SSY230 Learning dynamical systems using system identification

Project 2

Alfred Aronsson, Sotiris Koutsofta

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Introduction

Second project in SSY230 focused on developing functions for estimating transfer functions.

Question 1

(a) For the first task we where tasked with creating a the functions arxfit and uy2phi in order to be able to generate arxmodels from data.

The uy2phi function was validated by checking the produced results against expected results in the provided validation-script. The check confirmed that uy2phi worked correctly.

When it came to estimating arxfit, the validation-script was also used and confirmed that arxfit produced the expected regression-coefficients. Further, arxfit was validated by generating some noise free data passing it through a filter, and doing an arx model on on the output of the filter. The goal is to get the filter-coefficients from the arx model. The filter structure looks like this:

$$y = \frac{b_0 + \cdots + b_N}{1 + a_1 + \cdots + a_N} u \tag{1}$$

The filter we choose looks like this:

$$y = \frac{0.5}{1 - 0.5} u \tag{2}$$

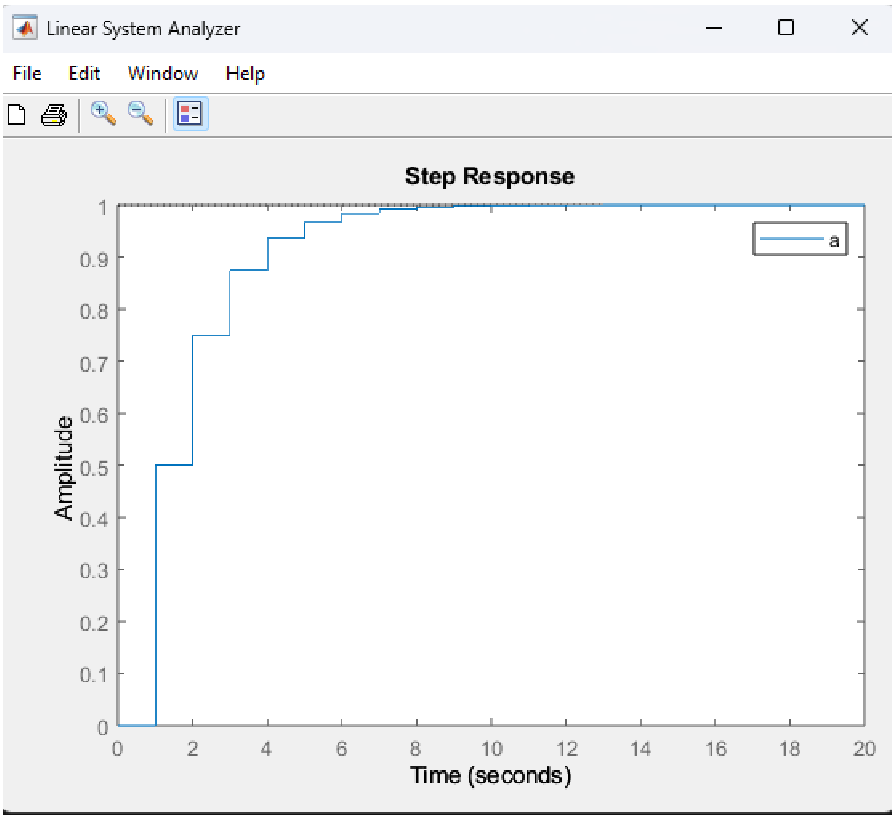
We choose an arbitrary input u of ones $\begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}$ and passed them through the filter. The output from the filter and the input was used to do an arxmodel using arxfit. The resulting model coefficients where:

$$b_0 = 0.5 \tag{3}$$

$$a_1 = 0.5 \tag{4}$$

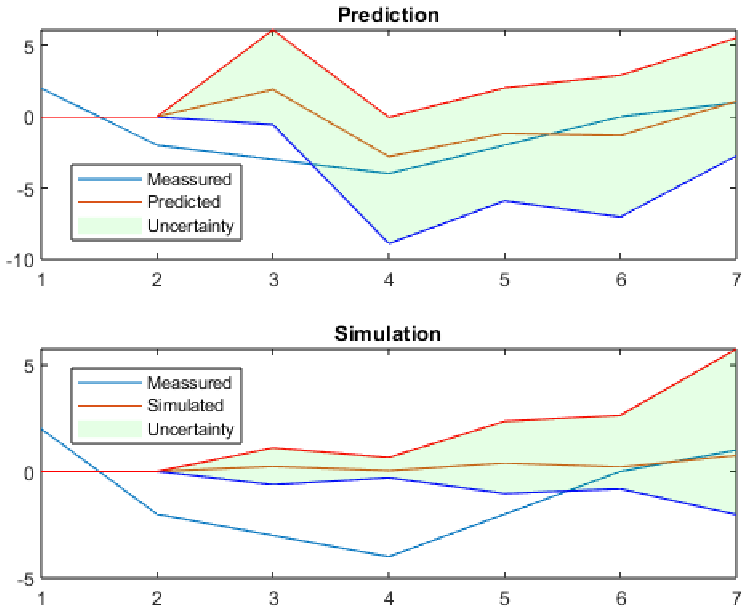
The difference in sign for the a_1 value can and should be attributed to the fact that we was following the convention specified in the validation script when generating the regressors for the uy2phi function.

(b) Here is the step response of the model estimated from the filter data avialable through the generated function id2tf:



(c) The functions idpredict and idsimulate was validated using the validation-script. There it was confirmed that both functions generate the expected results.

(d) Here is the plots produced by the function idcompare:



By comparing this to plots provided in the problem description we can see that they are identical which validates that the idcompare function works correctly.

Question 2

The OE (Output Error) model estimator, written in MATLAB as an `oefit` function, relates the input signals to output signals whilst considering the output error. Our function operates by initially estimating a high-order ARX model to capture the dynamic complexity of the system. The order of the ARX model is set to four times the order of the intended OE model to ensure that the system dynamics are captured. This simulated output representing an initial approximation of the system's response is then used to refine the OE model.Using this simulated output and the original input data, the `oefit` function fits the final OE model at the original specified order. In addition to that, our function categorizes the model type and returns the original model orders within the output structure.

The OE model estimated has showed high accuracy in parameter estimation, closely aligning with the true system coefficients, as shown in the following validation tests.

Results:

Data points = 10000, Noise variance = 0, and Input delay nk = 0

- **True B coefficients:** [1, 1]
- **Estimated B coefficients from OE model:** [1.0000, 1.0000]
- **Estimated B coefficients from ARX model:** [1.0000, 1.0000]
- **True A coefficients:** [-1.0000, -0.1000]
- **Estimated A coefficients from OE model:** [-1.0000, -0.1000]
- **Estimated A coefficients from ARX model:** [-1.0000, -0.1000]

Data points = 10000, Noise variance = 1, and Input delay nk = 0

- **True B coefficients:** [1, 1]
- **Estimated B coefficients from OE model:** [0.9979, 0.9291]
- **Estimated B coefficients from ARX model:** [1.0014, 0.0339]
- **True A coefficients:** [-1.0000, -0.1000]
- **Estimated A coefficients from OE model:** [-0.9292, -0.0983]
- **Estimated A coefficients from ARX model:** [-0.0365, 0.0213]

Question 3

Data Set Analysis included in Exercise1.mat

This part of the project aims to assess the performance of ARX and OE models on the given data set `z`, which has been constructed by merging u and y as [y u] (data in exercise1.mat). The models were evaluated based on their prediction and simulation capabilities, as well as their impulse and step responses, using horizon = 3 in all the plots below. The plots generated are for two different cases, the first one configured with parameters na = 2, nb = 2, and nk = 1 and the second one with na = 3, nb = 3, and nk = 1.

Results:

The following figures illustrate the performance and characteristics of the models:

1. **Prediction and Simulation Performance:** The prediction and simulation plots show the models' ability to forecast and adapt the observed data dynamics. These plots (as follows) provide a comparative analysis of measured versus predicted and simulated outputs, with confidence bounds.

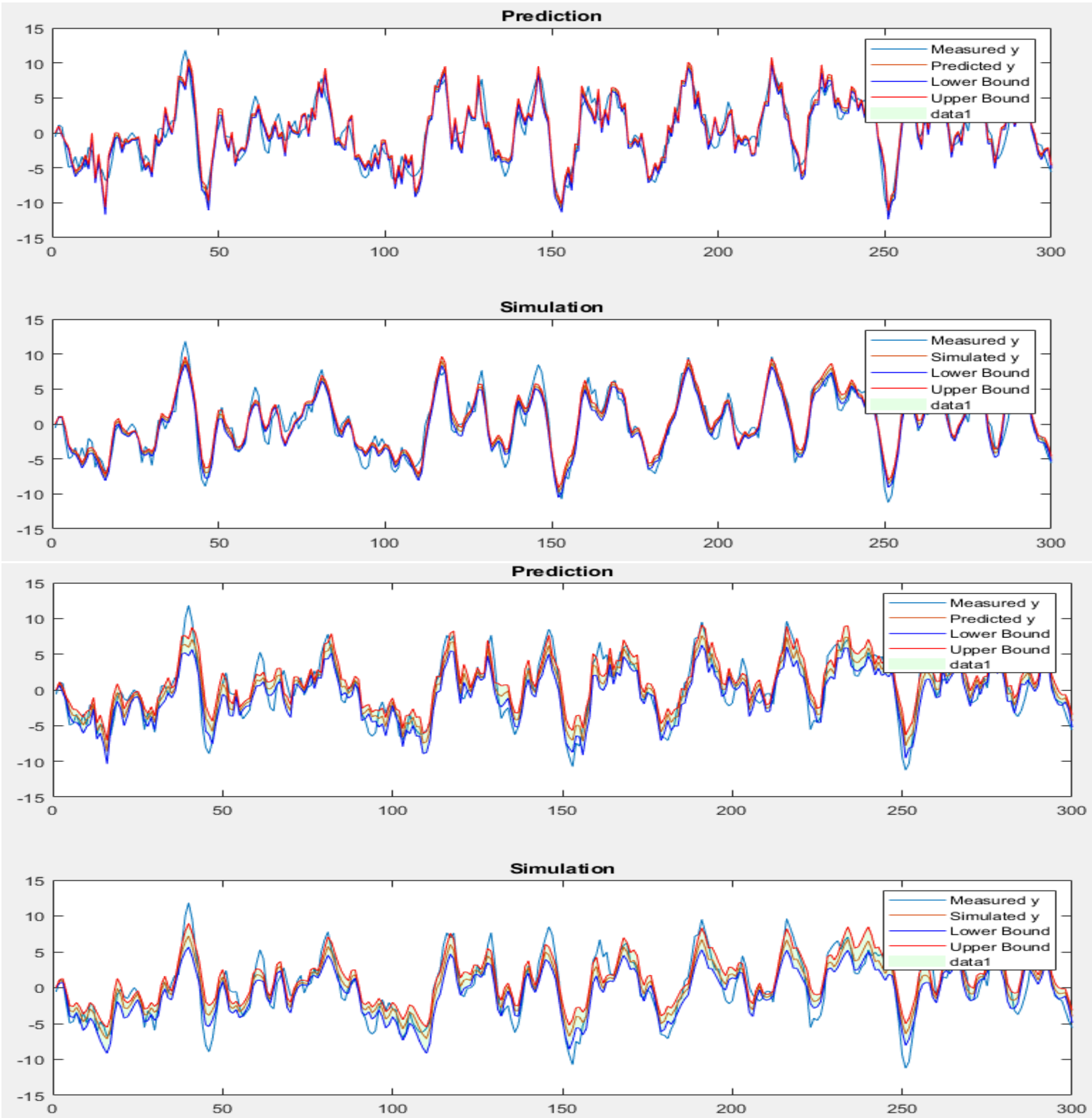


*Figure 1: Simulation performance of ARX and OE models on data set **z** using idcompare.*

2. **Impulse Response:** The impulse response plots compare the instantaneous reaction of the system to a brief external impact.

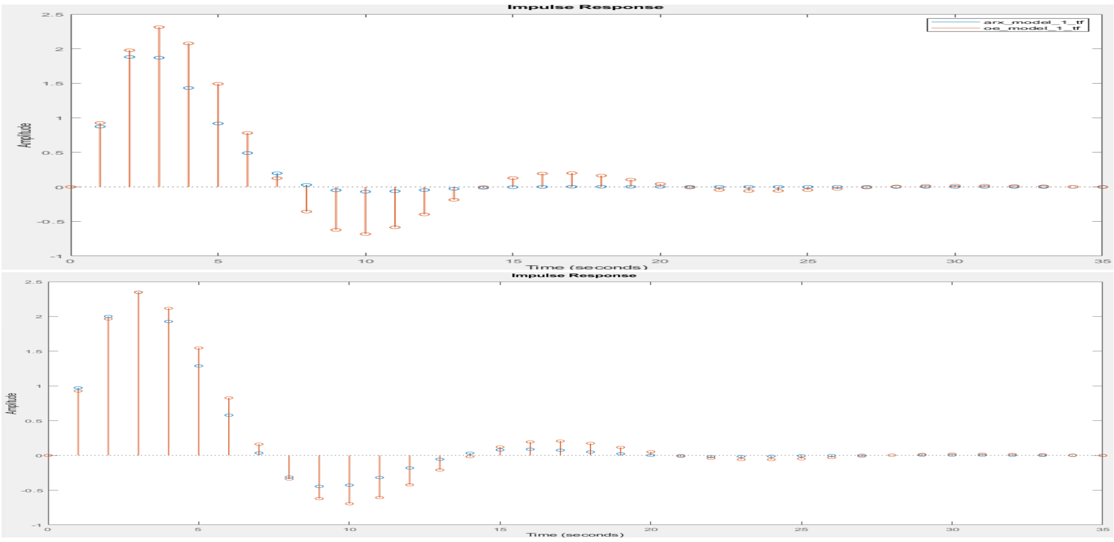


*Figure 2: Impulse response of ARX and OE models respectively for data set **z**.*

3. **Step Response:** The step response analysis helps in understanding how the system responds to a step input.

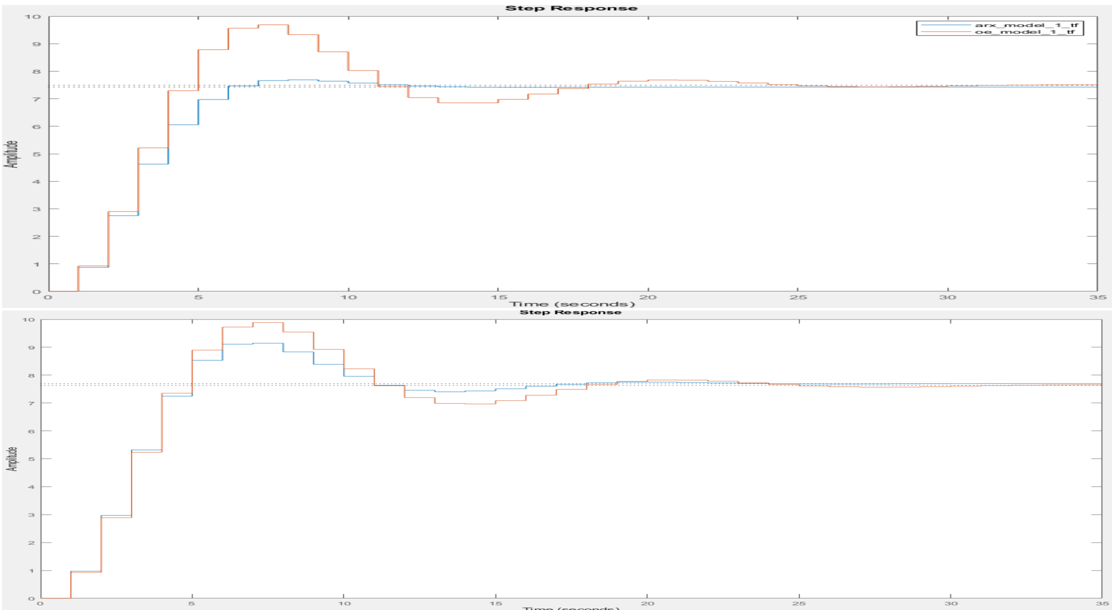


*Figure 3: Step response of ARX and OE models for data set **z**.*

Data Set Analysis included in Exercise2.mat

Data in `z1`

The file Exercise2.mat contains z1 and z2 variables. Models were configured with parameters na = 2, nb = 2, and nk = 1. The ARX and OE models were both trained on the first half of `z1` and `z2` and tested for their predictive accuracy and simulation capabilities on the second half (validation data).

Model Evaluation with data `z1`:

- **Prediction Accuracy**: Comparing the predicted outputs against actual measured outputs.
- **Simulation Fidelity**: Assessing how well the models simulate the system dynamics over the validation dataset.
- **Response Analyses**: Analyzing the step and impulse responses to understand the dynamic characteristics of the models.

Impulse and Step Response:

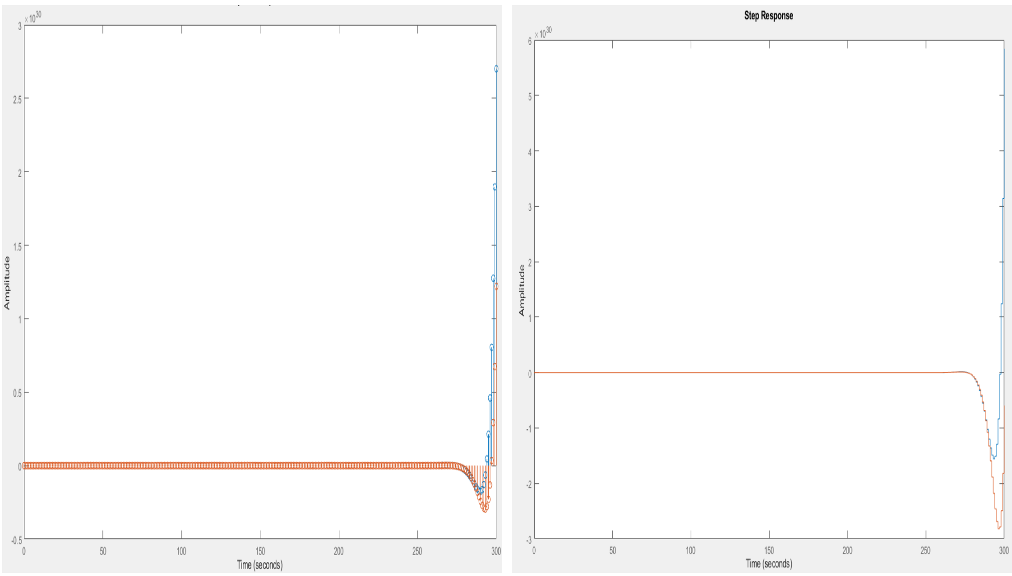


Figure 4: Left: Impulse response; Right: Step response of ARX and OE models using dataset `z1`.

Prediction and Simulation Results:

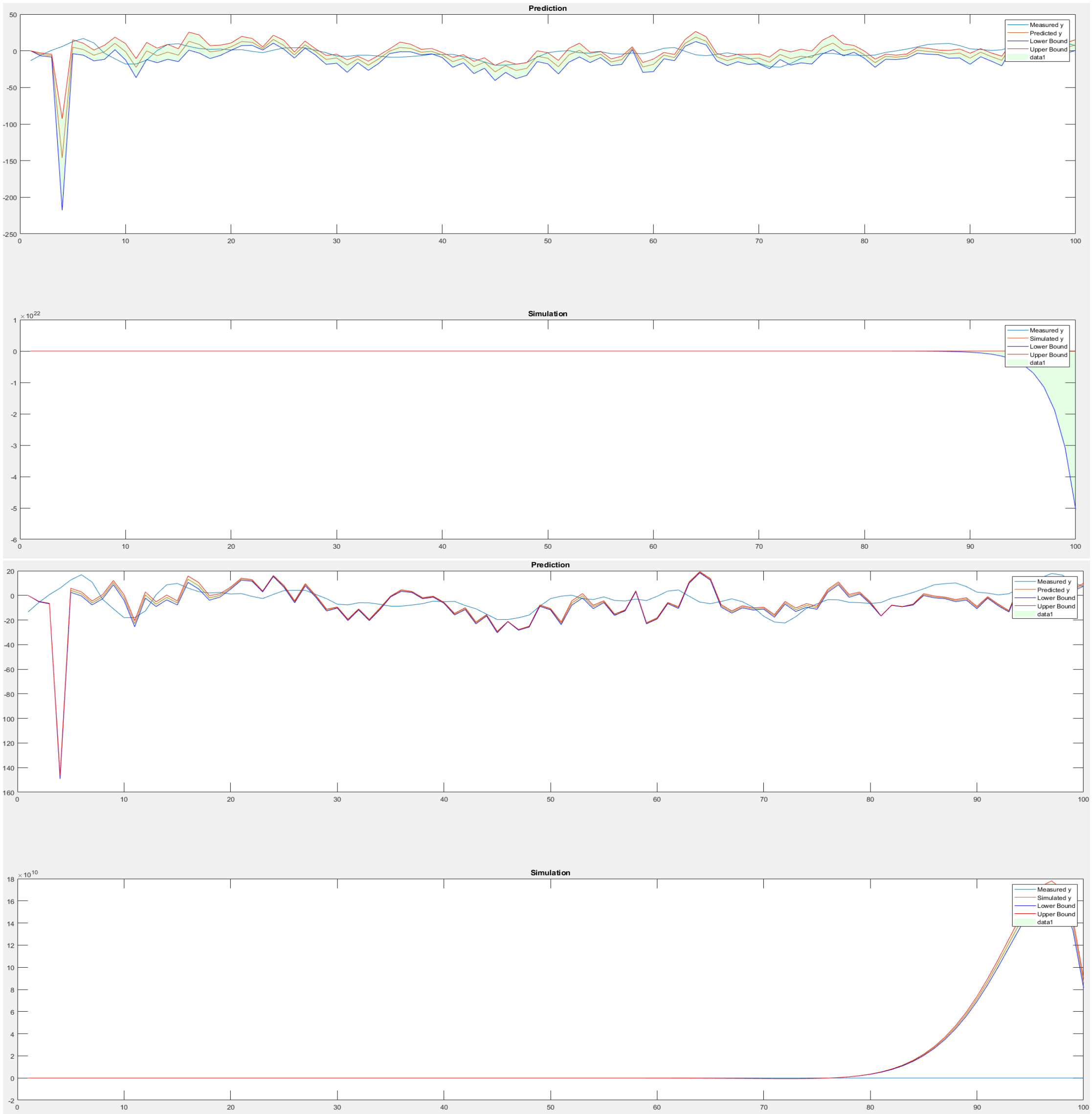


Figure 5: Prediction and Simulation performance for ARX and OE models on dataset `z1` *respectively.*

The evaluation shows the models' abilities to predict and simulate system behavior accurately. The ARX model tends to provide a close fit in simulation scenarios, while the OE model excels in capturing the dynamic nuances in prediction tasks, as indicated by the tighter confidence intervals.

Data in z2

Models were trained using the first half of `z2` and tested on the second half.

Evaluation Methods

- **Prediction Performance**: Assessing the models' ability to predict unseen data accurately.
- **Simulation Performance**: Evaluating the models' capability to simulate the system behavior based on historical data.
- **Dynamic Response**: Analyzing step and impulse responses to understand each model's characteristics under standard input conditions.

Results

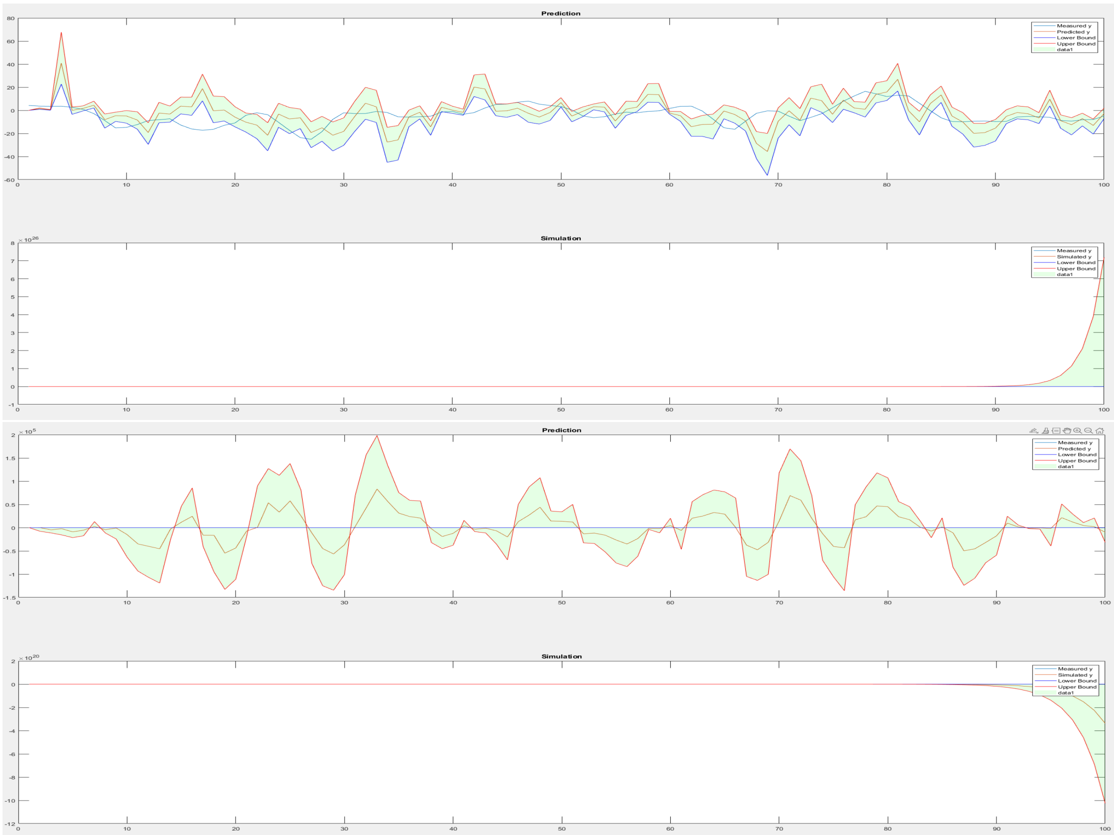


*Figure 6: Top: Prediction performance; Bottom: Simulation performance for ARX and OE models respectively on dataset **z2**.*

Prediction and Simulation Performance

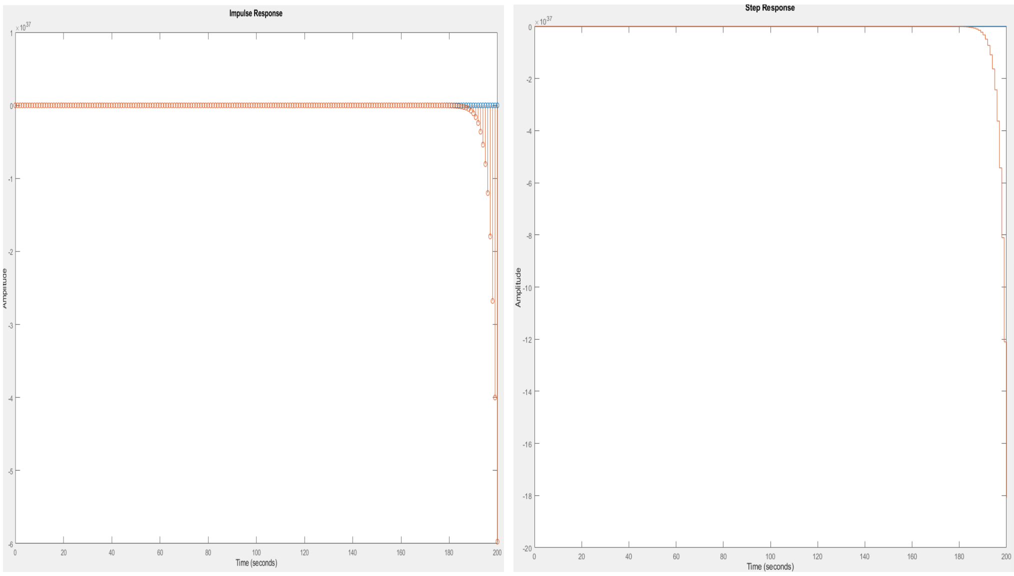


Figure 7: Left: Impulse response; Right: Step response of ARX and OE models for dataset **z2**.

Dynamic Responses The tests show that both ARX and OE models are capable of handling prediction and simulation tasks effectively, though differences in their dynamic responses suggest varying levels of suitability for different system behaviors. The OE model generally shows tighter confidence bounds in predictions, which indicate a more robust capture of the system dynamics.