text{Area} = \dfrac{1}{2} \times \dfrac{47\sin 27°}{\sin 119°} \times 47\sin 34°$	Do not round the answer at this stage.
Area = 321 m^2 (3 s.f.)	Area $\Delta ACD = \dfrac{1}{2} AD \times DC \times \sin D$; substitute AD from (1).

2

A surveyor of a lake measures AC = 225 m, and AB = 290 m, and $B\hat{A}C = 72°$ as shown in the diagram.

a Find BC.

b Find $\hat{C}$.



a $a^2 = b^2 + c^2 - 2bc\cos A$;

$CB^2 = 290^2 + 225^2 - 2 \times 225 \times 290 \times \cos 72°$

$CB = \sqrt{94398.2...}$

$CB = 307\,\text{m}$ (3 s.f.)

Use the Cosine Rule as two sides and an included angle are given. Substitute the given values for b, c, and angle $\hat{A}$ in the standard form of the rule.

b $\dfrac{\sin C}{290} = \dfrac{\sin 72°}{BC}$

$C = \sin^{-1}\left(\dfrac{290 \sin 72°}{307}\right)$

$= 63.9°$

Either Sine or Cosine Rule can be used as we now have 3 sides and 1 angle. Since the unknown angle is opposite the shorter side, you do not need to consider the Ambiguous Case.

3

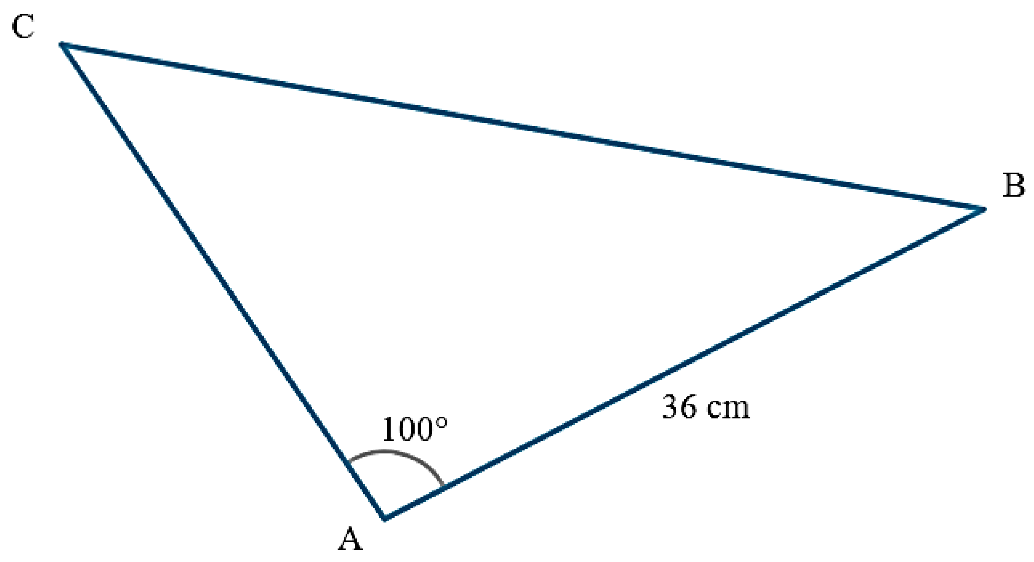


Diagram **NOT** accurately drawn.

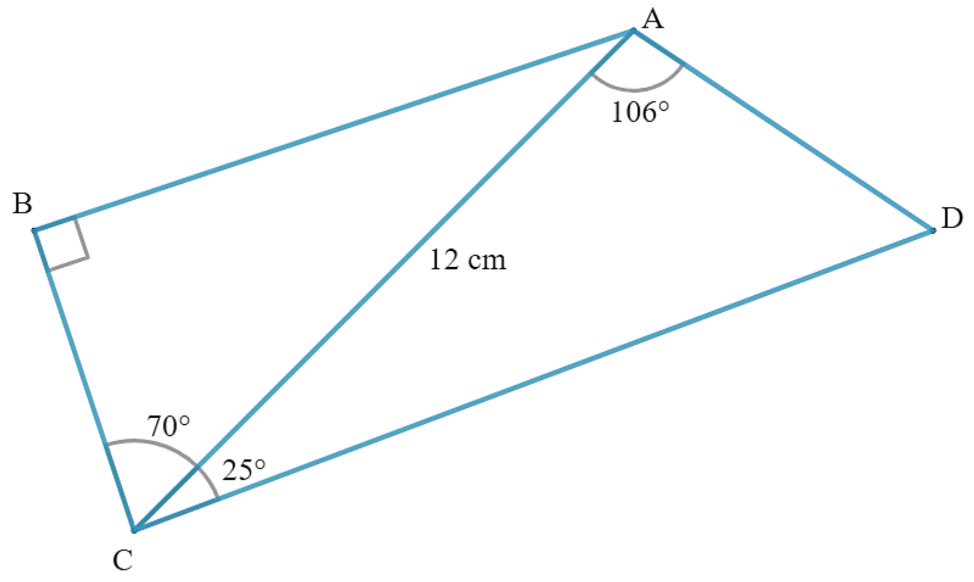
Triangle ABC has area 520 cm^2
Calculate the length of side AC.

$$\frac{1}{2} \times 36 \times AC \times \sin(100) = 520$$

$$AC = 29.3 \text{ cm}$$

4

Diagram **NOT** accurately drawn.



Find the area of the quadrilateral $ABCD$.

Find BC

$$\cos 70^\circ = \frac{BC}{12}$$

$$BC = 4.104$$

Find area of $ABC = \frac{1}{2} 4.104 \times 12 \sin 70 = 23.14$

Find $ADC = 180 - 106 - 25 = 49^\circ$

Find CD

$$\frac{CD}{\sin 106^\circ} = \frac{12}{\sin 49^\circ}$$

$$CD = 15.28$$

Find area of $ACD = \frac{1}{2} 15.28 \times 12 \sin 25 = 38.76$

Area of $ABCD = 38.76 + 23.14 = 61.90\,\text{cm}^2$

References

Oxford MAIHL Textbook

Exam.net