1. Dolittle's Method:-

is used to decompose matrix A into upper triangular and lower triangular where the digonal of Lower triangular is ristricted to 1.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$$ Decompose matrix A into L, U decomposition using Dolittle Method.

Sol

$$A = L \times U$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \times \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} U_{11}+0 & U_{12}+0 \\ l_{21}U_{11}+0 & l_{21}U_{12}+U_{22} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ l_{21}U_{11} & l_{21}U_{12}+U_{22} \end{bmatrix}$$

$U_{11} = 2$, $U_{12} = 4$

$l_{21}U_{11} = 3$ $\qquad$ $l_{21}U_{12} + U_{22} = 1$

$l_{21}(2) = 3$ $\qquad$ $\left(\frac{3}{2}\right)(4) + U_{22} = 1$

$l_{21} = \frac{3}{2}$ $\qquad$ $\left(\frac{12}{2}\right) + U_{22} = 1$

$U_{22} = 1 - \left(\frac{12}{2}\right) = 1 - 6 = -5$.

$$L = \begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix}$$

Decomposition of matrix A.

Varification....

$LU = A$

$$\begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 2+0 & 4+0 \\ (\frac{3}{\cancel{2}})(\cancel{2})+0 & \frac{3}{\cancel{2}}(\cancel{4})^{2}+1(-5) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$$

Decompose

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 3 \\ 4 & 3 & 6 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

CROUT'S METHOD:-

Crouts method is used to decompose a square matrix into upper and lower matrix where the digonal matrix of upper matrix is equal to 1.

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Q:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 4 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

Sol:-

$$A = L \times U$$

by using crouts method.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 4 & 2 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

$\boxed{l_{11} = 2}$ $\boxed{l_{21} = 3}$ $\boxed{l_{31} = 2}$

$\cancel{l_{11} U_{12} = 1}$
$\cancel{l_{11}(3) = 1}$
$\cancel{\boxed{l_{11} = 1/3}}$

$l_{11} U_{12} = 1$
$(2) U_{12} = 1$
$\boxed{U_{12} = 1/2}$

$l_{11} U_{13} = 1$
$(2) U_{13} = 1$
$\boxed{U_{13} = 1/2}$

$l_{21} U_{12} + l_{22} = 4$
$(3)(1/2) + l_{22} = 4$
$(3/2) + l_{22} = 4$
$l_{22} = 4 - 3/2$
$\boxed{l_{22} = 5/2}$

$l_{21} U_{13} + l_{22} U_{23} = 2$
$(3)(1/2) + 5/2\, U_{23} = 2$
$3/2 + 5/2\, U_{23} = 2$
$5/2\, U_{23} = 2 - 3/2$
$5/2\, U_{23} = 1/2$
$U_{23} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{5}$
$\boxed{U_{23} = 1/5}$

$l_{31} U_{12} + l_{32} = 3$
$(\cancel{2})(1/\cancel{2}) + l_{32} = 3$
$1 + l_{32} = 3$
$l_{32} = 3 - 1$
$\boxed{l_{32} = 2}$

$l_{31} U_{13} + l_{32} U_{23} + l_{33} = 5$
$(\cancel{2})(1/\cancel{2}) + (2)(1/5) + l_{33} = 5$
$1 + 2/5 + l_{33} = 5$
$7/5 + l_{33} = 5$
$l_{33} = 5 - 7/5$
$\boxed{l_{33} = \frac{18}{5}}$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}, \quad U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & \cancel{l_{22}} & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 5/2 & 0 \\ 2 & 2 & 18/5 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 1/5 \\ 0 & 0 & 1 \end{bmatrix}$$