Introduction to Higher Algebra

Aron Heleodoro

October 27, 2025

This is the plan for a course on Higher Algebra at SIMIS on Fall 2025. My plan is to follow very closely the presentation of Aaron Mazel-Gee's forthcoming book Higher Algebra, Chapter 0.

The goals for the course are:

1) give a modern presentation of the basics of homological algebra: definition of $h_0(\mathrm{Mod}_R)$ the homotopy category of the ∞-category of R-modules, via the dg-category of complexes of R-modules. Discussion of classical derived functors from functors between ∞-categories.

2) definition of a ∞-category via different models as: simplicial (or topological) categories, quasi-categories, relative categories and complete Segal spaces. Example of the ∞-category of spaces and construction of ∞-category of ∞-categories.

3) Discussion of basic categorical notions in the context of an ∞-category. Notion of monoid, commutative monoid objects and the notion of symmetric monoidal ∞-category.

4) Definition and properties of stable ∞-categories. Construction and discussion of examples: the derived ∞-category of R-modules and spectra.

5) How to formalize certain constructions in ∞-categories? This mainly would be via a discussion of the Grothendieck construction. The ∞-category of presentable stable ∞-categories and Lurie's tensor product.

6) Time permitting we would discuss some further topics, such as factorization homology, recollements, etc.

Plan

Week	Date	Topic	References
1	Sep. 18	Introduction and plan	??
2	Sep. 25	Chain complexes, homology, tensor products and homotopies	§2.1-3.1 [3]
3	Oct. 9	Homotopy (co)kernels and dg-category of complexes	§3.2-4.2 in [3]
4	Oct. 16	Resolutions and cellular approximations	§4.3-5.6 in [3]
5	Oct. 23	First discussion of (k-linear) ∞-categories	§6.1-6.2 in [3]
6	Oct. 30	The derived ∞-category Mod_R and derived functors	§6.3-7.2 in [3]
7	Nov. 6	Classical Ext and Tor, group (co)homology and Algebraic K-theory	§7.2-7.5 in [3]
8	Nov. 13	Model categories	§8 in [2]
9	Nov. 20	Basic notions in ∞-categories	§9 in [2]
10	Nov. 27	Monoids, Complete Segal spaces, and symmetric monoidal categories	§10 in [2]
11	Dec. 4	Stable ∞-categories	§11.5-11.7 in [3]
12	Dec. 11	The Grothendieck construction	§11 in [2]
13	Dec. 18	Limits and colimits of presentable ∞-categories	§12 in [2]
14	Dec. 25	Recollements and Factorization Homology (?)	[1, §1], §10.5 in [3]

Reality

Week	Date[1]	Topic	References
1	Sep. 18	Introduction and plan	??
2	Sep. 25	Chain complexes, homology, tensor products, homotopies, and homotopy cokernel	§2.1-3.2 [3]
3	Oct. 9	Homotopy kernels, exact sequences, and dg-category of complexes	§3.3-4.3 in [3]
4	Oct. 16	Resolutions and cellular approximations	§5.1-5.6 in [3]
5	Oct. 23	First discussion of (k-linear) ∞-categories	§6.1-6.2 in [3]

References

[1] David Ayala, Aaron Mazel-Gee, and Nick Rozenblyum. "Stratified Noncommutative Geometry". In: *Memoirs of the American Mathematical Society* 297.1485 (2024), pp. iii+260. ISSN: 0065-9266,1947-6221 (cit. on p. 1).

[2] Aaron Mazel-Gee. *An Invitation To Higher Algebra*. July 15, 2021. URL: `https://etale.site/teaching/w21/math-128-lecture-notes.pdf` (cit. on p. 1).

[3] Aaron Mazel-Gee. *Higher Algebra, Chapter 0*. June 2, 2023. URL: `https://etale.site/teaching/s23-128/math-128-s23-lecture-notes.pdf` (cit. on pp. 1, 2).

[1] You can access the recorded lectures by clicking on the dates below.