Mathematical Techniques to capture Grid System

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1 Introduction

This document outlines the mathematical techniques employed in the rugby analysis grid system, focusing on perspective transformation, motion detection, and geometric computations.

2 Perspective Transform

2.1 Homography Matrix

The system uses a homography matrix H to map points from the real-world plane to the image plane:

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Where (x', y') are image coordinates and (x, y) are world coordinates.

2.2 Grid Point Generation

Grid points are generated using the perspective transform:

$$p' = H \cdot p$$

where $p = (x, y, 1)^T$ represents a point in world coordinates.

3 Background Subtraction

3.1 Gaussian Mixture Model

The MOG2 background subtractor uses a Gaussian Mixture Model where each background pixel is modelled by a mixture of K Gaussian distributions:

$$P(X_t) = \sum_{i=1}^{K} w_{i,t} \cdot \mathcal{N}(X_t, \mu_{i,t}, \Sigma_{i,t})$$

Where:

- X_t is the pixel value at time t
- $w_{i,t}$ is the weight of the ith Gaussian
- $\mu_{i,t}$ is the mean of the ith Gaussian
- $\Sigma_{i,t}$ is the covariance matrix

4 Grid Marker Detection

4.1 Colour Space Transformation

HSV colour space transformation is used for marker detection:

$$\begin{bmatrix} H \\ S \\ V \end{bmatrix} = f\left(\begin{bmatrix} R \\ G \\ B \end{bmatrix}\right)$$

4.2 Circularity Measure

Marker circularity calculated using:

$$C = \frac{4\pi A}{P^2}$$

where A is the contour area and P is the perimeter.

5 Grid Occlusion

5.1 Mask Operations

Grid occlusion uses binary mask operations:

$$M_{final} = M_{grid} \wedge \neg(M_{foreground} \oplus B)$$

where:

- M_{grid} is the binary grid mask
- $M_{foreground}$ is the foreground mask
- B is the structuring element for dilation
- $\oplus$ denotes morphological dilation
- $\wedge$ denotes logical AND
- $\neg$ denotes logical NOT

6 Distance Metrics

6.1 Euclidean Distance

Used for point correspondence and tracking:

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

7 Implementation Considerations

7.1 Numerical Stability

To ensure numerical stability in the perspective transform:

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

NB: Care is required to handle cases where the denominator approaches zero.

8 Performance Optimisations

8.1 Mask Operations

Binary mask operations are optimised using lookup tables and vectorised operations:

$$\text{LUT}(i) = f(i), \quad i \in [0, 255]$$