NC Assignment Q1 , Q2 part a

Q No 1 part a)

$$y' = te^{3t} - 2y$$

$y(0) = 0$, $0 \leq t \leq 1$

$h = 0.5$

$t_0 = 0$, $y_0 = 0$

Solution:

$$y' = te^{3t} - 2y$$

$$\Rightarrow f(t_n, y_n) = t_n e^{3t_n} - 2y_n$$

Formula (Predictor)

$$\bar{y}_{n+1} = y_n + h[f(t_n + y_n)]$$

$$t_{n+1} = t_n + h$$

Formula (Corrector)

$$y_{n+1} = y_n + \frac{h}{2}\left[f(t_n, y_n) + f(\bar{t}_{n+1}, y_{n+1})\right]$$

$$t_{n+1} = t_n + h$$

For $n = 0$

$$\bar{y}_1 = y_0 + h[t_0 e^{3t_0} - 2y_0]$$

$$\bar{y}_1 = 0 + 0.5(0\, e^{3(0)} - 2(0)]$$

$$\boxed{\bar{y}_1 = 0}$$

$$\bar{t}_1 = t_0 + h$$
$$\bar{t}_1 = 0 + 0.5$$
$$\boxed{\bar{t}_1 = 0.5}$$

⇒ Corrector

$$y_1 = y_0 + \frac{h}{2}\left[(t_0 e^{3t_0} - 2y_0) + (\bar{t}_1 e^{3\bar{t}_1} - 2\bar{y}_1)\right]$$

$$y_1 = 0 + \frac{0.5}{2}\left[(0) + (0.5)e^{3(0.5)} - 2(0)\right]$$

$$\boxed{y_1 = 0.5602}$$

$$t_1 = t_0 + h$$
$$t_1 = 0 + 0.5$$
$$\boxed{t_1 = 0.5}$$

For $n = 1$

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Table

n	t_n	y_n	$\bar{t}_{n+1}$	$\bar{y}_{n+1}$	t_{n+1}	y_{n+1}
0	0	0	0.5	0	0.5	0.5602
1	0.5	0.5602	1	1.1204	1	5.3015

Q No 2 part b)

$$y(t) = \frac{1}{5} te^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t}$$

$0 \leq t \leq 1$, $h = 0.5$

$$y(0.5) = \frac{1}{5}(0.5) e^{3(0.5)} - \frac{1}{25} e^{3(0.5)} + \frac{1}{25} e^{-2(0.5)}$$

$$y(0.5) = 0.2836$$

Table

n	t_{n+1}	y_{n+1}	$y(t)$	Error
0	0.5	0.5602	0.2836	0.2766
1	1	5.3012	3.2191	2.0824