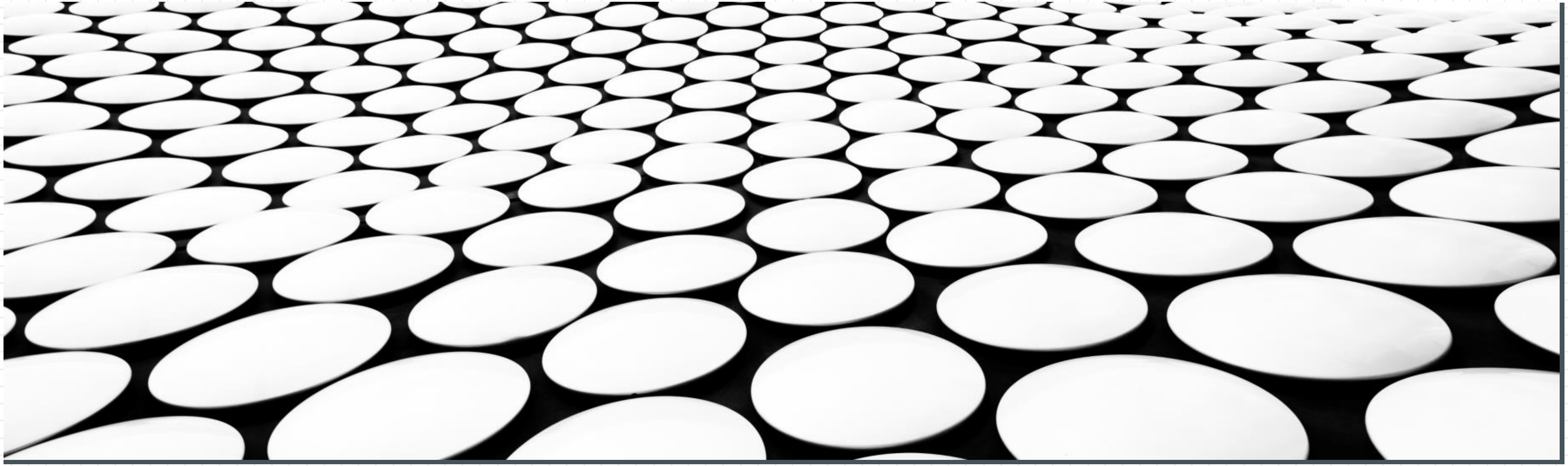
Function Notations

WEEK 3 – AUGUST 7 TO 11, 2023



Function Notation

Example:

$$f(x) = \underbrace{3x + 1}_{\text{output (range)}}$$

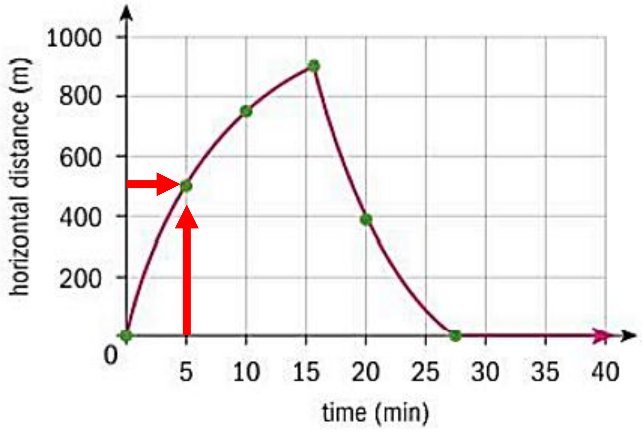
name of function
(f of x)

Input
(domain)

You can use **function notation** to concisely communicate a function's output for a given input by writing $y = f(x)$ or $f(x) = y$. You also say that y is the **image** of x under the function f.

Example

x	y
0	0
5	520
10	790
15	945
20	390
25	45



- From the given information, what distance corresponds to 5 minutes?
- How can we write the answer as a function notation?
- How can you see this from the graph?

Example 2

Raquel invests \$1200 in a savings account whose value increases over time. The future value, V, of the account is a function of the time t (in years) invested, represented by the equation $V(t) = 1200 \times (1.03)^t$ for $0 \le t \le 50$.

a Find

i $V(0)$ **ii** $V(50)$

Interpret each of these in context.

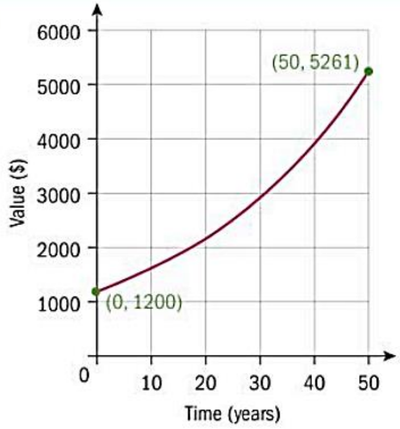
b If Raquel keeps her money invested for 50 years, determine how much she will earn on her initial \$1200.

c Sketch a graph of the function V for $0 \le t \le 50$.

d If Raquel invests her money in 2015, determine the year when the value of her account will reach \$2500.

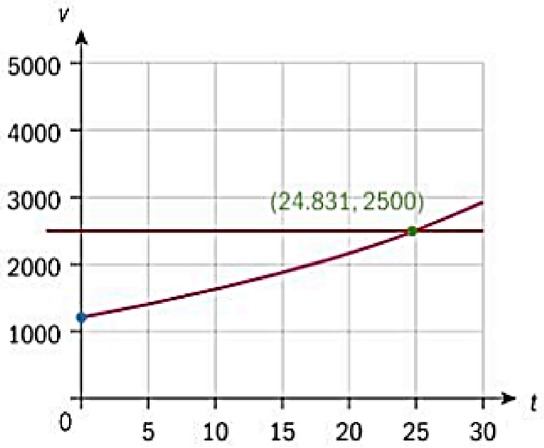
a **i** $V(0) = \$1200$ This shows that Raquel invested $1200 initially. **ii** $V(50) = \$5260$ (3 s.f.) This shows that Raquel will have $5260 in the account after 50 years. **b** $4060	Substitute the given inputs ($t = 0$ and $t = 50$) into the function. $V(0) = 1200 \times (1.03)^0 = 1200$ $V(50) = 1200 \times (1.03)^{50} = 5260.687\ldots$ Alternatively, graph $y = 1200 \times (1.03)^x$ and use the TRACE or TABLE function of your GDC to find the y-values corresponding to $x = 0$ and $x = 50$. Find $V(50) - V(0)$.

c



Graph the function $y = 1200 \times (1.03)^x$ with technology.

Choose an appropriate graphing window: since the given set of input values is $0 \le t \le 50$, set the x-axis to display from 0 to 50. From part **a**, the output grows to approximately $5300, so set the y-axis to display from 0 to 6000.

d $V(t) = 2500$	As you want to find the time when the value is \$2500, you are solving $V(t) = 2500$.
	Graph $y = 2500$ on the same screen as $y = 1200 \times (1.03)^x$. Use technology to find the intersection of these two graphs.
$t = 24.8$	
Raquel's account will reach a value of \$2500 during the year 2039.	Since $t = 0$ corresponds to the year 2015, Raquel's account value will reach \$2500 during the 24th year after 2015, or 2039.

Linear Functions

Objective

- Explore linear functions and the real-world situations they model

Linear Function $f(x) = mx + c$

- Where m and c are constants, represents a context with a **constant rate of change.**
- The constants in an equation are called **parameters.**
- A linear function has parameters gradient (m) and y-intercept (c).

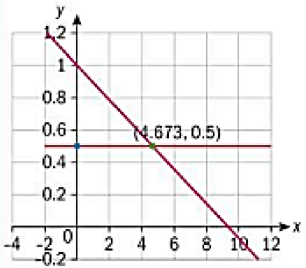
If you have travelled between lower and higher altitudes, you may have noticed that the air pressure changes. Air pressure at sea level (0 km) is defined as 1 atmosphere (atm). At an altitude of 5000 feet, or 1.524 km, above sea level, air pressure is 83.7% of the pressure at sea level, or 0.837 atm. Assume that the relationship between air pressure and altitude is linear.

a Find an equation to express air pressure P (in atm) as a function of altitude a (in km).

b Interpret the gradient and y-intercept of $P(a)$ in context.

c If $(k, 0.5)$ is a point on the graph of $P(a)$, find the value of k and interpret its meaning in context.

Oxford MAIHL, p.156

a $P(a) = 1 - 0.107a$	Use the given pairs of independent and dependent variables to calculate the gradient of the linear function: (0, 1) and (1.524, 0.837). The y-intercept in this case is given as one of the coordinates. So, $c = 1$ and $m = \dfrac{0.837-1}{1.524-0} = -0.107$.
b As the altitude increases, the air pressure reduces at a rate of 0.107 atm per kilometre. The atmospheric pressure at ground level (0 km) is 1 atm.	The gradient is the rate of change between the dependent variable (pressure in atm) and independent variable (altitude in km). The y-intercept occurs at $a = 0$.
c $k = 4.67$ The air pressure will be 50% of the pressure at sea level at an altitude of 4.67 km.	Method 1: Use technology to find the intersection of the function with the line $y = 0.5$:  Method 2: Substitute $P(a) = 0.5$ in the equation and solve for a: $0.5 = -0.107a + 1$ $a = \dfrac{-0.5}{-0.107} = 4.67$

Recall that there are three common ways to represent a linear equation:

- Gradient-intercept form: $y = mx + c$
- Point-gradient form: $y - y_1 = m(x - x_1)$, where $(x_1,\ y_1)$ is any point on the graph.
- Standard or general form: $ax + by + d = 0$, where a, b and d are constants.

The gradient-intercept form translates directly to the linear function $f(x) = mx + c$, but the other two forms can be useful starting points depending on the information given in the problem.

A water tank drains at a constant rate. It contains 930 litres of water 3.5 minutes after it starts to drain. It takes 50 minutes for the tank to empty. Let W be the amount of water in the tank (in litres) t minutes after it started to drain.

a Find a gradient–intercept model for $W(t)$, the amount of water in the tank with respect to time.

b Write down the amount of water in the tank when it starts to drain.

c Write down the rate at which the water tank is emptying.

d Use your model to find the amount of water after 30 minutes.

International-mindedness

The word "modeling" is derived from the Latin word "modellus", which means a human way of dealing with reality

a $W(t) = -20t + 1000$	If the tank drains at a constant rate then it is a linear function: $W(t) = mt + c$ From the information given you can create the ordered pairs (3.5, 930) and (50,0). Method 1: As you have two points, use these to calculate the gradient: $m = \frac{0-930}{50-3.5} = -20$ Substitute the gradient and one point into the point-gradient form: $y - 0 = -20(x - 50)$ Convert to gradient-intercept form. Method 2: Substituting each point into the equation $W(t) = mt + c$ gives: $\begin{aligned} 3.5m + c &= 930 \\ 50m + c &= 0 \end{aligned}$ Use technology to solve these simultaneous equations. $m = -20,\ c = 1000$

b 1000 litres	When it starts to drain, $t = 0$. This is the y-intercept of the function.
c The tank drains at 20 litres per minute.	The rate at which the tank drains (the rate of change) is given by the gradient, m.
d 400 litres	Substitute $t = 30$ into the formula: $W(30) = -20 \times 30 + 1000 = 400$

Siria reads her English textbook at a pace of 2 minutes per page and her Biology textbook at 3 minutes per page. She has two hours available to read.

a Write an equation that shows the relationship between the number of pages of English (x) and of Biology (y) that Siria can read in this time. Define all the variables.

b **i** Find the x- and y-intercepts of the graph of your equation.

ii Use these to sketch a graph of the equation.

iii Interpret each intercept in the context of the problem.

c Siria ends up reading 45 pages in total. Determine how many pages of English and of Biology she read.

a x = pages of English read

y = pages of Biology read

$2x + 3y = 120$

Siria will take $2x$ minutes to read x pages of English and $3y$ minutes to read y pages of Biology. These two times will add up to 2 hours (120 minutes).

b **i** y-intercept: $(0, 40)$

x-intercept: $(60, 0)$

To find the y-intercept, substitute $x = 0$ and solve for y. Similarly, for the x-intercept substitute $y = 0$ and solve for x.

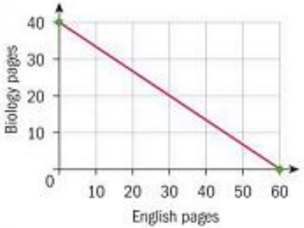
Note: The equation can also be converted to gradient-intercept form:

$$2x + 3y = 120$$

$$3y = 120 - 2x$$

$$y = 40 - \frac{2}{3}x$$

ii



Label bvoth axes.

Plot both intercepts and connect them with a line.

iii If Siria reads only English then she can read 60 pages in two hours. If she reads only Biology, she can read 40 pages.

c Siria read 15 pages of English and 30 pages of Biology.

Because the total number of pages must be 45, you have a second equation:

$$x + y = 45$$

Solving with technology,

$$\begin{cases} 2x + 3y = 120 \\ x + y = 45 \end{cases}$$

has the solution $(15, 30)$

International-mindedness

The development of functions bridged many countries including France (René Descartes), Germany (Gottfried Wilhelm Leibniz), and Switzerland (Leonhard Euler).

More Examples

1.

A function $T(d)$ gives the average daily temperature T, in °C, of a certain city during last January, where $d = 1$ corresponds to January 1st.

a Explain the meaning of $T(2) = 25$.

b State the domain of the function T.

More Examples

2.

Robbin is a mountain guide planning a two-week camping trip that will serve between three and eight clients. She knows that the total weight per person, w kg, of food that must be carried for p people (not including Robbin) can be calculated by the function

$$w(p) = 9 + \frac{15}{p+1}.$$

a Write down the reasonable domain for $w(p)$.

b Find and interpret $w(3)$ and $w(8)$.

c Find the associated reasonable range for $w(p)$.

d The company Robbin works for advertises that clients can expect to carry 11–12 kg of food each. Find the number of people Robbin can take on the trip in order to meet this requirement.

3. The value of Renee's car, in UK£, is changing as a function of time. This relationship can be expressed by the equation $V(t) = \dfrac{8500}{t+1} + 5(t-15)^2$, where t is the time in years since she purchased the car in 2008, $0 \le t \le 60$.

a Using the equation or a graphical representation of the function, find the values of a and b in the table. If more than one value is possible, list all possible values.

t	$V(t)$
0	a
1	b
3	2845
c	4000

b Find the year in which the car's value will drop below UK£1000.

c If Renee keeps her car long enough, it will become an antique and will begin to increase in value. Find the year in which her car's value will be as high as when she bought it.