HW 2 – 4803, Fall 2024
Each problem is worth 10 points

1 Part I – theoretical problems

1. Suppose we have a dataset that consists of n observations of the outcomes y_i of a Poisson process. For such a process, y_i must be a non-negative integer, and the probability distribution for any given y is the Poisson distribution
$$P(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!} \ ,$$
where $\lambda > 0$.

   Now suppose we want to estimate what the parameter λ from the above equation is for the Poisson process that generated our data.

   (a) Show that the conjugate prior to the Poisson distribution above is the Gamma distribution with hyperparameters α and β:
$$P(\lambda) = \frac{\lambda^{\alpha-1} e^{-\beta\lambda} \beta^{\alpha}}{\Gamma(\alpha)} \ ,$$
where $\Gamma(x)$ is the gamma function (but this is not so important here). That is, if we choose our prior for λ to be the Gamma distribution, then the posterior for λ will also be a Gamma distribution with some new hyperparameters α' and β'. Find formulas for α' and β'.

   (b) Suppose after forming our posterior $P(\lambda|y)$ we are then asked to predict the outcome z of another observation from this same Poisson process. Explain why a logical way to do so is via the *posterior predictive* distribution:
$$P(z|y) = \int_0^{\infty} P(z|\lambda) P(\lambda|y) d\lambda \ .$$
Compute the posterior predictive distribution in this case (hint: it is a negative binomial distribution).

2-5. From the Book "An Introduction to Statistical Learning"– 4.8 Exercises 6,7,8,12

6-8. From the Book "The Elements of Statistical Learning"– Chapter 3 Exercises 3.12, 3.28, and 3.29. These problems might be challenging!

2 Part II – programming

Programming Problems 1-2 From the Book "An Introduction to Statistical Learning"– 4.8 Exercises 14 and 16.