Math 347: Exam 1

Oct. 5, 2018

Name:

This exam has three questions and lasts 50 minutes. Make sure your exam is complete before start and don't forget to enter your name.

All of your answers need a justification, and when asked to prove a result you need to write a formal argument.

You can use the back pages for scratch work, but indicate if you want something there to be graded.

You will be given partial credit, so if you don't have a complete proof explain, in words, what you know how to do.

No textbook or notes are allowed during the exam.

Question	Points	Total
1		10
2		16
3		15
Total		41

1. (10 points) For any natural number $n \geq 1$, prove that

$$|\sum_{i=1}^{n} a_i| \leq \sum_{i=1}^{n} |a_i|,$$

where $a_i \in \mathbb{R}$, for all $1 \leq i \leq n$.

2. Suppose a, b, c and d are non-zero real numbers. Consider the function $f : \mathbb{R}\backslash\{-d/c\} \to \mathbb{R}$ given by

$$f(x) = \frac{ax + b}{cx + d}.$$

   (a) (10 points) Prove that f is injective if and only if $ad - bc \neq 0$.

   (b) (6 points) Is f surjective?

3. (15 points) Determine if the following are true or false. Only a brief justification is required.

   (a) For a function $f : A \to B$ and $T \subseteq A$ one has
   $$T = f^{-1}(f(T)).$$

   (b) For any sets A, B, C
   $$A \cap (B \cup C) = (A \cup B) \cap C.$$

   (c) A professor gives for each natural number n statements $P(n)$ and $Q(n)$. Then he states that
   $$(\forall n \in \mathbb{N})(P(n) \Rightarrow Q(n))$$
   holds. A student finds out that for $n = 4$, $P(4) \wedge \neg Q(4)$ is true. This implies that the professor is wrong.

   (d) The negation of the statement: "given any natural number n one can always find a number bigger than n", is:
   $$(\exists a \in \mathbb{N})(\forall b \in \mathbb{N})(b \geq a)$$

   (e) Consider the real numbers $x_1, \ldots, x_n$, for $n \geq 1$ a natural number. At least one of the x_i's is bigger than or equal to their average.