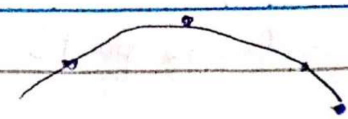
If we have 3 points.
$(x_0, y_0), (x_1, y_1), (x_2, y_2)$.



n data Points → Polynomial $(n-1)$ degree

INTERPOLATION (Unequal Space data)

Lagrange interpolation Method:-

x	y
x_1	y_1
x_2	y_2
x_3	y_3
⋮	⋮
x_n	y_n

Langrage function 1

$$P(x) = L_1(x)\, y_1 + L_2(x)\, y_2 + \ldots + L_n(x)\, y_n \quad \text{— (A)}$$

where

independent variable

from table

$$L_1(x) = \frac{(x - x_2)(x - x_3)\ldots(x - x_n)}{(x_1 - x_2)(x_1 - x_3)\ldots(x_1 - x_n)}$$

Similarly

$$L_2(x) = \frac{(x - x_1)(x - x_3)\ldots(x - x_n)}{(x_2 - x_1)(x_2 - x_3)\ldots(x_2 - x_n)}$$

Compressed form of above formulas.

$$P(x) = \sum_{i=1}^{n} L_i(x)\, y_i \quad \text{——— (A)}$$

where

$$L_i(x) = \prod_{\substack{J=1 \\ J \neq i}}^{n} \frac{(x - x_J)}{(x_i - x_J)} \quad \text{——— (B)}$$

Big pie its mean product

Q. Find the Lagrang polinomial for the given data.

	x	y	
x_1	1	1	y_1
x_2	3	5	y_2

Sol:-

Langrange interpolation.

$$P(x) = L_1(x)\, y_1 + L_2(x)\, y_2 \quad \text{——— (A)}$$

So,

$$L_1(x) = \frac{(x - x_2)}{(x_1 - x_2)} = \frac{x-3}{1-3} = \frac{x-3}{-2} \quad \text{——— (1)}$$

$$L_2(x) = \frac{(x - x_1)}{(x_2 - x_1)} = \frac{x-1}{3-1} = \frac{x-1}{2} \quad \text{——— (2)}$$

Put values in eq (A)

$$P(x) = \frac{x-3}{-2}(1) + \frac{x-1}{2}(5)$$

$$P(x) = \frac{-x+3}{2} + \frac{5x-5}{2}$$

$$= \frac{-x+3+5x-5}{2}$$

$$= \frac{4x-2}{2}$$

$$= \frac{\cancel{2}(2x-1)}{\cancel{2}}$$

$$= 2x-1$$

⇒ The above equation is used to find the points between points 1 and 2

Q2, Find the polinomial Langrange interpolation.

	x	y	
x_1	1	6	y_1
x_2	2	11	y_2
x_3	4	27	y_3

Sol:

$$P(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 \text{ ——— (A)}$$

So,

$$L_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}$$

$$= \frac{(x-2)(x-4)}{(1-2)(1-4)} = \frac{x^2-2x-4x+8}{(-1)(-3)} = \frac{x^2-6x+8}{3} \text{ ——— (1)}$$

=

$$L_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}$$

$$= \frac{(x-1)(x-4)}{(2-1)(2-4)} = \frac{x^2-1x-4x+4}{(1)(-2)}$$

$$= \frac{x^2-5x+4}{-2} \quad \text{—— } 2$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(x-1)(x-2)}{(4-1)(4-2)} = \frac{x^2-x-2x+2}{(3)(2)}$$

$$= \frac{x^2-3x+2}{6} \quad \text{—— } 3$$

Put above values in equation A.

$$P(x) = \frac{x^2-6x+8}{3}(6) + \frac{x^2-5x+4}{-2}(11) + \frac{x^2-3x+2}{6}(27)$$

$$= \frac{6x^2-36x+48}{3} + \frac{11x^2-55x+44}{-2} + \frac{27x^2-81x+54}{6}$$

$$= \frac{6x^2-36x+48}{3} + \frac{-11x^2+55x-44}{2} + \frac{27x^2-81x+54}{6}$$

$$= \frac{2(6x^2-36x+48)+3(-11x^2+55x-44)+27x^2-81x+54}{6}$$

$$= \frac{12x^2-72x+96-33x^2+165x-132+27x^2-81x+54}{6}$$

$$= \frac{6x^2+12x+18}{6}$$

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$$P(x) = \frac{\cancel{6}(x^2 + 2x + 3)}{\cancel{6}}$$

$$P(x) = x^2 + 2x + 3$$

Q Find the Langrange polinomial for the given data.

x	y
2	11
3	18
5	38

Sol:-

Langrange ~~Po~~ interpolation

$$P(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 \quad \text{——— (A)}$$

So,

$$L_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}$$

$$= \frac{(x-3)(x-5)}{(2-3)(2-5)}$$

$$= \frac{x^2-3x-5x+15}{(-1)(-3)}$$

$$= \frac{x^2-8x+15}{3} \quad \text{——— (1)}$$

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$$L_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}$$

$$= \frac{(x-2)(x-5)}{(3-2)(3-5)}$$

$$= \frac{x^2-2x-5x+10}{(1)(-2)}$$

$$= \frac{x^2-7x+10}{-2} \text{ ——— } (2)$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(x-2)(x-3)}{(5-2)(5-3)}$$

$$= \frac{x^2-2x-3x+6}{(3)(+2)}$$

$$= \frac{x^2-5x+6}{6} \text{ ——— } (3)$$

Put above values in equation (A)

$$P(x) = \frac{x^2-8x+15}{3}(11) + \frac{x^2-7x+10}{-2}(18) + \frac{x^2-5x+6}{6}(38)$$

$$= \frac{11x^2-88x+165}{3} + \frac{(-18x^2+126x+180)}{2} + \frac{38x^2-190x+228}{6}$$

$$= \frac{2(11x^2-88x+165)+3(-18x^2+126x-180)+38x^2-190x+228}{6}$$

$$= \frac{22x^2-176+330-54x^2+378x-540+38x^2-190x+228}{6}$$

$$P(x) = \frac{6x^2 + 12x + 18}{6}$$

$$= \frac{\cancel{6}(x^2 + 2x + 3)}{\cancel{6}}$$

$$P(x) = x^2 + 2x + 3$$

Q (A) Find the Langrange palinomial for the given data.
(B) Interpolate $x=2$

	x	y	
x_1	1	-3	y_1
x_2	3	13	y_2
x_3	4	39	y_3
x_4	5	85	y_4

Sol:-

Langrange interpolation.

$$P(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 + L_4(x)y_4 \quad \text{——— (A)}$$

So,

$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$= \frac{(x-3)(x-4)(x-5)}{(1-3)(1-4)(1-5)}$$

$$= \frac{(x-3)(x^2-4x-5x+20)}{(-2)(-3)(-4)}$$

$$= \frac{(x-3)(x^2-9x+20)}{-24}$$

$$= \frac{x^3-9x^2+20x-3x^2+27x-60}{-24}$$

$$= \frac{x^3-12x^2+47x-60}{-24}$$

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$$L_2(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$= \frac{(x-1)(x-4)(x-5)}{(3-1)(3-4)(3-5)}$$

$$= \frac{(x-1)(x^2-4x-5x+20)}{(2)(-1)(-2)}$$

$$= \frac{(x-1)(x^2-9x+20)}{4}$$

$$= \frac{x^3-9x^2+20x-x^2+9x-20}{4}$$

$$= \frac{x^3-10x^2+29x-20}{4}$$

$$L_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$= \frac{(x-1)(x-3)(x-5)}{(4-1)(4-3)(4-5)}$$

$$= \frac{(x-1)(x^2-3x-5x+15)}{(3)(1)(-1)}$$

$$= \frac{x^3-8x^2+15x-x^2+8x-15}{-3}$$

$$= \frac{x^3-9x^2+23x-15}{-3}$$

$$L_4(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$= \frac{(x-1)(x-3)(x-4)}{(5-1)(5-3)(5-4)}$$

$$= \frac{(x-1)(x^2-3x-4x+12)}{(4)(2)(1)}$$

$$= \frac{(x-1)(x^2-7x+12)}{8}$$

$$= \frac{x^3-7x^2+12x-x^2+7x-12}{8}$$

$$= \frac{x^3-8x^2+19x-12}{8}$$

$$P(x) = \frac{x^3-12x^2+47x-60}{-24}(-3) + \frac{x^3-10x^2+29x-20}{4}(13) +$$

$$\frac{x^3-9x^2+23x-15}{-3}(39) + \frac{x^3-8x^2+19x-12}{8}(85$$

$$= \frac{-3x^3+36x^2-141x+180}{-24} + \frac{13x^3-130x^2+377x-260}{4} +$$

$$\frac{39x^3-351x^2+897x-585}{-3} + \frac{85x^3-680x^2+1615x-1020}{8}$$

$$= \frac{-1(-3x^3+36x^2-141x+180)+6(13x^3-130x^2+377x-260)-8(39x^3-351x^2+897x-585)+3(85x^3-680x^2+1615x-1020)}{24}$$

$$= \frac{+3x^3-36x^2+141x-180+78x^3-780x^2+2262x-1560-312x^3+2808x^2-7176x+4680+255x^3-2040x^2+4845x-3060}{24}$$

$$P(x) = \frac{24x^3 - 48x^2 + 72x - 120}{24}$$

$$= \frac{\cancel{24}(x^3 - 2x^2 + 3x - 5)}{\cancel{24}}$$

$$P(x) = x^3 - 2x^2 + 3x - 5$$

Find $x = 2$

$$P(2) = (2)^3 - 2(2)^2 + 3(2) - 5$$

$$P(2) = 8 - 8 + 6 - 5$$

$$P(2) = 1.$$

NEWTON DIVIDED INTERPOLATION FORMULA

$$P(x) = f[x_1] + f[x_1, x_2](x-x_1) + f[x_1, x_2, x_3](x-x_1)(x-x_2) + f[x_1, x_2, x_3, x_4](x-x_1)(x-x_2)(x-x_3) + \dots + \text{upto } (n-1)^{th} \text{ term.}$$

Divided Difference Table:-

x	y	$f[\,,\,]$	$f[\,,\,,\,]$
x_1	y_1		
		$\frac{y_2-y_1}{x_2-x_1} = f[x_1,x_2]$	
			$\frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$
x_2	y_2		
		$\frac{y_3-y_2}{x_3-x_2} = f[x_2,x_3]$	
			$\frac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2}$
x_3	y_3		
		$\frac{y_4-y_3}{x_4-x_3} = f[x_3,x_4]$	
x_4	y_4		

Q1. Find the polinomial for give data set by using newton divided interpolation formula.

x	y
1	6
2	11
4	27

Sol:-

x	y	$f[\,,\,]$	$f[\,,\,,\,]$
1	6 $f[x]$		
		$f[x_1,x_2]$ $\frac{11-6}{2-1} = 5$	
			$f[x_1,x_2,x_3]$ $\frac{8-5}{4-1} = 1$
2	11		
		$f[x_2,x_3]$ $\frac{27-11}{4-2} = 8$	
4	27		

$\Rightarrow$ As in formula the top row entries will be used.

$$P(x) = f[x_1] + f[x_1,x_2](x-x_1) + f[x_1,x_2,x_3](x-x_1)(x-x_2)$$

$$P(x) = 6 + 5(x-1) + 1(x-1)(x-2)$$

$$P(x) = 6 + 5(x-1) + 1(x^2-3x+2)$$

$$P(x) = 6 + 5x - 5 + x^2 - 3x + 2$$

$$P(x) = x^2 + 2x + 3$$

Q2. Find the polinomial for the given data

x	y
1	-3
3	13
4	39
5	85

x	y $f[x_1]$	$f[,]$	$f[,,]$	$f[,,,]$
1	-3			
		$f[x_1,x_2]$ $\frac{13-(-3)}{3-1} = 8$		
			$f[x_1,x_2,x_3]$ $\frac{26-8}{4-1} = 6$	$f[x_1,x_2,x_3,x_4]$
3	13			$\frac{10-6}{5-1} = 1$
		$\frac{39-13}{4-3} = 26$		
			$\frac{46-26}{5-3} = 10$	
4	39			
		$\frac{85-39}{5-4} = 46$		
5	85			

$f[x_1] = -3$

$f[x_1, x_2] = 8$

$f[\cancel{x_3}, \cancel{f}x_2, \cancel{f x_3}] = 6$

$f[x_1, x_2, x_3, \cancel{x_4}] = 1$

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$$P(x) = f[x_1] + f[x_1, x_2](x - x_1) + f[x_1, x_2, x_3](x - x_1)(x - x_2) +$$
$$f[x_1, x_2, x_3, x_4](x - x_1)(x - x_2)(x - x_3)$$
$$= -3 + 8(x-1) + 6(x-1)(x-3) + 1(x-1)(x-3)(x-4)$$
$$= -3 + 8x - 8 + 6(x^2 - 4x + 3) + 1(x-1)(x^2 - 7x + 12)$$
$$= -3 + 8x - 8 + 6x^2 - 24x + 18 + x^3 - 7x^2 + 12x - x^2 + 7x - 12$$
$$= x^3 - 2x^2 + 3x - 5$$