Inverted pendulum lab revision

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LQR - Linear quadratic regulator

An LQR regulator uses state feedback, in the same way that pole placement does. But instead of us choosing all the poles the LQR finds the most optimal ones given a quadratic cost function that should be minimized. The cost function J can be seen below where the solution is found using the riccati equation.

$$J = \frac{1}{2}\int_0^{\infty}(x(t)^T Q_x x(t) + u(t)^T Q_u u(t))\, dt$$

In layman's terms, changing the values of Q_x and Q_u decides what the model prioritizes / decides to penalize. For example: if Q_u is high the model will penalize actuator movement, meaning that the system will avoid this by choosing a specific K_ct calculated by the LQR.

Task 1, LQR design:

The initial values given to us in the task was: $Q_x = diag([500, 500, 1, 1, 1])$, $Q_u = 0.2$, with covariance matrices Q_w and Q_v. We found that the system didn't stabilize for these values and that the system became insatiable within seconds. We changed the value of Q_x to $diag([300, 300, 1, 1, 1])$ to penalize the deviation of α and θ from the deviation point less. With these new parameters, the system was able to stabilize and withstand nudges.

When changing Q_u to value 0.05 and then 0.5 we notice that the system is more active for the smaller value, which is to be expected as actuators are less penalized in the cost function. For $Q_u = 0.5$ we noticed that the actuators are less active but have bigger movements to respond to the bigger deviation in position. We found that the best value for Q_u to be 0.2 as the actuators weren't too agitated and the system could withstand nudges well.

When the cost function is free of penalization ie $Q_x = diag([500, 500, 0, 0, 1])$, the system was found to behave a lot quicker but at the cost of stability and resistance to nudges.

Kalman filter

Kalman filters are a type of state observer that predicts and estimates unobservable states in a dynamic system. State observers are used when you have a system in which you can't measure every state. Instead of a measurement, a state observer estimates the state from a model. The Kalman filter also continuously corrects for its prediction by comparing a predicted output value of the system with the measured output value of the system. This is called innovation. The Kalman filter recursively adjusts its state estimates based on the innovation. The Kalman filter is tunable to which degree the observer should trust the predicted state values (dependent on process noise) and to which degree the observer should trust the measured state values (dependent on measurement noise).

The actual tuning is done by choosing different values for the covariance matrices Q_w and Q_v corresponding to the process noise and the measurement noise. Higher values in the matrix corresponding to the measurement noise means that the observer places increased trust in the predictive value with process noise. Vice versa is true for higher values in the matrix corresponding to the process noise.

Task 2, Design of Kalman filter

In the lab we were tasked with modifying the weights of the covariance matrices of our Kalman filter and observing the results. First we modified Q_w . With lower Q_w the pendulum had quite a large error. As Q_w increased, the error of the pendulum decreased. Next we tried changing Q_v. Small values for Q_v produced smaller errors. Big values for Q_v produced larger errors.

From analyzing different Q_w and Q_v we can draw certain conclusions. The error is smaller when the Kalman filter trusts the measured data more than the estimated data. (Which can be done by both increasing Q_w and decreasing Q_v) This is because our model for the system is linearized, which means that it is only accurate within a very small radius around the point it is linearized. When the pendulum is outside this radius the predictions become untrustworthy, at which point it is better to use the measured data.

The optimal values for Q_w and Q_v we found where:

$Q_w = 200 * diag([1, 1])$

$Q_v = 0.01 * diag([1, 1])$

LQG - Linear quadratic gaussian control

LQG is a method to design optimal controllers for linear systems with process and measurement noise, which is what we have in our system. The LQG consists of the Kalman filter for state estimation and a LQR for optimal control. An important aspect of the LQG is the separation theorem which states that the optimal observer and the optimal controller can be designed separately. This is important to us as we have, in the tests in the lab, adjusted the values for the LQR and the Kalman separately. This was so that we could find the optimal LQG for the system which we found to be:

$Q_x = diag([500, 500, 1, 1, 1])$, $Q_u = 0.2$, $Q_w = 200 * diag([1, 1])$
$Q_v = 0.01 * diag([1, 1])$

It should however be said that some theoretical understanding was missing in the lab when it came to the Kalman filter. It is thus a good idé to run trials on the covariance matrices again before concluding what the best model is.