Q NO 1) Use improved Eular method to approximate the solution for each of the initial-value-problems.

a) $y' = te^{3t} - 2y$

Given

$0 \leq t \leq 1$, $y(0) = 0$, $h = 0.5$

Solution

$t_0 = 0$, $y_0 = 0$

$y' = te^{3t} - 2y$

$f(t_n, y_n) = t_n e^{3t_n} - 2y_n$

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For n = 1
Predictor

For n = 0
Predictor:

$$\bar{y}_1 = y_0 + h[t_0 e^{3t_0} - 2y_0]$$
$$\bar{y}_1 = 0 + 0.5[0e^{3(0)} - 2(0)]$$
$$\boxed{\bar{y}_1 = 0}$$

$$\bar{t}_1 = t_0 + h$$
$$\bar{t}_1 = 0 + 0.5$$
$$\boxed{\bar{t}_1 = 0.5}$$

Corrector:

$$y_1 = y_0 + \frac{h}{2}\left[(t_0 e^{3t_0} - 2y_0) + (\bar{t}_1 e^{3\bar{t}_1} - 2\bar{y}_1)\right]$$
$$y_1 = 0 + \frac{0.5}{2}\left[(0) + (0.5)e^{3(0.5)} - 2(0)\right]$$
$$\boxed{y_1 = 0.5602}$$

$$t_1 = t_0 + h$$
$$t_1 = 0 + 0.5$$
$$\boxed{t_1 = 0.5}$$

For n = 1
Predictor:

$$\bar{y}_2 = y_1 + h[t_1 e^{3t_0} - 2y_1]$$
$$\bar{y}_2 = 0.5602 + 0.5[(0.5)e^{3(0.5)} - 2(0.5602)]$$
$$\boxed{\bar{y}_2 = 1.1204}$$

$$\bar{t}_2 = t_1 + h$$
$$\bar{t}_2 = 0.5 + 0.5$$
$$\boxed{\bar{t}_2 = 1}$$

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Corrector:

$$y_2 = y_1 + \frac{h}{2}\left[(t_1 e^{3t_1} - 2y_1) + (\bar{t}_2 e^{3\bar{t}_2} - 2\bar{y}_2)\right]$$

$$y_2 = 0.5602 + \frac{0.5}{2}\left[(1.1204) + (1e^{3(1)} - 2(1.1204))\right]$$

$$\boxed{y_2 = 5.3015}$$

$$t_2 = t_1 + h$$

$$t_2 = 0.5 + 0.5$$

$$\boxed{t_2 = 1}$$

Table:

n	t_n	y_n	$\bar{t}_{n+1}$	$\bar{y}_{n+1}$	t_{n+1}	y_{n+1}
0	0	0	0.5	0	0.5	0.5602
1	0.5	0.5602	1	1.1204	1	5.3015

QNO2 The actual solutions to the initial-value-problem in QNO1 is given here. Compare the actual error at each step to the error bond.

a) $y(t) = \frac{1}{5} te^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t}$

$$y(0.5) = \frac{1}{5}(0.5)e^{3(0.5)} - \frac{1}{25} e^{3(0.5)} + \frac{1}{25} e^{-2(0.5)}$$

$$\boxed{y(0.5) = 0.2836}$$

Table:

n	t_{n+1}	y_{n+1}	$y(t)$	error
0	0.5	0.5602	0.2836	0.2766
1	1	5.3012	3.2191	2.0824

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