Titel

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October 2015

1 Well-posed system

The 2-D Shrodinger equation

$$u_t = -iu_{xx} + -iu_{yy} + -iuF \tag{1}$$

Performing a time-dependent coordinate transformation of the form

$$\begin{aligned} x = x(\tau,\alpha,\beta), \quad y = y(\tau,\alpha,\beta), \quad t = \tau \\ \alpha = \alpha(t,x,y), \quad \beta = \beta(t,x,y) \quad t = \tau \end{aligned} \tag{2}$$

This leads to the transformation

$$u_\tau = -iu_{\alpha\alpha}\alpha_x^2 - iu_\alpha\alpha_{xx} - iu_{\beta\beta}\beta_y^2 - iu_\beta\beta_{yy} - u_\alpha\alpha_t - u_\beta\beta_t - iuF \tag{3}$$

The coordinate transformation is representing a box with changing boundaries. The boundaries of the box are parallel to the x and y-axis, Because of this,

$$\begin{aligned} x = x(\tau,\alpha), \quad y = y(\tau,\beta), \quad t = \tau \\ \alpha = \alpha(t,x), \quad \beta = \beta(t,y) \quad t = \tau \end{aligned} \tag{4}$$

and this leads to $\alpha_{xx} = \beta_{yy} = 0$. Equation (3) can then be simplified to

$$u_\tau = -iu_{\alpha\alpha}\alpha_x^2 - iu_{\beta\beta}\beta_y^2 - u_\alpha\alpha_t - u_\beta\beta_t - iuF \tag{5}$$

and in 1D

$$u_\tau = -iu_{\alpha\alpha}\alpha_x^2 - u_\alpha\alpha_t - iuF \tag{6}$$

Consider the coordinate transformation on the form

$$\alpha = h(t)x \tag{7}$$

where *h(t)* is a function of time. This gives $\alpha_x = h(t)$ and (6) becomes

$$u_\tau = -iu_{\alpha\alpha}h(t)^2 - u_\alpha h'(t) - iuF \tag{8}$$