

Q NO 1) Use improved Euler method to approximate the solution for each of the initial-value problems.

$$a) \quad y' = te^{3t} - 2y$$

Given

$$0 \leq t \leq 1, \quad y(0) = 0, \quad h = 0.5$$

Solution

$$t_0 = 0, \quad y_0 = 0$$

$$y' = te^{3t} - 2y$$

$$f(t_n, y_n) = t_n e^{3t_n} - 2y_n$$

For $n=0$

Predictor:

$$\bar{y}_1 = y_0 + h [t_0 e^{3t_0} - 2y_0]$$

$$\bar{y}_1 = 0 + 0.5 [0 e^{3(0)} - 2(0)]$$

$$\bar{y}_1 = 0$$

$$\bar{t}_1 = t_0 + h$$

$$\bar{t}_1 = 0 + 0.5$$

$$\bar{t}_1 = 0.5$$

Corrector:

$$y_1 = y_0 + \frac{h}{2} [(t_0 e^{3t_0} - 2y_0) + (\bar{t}_1 e^{3\bar{t}_1} - 2\bar{y}_1)]$$

$$y_1 = 0 + \frac{0.5}{2} [(0) + (0.5)e^{3(0.5)} - 2(0)]$$

$$y_1 = 0.5602$$

$$t_1 = t_0 + h$$

$$t_1 = 0 + 0.5$$

$$t_1 = 0.5$$

For $n=1$

Predictor:

$$\bar{y}_2 = y_1 + h [t_1 e^{3t_1} - 2y_1]$$

$$\bar{y}_2 = 0.5602 + 0.5 [(0.5)e^{3(0.5)} - 2(0.5602)]$$

$$\bar{y}_2 = 1.1204$$

$$\bar{t}_2 = t_1 + h$$

$$\bar{t}_2 = 0.5 + 0.5$$

$$\bar{t}_2 = 1$$

Corrector:

$$y_2 = y_1 + \frac{h}{2} \left[(t_1 e^{3t_1} - 2y_1) + (\bar{t}_1 e^{3\bar{t}_1} - 2\bar{y}_1) \right]$$

$$y_2 = 0.5602 + \frac{0.5}{2} \left[(1.1204) + (1e^{3(1)} - 2(1.1204)) \right]$$

$$y_2 = 5.3015$$

$$t_2 = t_1 + h$$

$$t_2 = 0.5 + 0.5$$

$$t_2 = 1$$

Table:

n	t_n	y_n	\bar{t}_{n+1}	\bar{y}_{n+1}	t_{n+1}	y_{n+1}
0	0	0	0.5	0	0.5	0.5602
1	0.5	0.5602	1	1.1204	1	5.3015

Q No 2 The actual solutions to the initial-val-
problem in Q No 1 is given here. Compare
the actual error at each step to the
error bond.

$$a) y(t) = \frac{1}{5} t e^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t}$$

$$y(0.5) = \frac{1}{5} (0.5) e^{3(0.5)} - \frac{1}{25} e^{3(0.5)} + \frac{1}{25} e^{-2(0.5)}$$

$$y(0.5) = 0.2836$$

Table:

n	t_{n+1}	y_{n+1}	$y(t)$	error
0	0.5	0.5602	0.2836	0.2766
1	1	5.3012	3.2191	2.0824