SSY130 - Project 1B - Acoustic Communication System

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Part A: Based on Matlab implementation

1 Bandwidth after interpolation and modulation

After interpolation the final sampling frequency, according to the task, is 16 kHz. Considering that the baseband signal is up-sampled by a factor 8, the bandwidth of the signal can be calculated by dividing the final sampling frequency by the factor of interpolation. Resulting in:

$$\text{Bandwidth} = \frac{16\text{ kHz}}{8} = 2\text{ kHz}$$

The baseband center frequency lies at 0 resulting in a bandwidth from -1 kHz to 1 kHz. It is also known that the interpolated signal is modulated with 4 kHz. This moves the center frequency of the baseband to 4 kHz. Resulting in the new bandwidth from $(-1+4)$ kHz to $(1+4)$ kHz $= 3$ kHz to $5kHz$.

2 Non-zero EVM

When using a high up-sampling rate L, and thus an equally large down-sampling rate M the filters H_{LP} will tend to an ideal low pass filter. This is true for both H_{LP} filters for the interpolation and decimation. The filters will however never be completely ideal, which leads to a loss of attenuation/amplitude for the signal. This is something that can't be completely counteracted by the equalization. This explains why there will always be a non-zero EVM for our system.

3 Parts of the OFDM system contributing to channel

- **Interpolation and decimation**

  As long as the bandwidth of the signal fulfills the Nyquist theorem of perfect reconstruction, interpolation or decimation doesn't add any noise to the channel. However, if the bandwidth does not fulfill the Nyquist theorem there will occur aliasing.

- **Modulation and demodulation**

  Modulation and demodulation do not contribute to the channel identification because the shift of the signal is reversible. The modulation in our effective channel initially shifts the signal while the demodulation will shift the signal back to the same phase after the physical channel.

- **Propagation over physical channel**

  Propagation over a physical channel contributes to the channel. A real world signal such as a sound wave can lose amplitude and get distorted. Furthermore the environment the sound wave is traveling in can add stochastic noise.

- **Real part of signal**

  Taking the real part of the signal won't contribute to the estimation of the channel if the bandwidth and the shift due to modulation are correctly determined. If they are not, the signal will be distorted and the channel will be estimated incorrectly. This is described in more detail in the task below.

4 Elimination of imaginary part for complex-valued signal

Our physical channel is in this case air pressure, which can be seen as a scalar field that can't transmit complex-values. To allow for transmission over the channel we have to remove the imaginary part of the complex-valued signal. This can be done by adding the signals complex conjugate to form the expression: $z_r(t) = \frac{1}{2}(y(t) + \overline{y}(t))$ which can be written as $Z_r(\omega) = \frac{1}{2}(Z(\omega) + \overline{Z}(\omega))$.

To understand why we safely can perform this operation, and therefore discard the imaginary part of the signal, we can analyze the signal $Z_r(\omega)$ in the complex plane. We can re-write the expression for $Z_r(\omega)$ as shown below, where $Z_i(\omega)$ is the interpolated signal before modulation and ω_m is the frequency the signal is modulated with.

$Z_r(\omega) = \frac{1}{2}(Z(\omega) + Z^*(-\omega)) = \frac{1}{2}(Z_i(\omega - \omega_m) + Z_i(-\omega + \omega_m))$

It can be seen that the transmitted signal $Z_r(\omega)$ is the sum of the signal we want to send: $Z(\omega) = Z_i(\omega - \omega_m)$ and the complex conjugate $Z_i(-\omega + \omega_m)$. As long as these two signals don't overlap in the frequency domain, the signal $Z(\omega)$ won't be distorted and no information will be lost. As seen in the terms above, the signal Z_i and its conjugate have a positive and conversely a negative argument, meaning that they will traverse and be modulated in different directions. This means that we can guarantee that no distortion will occur for a certain ω_m and bandwidth. This means that the original signal $Z(\omega)$ will be correctly sent within $Z_r(\omega)$.

5 Lowpass filter properties

In this task we will examine what lowpass filter properties are important / less important in the decimation and interpolation stages.

- **Passband ripple -** The passband ripple is a less important property of the lowpass filter for both decimation and interpolation. This is because its effect will be counteracted by equalization where the pilot data is used.

- **Stopband attenuation -** Stopband attenuation is an important property for both decimation and interpolation. In the interpolation, the stopband attenuation determines how well the filter rejects unwanted high-frequency components that have been introduced by the up-sampling. In the decimation, stop-band attenuation is important to effectively band-limit the signal to avoid aliasing by removing signals outside the new Nyquist limit before down-sampling.

- **Transition bandwidth -** The transition bandwidth is important in interpolation as it will determine how effective the filter is in removing unwanted signals created by the upsampling. If the transition bandwidth is too wide, we will have signal energy "leaking" outside the desired signal $Z(\omega)$. In the case of us also sending the conjugate $\overline{Z}(\omega)$, the signal "leakage" of the two signals might overlap and cause aliasing.

  After the demodulation in our effective channel, the filter in the decimation will remove the conjuagate signal $\overline{Z}(\omega)$. The transition bandwidth in the low-pass filter should primarily be small enough to attenuate frequencies outside the Nyquist frequency. In summary, a small transition bandwidth is important for interpolation to avoid signal overlap. It is not as important in decimation as the two signals $Z(\omega)$ and $\overline{Z}(\omega)$ should have space between them, signals over the Nyquist frequency should however be taken into consideration.

- **Phase Linearity -** To reconstruct a transmitted signal, phase linearity is usually quite important. However, since the phase shift can be compensated for by using the pilot message, it is not that important for neither interpolation nor decimation

Part B: Based on DSP-kit

6 Show that $\hat{H}$ and H has same amplitude and phase

The task stated that a channel can be estimated using the equation:

$$\hat{H} = \bar{T} \cdot R$$

We also know that the channel can be estimated using the equation:

$$H = \frac{R}{T}$$

This equation can be manipulated by multiplying with the conjugate of the transmitted signal in the numerator and demominator. Resulting in:

$$H = \frac{R}{T} = \frac{\bar{T}}{\bar{T}} \cdot \frac{R}{T} = \frac{\bar{T} \cdot R}{|T|^2}$$

Furthermore we know that the transmitted signal will be in QPSK-form.

$$T = \begin{cases} \sqrt{1/2}(1+j) \\ \sqrt{1/2}(1-j) \\ \sqrt{1/2}(-1+j) \\ \sqrt{1/2}(-1-j) \end{cases}$$

This entails that $|T|$ squared will always equal one. A conclusion can therefore be drawn that:

$$H = \frac{\bar{T} \cdot R}{1} = \bar{T} \cdot R = \hat{H}$$
$$\equiv$$
$$H = \hat{H}$$

We have now shown that when the transmitted signal is QPSK, the estimated channel using $\bar{T}$ will be equal to the estimated channel using $T \quad \rightarrow \quad \hat{H} = H$. Both channels will thus have the same amplitude and phase-shift.

7 Show that the phase of R_{eq} is correctly equalized

The task states that $R_{eq} = R \cdot \bar{\hat{H}}$. From the previous task we can substitute $\hat{H}$ with $\bar{T} \cdot R$. This grants us $R_{eq} = R \cdot \overline{\bar{T} \cdot R}$. Which simplifies into $R_{eq} = T \cdot |R|^2$. Because $|R|^2$ has no phase shift R_{eq} and T must have the same phase shift and is therefore be correctly equalized.

8 Sending messages with low and high amplitudes

From the constellation diagram with three different amplitudes we can clearly see that a higher amplitude gives a better result with a more similar assumed and received message. In the diagram with higher amplitude, the symbols stay in the correct quadrant while in the diagram with lower amplitude the symbols are distributed more randomly over the quadrants and the EVM increases as the amplitude decreases. This is reasonable since with a lower amplitude it will be harder for the dsp-kit to distinguish the assumed signal from the noise. The difference between high and low amplitude signals can be seen in the plots below.

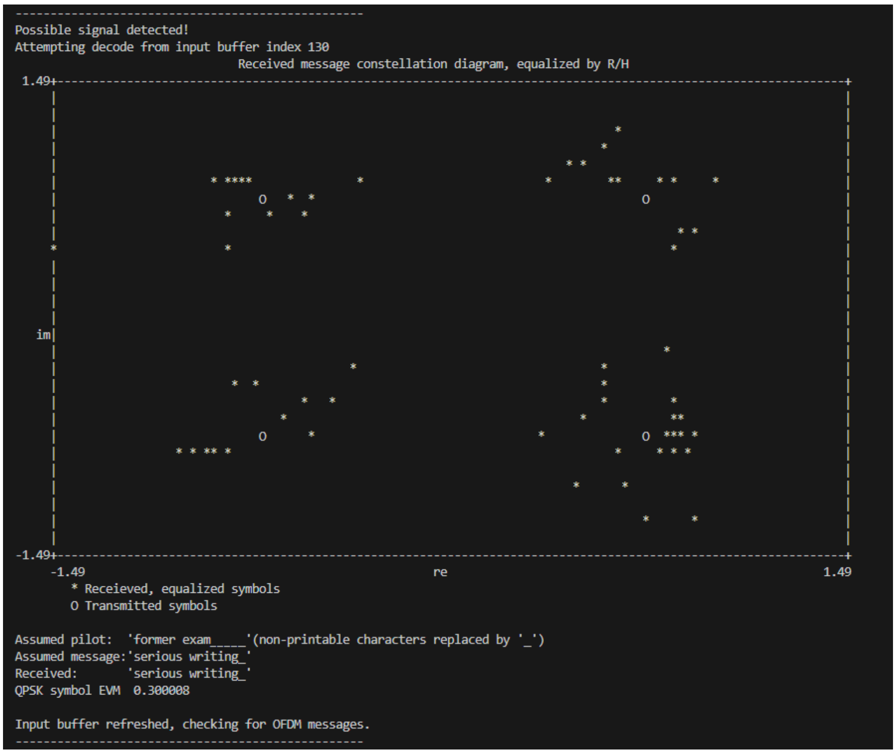

Figure 1: Signal with high amplitude

Figure 2: Signal with low amplitude