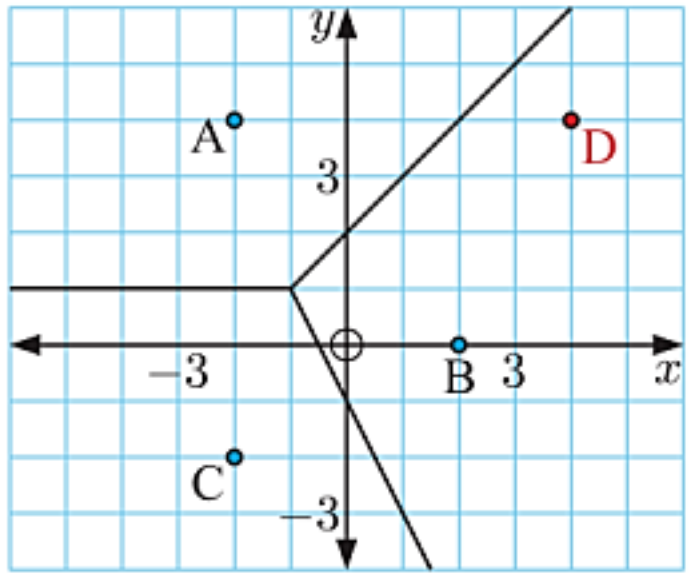
Adding a Site to a Voronoi Diagram

Example 1

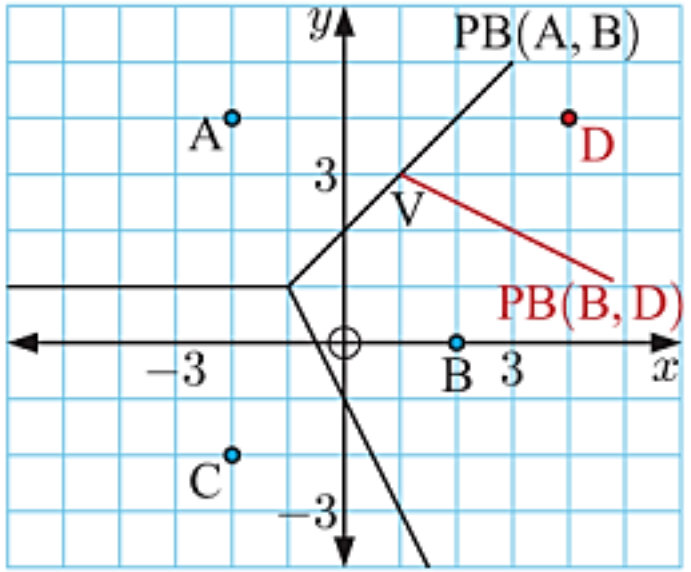
This Voronoi diagram shows the hospitals A, B, and C in a city.

A new hospital is being built at site D marked in red. When it is completed, the Voronoi diagram must be updated to include a cell corresponding to site D.

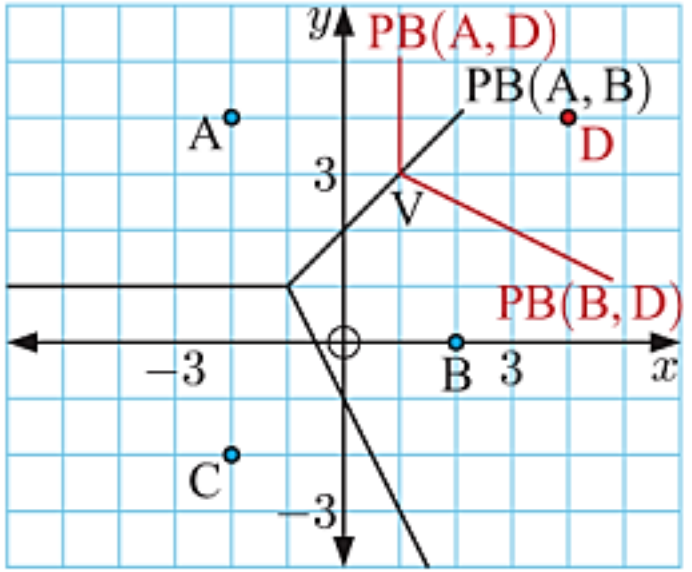
The new site currently lies in cell B, so some areas which were previously closest to Hospital B will now be closest to Hospital D.



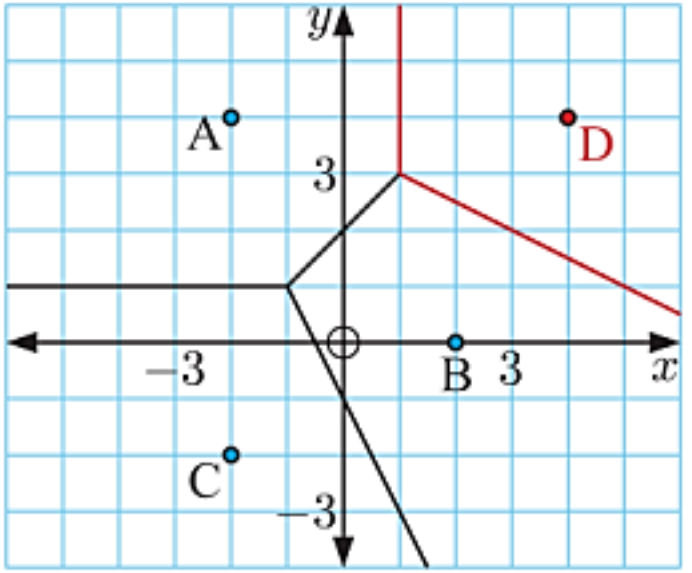
To establish the border between the new cell B and cell D, we draw the perpendicular bisector of [BD]. This line meets the edge PB(A, B) at point V.



V is equidistant from A, B, and D, so is a vertex of the new Voronoi diagram. We therefore need to add PB(A, D) to the diagram, starting at V as shown. This line does not meet any other existing edges, so no more new vertices are created. This tells us that the construction of cell D is complete.



Finally, we remove the part of the existing edge PB(A, B) which now lies within cell D. Notice that the cells for Hospital A and Hospital B were affected by the introduction of Hospital D. Hospital C is relatively further away from Hospital D, so cell C was not affected.



Incremental algorithm

In general, to add the cell for a new site X to an existing Voronoi diagram with sites P_1, P_2, P_3, ..., P_n, we follow these steps:

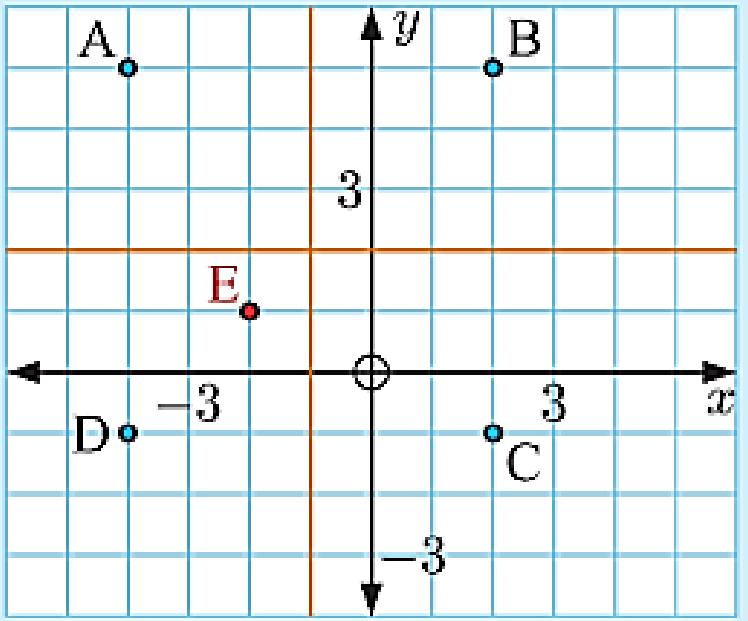
Step 1: Identify the site P_i whose cell contains the new site X. Construct $PB(P_i, X)$ within this cell. At any point where this line meets an existing edge, create a new vertex.

Step 2: For each site P_j whose cell is adjacent to a new vertex, construct $PB(P_j, X)$ within that cell through the vertex. Continue to create new vertices as in *Step 1*. Repeat this process until no more new vertices are created. At this time cell X is complete.

Step 3: Remove any segments of edges from the original Voronoi diagram which now lie within cell X.

Example 2

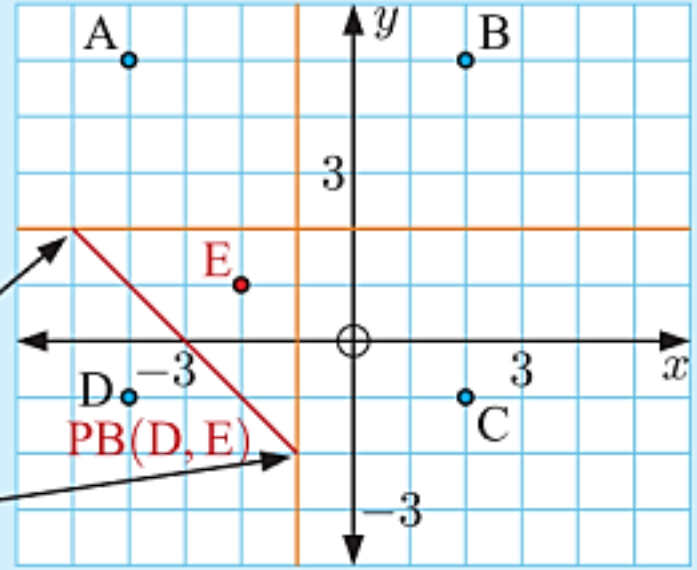
Redraw this Voronoi diagram with an additional site at E(−2, 1).



Step 1: E lies in the original cell D, so we construct PB(D, E) within this cell.

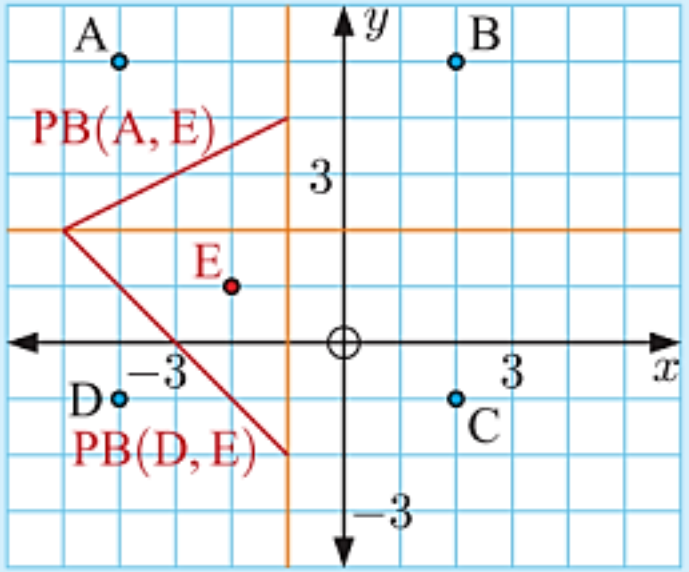
We create new vertices at $(-5, 2)$ and $(-1, -2)$.

In this case PB(D, E) creates *two* new vertices.

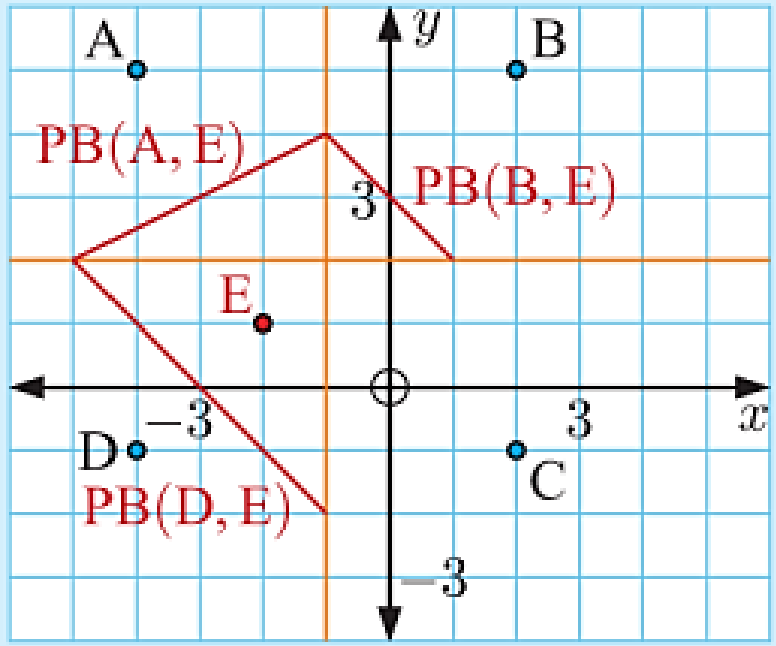


Step 2: Cell A is adjacent to the vertex $(-5, 2)$, so we construct PB(A, E) from $(-5, 2)$ through cell A.

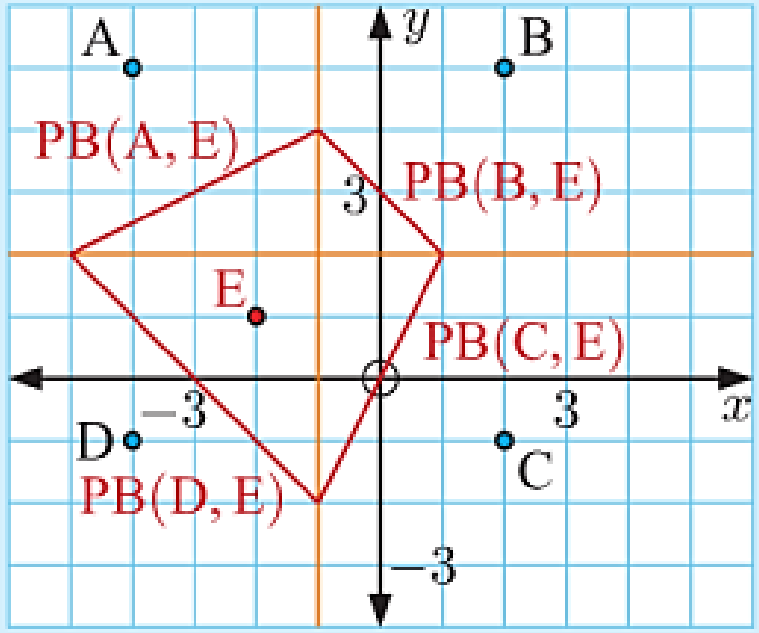
This creates a new vertex at $(-1, 4)$.



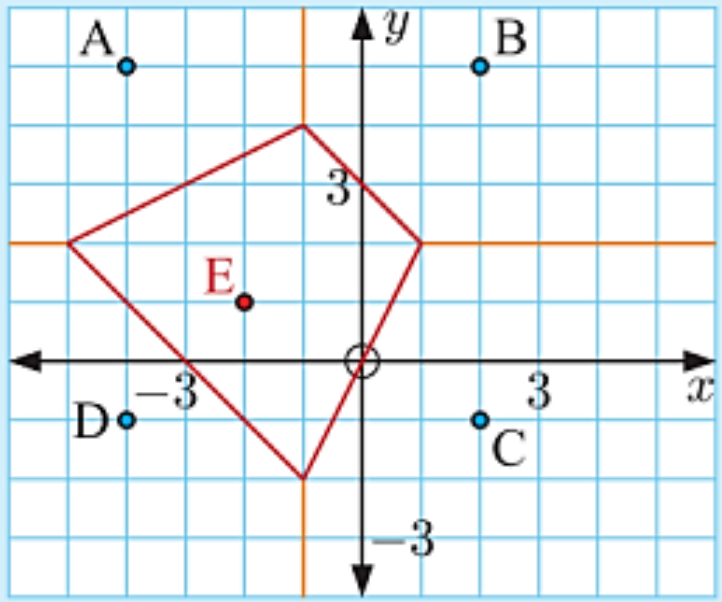
Cell B is adjacent to the vertex $(-1, 4)$, so we construct PB(B, E) from $(-1, 4)$ through cell B. This creates a new vertex at $(1, 2)$.



Cell C is adjacent to the vertex $(1, 2)$, so we construct PB(C, E) from $(1, 2)$ through cell C. This connects us back to the new vertex $(-1, -2)$.



Step 3: We remove the segments of edges from the original Voronoi diagram which now lie within cell E.



International-mindedness

Voronoi diagrams are used in computer graphics, epidemiology, geophysics, and meteorology.