Session 1

May 6, 2024, Monday
8:15 – 8:45 AM

Today…

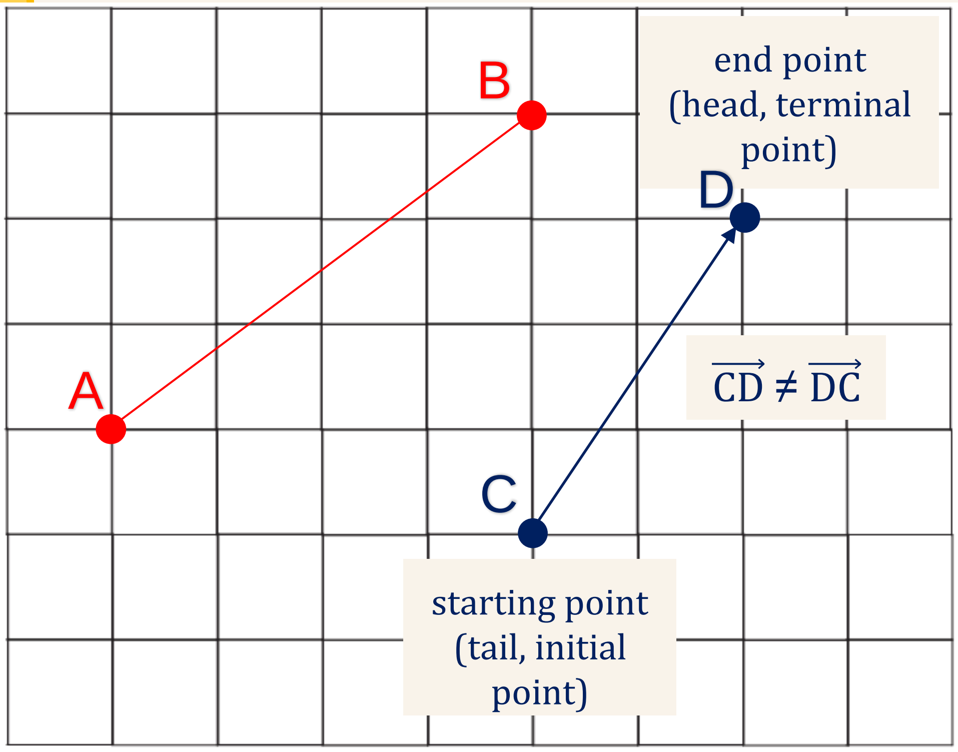
…you will be able to:

- define and explain terms related to vectors
- represent vectors geometrically
- add vectors

Introduction to Vectors

(Geometric construction
and operations)

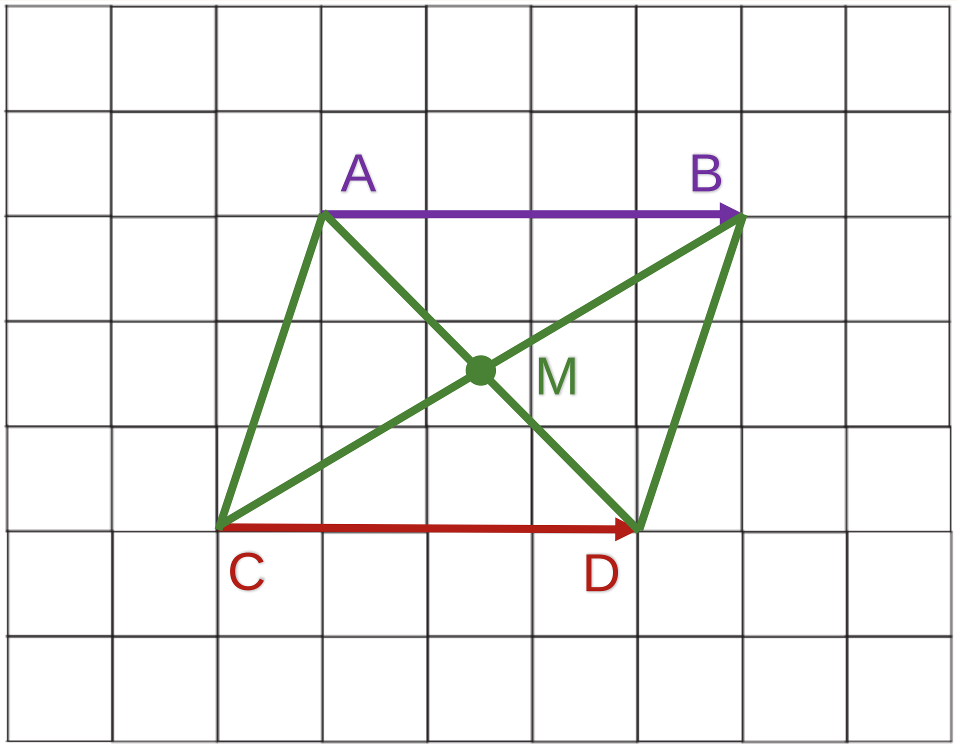
Let's draw!



line segment AB =
line segment BA

1. Line segment AB
2. **Directed** line segment CD

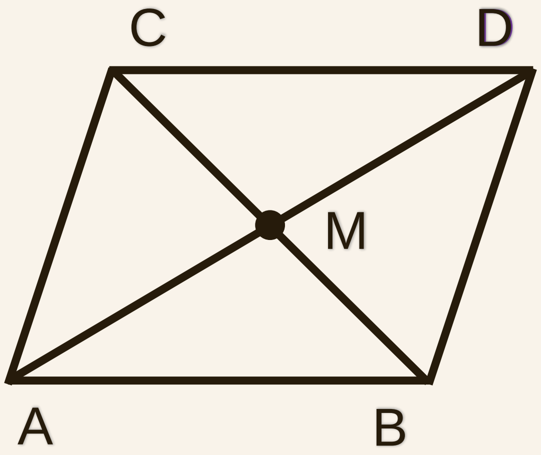
Equivalent Directed Line Segments



Is $\overrightarrow{AB}$ equivalent to $\overrightarrow{CD}$?

$\boldsymbol{Yes, \overrightarrow{AB} = \overrightarrow{CD}}$

Equivalent Directed Line Segments

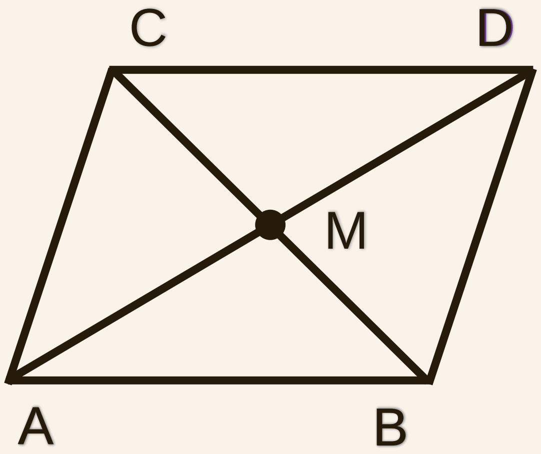


Is $\overrightarrow{AB}$ equivalent to $\overrightarrow{CD}$?

$\boldsymbol{Yes, \overrightarrow{AB} = \overrightarrow{CD}}$

They have the same direction and length.

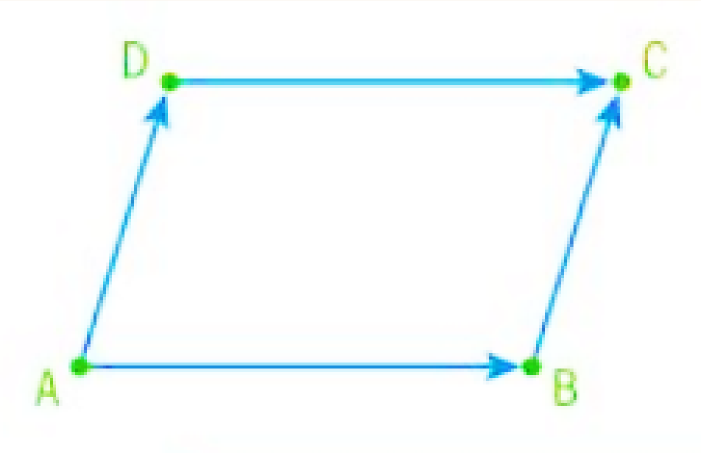
Equivalent Directed Line Segments



Is $\overrightarrow{AC}$ equivalent to $\overrightarrow{DB}$?

$\overrightarrow{AC}$ and $\overrightarrow{DB}$ are not equivalent.

They do not have the same direction.



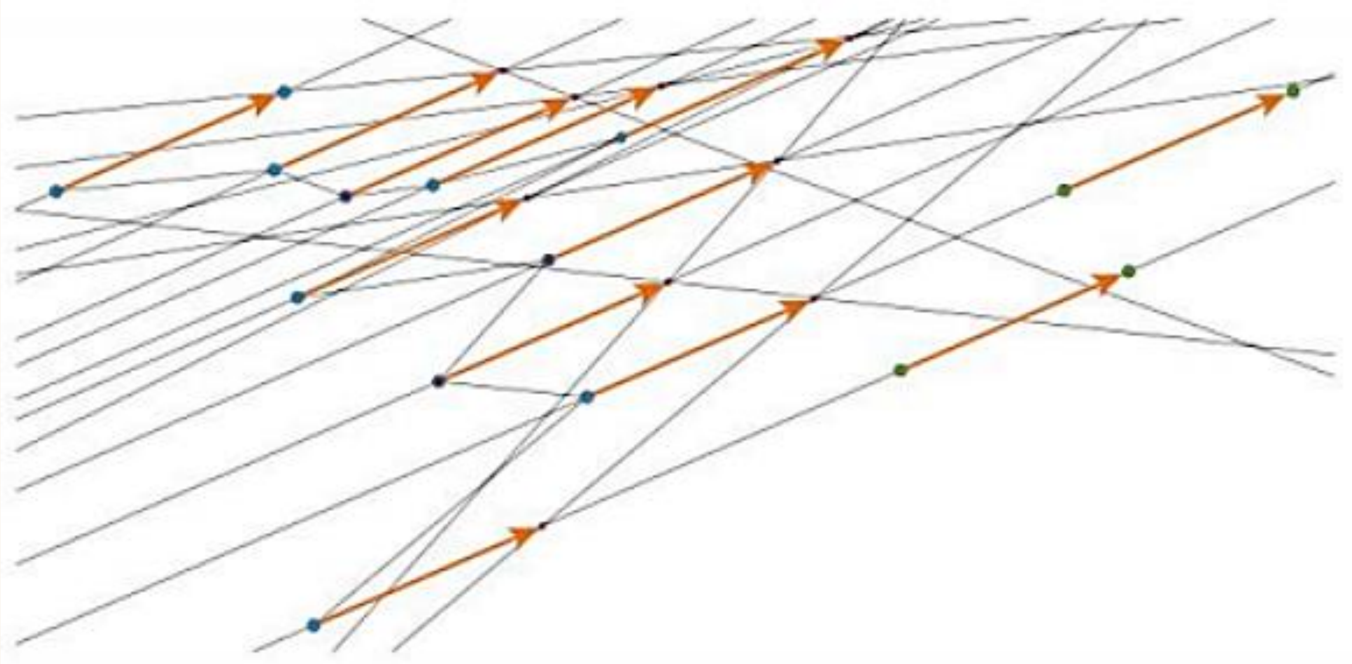
Name an equivalent directed line segment to $\overrightarrow{AB}$.

$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{DC}}$

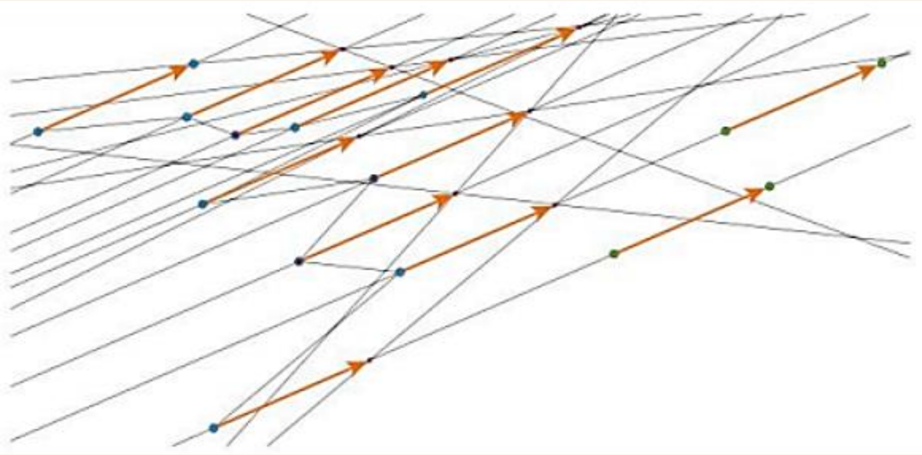
Name an equivalent directed line segment to $\overrightarrow{AD}$.

$\overrightarrow{\mathbf{AD}} = \overrightarrow{\mathbf{BC}}$

The figure shows equivalent directed line segments that represent the same vector.



What is a vector?



- It is a quantity that has magnitude (distance, length, size) and direction (right, left, north, etc.)

- It is represented by a small bold letter or a small letter with an arrow above ($\boldsymbol{a}$ or $\vec{a}$).

Vector or Not?

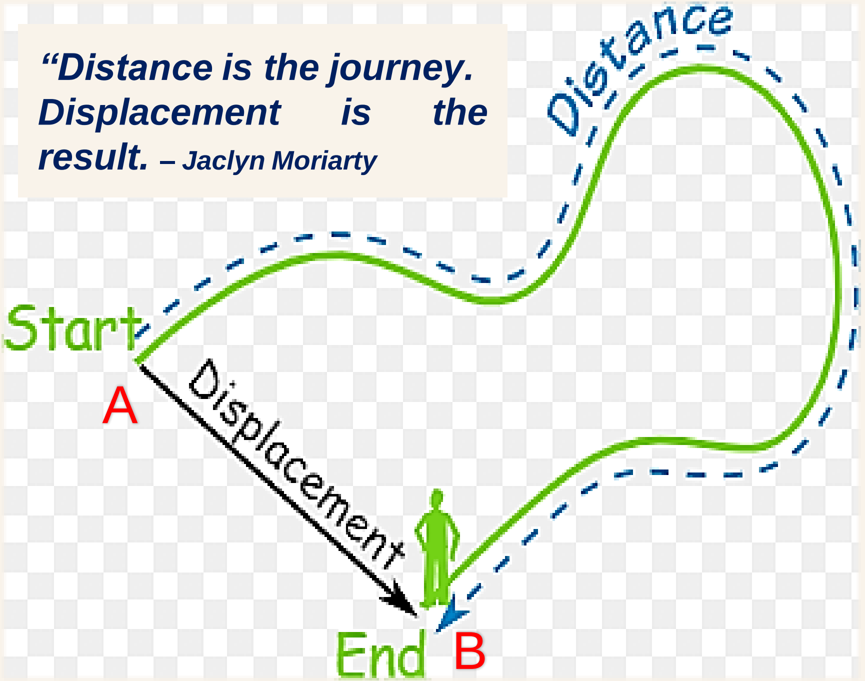
1) The football player was running 10 miles an hour towards the end zone.

Scalar

2) The volume of that box is 14 cubic feet.
3) The temperature of the room was 15 degrees Celsius.

4) The car accelerated north at a rate of 4 meters per second squared.

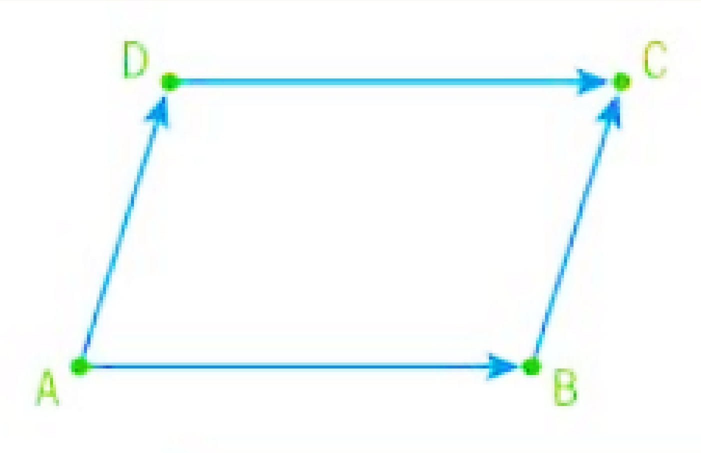
Displacement Vector



- This is special type of vector that is represented by particular directed line segments.

- It is represented by AB or $\overrightarrow{AB}$, where A is the initial or starting point and B is the terminal or end point.

Opposite Vectors

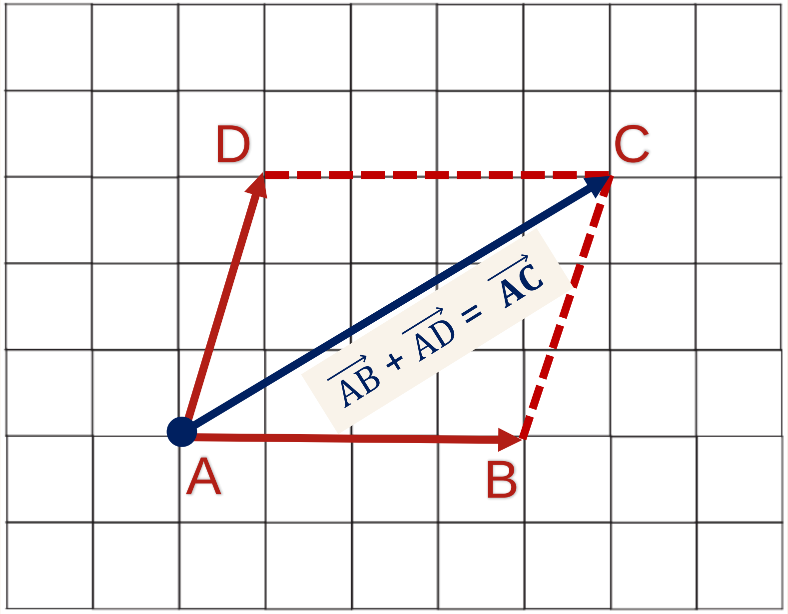


What can you say about the magnitude and direction of $\overrightarrow{AB}$ and $\overrightarrow{BA}$?

They have the same magnitude but different direction.
They are called opposite vectors.
$\overrightarrow{\mathbf{AB}} = -\overrightarrow{\mathbf{BA}}$

Addition of Vectors

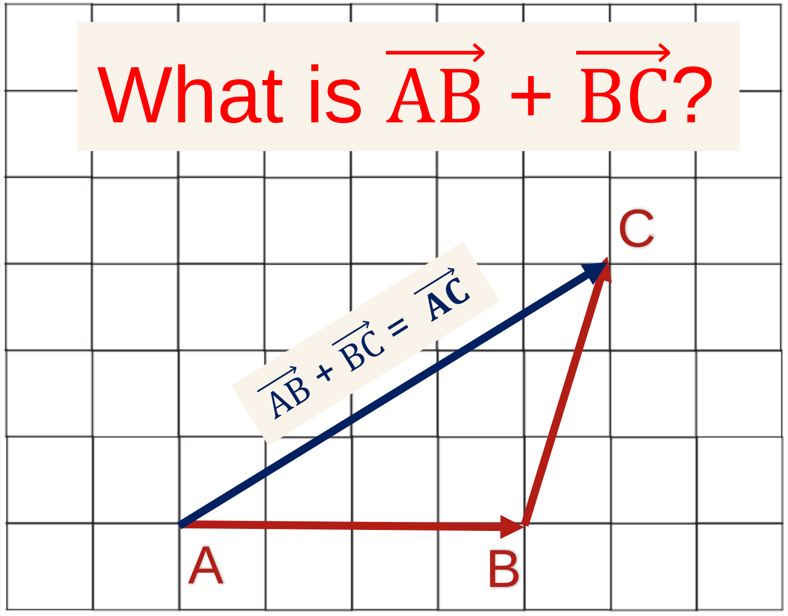
Parallelogram Law of Vector Addition



What is $\overrightarrow{AB} + \overrightarrow{AD}$?

1. Place both vectors at the same initial point.
2. Complete the parallelogram.
3. The diagonal of the parallelogram is the resultant vector.

Triangle Law of Vector Addition



1. Place the vectors with the head of the first vector connected to the tail of the last vector.
2. The resultant vector is formed by connecting the tail of the first vector to the head of the last vector.

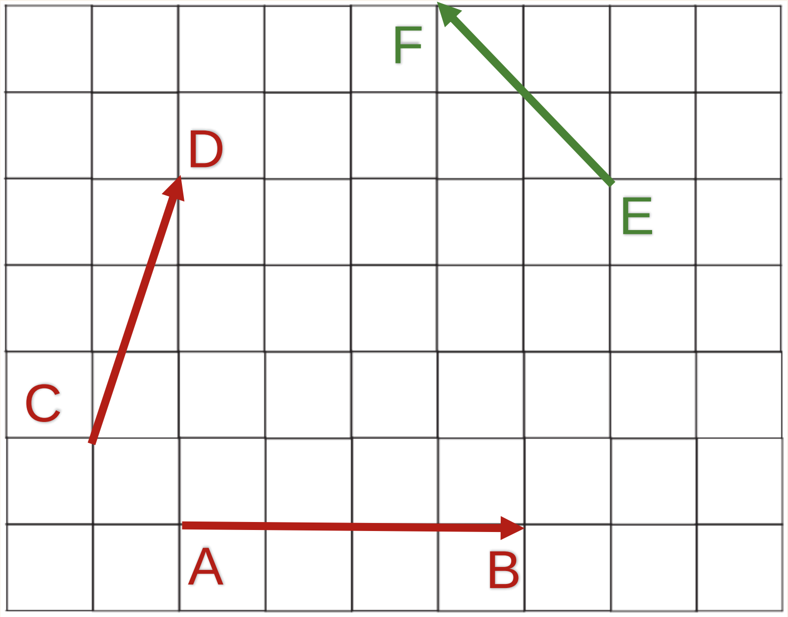
From the above we can derive the **triangle law of vector addition.**

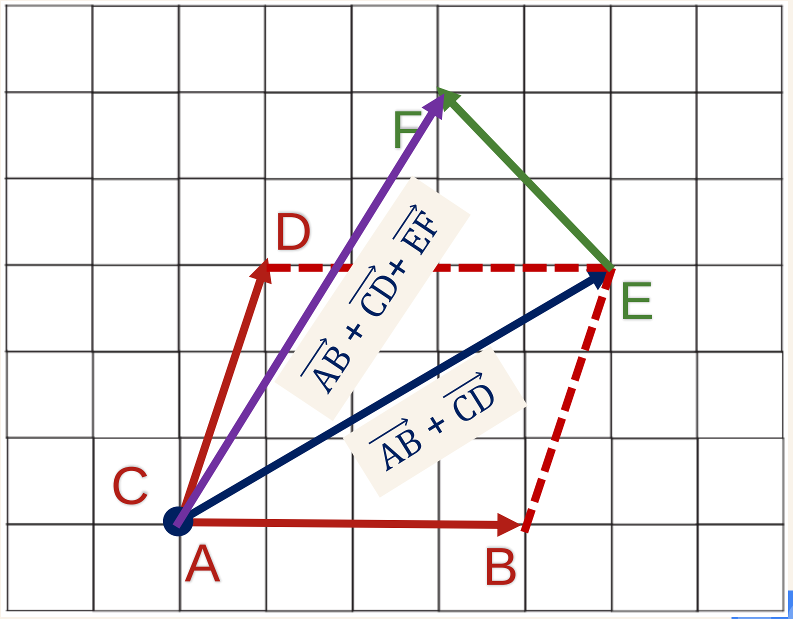
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

This can be extended to any number of vectors. A consequence of the law is that the sum of two of more displacement vectors is always equal to the final displacement and is independent of the route taken.

Let's Do This!

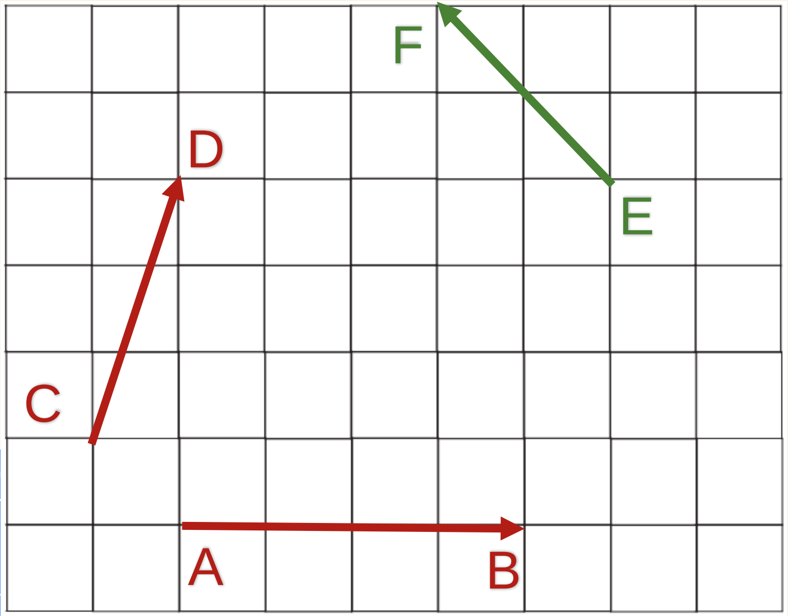
Given the diagram, use either parallelogram law or triangle law to draw the vector $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF}$.

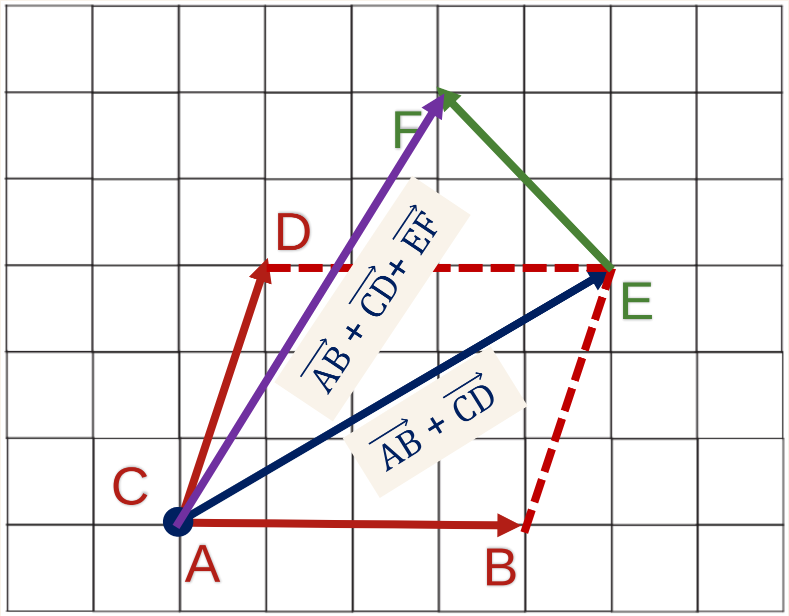




Therefore,

$$\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF}$$

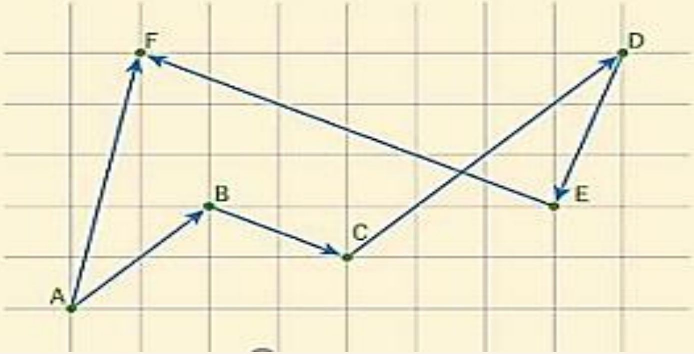




Try This!

Use either parallelogram law or triangle law to show that:

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{AF}$$



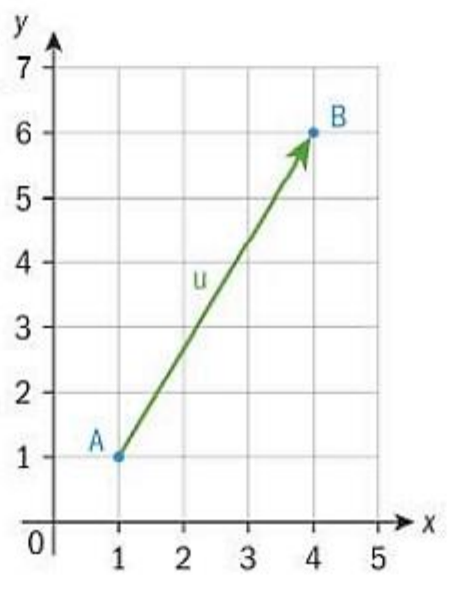
Session 2

9:15 – 9:45 AM

Now…

…you will be able to:

- write vectors as **column** vectors
- write vectors using the **base** vectors
- **add** vectors algebraically



Because the vector goes from A to B we can write it as $\overrightarrow{AB}$. Alternatively, we can give it a name such as $\boldsymbol{u}$. In print a vector is named using bold font, when handwritten it is written as $\underset{\sim}{u}$ or $\bar{u}$.

Vectors are normally described in **component** form. The vector shown can be written as a column vector $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ or, using the **base** vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ as $\overrightarrow{AB} = 3\boldsymbol{i} + 5\boldsymbol{j}$. In each case the first number, or component, indicates movement in the x-direction and the second movement in the y-direction.

The vector $\overrightarrow{BA} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ because to move from B to A you need to go 3 units to the left and 5 down.

It will always be the case that the vector $\overrightarrow{AB} = -\overrightarrow{BA}$

Addition of Vectors

Two vectors are added by adding the corresponding components.

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

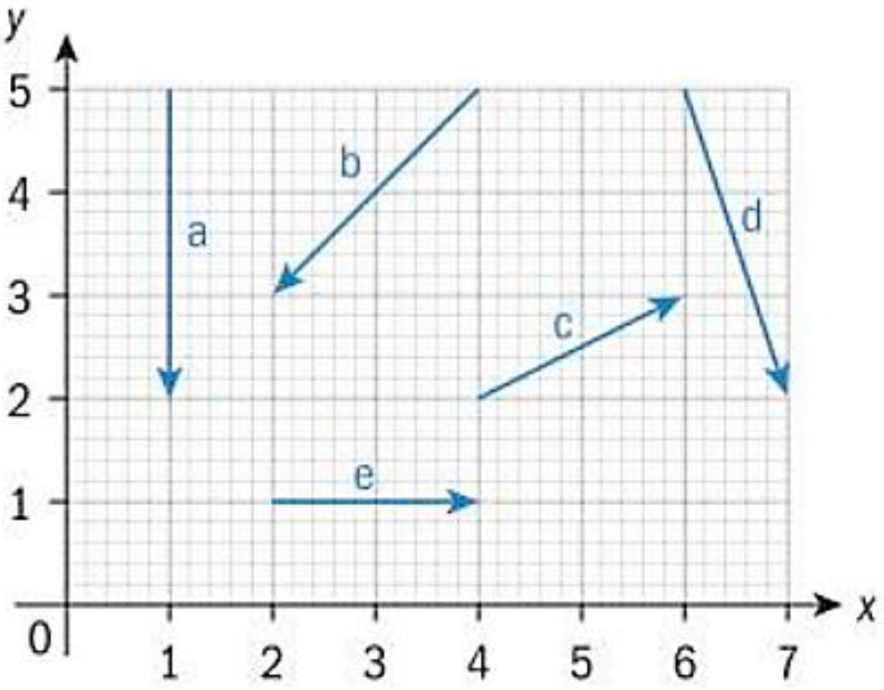
EXAM HINT

The choice of which notation to use may depend on the context but in an exam, both are equally valid.

Try!

Answer in your notebook.

1 Write the following vectors as column vectors and using ***i*** and ***j*** notation.



2 Find:

a $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

b $(3\boldsymbol{i} - \boldsymbol{j}) + (4\boldsymbol{i} + 5\boldsymbol{j})$

c $\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

d $(\boldsymbol{i} - 2\boldsymbol{j}) + 4\boldsymbol{i}$

3 Find

a $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

b $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

c Explain how you multiply a vector by a scalar.

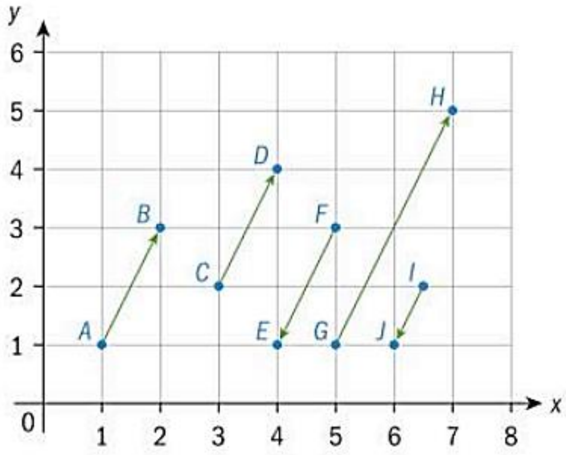
d $\boldsymbol{i}$ and $\boldsymbol{j}$ can also be written as

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Hence verify $3\boldsymbol{i} + 4\boldsymbol{j} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

→ In 2-D space, $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and O(0, 0).

4 Let $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ as shown.



a Write the following vectors in component form and in terms of the vector $\overrightarrow{AB}$.

i $\overrightarrow{CD}$ **ii** $\overrightarrow{FE}$

iii $\overrightarrow{GH}$ **iv** $\overrightarrow{IJ}$

b Comment on what can be deduced about parallel vectors.

5 State which of the following vectors are parallel to $5\boldsymbol{i} + 2\boldsymbol{j}$.

a $-5\boldsymbol{i} - 2\boldsymbol{j}$

b $25\boldsymbol{i} - 10\boldsymbol{j}$

c $-\boldsymbol{i} - 0.4\boldsymbol{j}$

d $\begin{pmatrix} 20 \\ 50 \end{pmatrix}$

e $\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

f $2\begin{pmatrix} -10 \\ 4 \end{pmatrix}$

g $2\begin{pmatrix} 3 \\ 3 \end{pmatrix} - 3\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

6 **a** Find p and q if

i $\begin{pmatrix} 4p \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ -2q \end{pmatrix} = \begin{pmatrix} -2p \\ 2 \end{pmatrix}$

ii $\begin{pmatrix} 3p \\ -2q \end{pmatrix} + \begin{pmatrix} 2q \\ p \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

b **i** Find p if $\begin{pmatrix} p+1 \\ 2p \end{pmatrix}$ is parallel to $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

ii Find q if $\begin{pmatrix} 2q-3 \\ q+6 \end{pmatrix}$ is parallel to $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel if and only if $\boldsymbol{b} = k\boldsymbol{a}$, where k is a scalar.

Reference

Oxford MAIHL, p. 104

THANK YOU!

Do you have any questions?
+62895-3841-52070