HW 5 – 4803, Fall 2024

Each problem is worth 10 points except programming problem 2, worth 20

1 Part I – theoretical problems

1. From the Book "An Introduction to Statistical Learning"– 10.10 Exercise 2. While this problem is from the Neural Networks chapter that we skipped, note that this problem does not depend on any knowledge of Neural Networks and relates to other course topics.

2-3. From the Book "An Introduction to Statistical Learning"– 12.6 Exercises 1 and 3

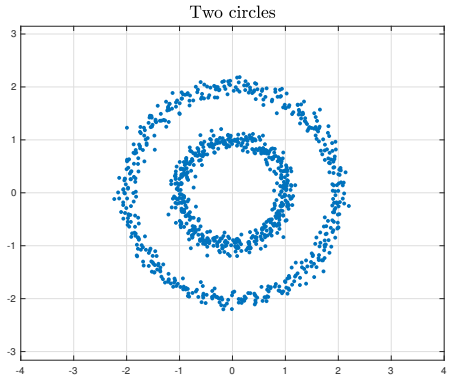
4. Consider an n-complete graph, with vertex $\{v_1, v_2, \ldots, v_n\}$. Every vertex is connected with the other $n-1$ vertices, and the connected edge has weight 1.

    1. What is the graph Laplacian?
    2. What are the eigenvalues and eigenvectors of the graph Laplacian?

2 Part II – programming

1. From the Book "An Introduction to Statistical Learning"– 12.6 Exercise 10

2. Generate the following data set near two circles. The circles are centered at the origin and have radius 1 and 2 respectively.



We will generate data in the following steps:

- First generate 200 samples on each circle. Let us parameterize the circles as $x_1(t) = r\cos t;\ x_2(t) = r\sin t$ where $r = 1, 2$ respectively. For each circle, 200 uniform samples of $t \in [0, 2\pi)$ give rise to 200 points on the circle.
- Add Gaussian noise to each sample above. The noise vector is $[n_1\ n_2]$ where $n_1, n_2 \sim \text{Normal}(0, \sigma^2)$, where $\sigma = 0.05$.

(a) Apply K means with $K = 2$, and display the clustering results.

(b) Apply spectral clustering and cluster this data set into 2 clusters. Construct the ϵ-neighborhood graph, and display the clustering results with three choices of ϵ. Try a large ϵ, a proper ϵ, and a small ϵ. What is a good range of ϵ such that we can cluster the two circles?

(c) Apply spectral clustering and cluster this data set into 2 clusters. Construct the k- nearest neighbor graph, and display the clustering results with three choices of k. Try a large k, a proper k, and a small k. What is a good range of k such that we can cluster the two circles?

(d) Repeat the experiments in (b) and (c) where the data set has larger noise: $\sigma = 0.2$.