Derivative Markets – Summary & Exercises

All exercises are taken from Hull, J. C.: Options, futures and other derivatives, 8th edition (Prentice Hall, 2011)

***1.9.*** *You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with a strike price of $30 costs $2.90. You have $5,800 to invest. Identify two alternative investment strategies, one in the stock and the other in an option on the stock. What are the potential gains and losses from each?*

*+You buy the call option. What is the breakeven ST?*

Breakeven means you neither win nor lose. Here there is no funding cost for the option premium, so 30+2.9=32.90 is the break even. If ST>32.90 then you will make a profit in a sense that you get more money back than your original amount. However, it is highly probable that you would have to pay at least the 3-month LIBOR as a funding cost on the 2.90 option premium, since the premium must be paid at the beginning. Thus, the breakeven would be 30 + 2.90 x (1 + LIBOR x 3/12).

*+You spend $5800 by purchasing 200 stocks. Your friend spends $5800 by purchasing call options on 2000 stock. What is the ST that makes both portfolios the same value?*

WARNING: This is not the same question, but looks very similar to the breakeven question!

200 x ST = 2000 x max(0;(ST-30)

ST>30 otherwise the option will be worthless and the friends’ portfolio would have zero value.

200 x ST = 2000 x (ST-30) // and ST>30

ST=33+1/3 ≈ $33.33

Funding cost is not relevant here since both you and your friend invested the same initial amount.

***1.25.*** *Suppose that USD/sterling spot and forward exchange rates are as follows:*

*Spot 1.4580*

*90-day forward 1.4556*

*180-day forward 1.4518*

*What opportunities are open to an arbitrageur in the following situations?*

*(a) A 180-day European call option to buy Ł1 for $1.42 costs 2 cents.*

*(b) A 90-day European put option to sell Ł1 for $1.49 costs 2 cents.*

***Hint: In-the-money options can’t be arbitrarily cheap!***

1. Call option is too cheap! Check the futures value of the following position:

**long call1.4200 & short forward1.4518** >= 1.4580 – 1.4200 = 0.0380 = 3.8 cents

you can buy this portfolio for 2 + 0 cents (2 cents is for the call, the forward position is free)

Unless there is a super-high risk free dollar yield, spending 2 cents today to get 3.8 cents or more 6 months later is a good deal.

1. Similar to the previous question: the put option is too cheap!

**long put1.4900 & long forward1.4556** >= 1.4900 – 1.4556 = 0.0344 = 3.44 cents

you can buy this portfolio for 2 + 0 cents (2 cents is for the put, the forward position is free)

*+Considering the quoted prices which currency has higher interest rate relative to the other: the GBP or the USD?*

S > F90days > F180days

F=S x Q / P

The base currency has higher rate: rGBP > rUSD

***4.5.*** *Suppose that zero interest rates with continuous compounding are as follows:*

*Maturity (months) Rate (% per annum)*

*3 8.0*

*6 8.2*

*9 8.4*

*12 8.5*

*15 8.6*

*18 8.7*

*Calculate forward interest rates for the second, third, fourth, fifth, and sixth quarters.*

Continously compounding rates are easy to calculate with. For example the continously compunded forward rate for the fifth quarter is:

15f\_log18 = (8.7% \* 18/12 – 8.6% \* 15/12)/(3/12) = 9.2%

*+What would be the fair price for a 3-month LIBOR in the 6x9 FRA?*

FRA is a derivative on the LIBOR, which is a linear (ACT/360) deposit rate.

First lets calculate the continously compunded rate for the 6month-9month time slice. Second lets convert it to a linear rate to get the answer.

6f\_log9 = (8.4% \* 9/12 – 8.2%\*6/12)/(3/12) = 8.8%

(1+3/12\*FRA6x9) = exp(8.8%\*3/12)

FRA6x9 = 8.897514% ≈ 8.90%

***4.13.*** *Suppose that the 6-month, 12-month, 18-month, and 24-month zero rates are 5%, 6%, 6.5%, and 7%, respectively. What is the 2-year par yield?*

By definition, **par** is the nominal interest paid by a fixed bond that makes the bond price 100% of its notional.

WARNING: This is a par yield for a semi-annually coupon paying bond.

k = nominal interest = coupon

The bond pays k/2 semiannually.

100% = k/2 \* DF0.5Y + k/2 \* DF1Y + k/2 \* DF1.5Y + k/2 \* DF2Y + 100% \* DF2Y

k/2 = (100% - DF2Y) / (DF0.5Y + DF1Y + DF1.5Y + DF2Y)

DF0.5Y=1/(1+5%)^(0.5) = 97.5900%

DF1Y=1/(1+6%)^(1) = 94.3396%

DF1.5Y=1/(1+5%)^(0.5) = 90.9862%

DF0.5Y=1/(1+5%)^(0.5) = 87.3439%

k/2 = 3.4182%

k = 6.8364% ≈ 6.84%

*+A bank has a long 2-years IRS (interest rate swap) position in which it receives the 6-months LIBOR and pays 6.55% fix on a 1 billion USD notional semi-annually. Does this swap has a positive or negative market value for the bank? What is the market value of this swap position?*

At this point we already know that the 2-years long semi-annually par rate is 6.84%. This is also the fair fix rate for a 2-years long semi-annually swap. In its particular swap the bank is paying 6.55% fix for the “6-month LIBOR for 2 years” cash flow package, when the market price for the “6-month LIBOR for 2 years” cash flow package is 6.84%, so this swap has **positive** market value for the bank at this very moment.

If the bank would close its long IRS by entering into a short IRS the following cash flow would occur semi-annually:

(+LIBOR – 6.55%) x 1 billion USD x 6/12 + (- LIBOR + 6.84%) x 1 billion USD x 6/12 =

= 1 billion x 6/12 x (6.84% - 6.55%) = 1 450 000 USD

So the bank would win 1.45 mio USD 6 months, 12 months, 18 months and 24 months from now. The total present value of this is:

1.45 mio x (97.5900% + 94.3396% + 90.9862% + 87.3439%) = 5 368 765,65 USD

*7.2. Company X wishes to borrow US dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies are subject to the following interest rates, which have been adjusted to reflect the impact of taxes:*

*Yen Dollars*

*Company X: 5.0% 9.6%*

*Company Y: 6.5% 10.0%*

*Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.*

Company X has an absolute advantage both in USD and JPY funding. However, Company Y is “less worse” in the USD, it has comparative advantage in USD borrowing.

Total benefit of the swap: (9.6%+6.5%) – (5%+10%) = 1.1%

Bank gets 0.50% = 50 basis point

Both X and Y will save 30 bp each.

10% USD

9.3% USD

10% USD

Company X

BANK

Company Y

5% JPY

6.2% JPY

5% JPY

Final payments:

Company X pays 9.3% USD which is 30 bp saving compared to 9.6%

Company Y pays 6.2% JPY which is 30 bp saving compared to 6.5%

Bank: +9.3% USD – 10% USD + 6.2% JPY – 5% JPY = -0.7% USD + 1.2% JPY ≈ 50 basis point as a total

The profit of the bank has FX risk! Of course it could be hedged, but that was not the question.

***7.5.*** *A currency swap has a remaining life of 15 months. It involves exchanging interest at 10% on Ł20 million for interest at 6% on $30 million once a year. The term structure of interest rates in both the United Kingdom and the United States is currently flat, and if the swap were negotiated today the interest rates exchanged would be 4% in dollars and 7% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (dollars per pound sterling) is 1.8500. What is the value of the swap to the party paying sterling? What is the value of the swap to the party paying dollars?*

From the viewpoint of the GBP paying party the cash flow is the following:

3 months: -10%\*20mio GBP + 6%\*30 mio USD

15 months: -(1+10%)\*20mio GBP + (1+6%)\*30 mio USD

Next step is calculating the present values and converting everything to USD:

Value = 1.85\*(-2mio)/(1+7%)^(3/12) + (1.8mio)/(1+4%)^(3/12) + 1.85\*(-22mio)/(1+7%)^(15/12)+ 31.8 mio/(1+4%)^(15/12)= -8 976 333,69 USD

What is the value for the other counterparty of the deal? It must be +8 976 333,69 USD, since the two values must add up to zero.

***5.10.*** *The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the 6-month futures price?*

F = S \* Q / P

Q = 1/(1+3.2%)^(6/12) = 98.4374% //Assuming that the index dividend yield is effective yield

P = exp(-7%\*6/12) = 96.5605%

F = 150 \* 98.4374% / 96.5605% = 152,92

If the dividend yield is also continuously compounded:

F = 150 \* exp((7%-3.2%)\*6/12) = 152.88

***5.14.*** *The 2-month interest rates in Switzerland and the United States are, respectively, 2% and 5% per annum with continuous compounding. The spot price of the Swiss franc is $0.8000. The futures price for a contract deliverable in 2 months is $0.8100. What arbitrage opportunities does this create?*

WARNING: the FX price is quoted inversely: 0.8 is the dollar price of one piece of CHF. (Normally the interbank market trades the USDCHF, not the CHFUSD)

S = 0.8000  
F\_theoretical = 0.8 \* exp((5%-2%)\*2/12) = 0,8040

Thus the F\_market = 0.8100 is too high!

Steps for arbitrage assuming we do the arbitrage with 1 mio CHF futures notional

#1) SELL 1 mio CHF futures @ 0.8100  
#2) BORROW 1 mio \* exp(-2%\*2/12) \*0.80 USD @ 5%  
#3) BUY 1 mio \* exp(-2%\*2/12) CHF spot @ 0.8000  
#4) LEND 1 mio \* exp(-2%\*2/12) CHF @ 2%

Today this strategy has no net cash flow, however 2 month later the following will happen:

futures settlement: **-1mio CHF** **+ 810 000 USD**USD loan must be paid back: -1 mio \* exp(-2%\*2/12) \*0.80 \* exp(5%\*2/12) **= -804 010,02 USD**  
we get back the CHF deposit: **+1 mio CHF**

We would have an approx. 6k USD profit 2 months from now.

***9.13.*** *Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.*

American = option can be exercised anytime before expiry

European = option can be exercised only at the expiry

Holder of an American option can always decide to wait until expiry (even if sometimes this is not the best she could do) so an American option can be used as if it were European.

***9.14.*** *Explain why an American option is always worth at least as much as its intrinsic value.*

American = option can be exercised anytime before expiry

Intrinsic value = the cash that the option would provide if it would be exercised immediately.

If the value of an American option would drop below its intrinsic value it would make sense exercising it immediately. Also if possible one should buy more of this option and exercise immediately after the purchase. This would generate free money.

***10.14.*** *The price of a European call that expires in 6 months and has a strike price of $30 is $2. The underlying stock price is $29, and a dividend of $0.50 is expected in 2 months and again in 5 months. The term structure is flat, with all risk-free interest rates being 10%. What is the price of a European put option that expires in 6 months and has a strike price of $30?*

Put-call parity helps here.

S – PV(DIV1) – PV (DIV2) – PV(K) = fwd = call – put

put = call – S + PV(DIV1) + PV (DIV2) + PV(K)

put = 2 – 29 + 0.5/(1+10%)^(2/12) + 0.5/(1+10%)^(5/12) + 30/(1+10%)^(6/12) = 2,58 dollars

***12.9.*** *A stock price is currently $50. It is known that at the end of 2 months it will be either $53 or $48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a 2-month European call option with a strike price of $49? Use noarbitrage arguments.*

53

stock: 50 48

4

call: ??? 0

delta = (4-0)/(53-48) = 0.8

To replicate the call lets buy 0.8 stock:

42,4 = 4 + 38,4

0.8 stock: 40 38,4 = 0 + 38,4

2 month later this 0.8 stock position will have the value of a callT + 38,4, so today the value of the portfolio must be the present value of a calltoday+38,4/(1+10%)^(2/12)

We also know that the 0.8 stock today worth 40 dollars.

40 = calltoday+38,4/(1+10%)^(2/12)

calltoday = 40 - 38,4/(1+10%)^(2/12) = 2,21 dollars

*+Pricing in the CRR (Cox-Ross-Rubinstein) binomial model. The current price of a non-dividend paying stock is 100, it follows a binomial process with the following parameters: u=1.25, d=1/u=0.80 and Δt=1 year. The risk free rate effective yield is 10%. What is the price of a K=100 strike price European straddle (call+put) which expires T=3 years later?*

Very useful in the CRR model that it is possible to price the straddle itself, we don’t have to calculate the call and the put separately.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| DF\_dt = | 0,9091 |  |  |  |
| q= | 0,6667 |  |  |  |
|  |  |  |  | 195,3125 |
|  |  |  | 156,3 | 125 |
|  |  | 125 | 100 | 80 |
| Stock | 100 | 80 | 64 | 51,2 |
|  |  |  |  |  |
|  |  |  |  | 95,3125 |
|  |  |  | 65,34 | 25 |
|  |  | 46,03 | 21,21 | 20 |
| straddle | **34,26** | 21,01 | 26,91 | 48,8 |

65,34 = (0,66\*95,3125 + (1-0,66) \* 25 ) \* 0,9091

The price of the straddle is 34,26 dollars.

*All the following 4 questions are about this formula:*

*u = exp(σ*

*σ = (1/ ln(u)*

*+What is the volatility (σ) of the stock if…  
… u=1.25 and Δt=1 year?*

*σ= ln(1.25) = 22,31%  
… u=1.01 and Δt=1 day? (assume 252 trading days per year)*

*σ= 1/(1/252)^(1/2)\* ln(1.01) = 15,80%*

*+What is the u parameter in the CRR model, if…  
… σ = 12% and Δt=1 year?*

*u = exp(12% =1,1275*

*… σ = 35% and Δt=1 hour? (assume 252 trading days per year and 8 hours per day)*

*u = exp(12% =1,0078*

***13.1.*** *What would it mean to assert that the temperature at a certain place follows a Markov process? Do you think that temperatures do, in fact, follow a Markov process?*

*+What is a martingale?*

Markov property: the next value of the stochastic process is only dependent on the current (very last) value of the process, any additional previous history is irrelevant. A Markov process has no memory since past values have no influence on future ones.

Martingale feature: the expected change of the process is zero. Hint: we do not expect the process to be

constant or unchanged! If we have a chance of winning or losing 1 dollar with 50% chance for both outcomes then the value of our portfolio will change for sure, however the expected change is zero.

*It is very probable that the temperature has some memory, trends can be found for example because previous changes could have still lasting causes. If temperature would follow Markov process it could be very crazy weather.*

***14.2.*** *The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one trading day?*

Assuming there are 252 trading days during a year:

*σyearly = σdaily \**

*σdaily = σyearly /* = 30% / = 1,89%

***14.3.*** *Explain the principle of risk-neutral valuation.*

By executing the dynamic delta hedging strategy the portfolio can as always (temporarily) made risk free. Thus the portfolio’s return must be equal to the risk free return. The risk neutral expected present value of its payout will be the price of the derivative.This is also the sum of the construction cost of the derivative by summing up all the delta-hedging efforts at present value. For an example see the exercise for the CRR model.

***14.6.*** *What is implied volatility? How can it be calculated?*

If an option price can be observed (say it is 8 dollars) one can find a volatility that would make the Black-Scholes formula to have the same result. In this sense the option price implies the volatility.

***16.4.*** *A currency is currently worth $0.80 and has a volatility of 12%. The domestic and foreign risk-free interest rates are 6% and 8%, respectively. Use a two-step binomial tree to value (a) a European four-month call option with a strike price of 0.79 and (b) an American four-month call option with the same strike price.*

Assuming the option price will be paid in dollar. Assuming dollar is the foreign currency.  
r\_USD = 8% PUSD = 1/(1+8%)^(2/12) = 98,73%  
r\_ccy = 6% Qccy = 1/(1+6%)^(2/12) = 99,03%

u = exp (12%\*) = 1.0502

d= 1/u = 1/1.05 = 0.9522

q = (Q/P – d) / (u – d) = 0,5188

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | 0,882 |
|  |  | 0,8402 | 0,8 |
| FX rate | 0,8 | 0,7618 | 0,725 |
|  |  |  |  |
|  |  |  | 0,092 |
|  |  | 0,0521 | 0,01 |
| European call | 0,0291 | 0,0051 | 0 |

So the European call worth 0,0291 dollars for each currency in the option notional.

An American call must worth at least this much, however it could worth even more. We should check for early exercise today and 2 month later.

Today exercising the call makes no sense: we would get 0,8-0,79 = 0,100 but the option has much higher value of 0,0291. Two month later if the spot is 0,7618 it makes no sense exercising since the option hhas no intrinsic value. If the spot fx is at 0,8402 then an early exercise would give 0,8402-0,7900 = 0,0502 value, however not exercising would give us 0,0521.

Since in this particular case there would never be an early exercise the American option should worth the same as the European.

***16.20.*** *What is the put–call parity relationship for European currency options?*

QS – PK = fwd = call - put

***18.5.*** *What is meant by the gamma of an option position? What are the risks in the situation where the gamma of a position is highly negative and the delta is zero?*

Gamma is the second derivative in respect to the spot price. Gamma is the change of delta if the spot is changing. If gamma is negative then our delta will change the opposite way as the spot moves. If spot moves up, our delta will decrease thus forcing a dynamic hedger to buy some. If spot goes down the negative gamma makes our delta positive thus forcing a dynamic hedger to sell some underlying. In case of a jump or drop this could be very inconvenient.

***18.15.*** *Under what circumstances is it possible to make a European option on a stock index both gamma neutral and vega neutral by adding a position in one other European option?*

For any vanilla option the following equation holds:

Vega = Gamma \* S2*σ2(T-t)*

So if all options would have the same T expiry time then making a portfolio vega-neutral automatically makes the portfolio gamma-neutral too.

WARNING: if options in the portfolio have different expiry dates it could easily occur that gamma and vega have different signs.

***18.21.*** *Does a forward contract on a stock index have the same delta as the corresponding futures contract? Explain your answer.*

Their deltas are close to each other, but most of the time not the same:

futures = F – K = S Q/P – K

delta\_futures = Q/P

forward = QS – PK

delta\_forward = Q

If P≠1 then the forward and the futures will have different deltas.

***18.22.*** *A bank’s position in options on the dollar/euro exchange rate has a delta of 30,000 and a gamma of 80,000. Explain how these numbers can be interpreted. The exchange rate (dollars per euro) is 0.90. What position would you take to make the position delta neutral? After a short period of time, the exchange rate moves to 0.93. Estimate the new delta. What additional trade is necessary to keep the position delta neutral? Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from*

*the exchange-rate movement?*

delta = 30000 means that at this moment the portfolio would behave as if it would be made of 30000 spot position. So if the market moves up from 0,90 to 0,91 it will make 300 dollars profit.

gamma = 80000 means that as the spot moves up, the delta will increase by 800 per 0.01 price changes. So if the delta is 30000 when the spot FX rate is 0,9 then the delta would be 30800 if the spot would move up to 0.91.

If we would like to make the position delta-neutral we should sell 30000 EUR. This position alone will have a delta of -30000.

If the market would move up to 0.93 our total delta would be approx. (30000+ (0,93-0,90)\*80000) - 30000 = 32400 – 30000 = 2400. It would make sense to sell additional 2400 EUR (assuming we have already sold 30000 EUR)

***18.23.*** *Use the put–call parity relationship to derive, for a non-dividend-paying stock, the*

*relationship between:*

*(a) The delta of a European call and the delta of a European put*

*(b) The gamma of a European call and the gamma of a European put*

*(c) The vega of a European call and the vega of a European put*

*(d) The theta of a European call and the theta of a European put.*

QS – PK = fwd = call – put

a)

delta\_fwd = delta\_call – delta\_put

delta\_put = delta\_call – delta\_fwd = QN(d1) – Q = Q\* (N(d1)-1)

b)

gamma\_fwd = gamma\_call – gamma\_put

gamma\_fwd = 0 //the forward is linear!

gamma\_call = gamma\_put

c)

vega\_fwd = vega\_call – vega\_put

vega\_fwd = 0 //the forward position value is independent from the volatility

vega\_call = vega\_put

d)

theta\_fwd = theta\_call – theta\_put

theta\_forward = qQS - rPK

theta\_put = theta\_call -theta\_fwd

theta\_put =