Probability

Agenda

Unit Outline

Concept development

Notes and examples

Practice

Homework

Lesson	IB Syllabus	Topic	Date
1	SL4.5 SL4.6	Practice with Simple probability Probability Rules Conditional Probability & Independence	October 21
2	SL4.7	Discrete Probability Distributions Expected Value Applications	October 23
3	AHL4.14	Combining Random Variables	October 25
4	SL4.8	Binomial Distribution	October 28
5	AHL4.17	Poisson Distribution	October 30
		Quiz 3	November 1
6	SL4.9	Normal Distribution	November 4
7	SL4.9 AHL4.15	Normal Distribution Central Limit Theorem	November 6

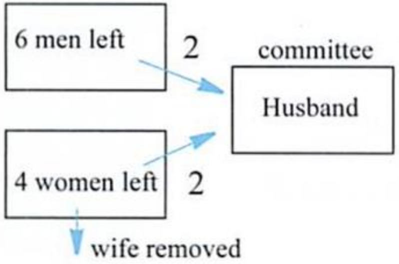
Think, pair, share

A committee of 3 men and 2 women is to be chosen from 7 men and 5 women. Within the 12 people there is a husband and wife. In how many ways can the committee be chosen if it must contain either the wife or the husband but not both?

Solution

Solution

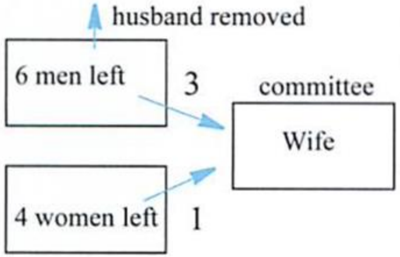
Case 1: Husband included



If the husband is included, the wife must be removed (so that she cannot be included). We then have to select 2 more men from the remaining 6 men and 2 women from the remaining 4 women.

This is done in ${}^{6}C_{2} \times {}^{4}C_{2} = 90$ ways

Case 2: Wife included



If the wife is included, the husband must be removed. We then have to select 3 men from the remaining 6 men and 1 woman from the remaining 4 women.

This is done is ${}^{6}C_{3} \times {}^{4}C_{1} = 80$ ways

Therefore there are a total of ${}^{6}C_{2} \times {}^{4}C_{2} + {}^{6}C_{3} \times {}^{4}C_{1} = 90 + 80 = 170$ possible committees.

Are They Correct?

Concept development

1. Emma claims:

> Tomorrow it will either rain or not rain. The probability that it will rain is 0.5.

Is she correct? Explain your answer fully:

2. Susan claims:

> If a family has already got four boys, then the next baby is more likely to be a girl than a boy.

Is she correct? Explain your answer fully:

Simple Probability

Know this

A **sample space** is the set of every possible outcome of an experiment.

An **event** is any subset of the sample space.

If an experiment has equally likely outcomes and, of those, A is defined then the **theoretical probability** of event A occurring is given by:

$$P(A) = \frac{n(A)}{n(U)}$$

$$= \frac{\text{number of outcomes in which A occurs}}{\text{total number of outcomes in the sample space}}$$

Definition: Two events are mutually exclusive (or disjoint) if they have no elements in common, that is if $A \cap B = \varnothing$.

Axioms of Probability

Know this

1. $0 \le P(A) \le 1$

2. $P(\varnothing) = 0$ and $P(U) = 1$

   that is, if $A = \varnothing$, then the event can never occur

   $A = U$, then the event A is certain to occur

3. If A and B are both subsets of U, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4. If A and B are both subsets of U and are mutually exclusive, then

   $P(A \cup B) = P(A) + P(B)$.

5. A' is the complement of an event such that P(A) + P(A') = 1

Know this

The following diagrams can be used to solve probability problems:

1. Venn diagrams
2. Tree diagrams
3. Lattice diagrams or grids
4. Probability tables

Example: Find the probability of getting a sum of 6 on two throws of a die.

6						
5						
4						
3						
2						
1						
	1	2	3	4	5	6

Example: If A and B are any two events with, $P(A)=\frac{3}{8}$, $P(B)=\frac{5}{12}$ and $P(A\cap B)=\frac{1}{4}$, find $P(A\cup B)$.

Example: An unbiased die is thrown three times.
Find the probability of obtaining

a) exactly two sixes

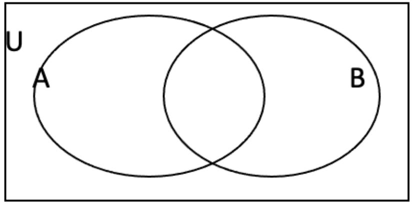
b) three sixes

c) at least one six

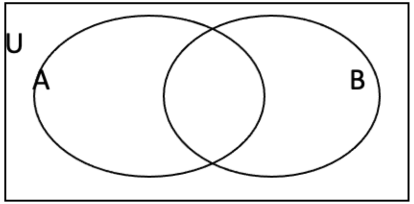
Example: A card is randomly selected from an ordinary pack of 52 playing cards. Find the probability that it is either a red card or an ace.

Venn Diagrams

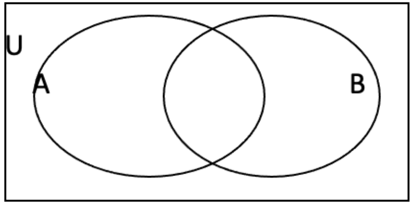
Know this



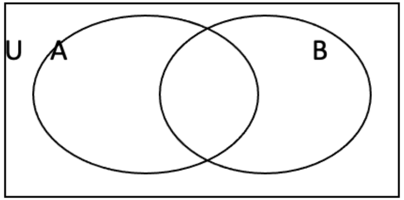
$A \cup B$ is shaded . Union



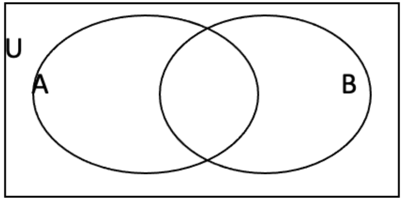
$A \cap B$ is shaded . Intersection



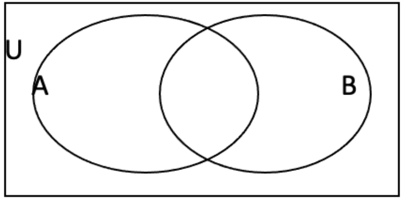
$A - B$ is shaded . Difference



A' is shaded . Complement



$(A \cup B)' = A' \cap B'$ is shaded. De Morgan's Law



$(A \cap B)' = A' \cup B'$ is shaded. De Morgan's Law

Daniel has 18 toys. 12 are made of plastic and 13 are red. 2 are neither red nor plastic.

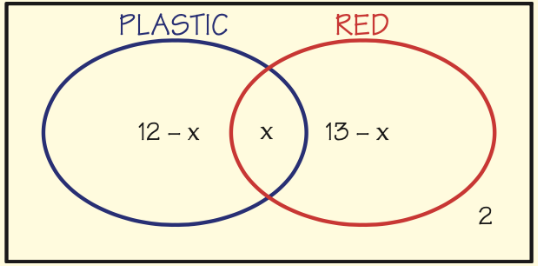
Daniel chooses a toy at random.

(a) Find the probability that it is a red plastic toy.

(b) If it is a red toy, find the probability that it is plastic.

Solution

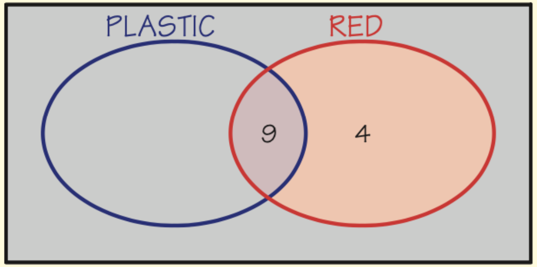
(a)



$$(12 - x) + x + (13 - x) + 2 = 18 \Leftrightarrow 27 - x = 18$$
$$\Leftrightarrow x = 9$$

$$\therefore P(\text{plastic and red}) = \frac{9}{18} = \frac{1}{2}$$

(b)



9 out of 13 red toys are plastic

$$\therefore P(\text{plastic} \mid \text{red}) = \frac{9}{13}$$

Venn Diagrams

Breakout
discussion

Group work: Venn Diagram Practice https://www.math.tamu.edu/~kahlig/venn/toons/toons.html

Cartoons

Fill in the Venn Diagram that would represent this data.

A study was made of 200 students to determine what TV shows they watch.

- 22 students don't watch these cartoons.
- 73 students watch only Tiny Toons.
- 136 students watch Tiny Toons.
- 14 students watch only Animaniacs and Pinky & the Brain.
- 31 students watch only Tiny Toons and Pinky & the Brain.
- 63 students watch Animaniacs.
- 135 students do not watch Pinky & the Brain (for some completely incomprehensible reason).

Conditional Probability
Bayes' Theorem

Conditional Probability

Know this

Conditional probability is used if information about the outcome of the experiment has been **given.**

If A and B are two events then the conditional probability of event A **given** event B is: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)},\ P(B) \neq 0$

Note: If A and B are mutually exclusive then $P(A \mid B) = 0$

Rearranging gives $P(A \cap B) = P(A \mid B) \cdot P(B)$

Note: Marbles could be selected from a bag “with replacement” or “without replacement”

Independence

Know this

Two events are independent if the probability of one occurring does not influence the probability of the other occurring.

1. Two events A and B are independent if, and only if,

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

2. Two events A and B are independent if, and only if,

$$P(A \cap B) = P(A) \cdot P(B)$$

Example: A coin is tossed twice in succession. Let A be the event that the first toss is heads and let B be the event that the second toss is heads. Find:

a) P(A)

b) P(B)

c) $P(A \cap B)$

d) $P(A|B)$

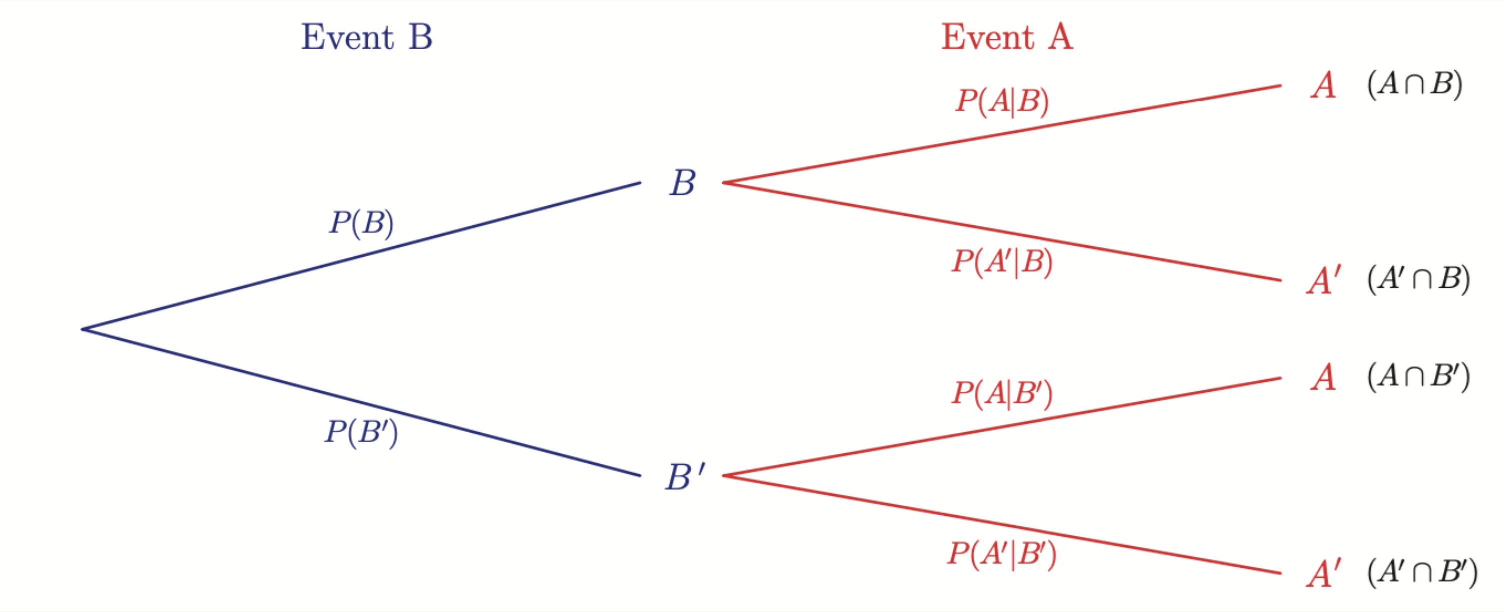
Bayes' Theorem

Example: Two machines A and B produce 60% and 40% respectively of the total output of a factory. Of the parts produced by machine A, 3% are defective and of the parts produced by machine B, 5% are defective. A part is selected at random from a day's production and found to be defective. What is the probability that it came from machine A?

Using the diagram,write down:

Know this

(a) probability of event A occurring, P(A)
(b) probability of event A occurring given that event B has occurred, P(A|B)
(c) substitute P(A) into your formula for P(A|B).



Bayes' theorem:

Bayes' theorem	$P(B \mid A) = \dfrac{P(B)\,P(A \mid B)}{P(B)\,P(A \mid B) + P(B')\,P(A \mid B')}$ $P(B_i \mid A) = \dfrac{P(B_i)P(A \mid B_i)}{P(B_1)P(A \mid B_1) + P(B_2)P(A \mid B_2) + P(B_3)P(A \mid B_3)}$

Example:

The Adelaide Eagles want to be sponsored by the International Baccalaureate©. If they come first in the league there is a 90% chance that they will be sponsored. If they come second there is a 20% chance that they will be sponsored and if they come lower than second there is a 5% chance that they will be sponsored. There is a 30% chance that they will come first in the league and a 20% chance that they will come second. At the end of the season they are sponsored by the International Baccalaureate©. What is the probability that they came first in the league?

Solution

Let Sp be the event 'being sponsored'

$$P(1^{st}\,|\,Sp) = \frac{P(Sp\,|\,1^{st})P(1^{st})}{P(1^{st})P(Sp\,|\,1^{st}) + P(2^{nd})P(Sp\,|\,2^{nd}) + P(<2^{nd})P(Sp\,|<2^{nd})}$$

$$= \frac{0.9 \times 0.3}{0.3 \times 0.9 + 0.2 \times 0.2 + 0.5 \times 0.05}$$

$$= 0.806$$

Mathematics Higher Level (2012), Cambridge University Press

Thanks

Do you have any question?

addyouremail@freepik.com
+91 620 421 838
yourcompany.com