Problem Set 1

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All code and raw results are available at `https://github.com/Arongil/6.S098/tree/main/pset1`.

1. (Bonds) The idea is to formulate a linear program which encodes the constraints on the price discounting. Then to find $d^{\max}$ and $d^{\min}$, we solve $2n$ linear programs of the form

$$\begin{aligned} &\max\ [0 \cdots \pm 1 \cdots 0]^T p \\ &\text{subject to } C^T p = b,\ Dp \geq 0, \end{aligned}$$

   where C is the bond matrix (rows are bonds, columns are payment schedule) and D is the matrix

$$D = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix},$$

   which serves to enforce both the nonincreasing and nonnegativity conditions.

2. (Permutations) We use the constraints

$$\begin{aligned} &P \geq 0, \\ &P\mathbb{1} = \mathbb{1}, \\ &P^T\mathbb{1} = \mathbb{1}, \\ &PA = BP, \end{aligned}$$

   where the final constraint is a rewriting of $PAP^{-1} = B$ that makes use of $P^T = P^{-1}$ for permutation matrices. The resulting permutation vector is (6. 22. 27. 19. 15. 3. 11. 25. 21. 23. 9. 4. 26. 16. 12. 17. 30. 1. 20. 7. 5. 24. 18. 8. 10. 13. 2. 28. 14. 29.) For proof that matrix elements were near to integers, we report $\max p_{ij}(1 - p_{ij}) = 1.87 \times 10^{-10}$.

3. (Reformulations) We present our reformulations. We have tested each and successfully shown them all to be DCP.

   (a) Use constraints $x \geq 0, y \geq 0, cp.harmonic\_mean(cp.hstack([x, y])) \geq 2$.

   (b) Use constraints $x \geq 0, y \geq 0, cp.inv\_prod(cp.hstack([x, y])) \leq 1$.

   (c) Use constraints $y \geq 0, cp.quad\_over\_lin(x + y, cp.sqrt(y)) \leq x - y + 5$.

   (d) Let $GM(x, y)$ denote $cp.geo\_mean(cp.hstack([x, y]))$.
   Use constraints $x \geq 0, y \geq 0, x + z \leq 1 + GM(GM(x, y) - z, GM(x, y) + z)$.

4. (Runtime) We report the desired data in a table. From top to bottom, the rows are for the exponential, linear, and built-in constraints. From left to right, the columns are for $n = 2, 5, 10$.

0.000016	0.000074	0.004215
0.000018	0.000031	0.000046
0.000022	0.000030	0.000047