SSY130 - Project 1 - Acoustic Communication System

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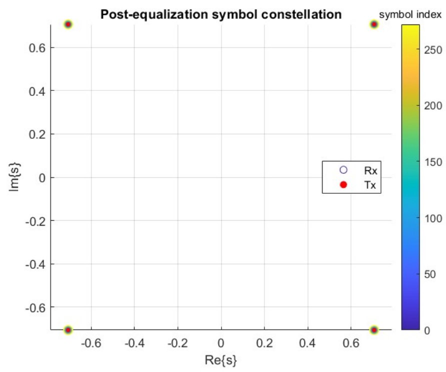
1 System with linear channel

(a) For this task the ideal known channel h_1 is used, no noise is added and there is no synchronization error. Since h_1 is ideal the received symbols are exactly the same before and after equalization because there is nothing to equalize as there is no gain in an ideal channel. The EVM for this case is 3.01e-16 and the BER is 0, if N = 272.



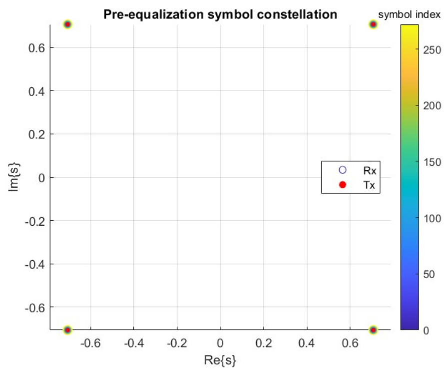


Figure 1: Pre- & Post-equalization symbol constellation for h_1

(b) The cyclic prefix doesn't have any influence on our communication system as long as the channel is ideal and there is no noise. In an ideal channel, the messages don't have to be protected. Therefore the cyclic prefix is redundant. This can be seen in the program as adding a cycling prefix doesn't change the EVM or BER. The cyclic prefix is added to guard against symbol interference by adding a copy of the signals end to the beginning to protect against delays and noise. By choosing the cyclic prefix to the channel's length - 1 we ensure that the cyclic prefix is sufficient to cover the maximum expected delay.

(c) In h_2 the pre-equalization values are scaled down by a factor 2, thereby $\alpha = 0.5$. This can be seen in the right part of *Figure 2* below. The EVM is equal to 3.01e-16 and the BER is 0, i.e. the same values as for h_1. This is true for N = 272.

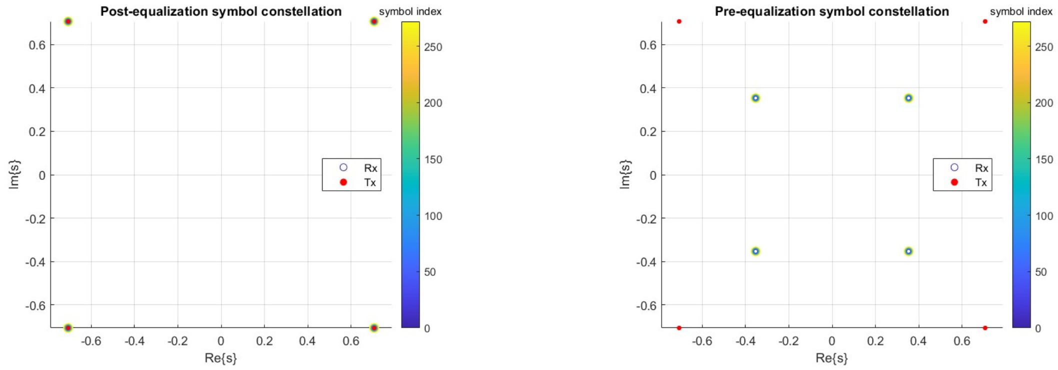


Figure 2: Pre- & Post-equalization symbol constellation for h_2

For the system with the channel h_3 the pre-equalization values are phase shifted by 1/2 radians, thereby $\alpha = e^{j/2}$. This can be seen in *Figure 3* below. It can also be noticed that there is no amplification occurring within the channel as $|\alpha| = 1$. For this system, the EVM is equal to 3.31e-16 and BER is 0 for N=272. Since h(k) is known and there is no noise, the post-equalization will always match the transmitted signal since we can just revert its impact.

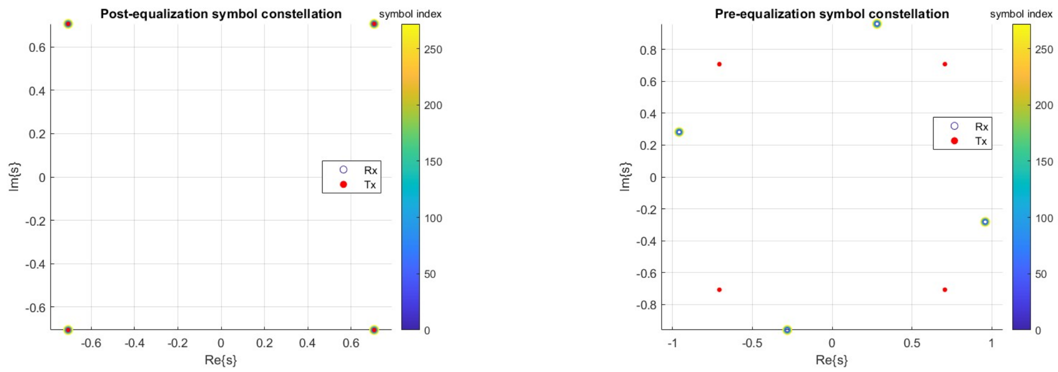


Figure 3: Pre- & Post-equalization symbol constellation for h_3

(d) As hinted in the task the first and last bits of the message is correctly transmitted for $n_{se} = \pm 1$, while the first, middle and last part is transmitted correctly for $n_{se} = \pm 2$. It's noteworthy to mention that the correctly transmitted parts are smaller for $n_{se} = \pm 2$ compared to for $n_{se} = \pm 1$.

This can be explained by analyzing the phase shift, which is defined as $e^{-j2\pi n_{se} f/f_s}$, where n_{se} is the synchronization error. It can firstly be noted that if n_{se} is positive it will result in a negative exponent and thus leading to a delay, while if n_{se} is negative it results in a positive exponent and thus leading to early reception. Furthermore n_{se} will determine how large period the phase shift will have. If $n_{se} = \pm 1$ the period is 2π or 1 with respect to the relative frequency. If $n_{se} = \pm 2$ the period is π or $\frac{1}{2}$ with respect to the relative frequency.

Given that the relative frequency f/f_s takes values $[0/N, 1/N, 2/N, ...(N-1)/N]$ which corresponds to the interval [0, 1) for large N, the phase shift is small for subcarrier frequencies close to the phase shifts periods boundary's. Subcarrier frequencies that are in between the phase shifts period are conversely mangled. This demonstrates why the system has three correctly transmitted zones for $n_{se} = \pm 2$ and only two for $n_{se} = \pm 1$.

Considering the system is QPSK a phaseshift more than $\frac{\pi}{4}$ away from the middle intended quadrant will result in the signal being magled. If the frequency f is in the intended quadrant it will be in the interval shown below:

$$-\frac{\pi}{4} \leq 2\pi n_{se}\frac{x}{N} + 2\pi n \leq \frac{\pi}{4}, \quad n \in \mathbb{Z}$$

where x is frequency component $x \in [0, 1, 2, ..., N-1]$ and $n2\pi$ is a term used for the period.

(e) We see that both EVM and BER become worse when the signal-to-noise ratio decreases which is reasonable since the noise may make the bits change quadrants which increases the BER and since this also increases the error of the received signal, the EVM will increase. Since the noise is added to the signal after the channel is applied, some values might change quadrants which causes them to be translated to the wrong bits. We see that when the SNR is 30 no bits are translated wrong, while some are translated wrong when SNR is 5.

SNR	EVM	BER
30	0,0275	0
5	0,508	0,0368

2 Part 2

(a) We can somewhat see that we get 4 circles in the pre-equalization plot. Since the amplitude of H(1)=5 and the phase of H(1)=0, which means that the first symbol will only be multiplied by 5 and we will always find 1 symbol at one of the combinations from $5\sqrt{\frac{1}{2}}(\pm 1 \pm j)$ for any sent message. For our case, the coordinate was found at $5\sqrt{\frac{1}{2}}(-1, +j)$.

We also see that some of the early and late values of h(k) have a higher amplitude which is seen in both the $|H(K)|$ and pre-equalization-plot as the symbols in the pre-equalization-plots are further away. in the $Arg(H(k))$ plot we see a phase shift that corresponds to the rotation of the symbols in the pre-equalization-plot, both in a positive and negative direction. An example of a conclusion that can be drawn from the $|H(K)|$ & $Arg(h(k))$ plots is that the phase shift is zero and the amplitude is low for $k = 150$. This is seen in the pre-equalization symbol constellation as values near $k = 150$ are in the correct quadrant and close to $(0, 0)$.

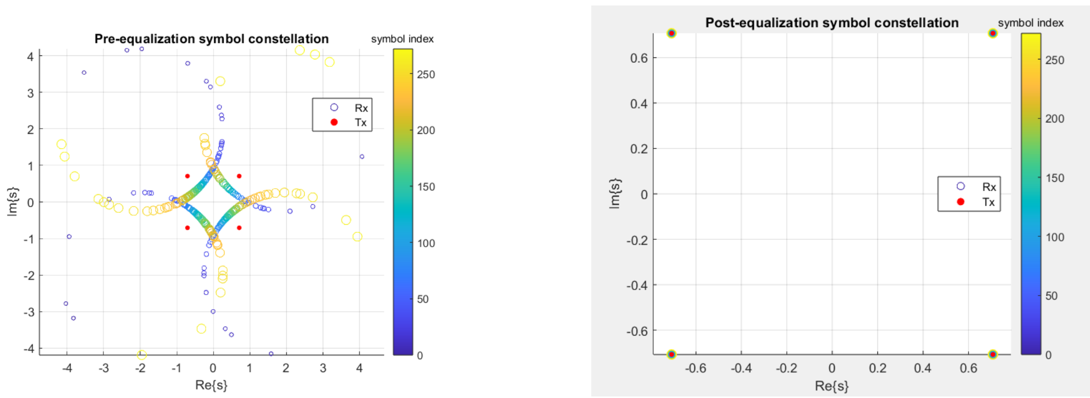


Figure 4: Pre- & Post-equalization symbol constellation for h_4

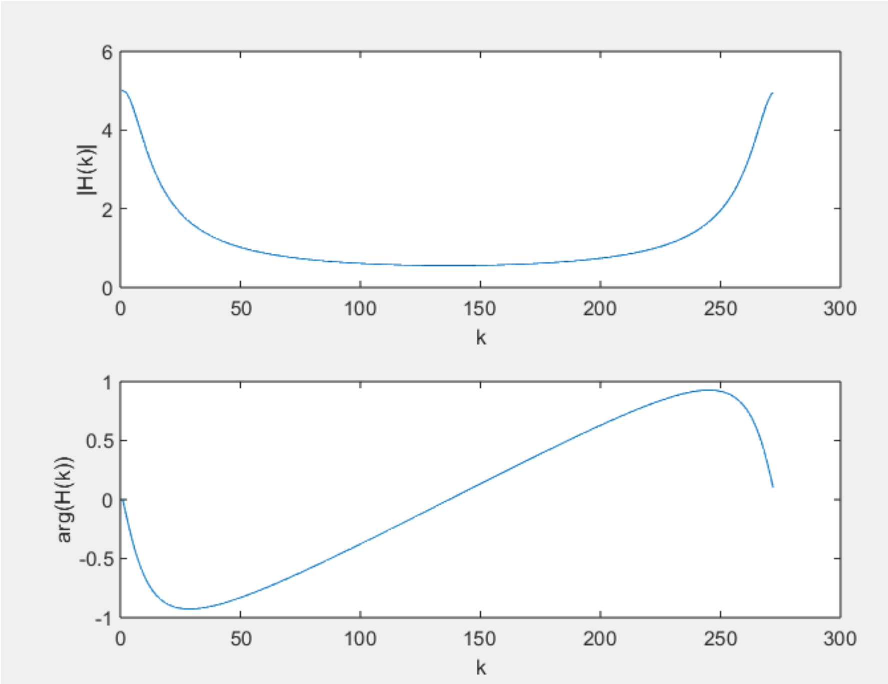


Figure 5: Argument and amplitude for h_4

(b) We see here that the EVM reaches close to zero at $N_{cp} = 59$. The EVM decreases drastically down to $6.52 * 10^{-16}$ which is within the machine accuracy of zero. By choosing the cyclic prefix to the channel's length - 1 we ensure that the cyclic prefix is sufficient to cover the maximum expected delay. BER doesn't change anything by the choice of N_{cp} and remains at zero with the given parameters.

N_{cp}	EVM	BER
0	0,469	0
10	0,011	0
20	$6,11 * 10^{-4}$	0
30	$6,45 * 10^{-5}$	0
50	$3,5 * 10^{-7}$	0
58	$1,69 * 10^{-8}$	0
59	$6,52 * 10^{-16}$	0
60	$6,51 * 10^{-16}$	0
80	$6,63 * 10^{-16}$	0

(c) Here we see that the value of N_{cp} is much more important since both EVM and BER are much higher at lower values. It should however be noted that the EVM is within the machine accuracy for $N_{cp} = 59$.

N_{cp}	EVM	BER
10	0,628	0,0404
20	0,471	0,0129
50	0,102	0
58	$6,4 * 10^{-3}$	0
59	$3,34 * 10^{-15}$	0
60	$3,39 * 10^{-15}$	0
70	$3,39 * 10^{-15}$	0
80	$3,62 * 10^{-15}$	0

3 Part 3

(a) The case with the unknown channel performs better than the case with a known channel. As stated before the known ideal channel is sensitive to phase shifts due to synchronization errors. All phase shifts greater than $\pi/4$ result in an error in transmission. The difference with the unknown channel is that the transmission symbols are initiated with pilot messages. These pilot messages introduce possible synchronization errors and other characteristics of the channel. Then the unknown channel can then adapt to the phase shift caused by the synchronization error and transmit the symbols correctly.

Channel known	EVM	BER
True	1,4	0,496
False	0,132	0

(b) We still need the same cyclic prefix since the length of h remains the same. It can be seen in the table below that machine accuracy is achieved for $N_{cp} = 59$.

N_{cp}	EVM	BER
10	0,0123	0
20	$1,52 * 10^{-3}$	0
30	$1,98 * 10^{-4}$	0
58	$2,62 * 10^{-7}$	0
59	$7,42 * 10^{-16}$	0
60	$7,48 * 10^{-16}$	0

It can be seen in the table below that the system now is more sensitive to noise. This is reasonable since if we don't know which impulse response is used, a greater noise will make it harder to map.

Channel known	SNR	N_{cp}	EVM	BER
False	5	60	2,1	0,248
False	10	60	1,03	0,13
False	30	60	0,085	0
False	50	60	0,00867	0
True	5	60	1,1	0,145
True	10	60	0,647	0,0588
True	30	60	0,0635	0

(c) With h(1)=0,5 and h(9)=0,5 we get the result below:

N_{cp}	EVM	BER
1	0.756	0.0294
5	0.727	0.0221
7	0.36	0.0184
8	0.545	0.011
10	0.545	0.011
60	0.545	0.011
200	0.545	0.011

When we use the original h_5 it does not matter how high N_{cp} we use we will always have some BER and high EVM. If we look at the received message every eighth or ninth character gets translated wrong. This is due to the fact that $|H(k)| = 0$ for some ks. These k:s can be described by the formula: $18 + n34, \quad n \in \mathbb{N}$. This means we will always have some BER.

With h(1)=0,5 and h(9)=0,4 we get the result below:

N_{cp}	EVM	BER
1	0.307	0.00551
5	0.2	0
7	0.11	0
8	$5,8 * 10^{-16}$	0
9	$5,8 * 10^{-16}$	0
60	$5,8 * 10^{-16}$	0

When using h_5' We can choose N_{cp} as 8 to get a good response since there is a delay in h(9). If we again look at the arg(H(k)) plot, the phase shift is not as instant anymore and since $|H(k)| \neq 0$ we can use the cyclic prefix to remove the BER.

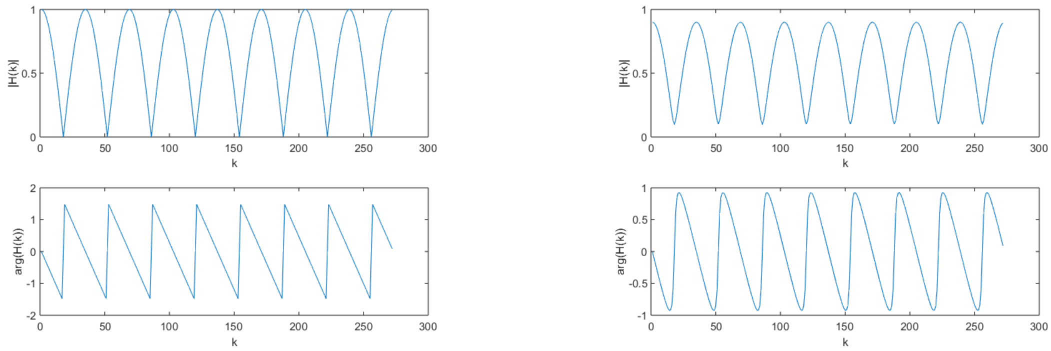


Figure 6: Argument and amplitude for h_5 (to the left) and h_5' (to the right)