

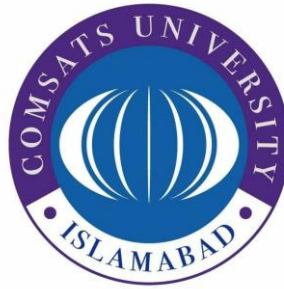
# **INTRODUCTION TO DATA SCIENCE**

## **ASSIGNMENT # 04**

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### Question: 1

Compute BoW, TF, IDF, and then TF.IDF values for each term in the following three sentences.

S1: “data science is one of the most important courses in computer science”

S2: “this is one of the best data science courses”

S3: “the data scientists perform data analysis”

### Answer:

#### 1. Bag of Words:

First we generate vocabulary of words:

Vocabulary: [analysis, best, computer, courses, data, important, perform, science, scientists, the, this, one, is, most, in, of]

	analysis	best	computer	courses	data	important	perform	science	scientists	the	this	one	is	most	in	of
S1	0	0	1	1	1	1	0	2	0	1	0	1	1	1	1	1
S2	0	1	0	1	1	0	0	1	0	1	1	1	1	0	0	1
S3	1	0	0	0	2	0	1	0	1	1	0	0	0	0	0	0

#### BoW Vectors:

S1: [0, 0, 1, 1, 1, 1, 0, 2, 0, 1, 0, 1, 1, 1, 1, 1]

S2: [0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1]

S3: [1, 0, 0, 0, 2, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0]

#### 2. TF

#### 3. IDF

#### 4. TF.IDF

### Question: 2

Compute the similarity between S1, S2, and S3 using cosine, manhattan, and euclidean distances.

### Answer:

#### Cosine Similarities:

Cosine Similarity between S1 and S2:

$$\begin{aligned}
\text{Cosine similarity} &= \frac{S1.S2}{|S1|. |S2|} \\
&= \frac{(0 \times 1) + (0 \times 0) + (1 \times 0) + (1 \times 1) + (1 \times 1) + (1 \times 0) + (0 \times 0) + (2 \times 1) + (0 \times 0) + (1 \times 1) + (0 \times 1) + (1 \times 1) + (1 \times 1) + (1 \times 0) + (1 \times 0) + (1 \times 1)}{\sqrt{14} \times \sqrt{9}} \\
&= \frac{0 + 0 + 0 + 1 + 1 + 0 + 0 + 2 + 0 + 1 + 0 + 1 + 1 + 0 + 0 + 1}{\sqrt{14} \times \sqrt{9}} \\
&= \frac{8}{\sqrt{14} \times \sqrt{9}} \\
&= \frac{8}{11.224972} \\
&= 0.71269
\end{aligned}$$

### Cosine Similarity between S1 and S3:

$$\begin{aligned}
\text{Cosine similarity} &= \frac{S1.S3}{|S1|. |S3|} \\
&= \frac{(0 \times 1) + (0 \times 0) + (1 \times 0) + (1 \times 0) + (1 \times 2) + (1 \times 0) + (0 \times 1) + (2 \times 0) + (0 \times 1) + (1 \times 1) + (0 \times 0) + (1 \times 0) + (1 \times 0) + (1 \times 0) + (1 \times 0) + (1 \times 0)}{\sqrt{14} \times 2\sqrt{2}} \\
&= \frac{0 + 0 + 0 + 0 + 2 + 0 + 0 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0 + 0}{\sqrt{14} \times 2\sqrt{2}} \\
&= \frac{3}{\sqrt{14} \times 2\sqrt{2}} \\
&= \frac{3}{10.58300} \\
&= 0.28347
\end{aligned}$$

### Cosine Similarity between S3 and S2:

$$\begin{aligned}
\text{Cosine similarity} &= \frac{S2.S3}{|S2|. |S3|} \\
&= \frac{(0 \times 1) + (1 \times 0) + (0 \times 0) + (1 \times 0) + (1 \times 2) + (0 \times 0) + (0 \times 1) + (1 \times 0) + (0 \times 1) + (1 \times 1) + (1 \times 0) + (1 \times 0) + (1 \times 0) + (0 \times 0) + (0 \times 0) + (1 \times 0)}{\sqrt{9} \times 2\sqrt{2}} \\
&= \frac{0 + 0 + 0 + 0 + 2 + 0 + 0 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0 + 0}{\sqrt{9} \times 2\sqrt{2}} \\
&= \frac{3}{\sqrt{9} \times 2\sqrt{2}} \\
&= \frac{3}{8.48528} \\
&= 0.35355
\end{aligned}$$

## Manhattan Distance

### Manhattan distance between S1 and S2:

$$\begin{aligned}
&= |0-0| + |0-1| + |1-0| + |1-1| + |1-1| + |1-0| + |0-0| + |2-1| + |0-0| + |1-1| + |0-1| + |1-1| + |1-1| + |1-0| + |1-0| + |1-1| \\
&= 1 + 1 + 1 + 1 + 1 + 1 + 1 \\
&= 7
\end{aligned}$$

### Manhattan distance between S1 and S3:

$$\begin{aligned} \mathbf{S1}: & [0, 0, 0, 1, 1, 1, 1, 0, 2, 0, 1, 0, 1, 1, 1, 1, 1] \\ \mathbf{S3}: & [1, 0, 0, 0, 0, 2, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0] \\ & = |0 - 1| + |0 - 0| + |1 - 0| + |1 - 0| + |1 - 2| + |1 - 0| + |0 - 1| + |2 - 0| + |0 - 1| + |1 - 1| + |0 - 0| + |1 - 0| + |1 - 0| + |1 - 0| + |1 - 0| \\ & = 1 + 1 + 1 + 1 + 1 + 1 + 2 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \\ & = 14 \end{aligned}$$

### Manhattan distance between S2 and S3:

$$\begin{aligned}
&= |0-1| + |1-0| + |0-0| + |1-0| + |1-2| + |0-0| + |0-1| + |1-0| + |0-1| + |1-1| + |1-0| + |1-0| + |1-0| + |0-0| + |0-0| + |1-0| \\
&= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \\
&= 11
\end{aligned}$$

## Euclidean Distance

**Euclidean distance between S1 and S2:**

$$\begin{aligned} &= \sqrt{(0-0)^2 + (0-1)^2 + (1-0)^2 + (1-1)^2 + (1-1)^2 + (1-0)^2 + (0-0)^2 + (2-1)^2 + (0-0)^2 + (1-1)^2 + (0-1)^2 + (1-1)^2 + (1-1)^2 + (1-0)^2 + (1-0)^2 + (1-1)^2} \\ &= \sqrt{1+1+1+1+1+1+1+1} \\ &= \sqrt{7} = 2.6457 \end{aligned}$$

**Euclidean distance between S1 and S3:**

$$\begin{aligned}
&= \sqrt{(0-1)^2 + (0-0)^2 + (1-0)^2 + (1-0)^2 + (1-2)^2 + (1-0)^2 + (0-1)^2 + (2-0)^2 + (0-1)^2 + (1-1)^2 + (0-0)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2} \\
&= \sqrt{1+1+1+1+1+1+4+1+1+1+1+1+1+1+1} \\
&= \sqrt{16}
\end{aligned}$$

$$= 4$$

**Euclidean distance between S2 and S3:**

$$= \sqrt{(0-1)^2 + (1-0)^2 + (0-0)^2 + (1-0)^2 + (1-2)^2 + (0-0)^2 + (0-1)^2 + (1-0)^2 + (0-1)^2 + (1-1)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2 + (0-0)^2 + (0-0)^2 + (1-0)^2}$$

$$= \sqrt{1+1+1+1+1+1+1+1+1+1+1+1+1+1+1}$$

$$= \sqrt{11} = 3.31662$$

## 2. Term Frequency (tf):

$$\text{Term frequency} = \frac{\text{Number of times the term appears in a sentence}}{\text{Total number of terms in the sentence}}$$

**Sentence 1:**

*“data science is one of the most important courses in computer science”*

Tf value of <b>‘data’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘science’</b>	$= \frac{2}{12} = \frac{1}{6} = 0.16666$
Tf value of <b>‘is’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘one’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘of’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘the’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘most’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘important’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘courses’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘in’</b>	$= \frac{1}{12} = 0.083333$
Tf value of <b>‘computer’</b>	$= \frac{1}{12} = 0.083333$

**Sentence 2:**

*“this is one of the best data science courses”*

$$\begin{aligned}
\text{Tf value of 'this'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'is'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'one'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'of'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'the'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'best'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'data'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'science'} &= \frac{1}{9} = 0.11111111 \\
\text{Tf value of 'courses'} &= \frac{1}{9} = 0.11111111
\end{aligned}$$

### Sentence 3:

*“the data scientists perform data analysis”*

$$\begin{aligned}
\text{Tf value of 'the'} &= \frac{1}{6} = 0.16666666 \\
\text{Tf value of 'data'} &= \frac{2}{6} = \frac{1}{3} = 0.33333333 \\
\text{Tf value of 'scientists'} &= \frac{1}{6} = 0.16666666 \\
\text{Tf value of 'perform'} &= \frac{1}{6} = 0.16666666 \\
\text{Tf value of 'analysis'} &= \frac{1}{6} = 0.16666666
\end{aligned}$$

### Calculating Vectors:

	analysis	best	computer	courses	data	important	perform	science	scientists	the	this	one	is	most	in	of
S1	0	0	0.083333	0.083333	0.083333	0.083333	0	0.166666	0	0.083333	0	0.083333	0.083333	0.083333	0.083333	0.083333
S2	0	0.111111	0	0.111111	0.111111	0	0	0.111111	0	0.111111	0.111111	0.111111	0.111111	0	0	0.111111
S3	0.166666	0	0	0	0.333333	0	0.166666	0	0.166666	0.166666	0	0	0	0	0	0

S1 = [0, 0, 0.083333, 0.083333, 0.083333, 0.083333, 0, 0.166666, 0, 0.083333, 0, 0.083333, 0.083333, 0.083333, 0.083333, 0.083333, 0.083333]

S2 = [0, 0.111111, 0, 0.111111, 0.111111, 0, 0, 0.111111, 0, 0.111111, 0.111111, 0.111111, 0.111111, 0, 0, 0.111111]

S3 = [0.166666, 0, 0, 0, 0.333333, 0, 0.166666, 0, 0.166666, 0.166666, 0, 0, 0, 0, 0, 0, 0]

## Cosine Similarities:

### Cosine Similarity between S1 and S2:

$$\text{Cosine similarity} = \frac{S1.S2}{|S1|. |S2|}$$

$$= \frac{(0 \times 0) + (0 \times 0.1111) + (0.083333 \times 0) + (0.083333 \times 0.1111) + (0.083333 \times 0.1111) + (0.083333 \times 0) + (0 \times 0) + (0.16666 \times 0.1111) + (0 \times 0) + (0.083333 \times 0.1111) + (0 \times 0.1111) + (0.083333 \times 0.1111) + (0.083333 \times 0.1111) + (0.083333 \times 0) + (0.083333 \times 0) + (0.083333 \times 0.1111)}{0.31180 \times 0.33333}$$

$$= 0.833333$$

### Cosine Similarity between S1 and S3:

$$\text{Cosine similarity} = \frac{S1.S3}{|S1|. |S3|}$$

$$= \frac{(0 \times 0.16666) + (0 \times 0) + (0.083333 \times 0) + (0.083333 \times 0.33333) + (0.083333 \times 0) + (0 \times 0.16666) + (0.16666 \times 0) + (0 \times 0.16666) + (0.083333 \times 0.16666) + (0 \times 0) + (0.083333 \times 0) + (0.083333 \times 0) + (0.083333 \times 0) + (0.083333 \times 0) + (0.083333 \times 0)}{0.31180 \times 0.471399}$$

$$= 0.36514$$

### Cosine Similarity between S3 and S2:

$$\text{Cosine similarity} = \frac{S2.S3}{|S2|. |S3|}$$

$$= \frac{(0.16666 \times 0) + (0 \times 0.1111) + (0 \times 0) + (0 \times 0.1111) + (0.33333 \times 0.1111) + (0.083333 \times 0) + (0 \times 0.16666) + (0.16666 \times 0) + (0 \times 0.16666) + (0.083333 \times 0.16666) + (0 \times 0) + (0.083333 \times 0) + (0.083333 \times 0) + (0.083333 \times 0) + (0.083333 \times 0)}{0.471399 \times 0.33333}$$

$$= 0.182574$$

## Manhattan Distance

### Manhattan distance between S1 and S2:

$$\begin{aligned} &= |0 - 0| + |0 - 0.1111| + |0.083333 - 0| + |0.083333 - 0.1111| + |0.083333 - 0.1111| + |0.083333 - 0| + |0 - 0| + |0.16666 - 0.1111| \\ &\quad + |0 - 0| + |0.083333 - 0.1111| + |0 - 0.1111| + |0.083333 - 0.1111| + |0.083333 - 0.1111| + |0.083333 - 0| \\ &\quad + |0.083333 - 0| + |0.083333 - 0.1111| \end{aligned}$$

$$= 0.333333$$

### Manhattan distance between S1 and S3:

$$\begin{aligned} &= |0 - 0.166666| + |0 - 0| + |0.083333 - 0| + |0.083333 - 0| + |0.083333 - 0.33333| + |0.083333 - 0| + |0 - 0.166666| + |0.16666 - 0| \\ &\quad + |0 - 0.166666| + |0.083333 - 0.166666| + |0 - 0| + |0.083333 - 0| + |0.083333 - 0| + |0.083333 - 0| \\ &\quad + |0.083333 - 0| + |0.083333 - 0| \end{aligned}$$

$$= 1.333333$$

### Manhattan distance between S2 and S3:

$$\begin{aligned}
&= |0.166666 - 0| + |0 - 0.1111| + |0 - 0| + |0 - 0.1111| + |0.33333 - 0.1111| + |0.083333 - 0| + |0 - 0.166666| + |0.16666 - 0| \\
&\quad + |0 - 0.166666| + |0.083333 - 0.166666| + |0 - 0| + |0.083333 - 0| + |0.083333 - 0| + |0.083333 - 0| + |0.083333 - 0| \\
&\quad + |0.083333 - 0| \\
&= 1.66666667
\end{aligned}$$

## Euclidean Distance

### Euclidean distance between S1 and S2:

$$\begin{aligned}
&= \sqrt{(0-0)^2 + (0-0.1111)^2 + (0.083333-0)^2 + (0.083333-0.1111)^2 + (0.083333-0.1111)^2 + (0.083333-0)^2 + (0-0)^2 + (0.16666-0.1111)^2 + (0-0)^2 + (0.083333-0.1111)^2 + (0-0.1111)^2 + (0.083333-0.1111)^2 + (0.083333-0.1111)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0.1111)^2} \\
&= 0.23570
\end{aligned}$$

### Euclidean distance between S1 and S3:

$$\begin{aligned}
&= \sqrt{(0-0.166666)^2 + (0-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0.33333)^2 + (0.083333-0)^2 + (0-0.166666)^2 + (0.16666-0)^2 + (0-0.166666)^2 + (0.083333-0.166666)^2 + (0-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0)^2} \\
&= 0.48304589
\end{aligned}$$

### Euclidean distance between S2 and S3:

$$\begin{aligned}
&= \sqrt{(0.166666-0)^2 + (0-0.1111)^2 + (0-0)^2 + (0-0.1111)^2 + (0.33333-0.1111)^2 + (0.083333-0)^2 + (0-0.166666)^2 + (0.16666-0)^2 + (0-0.166666)^2 + (0.083333-0.166666)^2 + (0-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0)^2 + (0.083333-0)^2} \\
&= 0.547722
\end{aligned}$$

## 3. IDF

$$IDF = \log\left(\frac{\text{Total Number of documents}}{\text{Number of documents containing term}}\right)$$

Analysis	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$
Best	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$
Computer	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$
Courses	$= \log\left(\frac{3}{2}\right) = 0.4054651081081644$
data	$= \log\left(\frac{3}{3}\right) = 0.0$
important	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$
in	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$
is	$= \log\left(\frac{3}{2}\right) = 0.4054651081081644$
most	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$
of	$= \log\left(\frac{3}{2}\right) = 0.4054651081081644$
one	$= \log\left(\frac{3}{2}\right) = 0.4054651081081644$
perform	$= \log\left(\frac{3}{1}\right) = 1.0986122886681098$



science  $= \log\left(\frac{3}{2}\right) = 0.4054651081081644$   
 scientists  $= \log\left(\frac{3}{1}\right) = 1.0986122886681098$   
 the  $= \log\left(\frac{3}{3}\right) = 0.0$   
 this  $= \log\left(\frac{3}{1}\right) = 1.0986122886681098$

#### 4. Tf.IDF

##### For Sentence 1:

{'data': 0.0, 'science': 0.06757751801802739, 'is': 0.06757751801802739, 'one': 0.06757751801802739, 'of': 0.06757751801802739, 'the': 0.0, 'most': 0.0, 'important': 0.0, 'courses': 0.0, 'in': 0.0, 'computer': 0.0}

##### For Sentence 2:

{'this': 0.1831020481113516, 'is': 0.06757751801802739, 'one': 0.06757751801802739, 'of': 0.06757751801802739, 'the': 0.0, 'best': 0.0, 'data': 0.0, 'science': 0.06757751801802739, 'courses': 0.0}

##### For Sentence 3:

{'the': 0.0, 'data': 0.0, 'scientists': 0.21972245773362198, 'perform': 0.21972245773362198, 'analysis': 0.21972245773362198}

#### Calculating Vectors:

	analysis	best	computer	courses	data	important	perform	science	scientists	the	this	one	is	most	in	of
S1	0	0	0	0	0	0	0	0.067577	0	0	0	0.067577	0.067577	0	0	0.067577
S2	0	0	0	0	0	0	0	0.067577	0	0	0.183102	0	0.067577	0	0	0.067577
S3	0.219722	0	0	0	0	0	0.219722	0	0.219722	0	0	0	0	0	0	0

S1 = [0, 0, 0, 0, 0, 0, 0, 0, 0.067577, 0, 0, 0, 0.067577, 0.067577, 0, 0, 0.067577]

S2 = [0, 0, 0, 0, 0, 0, 0, 0, 0.067577, 0, 0, 0.183102, 0, 0.067577, 0, 0, 0.067577]

S3 = [0.219722, 0, 0, 0, 0, 0, 0, 0.219722, 0, 0.219722, 0, 0, 0, 0, 0, 0, 0]

#### Cosine Similarities:

##### Cosine Similarity between S1 and S2:

$$\text{Cosine similarity} = \frac{S1.S2}{|S1|.|S2|}$$

$$= \frac{(0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0.0667577 \times 0.0667577) + (0 \times 0) + (0 \times 0) + (0 \times 0.183102) + (0.0667577 \times 0) + (0.0667577 \times 0.0667577) + (0 \times 0) + (0 \times 0) + (0.0667577 \times 0.0667577)}{0.135154 \times 0.2173}$$

$$= 0.57732$$

### Cosine Similarity between S1 and S3:

$$\text{Cosine similarity} = \frac{S1.S3}{|S1|.|S3|}$$

$$= \frac{(0 \times 0.219722) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0.219722) + (0.0667577 \times 0) + (0 \times 0.219722) + (0 \times 0) + (0 \times 0) + (0.0667577 \times 0) + (0.0667577 \times 0) + (0 \times 0) + (0 \times 0) + (0.0667577 \times 0)}{0.135154 \times 0.3805}$$

$$= 0.15733$$

### Cosine Similarity between S3 and S2:

$$\text{Cosine similarity} = \frac{S2.S3}{|S2|.|S3|}$$

$$= \frac{(0 \times 0.219722) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0) + (0 \times 0.219722) + (0.0667577 \times 0) + (0 \times 0.219722) + (0 \times 0) + (0.183102 \times 0) + (0 \times 0) + (0.0667577 \times 0) + (0 \times 0) + (0 \times 0) + (0.0667577 \times 0)}{0.2173 \times 0.3805}$$

$$= 0.20321$$

## Manhattan Distance

### Manhattan distance between S1 and S2:

$$S1 = [0, 0, 0, 0, 0, 0, 0, 0, 0.067577, 0, 0, 0, 0.067577, 0.067577, 0, 0, 0.067577]$$

$$S2 = [0, 0, 0, 0, 0, 0, 0, 0, 0.067577, 0, 0, 0.183102, 0, 0.067577, 0, 0, 0.067577]$$

$$S3 = [0.219722, 0, 0, 0, 0, 0, 0.219722, 0, 0.219722, 0, 0, 0, 0, 0, 0, 0]$$

$$= |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0.0667577 - 0.0667577| + |0 - 0| + |0 - 0| + |0 - 0.183102| \\ + |0.0667577 - 0| + |0.0667577 - 0.0667577| + |0 - 0| + |0 - 0| + |0.0667577 - 0.0667577|$$

$$= 2.735564$$

### Manhattan distance between S1 and S3:

$$= |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0.0667577 - 0.0667577| + |0 - 0| + |0 - 0| + |0 - 0.183102| \\ + |0.0667577 - 0| + |0.0667577 - 0.0667577| + |0 - 0| + |0 - 0| + |0.0667577 - 0.0667577|$$

$$= 4.60822$$

### Manhattan distance between S2 and S3:

$$= |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0 - 0| + |0.0667577 - 0.0667577| + |0 - 0| + |0 - 0| + |0 - 0.183102| \\ + |0.0667577 - 0| + |0.0667577 - 0.0667577| + |0 - 0| + |0 - 0| + |0.0667577 - 0.0667577|$$

= 4.1455

Euclidean Distance

Euclidean distance between S1 and S2:

=  $\sqrt{(0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0.0667577-0.0667577)^2 + (0-0)^2 + (0-0)^2 + (0-0.183102)^2 + (0.0667577-0)^2 + (0.0667577-0.0667577)^2 + (0-0)^2 + (0-0)^2 + (0.0667577-0.0667577)^2}$   
= 091943005

Euclidean distance between S1 and S3:

=  $\sqrt{(0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0.0667577-0.0667577)^2 + (0-0)^2 + (0-0)^2 + (0-0.183102)^2 + (0.0667577-0)^2 + (0.0667577-0.0667577)^2 + (0-0)^2 + (0-0)^2 + (0.0667577-0.0667577)^2}$   
= 1.2982

Euclidean distance between S2 and S3:

=  $\sqrt{(0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0-0)^2 + (0.0667577-0.0667577)^2 + (0-0)^2 + (0-0)^2 + (0-0.183102)^2 + (0.0667577-0)^2 + (0.0667577-0.0667577)^2 + (0-0)^2 + (0-0)^2 + (0.0667577-0.0667577)^2}$   
= 1.26236