Group Homework 01

Multi-class Classification

In this exercise, you will implement one-vs-all logistic regression to recognize hand-written digits (from 0 to 9). Automated handwritten digit recognition is widely used today - from recognizing zip codes (postal codes) on mail envelopes to recognizing amounts written on bank checks. This exercise will show you how the methods you've learned can be used for this classification task. In this exercise, you will extend your previous implementation of logistic regression and apply it to one-vs-all classification.

Throughout the exercise, you will be using the scripts **ghw1.m**. This script sets up the dataset for the problems and makes calls to functions that you will write. You do not need to modify this script. You are only required to modify functions in other files, by following the instructions in this assignment.

Part 1: Dataset (0 points)

You are given a data set in **ghw1data.mat** that contains 5000 training examples of handwritten digits. The .mat format means that that the data has been saved in a native Octave/Matlab matrix format, instead of a text (ASCII) format like a csv file. These matrices can be read directly into your program by using the load command. After loading, matrices of the correct dimensions and values will appear in your program's memory. The matrix will already be named, so you do not need to assign names to them. There are 5000 training examples in **ghw1data.mat**, where each training example is a 20 pixel by 20 pixel grayscale image of the digit. Each pixel is represented by a floating point number indicating the grayscale intensity at that location. The 20 by 20 grid of pixels is "unrolled" into a 400-dimensional vector. Each of these training examples becomes a single row in our data matrix **X**. This gives us a 5000 by 400 matrix **X** where every row is a training example for a handwritten digit image.

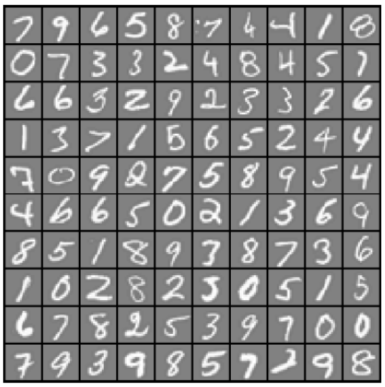
$$X = \begin{bmatrix} — (x^{(1)})^T — \\ — (x^{(2)})^T — \\ \vdots \\ — (x^{(m)})^T — \end{bmatrix}$$

The second part of the training set is a 5000-dimensional vector **y** that contains labels for the training set. To make things more compatible with Octave/Matlab indexing, where there is no zero index, we have mapped the digit zero to the value ten. Therefore, a "**0**" digit is labeled as "**10**", while the digits "**1**" to "**9**" are labeled as "**1**" to "**9**" in their natural order.

Part 2: Visualizing the data (0 points)

You will begin by visualizing a subset of the training set. In Part 1 of **ghw1.m**, the code randomly selects 100 rows from **X** and passes those rows to the **displayData** function. This

function maps each row to a 20 pixel by 20 pixel grayscale image and displays the images together. We have provided the **displayData** function, and you are encouraged to examine the code to see how it works. After you run this step, you should see an image like this:



Part 3: Vectorizing the Cost Function (10 points)

You will be using multiple one-vs-all logistic regression models to build a multi-class classifier. Since there are 10 classes, you will need to train 10 separate logistic regression classifiers. To make this training efficient, it is important to ensure that your code is well vectorized. In this section, you will implement a vectorized version of logistic regression that does not employ any for loops. You can use your code in the last exercise as a starting point for this exercise.

We will begin by writing a vectorized version of the cost function. Recall that in (unregularized) logistic regression, the cost function is

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\log(h_\theta(x^{(i)})) - (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right]$$

To compute each element in the summation, we have to compute theta $h_\theta\left(x^{(i)}\right)$ for every example i, where $h_\theta\left(x^{(i)}\right) = g\left(\theta^T x^{(i)}\right)$ and $g(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function. It turns out that we can compute this quickly for all our examples by using matrix multiplication. Let us denote:

$$X = \begin{bmatrix} — (x^{(1)})^T — \\ — (x^{(2)})^T — \\ \vdots \\ — (x^{(m)})^T — \end{bmatrix} \quad \text{and} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

Then, by computing the matrix product, we have

$$X\theta = \begin{bmatrix} — (x^{(1)})^T\theta — \\ — (x^{(2)})^T\theta — \\ \vdots \\ — (x^{(m)})^T\theta — \end{bmatrix} = \begin{bmatrix} — \theta^T(x^{(1)}) — \\ — \theta^T(x^{(2)}) — \\ \vdots \\ — \theta^T(x^{(m)}) — \end{bmatrix}$$

Your job is to write the unregularized cost function in the file **lrCostFunction.m.** Your implementation should use the strategy we presented above. You should also use a vectorized approach for the rest of the cost function. A fully vectorized version of **lrCostFunction.m** should not contain any loops. (Hint: You might want to use the element-wise multiplication operation (**.***) and the sum operation sum when writing this function).

Part 4: Vectorizing the gradient (10 points)

Recall that the gradient of the (unregularized) logistic regression cost is a vector where the jth element is defined as

$$\frac{\partial J}{\partial \theta_j} = \frac{1}{m}\sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}\right)$$

To vectorize this operation over the dataset, we start by writing out all the partial derivatives explicitly for all theta,

$$\begin{bmatrix} \frac{\partial J}{\partial \theta_0} \\ \frac{\partial J}{\partial \theta_1} \\ \frac{\partial J}{\partial \theta_2} \\ \vdots \\ \frac{\partial J}{\partial \theta_n} \end{bmatrix} = \frac{1}{m}\begin{bmatrix} \sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)}\right) \\ \sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x_1^{(i)}\right) \\ \sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x_2^{(i)}\right) \\ \vdots \\ \sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x_n^{(i)}\right) \end{bmatrix}$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x^{(i)}\right)$$

$$= \frac{1}{m}X^T(h_\theta(x) - y).$$

where

$$h_\theta(x) - y = \begin{bmatrix} h_\theta(x^{(1)}) - y^{(1)} \\ h_\theta(x^{(2)}) - y^{(2)} \\ \vdots \\ h_\theta(x^{(1)}) - y^{(m)} \end{bmatrix}.$$

Note that $x^{(i)}$ is a vector, while $h_\theta\left(x^{(i)}\right) - y^{(i)}$ is a scalar (single number). To understand the last step of the derivation, let $\beta_i = (h_\theta(x^{(i)}) - y^{(i)})$ and observe that:

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$$\sum_i \beta_i x^{(i)} = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix} = X^T \beta,$$

where the values $\beta_i = (h_\theta(x^{(i)}) - y^{(i)})$.

The expression above allows us to compute all the partial derivatives without any loops. If you are comfortable with linear algebra, we encourage you to work through the matrix multiplications above to convince yourself that the vectorized version does the same computations. You should now implement the partial derivative equation to compute the correct vectorized gradient. Once you are done, complete the function **lrCostFunction.m** by implementing the gradient.

Part 5: Vectorizing regularized logistic regression (10 points)

After you have implemented vectorization for logistic regression, you will now add regularization to the cost function. Recall that for regularized logistic regression, the cost function is defined as:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left[-y^{(i)} \log(h_\theta(x^{(i)})) - (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_j^2$$

Note that you should not be regularizing θ_0 which is used for the bias term.

Correspondingly, the partial derivative of regularized logistic regression cost for θ_j is defined as

$$\frac{\partial J(\theta)}{\partial \theta_0} = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} \qquad \text{for } j = 0$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \left(\frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} \right) + \frac{\lambda}{m} \theta_j \qquad \text{for } j \geq 1$$

Now commend out the code for unregularized logistic regression (still need the code for grading) modify your code in **lrCostFunction** to account for regularization. Once again, you should not put any loops into your code.

Part 6: One-vs-all Classification (10 points)

In this part of the exercise, you will implement one-vs-all classification by training multiple regularized logistic regression classifiers, one for each of the K classes in our dataset. In the handwritten digits dataset, K = 10, but your code should work for any value of K. You should now complete the code in **oneVsAll.m** to train one classifier for each class. In particular, your code should return all the classifier parameters in a matrix $\Theta \in \Re^{K \times (N+1)}$, where each row corresponds to the learned logistic regression parameters for one class. You can do this with a

"**for**"-loop from 1 to K, training each classifier independently. Note that the **y** argument to this function is a vector of labels from 1 to 10, where we have mapped the digit "0" to the label 10 (to avoid confusions with indexing).

When training the classifier for class $k \in \{1,\ldots,K\}$, you will want a m-dimensional vector of labels **y**, where $y_j \in \{0,1\}$ indicates whether the j-th training instance belongs to class k ($y_j = 1$), or if it belongs to a different class ($y_j = 0$). You may find logical arrays helpful for this task. Logical arrays in Octave are arrays which contain binary (0 or 1) elements. In Octave, evaluating the expression a == b for a vector a (of size $m \times 1$) and scalar b will return a vector of the same size as a with ones at positions where the elements of a are equal to b and zeroes where they are different. To see how this works for yourself, try the following code in Octave:

```
a = 1:10; % Create a and b
b = 3;
a == b % You should try different values of b here
```

Furthermore, you will be using **fmincg** for this exercise (instead of **fminunc**). **fmincg** works similarly to **fminunc**, but is more efficient for dealing with a large number of parameters. After you have correctly completed the code for **oneVsAll.m**, the script **ghw1.m** will continue to use your **oneVsAll** function to train a multi-class classifier.

Part 7: One-vs-all Prediction (10 points)

After training your one-vs-all classifier, you can now use it to predict the digit contained in a given image. For each input, you should compute the "probability" that it belongs to each class using the trained logistic regression classifiers. Your one-vs-all prediction function will pick the class for which the corresponding logistic regression classifier outputs the highest probability and return the class label (1, 2, ... , or K) as the prediction for the input example.

You should now complete the code in **predictOneVsAll.m** to use the one-vs-all classifier to make predictions. Once you are done, **ghw1.m** will call your **predictOneVsAll** function using the learned value of theta. You should see that the training set accuracy is about **94.9%** (i.e., it classifies 94.9% of the examples in the training set correctly).

Submission:

To submit, turn in the following files on Canvas:

- **lrCostFunction.m**
- **oneVsAll.m**
- **predictOneVsAll.m**