Algorithms

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Outline

- Motivation
- Sorting
- Application to xNNs
- References

Disclaimer

- This set of slides is more accurately titled "A brief refresher of a subset of algorithms for people already somewhat familiar with the topic followed by it's specific application to xNN related items needed by the rest of the course"
- However, that's not very catchy so we'll just stick with "Algorithms"
- In all seriousness, recognize that algorithms is a very broad and deep topic that has and will continue to occupy many lifetimes of work; if interested in learning more, please consult the references to open a window into a much larger world

Motivation

Layers And Post Processing

- An algorithm is a set of rules for solving a problem
  - An exceedingly brief look at ~ 1 example for 1 problem is covered here
  - So in addition to the earlier disclaimer, calling this set of slides “Algorithms” was still optimistic by 1 letter too many
- Max is a a key component of a common CNN layer and a subset of sorting
  - Max pooling
  - Also as part of spatial pyramid and region of interest pooling variants
- Sorting is a key component of common post processing methods
  - Median and rank order filtering
  - Non maximal suppression

Sorting

Definition

- From Wikipedia: arranging items in a sequence which is ordered based on some criteria

Comparison Sort

- A type of sorting algorithm that applies a comparison operator to elements in a list

- The comparison operator satisfies 2 properties
  - Transitivity: if $a \leq b$ and $b \leq c$ then $a \leq c$
  - Totalness: $\forall$ a, b either $a \leq b$ or $b \leq a$

Comparison Sort

- Optimal comparison sorts (in the sense of minimizing the number of comparisons) require $O(N \log_2(N))$ comparisons where N is the length of the sequence

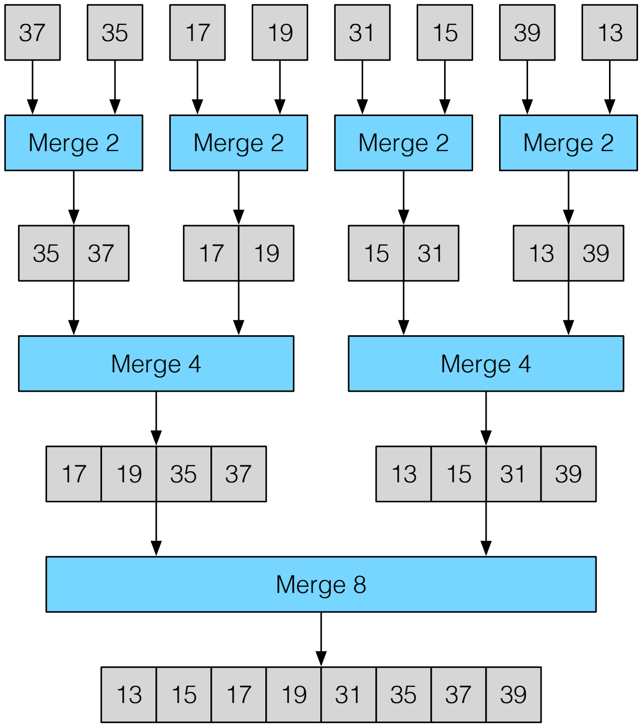
- A standard (quasi) proof
  - There are N! possible arrangements of a sequence of length N
  - C comparisons can distinguish between 2^C different arrangements
  - To distinguish between all possible arrangements requires $2^C \geq N!$
    - $C \geq \log_2(N!)$ $\approx N \log_2(N) - N \log_2(e) + O(\log_2(N))$, via Stirling's approximation
      $= O(N \log_2(N))$

Comparison Sort

- An information theory based (quasi) proof of the same bound
  - There are N! possible arrangements of a sequence of length N
  - View the arrangements as a random variable X(s)
    - The probability of each arrangement is 1/N!
    - Uniform probability mass function with support of size N!
  - The entropy (information) of a realization of this random variable
    - $H(X(s)) = -\Sigma\ (1/N!) \log_2(1/N!) = \log_2(N!)$
  - Each comparison in a comparison sort gives at most 1 bit of information
  - To reduce the entropy to 0 with C comparisons need $\log_2(N!) - C \leq 0$
    - $C \geq \log_2(N!) \approx O(N \log_2(N))$

Sequential Merge Sort

- The previous slides discussed comparison sorts and bounds on the minimum number of comparisons in theory
- The sequential merge sort is an example sorting algorithm that achieves the bound in practice
  - Exploits that 2 sorted lists of size N/2 can be merged into a sorted list of size N using N – 1 comparisons
  - Recursively divides a list into 1/2 size lists then applies the exploit
  - It’s a divide and conquer style algorithm a la the FFT



Sequential Merge Sort

- The number of comparisons in sequential merge sort
  - Depth
    - Total depth $D = \log_2(N)$
    - Depth index $d = 0, ..., D - 1$
  - Merges
    - Number inputs to a merge at depth d $M_d = 2^{d+1}$
    - Comparisons per merge at depth d $M_d - 1$
  - Comparisons
    - At depth d $(N/M_d)\ (M_d - 1) = N\ (M_d - 1) / M_d \approx N$
    - Total $O(N\ D) = O(N \log_2(N))$

Going Faster: Exploit Information

- It's possible to sort a list faster than a comparison sort using **less than** $O(N \log_2(N))$ comparisons if there's known information about the list that's exploitable
  - Remember that a uniform distribution is an entropy maximizing distribution
  - Thinking about sorting from an information theoretic approach, if the probability of the N! possible arrangements is not uniform then $H(X(s)) < \log_2(N!)$ and fewer operations are needed to reduce the information to below 0
  - Maybe apply the known information exploit recursively
  - Maybe clean up approximate arrangement at the end with a comparison sort

- Example
  - Consider sorting 1M last names of random Dallas residents
  - Approximate statistics are known ahead of time as to where a given last name will end up in the final list

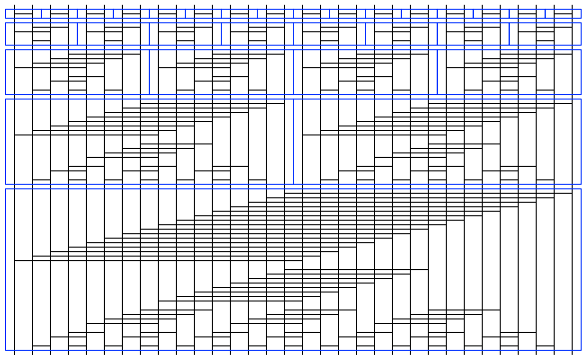
Going Faster: Parallel Comparisons

- It's possible to sort a list faster than a comparison sort using **more than** $O(N \log_2(N))$ comparisons (whaaaaat?)

- A few comments on the direct parallelization of sequential merge sort
  - Separate merges at a given depth are disjoint and can run in parallel (good)
  - Within a merge operation comparisons are done sequentially (bad)
  - Put another way the elements involved in a comparison are dependent on the input (still bad)

- Strategy for going faster on hardware that is optimized for parallel computation: instead of optimizing to minimize the number of comparisons, now optimize to minimize the number of sequential dependencies

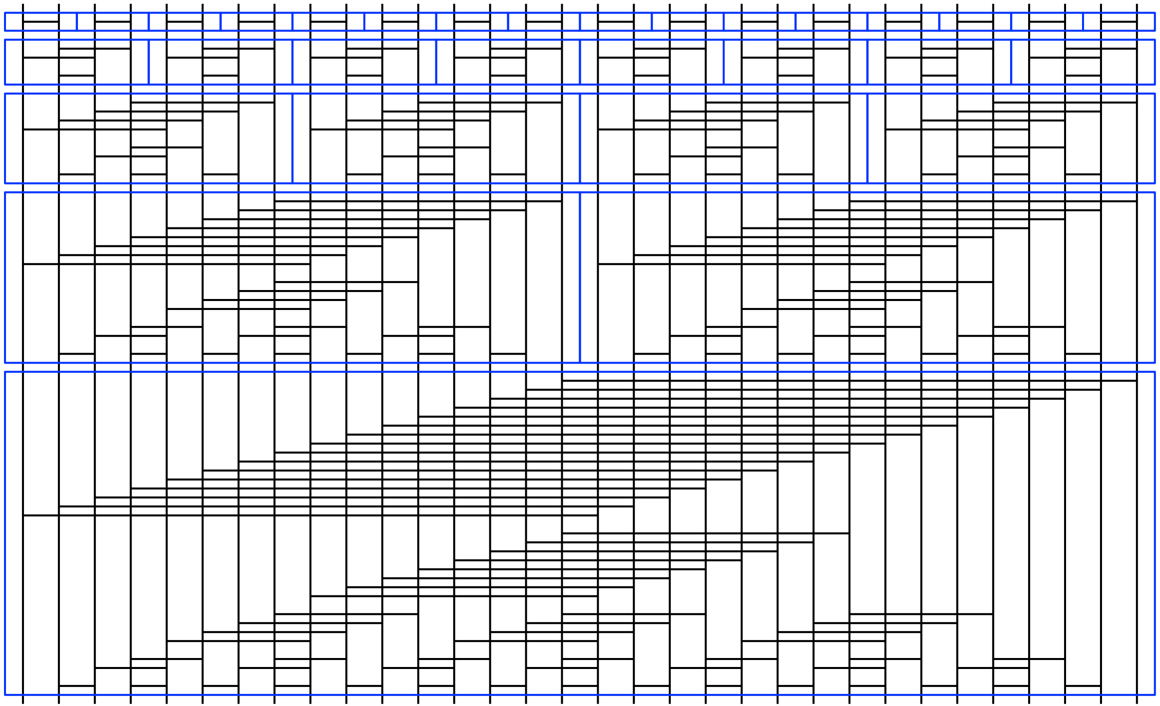
Parallel Merge Sort Network



- Sorting networks
  - Loosely: a fixed structure of comparisons (with swaps) that sorts inputs
  - Cookbook rules for creation of some types
  - No input dependence
  - Prove correctness via sorting all 0 / 1 sequences

- Odd even merging network
  - A merge sort style sorting network
  - $O(N \log_2(N) \log_2(N))$ comparisons
    - More than a sequential merge sort
  - $\log_2(N) (\log_2(N) + 1) / 2$ sequential steps
    - Less than a sequential merge sort, this is the key
    - If time: story of Gauss and a misbehaving class

Parallel Merge Sort Network



16x merge 2
(16 comparisons, 1 substage)

8x merge 4
(24 comparisons, 2 substages)

4x merge 8
(36 comparisons, 3 substages)

2x merge 16
(50 comparisons, 4 substages)

1x merge 32
(65 comparisons, 5 substages)

Application To xNNs

Pooling Layers

- It's common in the encoder portion of CNNs to gradually reduce spatial resolution to increase the receptive field size and reduce the data volume (complexity)
  - CNN style 2D convolution with down sampling (striding) is 1 common way of doing this
  - Pooling is another common way of doing this

- Pooling is a spatial operation that works on individual feature maps (not across feature maps) and maps inputs to down sampled outputs
  - Some variants use max operations and finding the max is a subset of sort hence it's inclusion here

- Common pooling layers (pooling size / stride)
  - Max pooling 3 x 3 / 2 and 2 x 2 / 2
  - Average pooling 3 x 3 / 2 and 2 x 2 / 2 // doesn't need sorting
  - Global average pooling L_r x L_c / L_r x L_c // doesn't need sorting
  - Spatial pyramid pooling R_r x R_c / D_r x D_c to produce a fixed number of $(R_r/D_r)(R_c/D_c)$ elements

Pooling Layers

31	21	33	34	5	2	15
10	29	32	6	27	16	13
7	4	28	20	24	30	26
25	18	14	35	22	1	3
17	23	12	8	19	9	11

Max pool ↓ 3x3 / 2

33	34	30
28	35	30

32	36	68	56	72	48
8	40	64	84	80	12
28	96	92	76	16	4
52	88	44	20	60	24

Avg pool ↓ 2x2 / 2

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66	58	26

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Median And Rank Order Filtering

- 1D and 2D convolution (correlation)
  - Define filter coefficients $h(\tau)$, $\tau = 0, \ldots, L - 1$
  - Generate outputs $y(n)$ from inputs $x(n)$ for the 1D correlation case as

$$y(n) = \Sigma_{\tau} h(\tau)\, x(n + \tau),\ n = 0, \ldots, N - L$$

  - Linear filter with trainable parameters

- 1D and 2D rank order filtering
  - Define filter length L and rank R
  - Generate outputs $y(n)$ from inputs $x(n)$ for the 1D rank order case as

$$y(n) = \text{select}_R(\text{sort}(x(n), \ldots, x(n + L - 1))),\ n = 0, \ldots, N - L$$

  - Nonlinear filter with 1 parameter R that selects the Rth element from the sorted array

Median And Rank Order Filtering

- Mathematical morphology
  - Erosion
    - 2D rank order filtering where the smallest element is selected
    - Most commonly used on binary images but also applicable to gray scale images
  - Dilation
    - 2D rank order filtering where the largest element is selected
    - Most commonly used on binary images but also applicable to gray scale images
  - Opening
    - Erosion followed by dilation
  - Closing
    - Dilation followed by erosion

Non Maximal Suppression

- Vision
  - Multiple object detection networks frequently have a common convolutional tail and body of a network that takes inputs and generates strong features and 2 heads that work together to do multiple object detection
  - Head 1 to produce region proposals
    - Box coordinates vs anchors and a score indicating likelihood of an object of any class
    - Non maximal suppression can be applied here to keep R best region proposals
  - Head 2 to classifies the R best region proposals each to 1 of C classes
    - Could use spatial pyramid pooling to extract
    - Class 0 ends up with R_0 possibilities (box coordinates + class probability)
    - Class 1 ends up with R_1 possibilities (box coordinates + class probability)
    - ...
    - Non maximal suppression can be applied here to each class individually to keep the N_c best estimates for that class
    - A minimum probability threshold for each class is typically also used

Non Maximal Suppression

- Non maximal suppression algorithm
  - Inputs contain scores and box locations
  - Create a score list by sorting the entries based on the score
    - This is why non maximal suppression is being discussed here
  - Repeat the following until no more entries with scores above a threshold are in the score list
    - Start with the entry with the best score on the score list
    - Remove other entries (suppress non maximals) from the score list with significant overlap
    - Add the best entry from the score list to the prediction list
    - Remove the best entry from the score list
- Variants
  - Can play games like averaging a few together, instead of removing others penalize them by reducing their score, ...
  - A difficulty is finding balance between suppressing windows and detecting close together objects (overlap parameter choice)

References

Sorting

- Sorting networks and their applications
    - https://dl.acm.org/citation.cfm?id=1468121
- Batcher’s odd-even merging network
    - http://bekbolatov.github.io/sorting/
- Batcher's odd-even merging network
    - http://sparkydots.blogspot.com/2015/05/batchers-odd-even-merging-network.html

Application To xNNs

- Soft-NMS - Improving Object Detection With One Line of Code
  - https://arxiv.org/abs/1704.04503
- Learning non-maximum suppression
  - https://arxiv.org/pdf/1705.02950.pdf