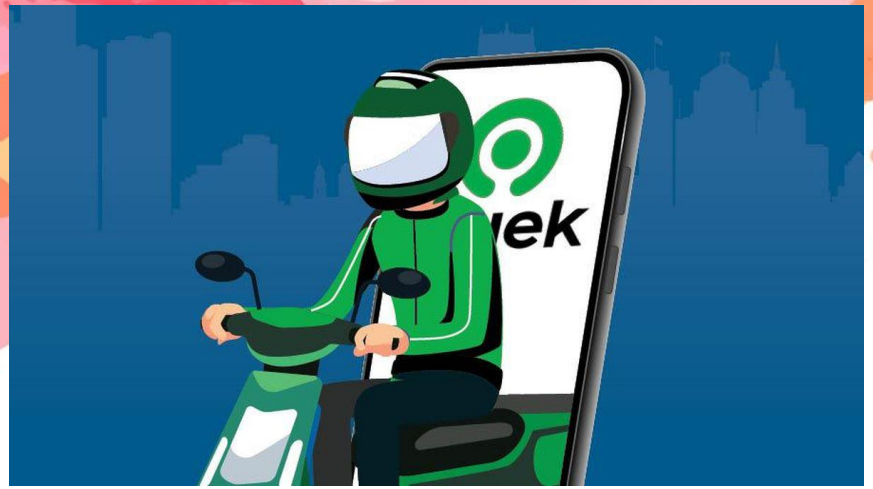


How do these delivery options know what area each restaurant serves?

How do they choose where to locate their restaurants?

Think of similar strategical location problems.

How do you find a point that is equidistant from other three points? Or more?

Voronoi Diagrams

Contents

01

What are Voronoi Diagrams?

Georgy Voronoy (1868-1908)

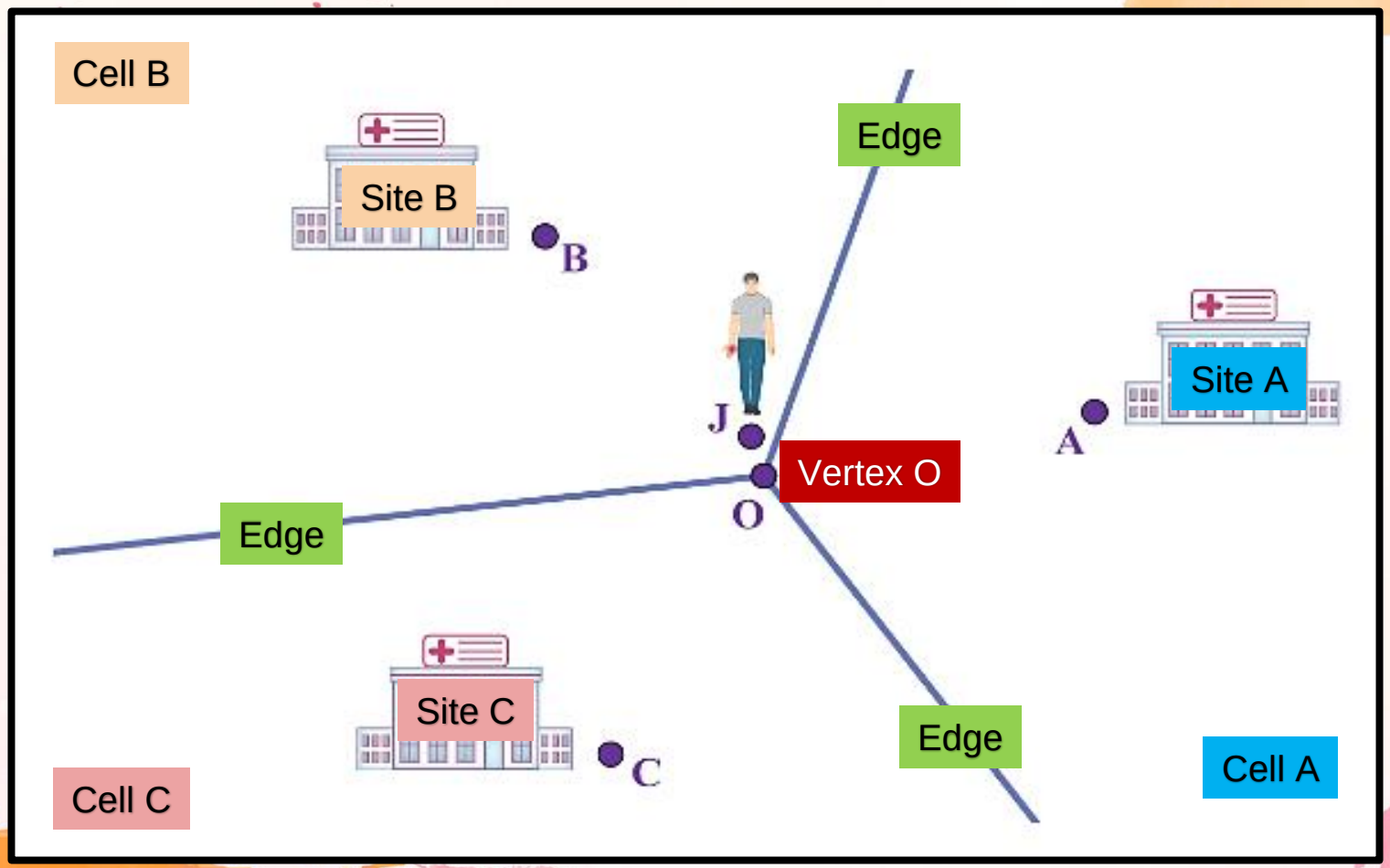
He was a Ukranian mathematician best known for the *Voronoi diagram* which is a partitioning of a plane into regions based on distance to a finite set of points.

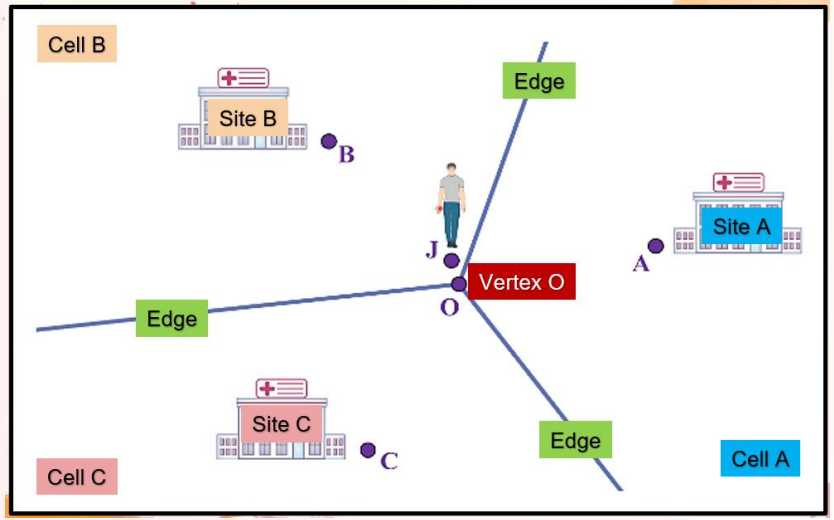
https://mathshistory.st-andrews.ac.uk/Biographies/Voronoy/

Voronoi diagrams are tools for analyzing spaces, locations, and paths with respect to established points, known as sites. They can identify optimum locations and the relationship between positions.

How would you identify the optimum location for a new hospital? A new school? Or a new restaurant?

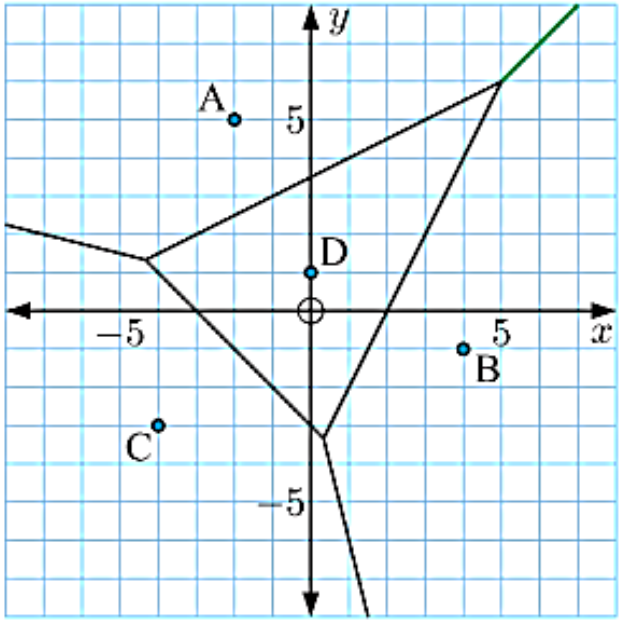
How does analyzing the territories of animals help to preserve wildlife?


Cell B
Site B
B
Edge
J
Site A
A
Vertex O
O
Edge
Site C
C
Edge
Cell A
Cell C



- Important locations are called sites.
- Each site is surrounded by a region called cell, which contains the points which are closer to that site than any other site. The cells are labelled according to the site which they contain.
- The lines which separate the cells are called edges. Each point on an edge is equally closest to the two sites whose cells are adjacent to that edge.
- The point at which the edges meet is called a vertex. Each vertex is equally closest to the sites whose cells meet at that vertex.

Consider this Voronoi diagram for the sites A, B, C, and D.



a How many cells does the diagram contain?

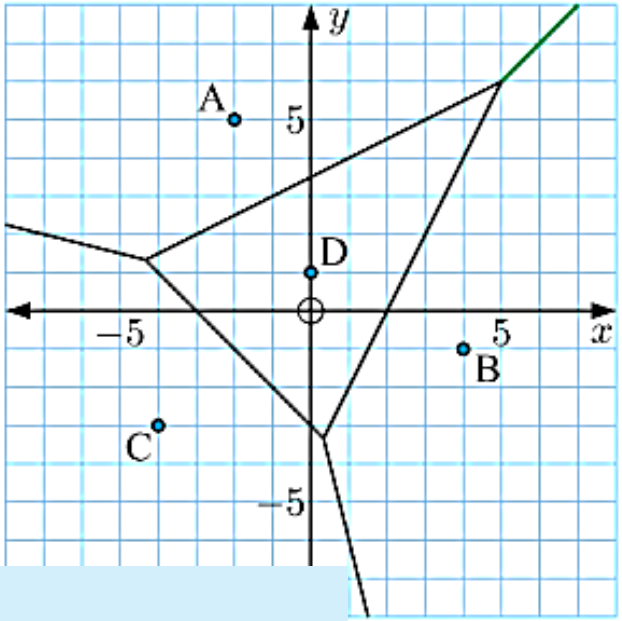
b How many vertices does the diagram contain?

c Identify the site(s) closest to:

i $(2, -3)$ **ii** $(1, 5)$ **iii** $(-1, -2)$

d What can we say about points which lie on the green edge?

Consider this Voronoi diagram for the sites A, B, C, and D.



a How many cells does the diagram contain?

b How many vertices does the diagram contain?

c Identify the site(s) closest to:

i $(2, -3)$ **ii** $(1, 5)$ **iii** $(-1, -2)$

d What can we say about points which lie on the green edge?

a The diagram contains 4 cells, one for each site.

b The diagram contains 3 vertices.

c **i** $(2, -3)$ lies in cell B, so it is closest to site B.

ii $(1, 5)$ lies in cell A, so it is closest to site A.

iii $(-1, -2)$ lies on the edge adjacent to cells C and D, so it is equally closest to sites C and D.

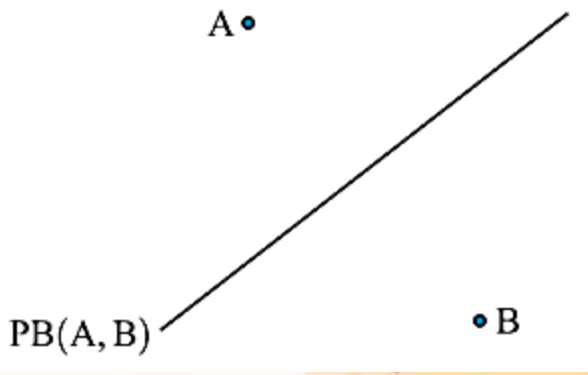
d The green edge is adjacent to cells A and B, so points which lie on this edge are equally closest to sites A and B.

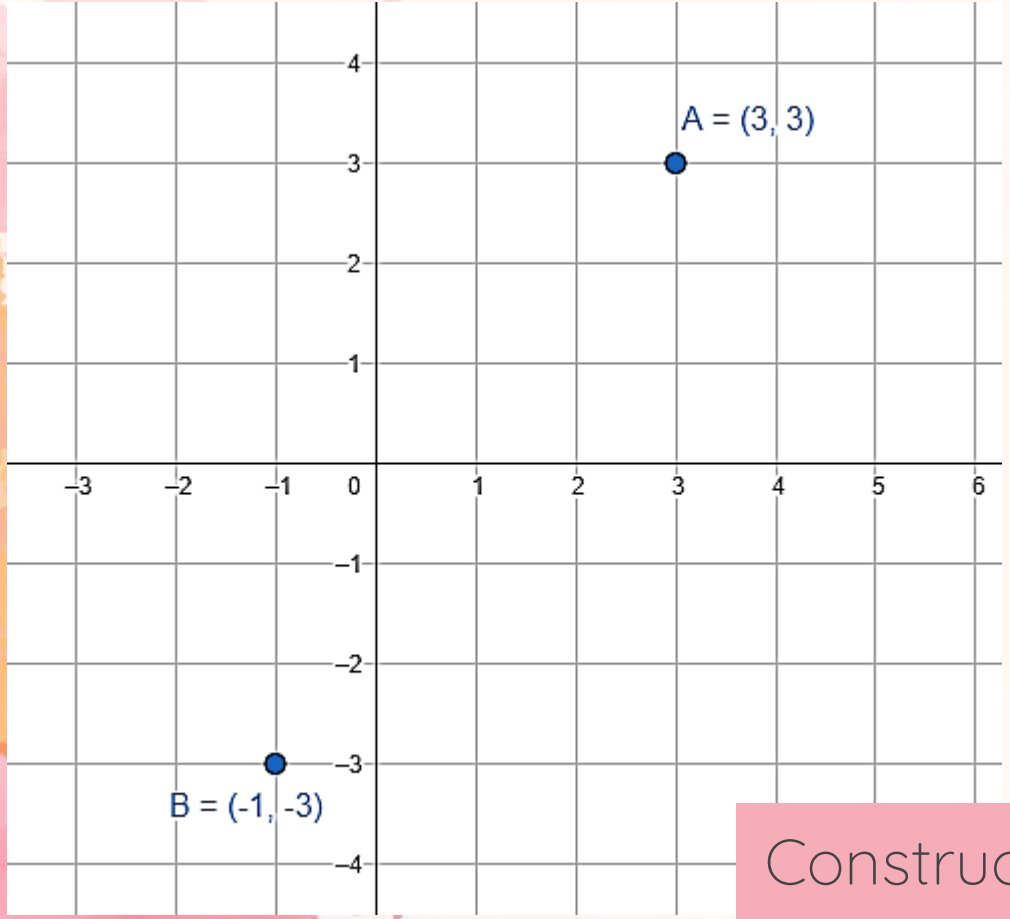
02

Constructing a Voronoi Diagram

Voronoi Diagram with two sites

For a plane with two sites A and B, the Voronoi diagram simply consists of the perpendicular bisector of [AB], which we will call PB(A, B).

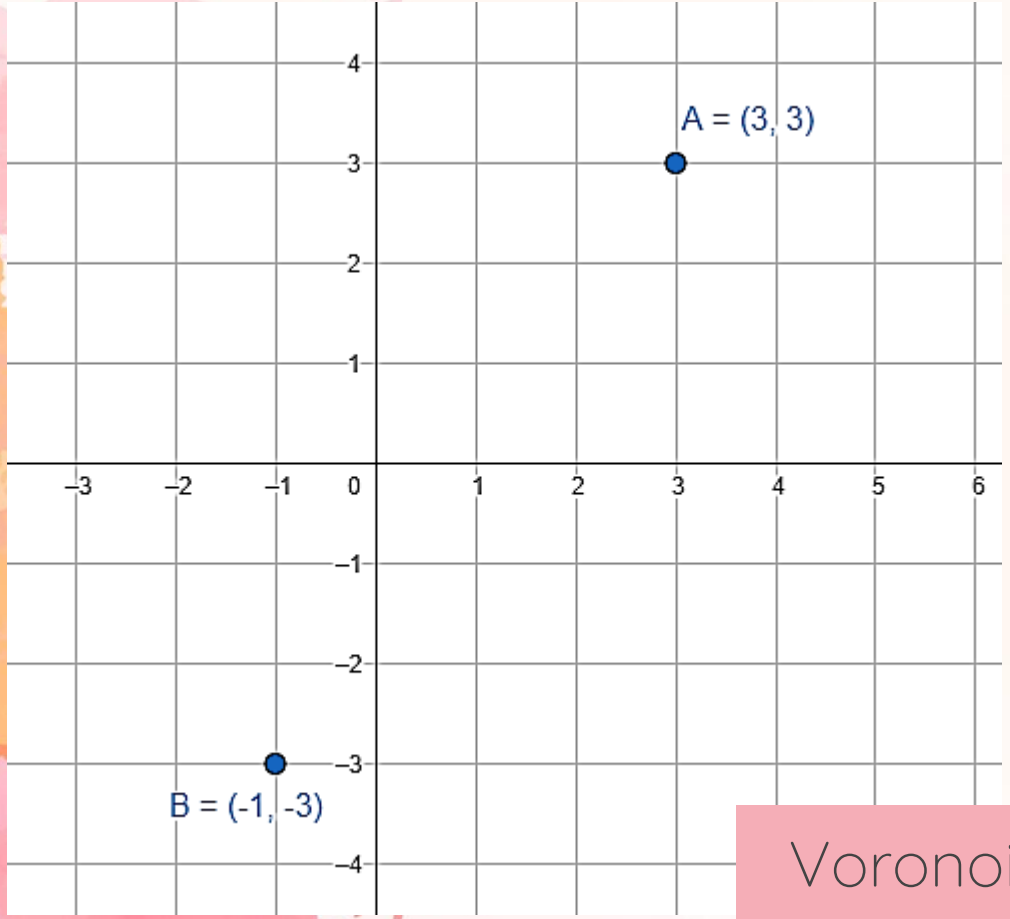




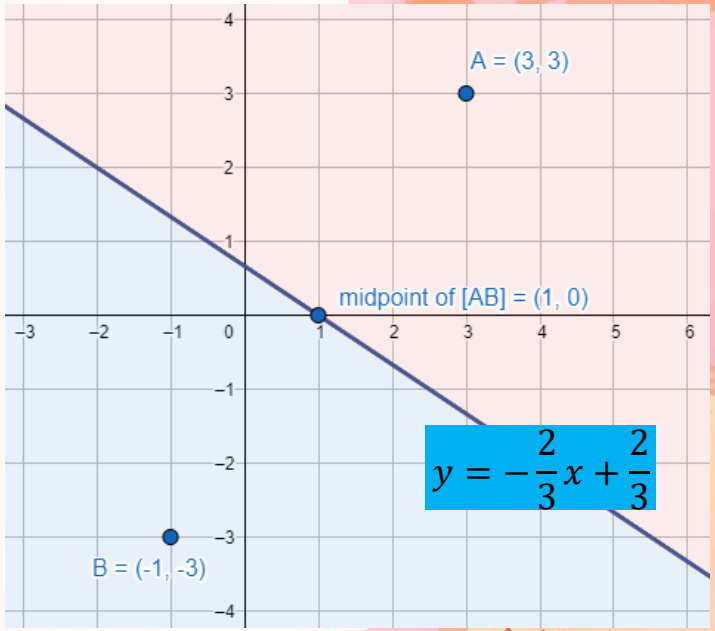
What to do:

- Find the midpoint and gradient of [AB].
- Draw the perpendicular bisector of [AB].

Construct a Voronoi Diagram for the sites A and B



$$gradient\ of\ [AB] = \frac{-3-3}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$$

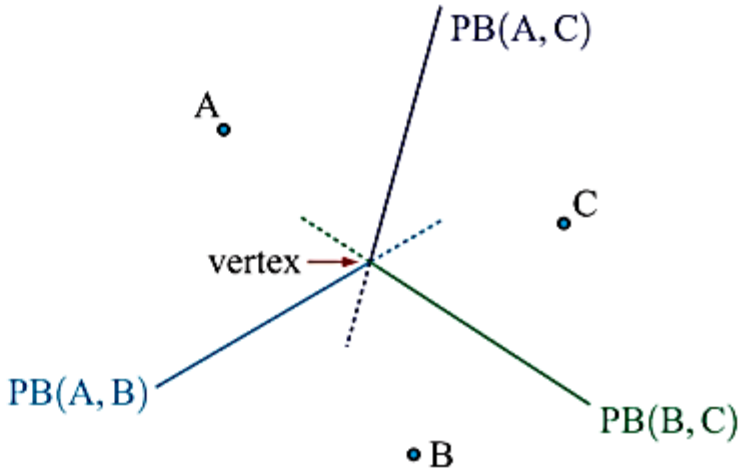


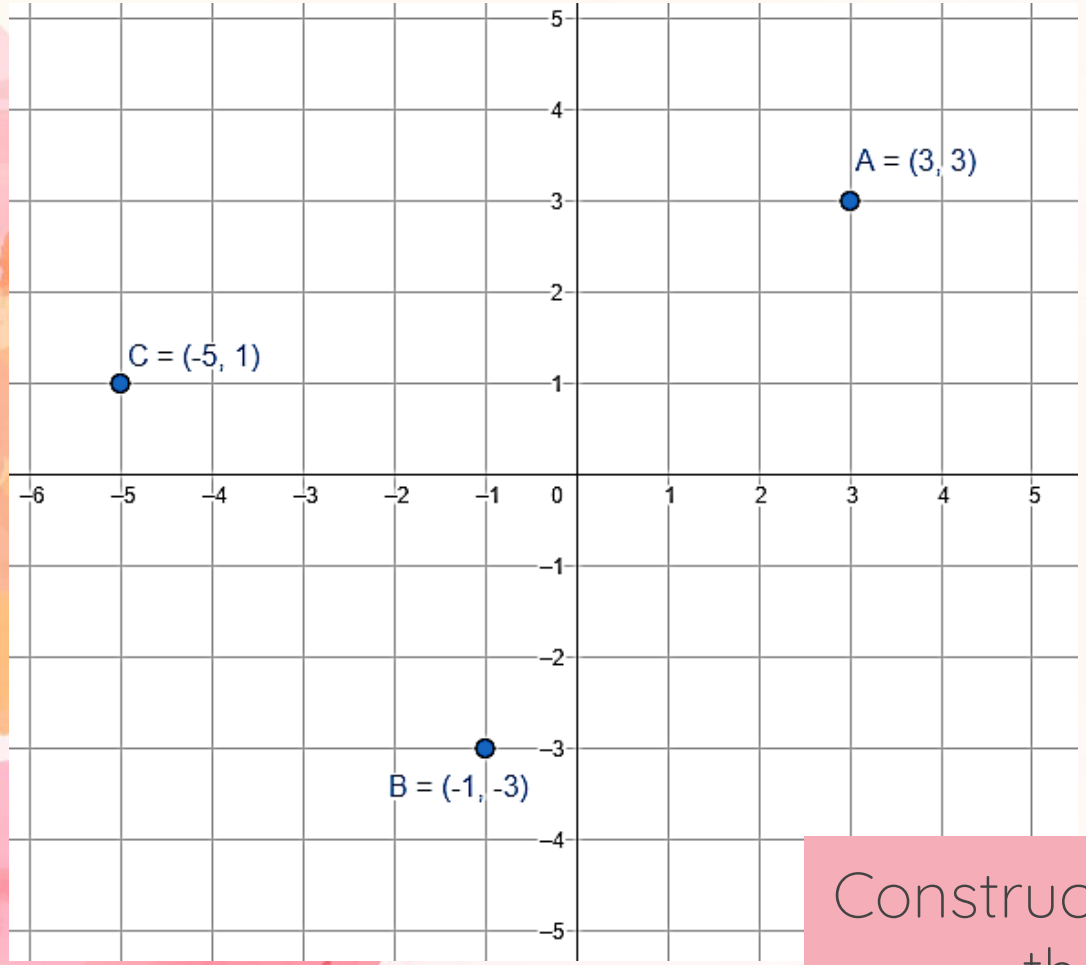
Voronoi Diagram with two sites (A and B)

Voronoi Diagram with three sites

For a plane with three sites A, B, and C, we draw the perpendicular bisectors of [AB], [AC], and [BC]. Provided A, B, and C are not collinear, these lines will meet at a single point equidistant from A, B, and C. This point is a vertex of the Voronoi diagram.

However, notice that only a section of each perpendicular bisector is included as a Voronoi edge. For example, the points on the dotted blue section of PB(A, B) are *not* part of the Voronoi edge. This is because, although they are equidistant from sites A and B, their *closest* site is site C.

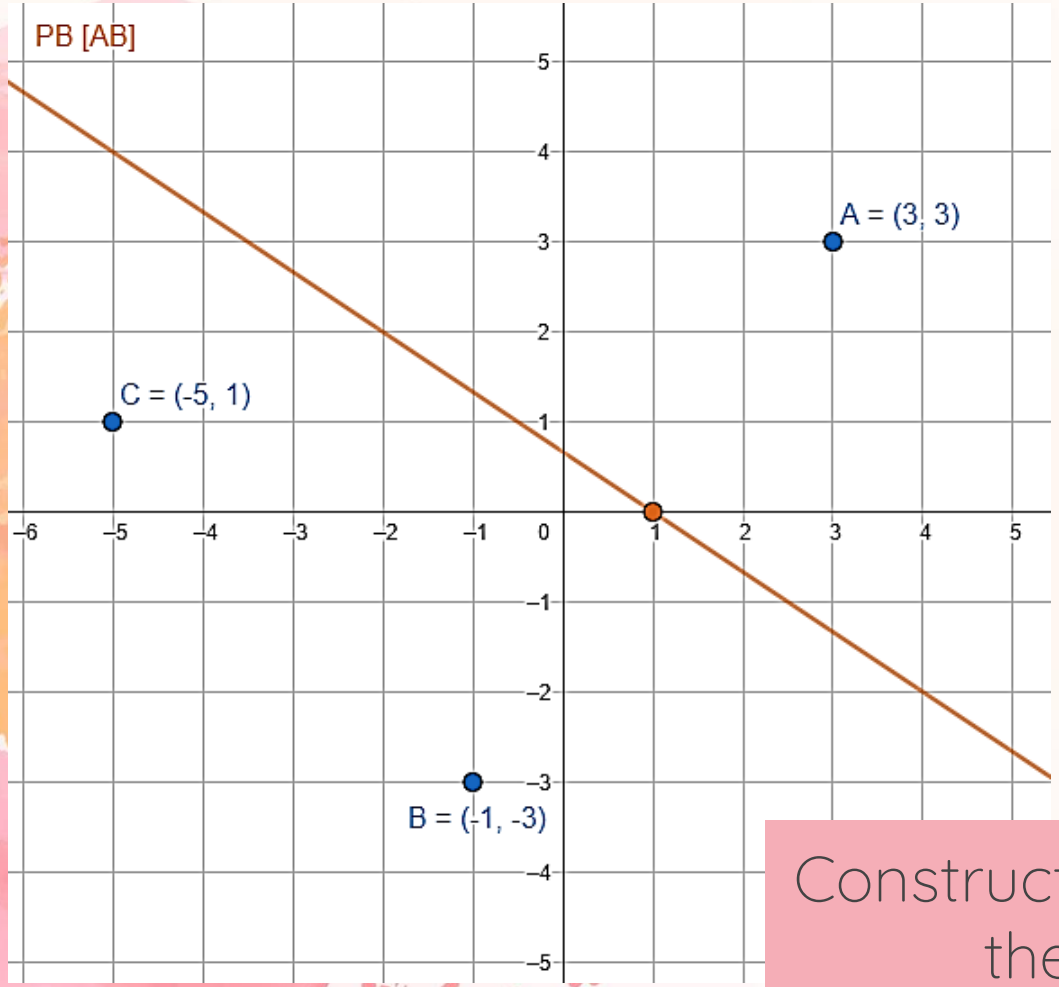




What to do:

- Find the midpoint and gradient of [AB], [BC], and [AC].
- Draw the perpendicular bisectors.

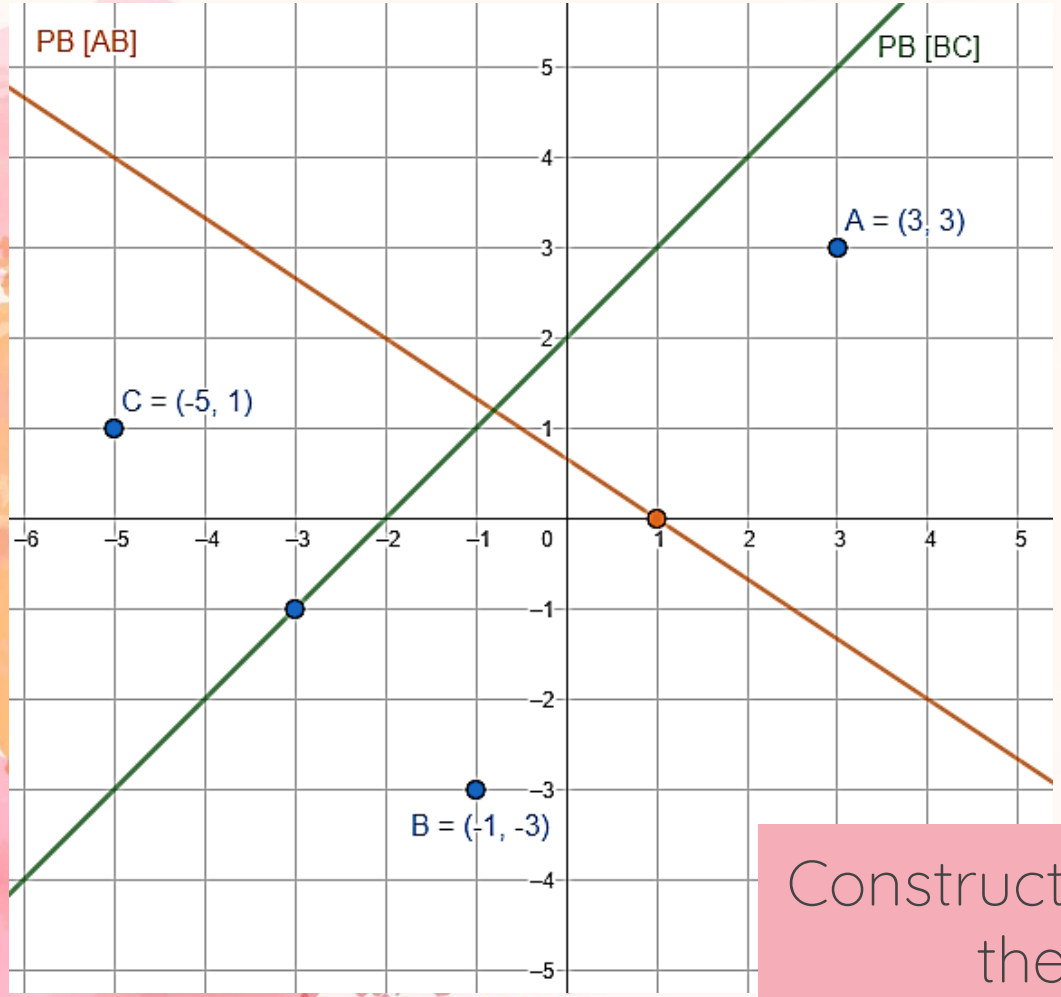
Construct a Voronoi Diagram for the sites A, B, and C.



What to do:

- Find the midpoint and gradient of [AB], [BC], and [AC].
- Draw the perpendicular bisectors.

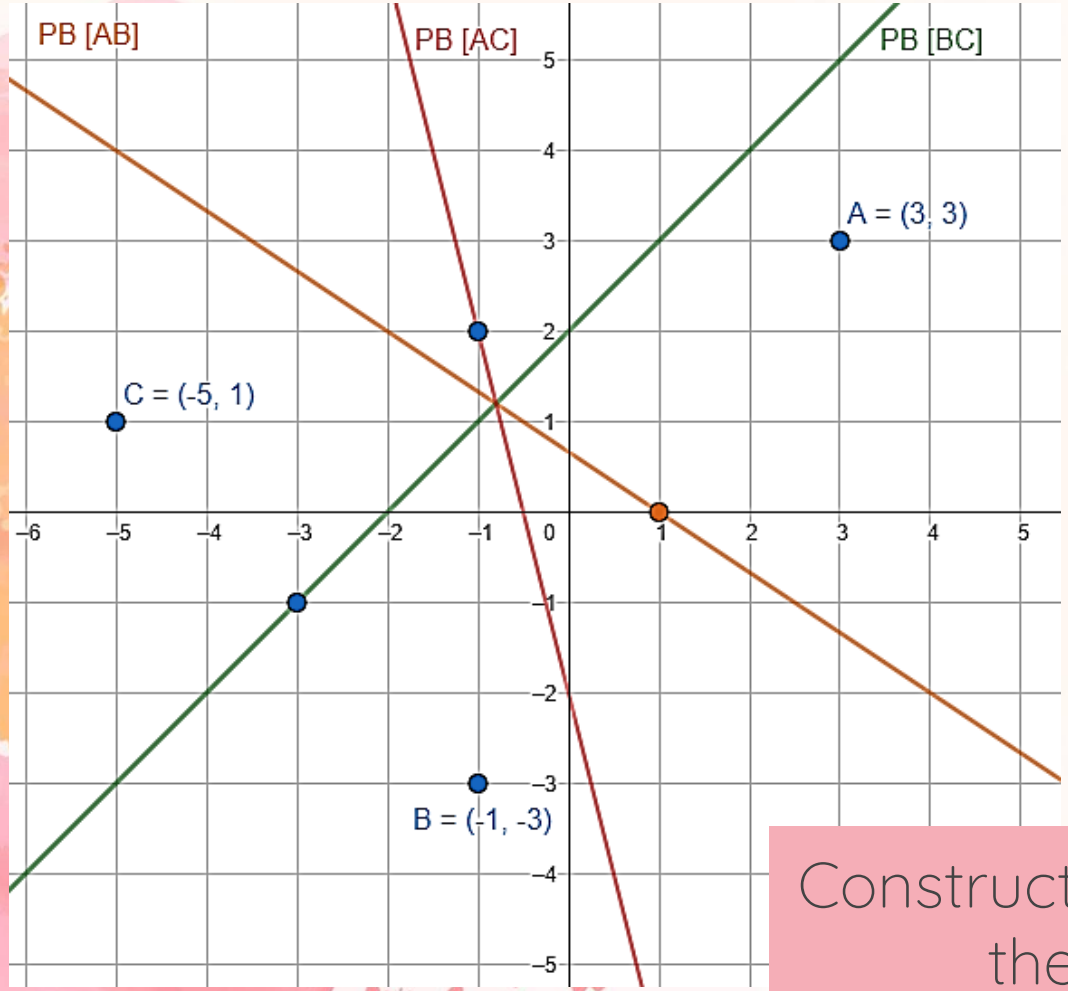
Construct a Voronoi Diagram for the sites A, B, and C.



What to do:

- Find the midpoint and gradient of [AB], [BC], and [AC].
- Draw the perpendicular bisectors.

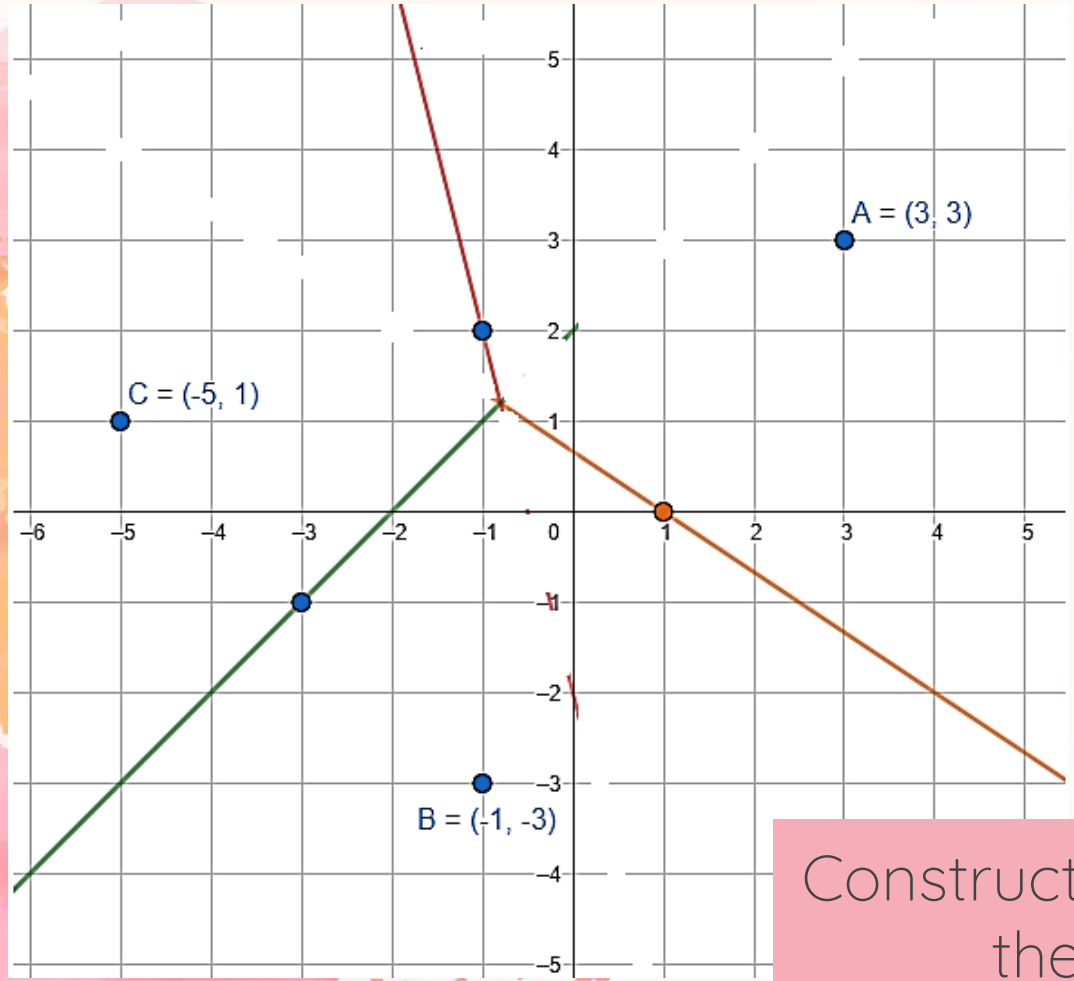
Construct a Voronoi Diagram for the sites A, B, and C.



What to do:

- Find the midpoint and gradient of [AB], [BC], and [AC].
- Draw the perpendicular bisectors.

Construct a Voronoi Diagram for the sites A, B, and C.



HINT

Normally a final version of the Voronoi diagram will have the perpendicular bisectors removed so only the edges of the regions remain as in the diagram shown.

Construct a Voronoi Diagram for the sites A, B, and C.