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C-335

· HW2

1. Base: $N=0$, $r^0-1=0$, which is true

Assume $N=k$, max is ~~=~~ r^k-1

prove $N=k+1$ which equal to $r^{k+1}-1$.

$$\sum_{i=0}^{k}(r-1)r^i$$
$$=(r-1)r^0+(r-1)r+(r-1)r^2+\ldots+(r-1)r^k$$
$$=r^{k-1}+(r-1)r^k$$
$$=r^{k+1}-1 \text{, which is true}$$

Thus, prove holds.

2. For Radix-r, the largest number with one digit is $r-1$,

If the carry bit is 2, we can say $r-1+r-1 \geq 2r$

$2r-2 \geq 2r$

$-2 \geq 0$ which is false.

Thus, any r digits number, the carry could be 0, 1 can't over 1.

3. The minimum is $\underbrace{1\,0000\ldots0}_{n-1}$, which is -2^{n-1}

The maximum is $\underbrace{0\,111\ldots1}_{n-1}$, which is $2^{N}-1$

4. 2's complement less than 2^{N-1} is $00\,b_{N-3}\,b_{N-4}\ldots b_0$,

$11\,b_{N-3}\,b_{N-4}\ldots b_0$.

add 1, we get $10000\ldots0$,

when keep N bits, $\Rightarrow 00000\ldots0$, which is 0.

$-0=0 \Rightarrow$ The prove is correct.

5. a) 121

121	60	30	15	7	3	1
1	0	0	1	1	1	1

$(121)_{10}=(1111001)_2$

b) 1537

1537	768	384	192	76	48	24	12	6	3	1
1	0	0	0	0	0	0	0	0	1	1

$1537 = 11000000001$

c. 31333 31333 156666 7833 3916 1958 979 487 244 122
1 0 1 0 0 1 1 0 0
61 30 15 7 3 1
1 0 1 1 1 1
31333 = 111101000110010 1

d) 97 97 48 24 12 6 3 1
1 0 0 0 0 1 1
97 = 1100001

7. 121-41

$$\begin{array}{r} 121 \\ -41 \\ \hline 80 \end{array} \quad \begin{array}{r} 121 \\ +\ (-41) \\ \hline \end{array} \Rightarrow \begin{array}{r} 121 \\ +959 \\ \hline 80 \end{array}$$

we get 80

b) 1022-35

$$\begin{array}{r} 1022 \\ -35 \\ \hline 987 \end{array} \quad \begin{array}{r} 1022 \\ +9965 \\ \hline 0987 \end{array}$$

we get 987

c) 151-90

$$\begin{array}{r} 151 \\ -90 \\ \hline 61 \end{array} \quad \begin{array}{r} 151 \\ +910 \\ \hline 061 \end{array}$$

we get 61

d) 2120-101

$$\begin{array}{r} 2120 \\ -101 \\ \hline 2019 \end{array} \quad \begin{array}{r} 2120 \\ +9899 \\ \hline 2019 \end{array}$$

we get 2017

8) a) -121 $(121)_{10} = (1111001)_2$

$(\overline{121})_{10} = 10000110$

$$\begin{array}{r} 10000110 \\ +\quad 1 \\ \hline 10000111 \end{array}$$

$(-121)_{10} = (10000111)_2$

b) -57

$(51)_{10} = (110011)_2$

$\overline{(00110011)}_2 = 11001100$ $\quad -(57)_{10} = (11001101)_2$

$+ \quad\quad 1$

11001101

c) -104 $\quad (104)_{10} = (1101000)_2$

$\overline{(01101000)}_2 = 10010111$ $\quad -(104)_{10} = (10011000)_2$

$+ \quad\quad 1$

10011000

d) 115 $\quad (115)_{10} = (1110011)_2$

$(115)_{10} = (01110011)_2$

e) 127 $\quad (127)_{10} = (1111111)_2$

$(127)_{10} = (01111111)_2$