Math 347: Exam 2
Oct. 29, 2018

Name:

This exam has five questions and lasts 50 minutes. Make sure your exam is complete before the start and don't forget to enter your name.

All of your answers need a justification, and when asked to prove a result you need to write a formal argument.

You can use the back page for scratch work, but indicate if you want something there to be graded.

You will be given partial credit, so if you don't have a complete proof explain, in words, what you know how to do.

No textbook, electronics, or notes are allowed during the exam.

Question	Points	Total
1		32
2		20
3		14
4		14
5		20
Total		100

1. Let (a_n) be a sequence of real numbers.

   a) Define the statement:

   "The sequence (a_n) converges to L."

   b) Define the statement:

   "(a_n) is a Cauchy sequence".

c) Check using the definition that the sequence

$$a_n = \frac{1}{n^2}$$

converges to 0.

d) Check directly using the definition, that is without using Cauchy's theorem, that the sequence

$$a_n = \frac{1}{n^2}$$

is a Cauchy sequence.

2. a) Let C be an infinite set. Define the statement: "C is countable".

b) Suppose that C is infinite and countable, prove that $C \times C$ is countable.

3. a) State the monotone convergence theorem.

b) Prove that the sequence

$$a_n = \frac{1}{n^2} \sum_{i=1}^{n} i$$

converges.

4. Consider (a_n) a sequence of real numbers.

   (a) State Cauchy's convergence theorem.

   (b) Prove that if (a_n) converges to a number L, then (a_n) is a Cauchy sequence.

5. Determine if the following are true or false, only a brief explanation is necessary.

   a) Every sequence has a convergent subsequence.

   b) If (a_n) converges, then (a_n) is monotone.

   c) Every bounded sequence (a_n) has a convergent subsequence.

   d) If (a_n) converges then any subsequence converges.

   e) If (a_n) is a Cauchy sequence, then (a_n) is bounded.