Assignment 0x03

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- 1 Explain why the integers Z form a ring, and the rationals Q form a field.
- 1.1 Definition: ring

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2 Task 1: Number Theory and Algebra

2.1 Explain why the integers Z form a ring, and the rationals Q form a field.

A ring is a set R with two operations "+" and "*". (R,+) is an abelian group, that means it is associative, commutative and has a neutral and inverse element. (R,*) is a monoid, that means it is associative and has a neutral element. Lastly multiplication is distributive with respect to addition. If all axioms hold, a set is a ring. And they hold for Z.

A field is a ring but with the special case that (R

{0}, *) is an abelian group. Meaning that associativity and commutativity hold for all elements except 0 and that there also exists a neutral and inverse element for every element of the field.

2.2 Find the inverses of all elements in \mathbb{Z}_7^* . Why do all numbers between 1 and 6 have an inverse?

```
Z_7^* = 1, 2, 3, 4, 5, 6
```

A number is an inverse, if number mod inverse = 1 (the neutral element)

Number: 1 2 3 4 5 6 Inverse: 1 4 5 2 3 6

If n is a prime, then all numbers between 1 and n-1 have an inverse with the modulo-Operation. This is due to n not being a multiple of any of the elements in the set.

2.3 As an exercise, calculate the inverse of 331 in Z_{1234} using the extended euclidean algorithm.

GCD = 1, if Rest is 0 in the end 1234 == 3 * 331 + 241 331 == 1 * 241 + 90 241 == 2 * 90 + 61 90 == 1 * 61 + 2961 == 2 * 29 + 3

$$29 == 9 * 3 + 2$$

$$3 == 1 * 2 + 1$$

$$2 == 2 * 1 + 0$$

Applying the algorithm:

```
1 == 3 - 1 * 2
1 == 3 - (29 - 9 * 3)
1 == 10 * 3 - 1 * 29
1 == 10 * (61 - 2 * 29) - 1 * 29
1 == 10 * 61 - 21 * 29
1 == 10 * 61 - 21 * (90 - 1 * 61)
1 == 31 * 61 - 21 * 90
1 == 31 * (241 - 2 * 90) - 21 * 90
1 == 31 * 241 - 83 * 90
1 == 31 * 241 - 83 * (331 - 241 * 1)
1 == 114 * 241 - 83 * 331
1 == 114 * (1234 - 3 * 331) - 83 * 331
1 == 114 * 1234 - 425 * 331
```

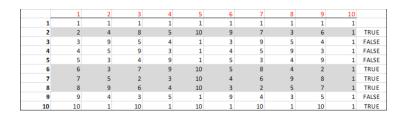
Inverse of 331 is -425. Since it is not in Z we add 1234 to it until it is: -425 + 1234 = 809 = 809 is the inverse of 331 in canonical form.

Test:
$$331 * 809 = 267779$$

267779%1234 = 1

So the calculations are correct.

2.4 Find all generators of Z_{11}



ExcelFormula = MOD((linke Spalte Obere Zeile);11) The marked lines (2, 6, 7, 8) are generators.

2.5 Calculate 42^{497} in Z_{1361} using fast exponentiation and a handheld calculator (not the one running on your computer or a programming language). Document a, e and n for each step.

Fastexp(a=42, n=497) 1st Iteration:

```
e = 1
n is odd:
e = (42 * 1) \mod 1361 = 42
a = 42 \mod 1361 = 1764 \mod 1361 = 403
n = 497 / 2 = 248
   2):
   e = 42
n is even:
a = 403 \mod 1361 = 450
n = n/2 = 124
   3)
   e = 42
n is even:
a=450 \bmod 1361=1072
n = 124 / 2 = 62
   4)
   e = 42
n is even:
a = 1072 \mod 1361 = 500
n = 62 / 2 = 31
   5)
   e = 42
n is odd:
e = (500 * 42) \mod 1361 = 585
a = 500 \mod 1361 = 937
n = 31 / 2 = 15
   6)
   e = 1072
n is odd:
e = (937 * 585) \mod 1361 = 1023
a = 937 \bmod 1361 = 124
n = 15 / 2 = 7
   7)
```

```
e = 46
n is odd:
e = (124 * 1023) \mod 1361 = 279
a = 124 \mod 1361 = 405
n = 7 / 2 = 3
  8)
  e = 260
n is odd:
e = (405*279) \mod 1361 = 32
a = 405 \bmod 1361 = 705
n = 3 / 2 = 1
  9)
  e = 503
n is odd:
e = (705*32) \mod 1361 = 784
a = 705 \mod 1361 = 260
n = 1 / 2 = 0
  10)
  n = 0 =  return e = 784
  =>42497 \mod 1361=784
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```

2.6 Definition: field

===== $\clip{2.25}$ 621161c88ec086a6b093522fd76a88451fabbb7a

3 Task 2: The RSA cryptosystem

3.1 Using that private key, decrypt c

$$PHI(n) = (p-1)(q-1) = (17-1) * (23-1) = 16 * 22 = 352$$

$$e = 17; Z_{352}^*$$

(e * d) mod $352 = 1$

Using Excel we found the inverse using the formula above: d = 145

$$\mathbf{m} = c^d \text{ mod } \mathbf{n} = 282^{145} \text{ mod } 391 = 197$$

3.2 Did Donald choose reasonable parameters? Explain!

When e = 1, the message itself is encrypted just by applying the modulo with the public Key part N. Resulting in $c = m \mod N$, which means that if m is smaller than N, the encrypted message is just the plain message => c = m

3.3 Find a value of e for which RSA encryption is the identity function