Math 347: Final Participation

November 26, 2018

Description: The goal of this last presentation is to have students work in groups to understand a bit of mathematical theory, apply it to a problem, and then present it to the class. Your group should come up with a 20 minutes presentation, which should include the definition of any concept that didn't show up in class before with examples. You should state the problem you propose to solve and give a brief explanation of the solution. Every member of the group should contribute to the presentation.

Instructions: Each group should pick a single problem to present.

The schedule is the following:

Problems from Chapter 5: **December 3**

Group 1: Jingquan Fu, Ruisong Li, Houyi Du and Kaiwen Hu (Problem I from Chapter 5).

Group 2: Amr Ellayyan, Danyu Sun and Albert Cao (Problem II from Chapter 5).

Problems from Chapter 6: **December 5**

Group 3: Jacob Elling, Ismail Dayan and Bryan Ulziisaikhan (Problem I from Chapter 6).

Group 4: Bangzheng Li, Dongfan Li and Zhenghong Huang (Problem II from Chapter 6).

Problems from Chapter 7: **December 7**

Group 5: Daniel Coonley, Adreas Ruiz-Gehrt and Nishant Dalmia (Problem I from Chapter 7).

Group 6: Jinsoo Oh, Vetrie Senthilkumar, and Thomas Varghese (Problem II from Chapter 7).

Problems from Chapter 5

I. The number of transpositions needed to sort the word form of a permutation of $[n]$ is $n - k$, where k is the number of cycles in its cycle description.

1) Define the problem, that is explain what all the words in the statement mean.
2) Discuss different representations of a permutation, with examples.
3) Give an intuitive idea of why the statemet is true, again in an example.
4) Present the formal argument to make 3) precise.

II. For $k \in \mathbb{N}$, the value of $\sum_{i=1}^{n} i^k$ is a polynomial in n with leading term $\frac{n^{k+1}}{k+1}$ and next term $\frac{n^k}{2}$.

1) Explain why one expects the above behaviour.
2) Define leading term and next term of a polynomial, give examples.
3) Give a sketch of how to prove this result, that is present the logic of the proof and the main claims that are used.
4) Eventually explain in more details why each of the claims is true and how the whole proof works.

Problems from Chapter 6

I. (Division algorithm for polynomials.) Let $\mathbb{R}[x]$ be the set of polynomials with real coefficients. Prove that if $a, b \in \mathbb{R}[x]$ and $b \neq 0$. Then there exist unique $q, r \in \mathbb{R}[x]$ such that

$$a = qb + r \quad \text{and} \quad r = 0 \text{ or } \deg(r) < \deg(b).$$

1) Explain the meaning of all the words and symbols in the statement.
2) Give the proof of the statement.
3) Discussion of the relation to the statement about integer numbers.

II. Every ideal in $\mathbb{R}[x]$ is a principal ideal.

1) Define what an ideal is and give examples.
2) Define what a principal ideal is and give examples and non-examples.
3) Prove the result.

Problems from Chapter 7

I. If p is prime then $(\mathbb{Z}/p\mathbb{Z})\backslash\{0\}$ is a group.

1) Define what a group is and give examples.
2) Discuss the set we are considering and examples for small numbers, p prime and non-prime.
3) Prove the statement.

II. (Wilson's theorem) $(n-1)! \equiv -1 \bmod n$ if and only if n is a prime.

1) Explain why intuitively the result should be true.
2) Explain which proof technique you want to use.
3) Present the logic of the proof and its ingredients.
4) Time permitting give the details of the proof.