

# Introduction to Higher Algebra

Aron Heleodoro

October 31, 2025

This is the plan for a course on Higher Algebra at SIMIS on Fall 2025. My plan is to follow very closely the presentation of Aaron Mazel-Gee's forthcoming book Higher Algebra, Chapter 0.

The goals for the course are:

- 1) give a modern presentation of the basics of homological algebra: definition of  $h_0(\text{Mod}_R)$  the homotopy category of the  $\infty$ -category of  $R$ -modules, via the dg-category of complexes of  $R$ -modules. Discussion of classical derived functors from functors between  $\infty$ -categories.
- 2) definition of a  $\infty$ -category via different models as: simplicial (or topological) categories, quasi-categories, relative categories and complete Segal spaces. Example of the  $\infty$ -category of spaces and construction of  $\infty$ -category of  $\infty$ -categories.
- 3) Discussion of basic categorical notions in the context of an  $\infty$ -category. Notion of monoid, commutative monoid objects and the notion of symmetric monoidal  $\infty$ -category.
- 4) Definition and properties of stable  $\infty$ -categories. Construction and discussion of examples: the derived  $\infty$ -category of  $R$ -modules and spectra.
- 5) How to formalize certain constructions in  $\infty$ -categories? This mainly would be via a discussion of the Grothendieck construction. The  $\infty$ -category of presentable stable  $\infty$ -categories and Lurie's tensor product.
- 6) Time permitting we would discuss some further topics, such as factorization homology, recollements, etc.

## Plan

Week	Date	Topic	References
1	Sep. 18	Introduction and plan	??
2	Sep. 25	Chain complexes, homology, tensor products and homotopies	§2.1-3.1 [3]
3	Oct. 9	Homotopy (co)kernels and dg-category of complexes	§3.2-4.2 in [3]
4	Oct. 16	Resolutions and cellular approximations	§4.3-5.6 in [3]
5	Oct. 23	First discussion of ( $k$ -linear) $\infty$ -categories	§6.1-6.2 in [3]
6	Oct. 30	The derived $\infty$ -category $\text{Mod}_R$ and derived functors	§6.3-7.2 in [3]
7	Nov. 6	Classical Ext and Tor, group (co)homology and Algebraic K-theory	§7.2-7.5 in [3]
8	Nov. 13	Model categories	§8 in [2]
9	Nov. 20	Basic notions in $\infty$ -categories	§9 in [2]
10	Nov. 27	Monoids, Complete Segal spaces, and symmetric monoidal categories	§10 in [2]
11	Dec. 4	Stable $\infty$ -categories	§11.5-11.7 in [3]
12	Dec. 11	The Grothendieck construction	§11 in [2]
13	Dec. 18	Limits and colimits of presentable $\infty$ -categories	§12 in [2]
14	Dec. 25	Recollements and Factorization Homology (?)	[1, §1], §10.5 in [3]

## Reality

Week	Date <sup>1</sup>	Topic	References
1	Sep. 18	Introduction and plan	??
2	Sep. 25	Chain complexes, homology, tensor products, homotopies, and homotopy cokernel	§2.1-3.2 [3]
3	Oct. 9	Homotopy kernels, exact sequences, and dg-category of complexes	§3.3-4.3 in [3]
4	Oct. 16	Resolutions and cellular approximations	§5.1-5.6 in [3]
5	Oct. 23	First discussion of ( $k$ -linear) $\infty$ -categories	§6.1-6.2 in [3]
6	Oct. 30	The derived $\infty$ -category $\mathrm{Mod}_R$ and derived functors	§6.3 in [3]

## References

- [1] David Ayala, Aaron Mazel-Gee, and Nick Rozenblyum. “Stratified Noncommutative Geometry”. In: *Memoirs of the American Mathematical Society* 297.1485 (2024), pp. iii+260. ISSN: 0065-9266,1947-6221 (cit. on p. 1).
- [2] Aaron Mazel-Gee. *An Invitation To Higher Algebra*. July 15, 2021. URL: <https://etale.site/teaching/w21/math-128-lecture-notes.pdf> (cit. on p. 1).
- [3] Aaron Mazel-Gee. *Higher Algebra, Chapter 0*. June 2, 2023. URL: <https://etale.site/teaching/s23-128/math-128-s23-lecture-notes.pdf> (cit. on pp. 1, 2).

---

<sup>1</sup>You can access the recorded lectures by clicking on the dates below.