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THE ROLE OF VALONS IN LOW-p_T PHYSICS*

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ABSTRACT

Hadron structure is described in the framework in which a nucleon is treated as a composite system of three valence quark clusters, called valons. Their momentum distribution is extracted from deep inelastic scattering data. The valon representation provides a quantitative description of the recombination function, which characterizes the hadronization of quarks. A formalism is then developed in terms of valons and quarks such that the inclusive distributions for hadronic reactions at low-p_T can be calculated without free parameters.

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I. INTRODUCTION

In applying the quark-parton model to the description of low-p_T hadronic reactions, some of the questions one should bear in mind are: Is the constituent picture reliable when there is no large Q^2 in the problem? Are the relevant quarks the so-called "constituent quarks" in the quark model, or are they the "current quarks" as probed in electroproduction? How do the quarks hadronize? Any treatment of low-p_T physics in the framework of quarks must provide some answers to these questions.

There are hints that suggest the relevance of the quark-parton model. The observation by Ochs[1] that $(x/\sigma)(d\sigma/dx)$ at low-p_T looks similar to $\nu W_2(x)$ is certainly suggestive. The recombination model provides further support at a quantiative level.[2,3] In the fragmentation region of one of the incident particles, the hadrons produced have a distribution that depends only on the nature of the fragmenting hadrons, and not the other. In the framework of the quark-parton model the fragmentation process is to be described in terms of quarks in the intermediate state between the initial hadron and the final detected particle. It therefore means that we must at least know hadron structure as the first part of the process as well as quark hadronization as the final part of the process. What goes on in between is the effect of the collision.

In the following we shall first discuss the problem of structure and hadronization which are intimately related once the proper representation is found. It is here that the valons play a crucial role. The problem of low-p_T reactions in the fragmentation region is then formulated. The result of a specific calculation for proton fragmenting into a π^+ will be presented.

It should be mentioned that the relationship between our approach here which is basically recombination and the approach of Lund, Orsay, and Saclay[4] (LOS) which is basically fragmentation is at the present stage an open question. Our approach is confined to the fragmentation region, while the LOS approach is an extension from the central region (where Regge theory is used) to the fragmentation region using a phenomenological fragmentation function without addressing the basic questions concerning hadronization.

II. HADRON STRUCTURE

In the quark model or bag model suitable for describing bound-state problems, there are only three (constituent) quarks in a proton. In the parton model suitable for describing deep-inelastic scattering processes, there are three valence quarks plus an infinity of sea quarks and gluons. How can these two picures of the nucleon be reconciled? A host of people[5] have suggested the idea that the constituent quarks are clusters of partons (quarks, antiquarks, and gluons) or dressed quarks. For brevity as well as for emphasis that they are not the point-like quarks probed in deep-inelastic scattering, we shall refer to these valence quark clusters as valons. Later in our discussion we shall give a more precise mathematical definition of the valons, which may differ somewhat from other definitions in Ref.5. But physically we may think of them as the constituent quarks in the bound-state problem.

It has sometimes been asked from the standpoint of the parton model why a nucleon should have only three clusters of partons. Why not more? Why shouldn't there be gluon clusters? The answer is, of course, that we are primarily interested in building a bridge between the bound-state and scattering problems. From the parton model alone one wouldn't know that there are only three. The picture can best be described by considering instead the deuteron problem for a moment. A first-order view (in some sense) of the deuteron is that it has a proton and a neutron. Pions that are exchanged between the nucleons to provide the binding are not regarded as a third constituent or cluster of pions. The wave funciton of the deuteron in terms of the nucleons alone describes the binding effect without explicit reference to the pion glue. This picture is, of course, incomplete especially at short distance between the nucleons. But generally the picture is adequate, i.e. the proton-neutron Fock space is nearly complete for the description the deuteron. It is suitable even for scattering at high energy so long as the composite nature of the nucleons is recognized. Indeed, it is this picture of the deuteron that provides the determination of the structure function of the neutron in deep-inelastic e-d scattering.

The situation is very similar in the case of the nucleon structure. A nucleon has three valons, the detail internal structure of which cannot be resolved at low Q^2. The probability for a valon to have momentum fraction y in the hadron (nucleon) is $G_{v/h}(y)$. In deep-inelastic scattering the virtual photon at high Q^2 has the resolution to probe the partons in a valon, the structure function of which is denoted by $F^v(z,Q^2)$. If Q^2 is high enough, the binding energy of the valons is small compared to Q^2, so we may use impulse approximation to justify the neglect of spectators in a deep-inelastic process and write the structure function of the nucleon as a convolution[6,7,8]

$$F^h(x,Q^2) = \sum_v \int_x^1 dy\ G_{v/h}(y) F^v(x/y,Q^2) \tag{2.1}$$

If we have reliably solved the bound-state problem of the nucleon, we would know $G_{v/h}(y)$. Using perturbative QCD we can determine $F^v(x/y,Q^2)$ at high Q^2. Combining the two in (2.1) would then enable us to determine the nucleon structure function from first principle. Unfortunately, the confinement problem is still unsolved, and the so-called "hadronic complication" remains untractable. However, we can turn the procedure around by using the experimental data for $F^h(x,Q^2)$ as input and derive through (2.1) a semi-phenomenological description for $G_{v/h}(y)$. That is what we have done[7-9] using muon as well as neutrino scattering data at high Q^2 as input.

For details of the extraction of $G_{v/h}(y)$ the reader is referred to Refs.7-9. Before the results are exhibited, it is important to state the approximation made so that the mathematical definition of the valons can be made precise. In applying (2.1) we have used only the leading-order result for $F^v(z,Q^2)$. That is, for the moments $M^v(n,Q^2)$ of $F^v(z,Q^2)$ we have used for the nonsinglet part

$$M^v_{NS}(n,Q^2) = \exp\left(-d_n^{NS}\ s\right) \tag{2.2}$$

where d_n^{NS} is the anomalous dimension and s is the evolution parameter

$$s = \ln \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \tag{2.3}$$

The singlet part has a combination of exponentials as in (2.2). For ease of discussion let us focus only on the non-singlet component here. Because of non-leading order and higher-twist contributions, $M^v_{NS}(n,Q^2)$ deviates from the form given in (2.2) when Q^2 is small. But when Q^2 is large, the exponential form in (2.2) is reliable. We adjust Q_0^2 and Λ^2 such that the data at high Q^2 can be fitted. According to (2.1) $G_{v/h}(y)$ is to be identified with $F^h(y)$ if $F^v(z,Q^2)$ is $\delta(z - 1)$. The δ function is just what $M^v_{NS}(n,Q^2)$ would correspond to (i.e. equal to one for all n) if (2.2) is extrapolated to $Q^2 = Q_0^2$. Thus the valons are effective constituents of the hadron at Q_0^2, effective in the precise sense that leading-order expressions such as (2.2) are extrapolated to Q_0^2 (beyond its region of validity). Since there exists no reliable method to extrapolate accurately to the low Q^2 region without first solving the confinement problem (which is what we want to circumvent) the inclusion of the next-to-the-leading-order terms in (2.2) would considerlably complicate the matter without yielding a more illuminating description of the constituent quarks. Since Q_0 turns out to be 0.8 GeV[9] which is quite reasonable (larger than the inverse size of the nucleon but smaller than the onset of precocious scaling), the valons described by $G_{v/h}(y)$ may be thought of as an approximation of the constituent quarks. The important point is that the wave function of the nucleon in terms of the valons, or more precisely its absolute square, is mathematically defined by (2.1) and (2.2), and can be extracted from data.

We now give the result.[9] Let the exclusive three-valon distribution in a proton have the form

$$G_{UUD/p}(y_1,y_2,y_3,) = \alpha(y_1y_2)^a y_3^b \, \delta(y_1 + y_2 + y_3 - 1) \quad , \tag{2.4}$$

where α is a normalization constant that is fixed by

$$\int dy_1 dy_2 dy_3 \; G_{UUD/p}(y_1y_2y_3) = 1 \quad . \tag{2.5}$$

All high-Q^2 data ($Q^2 > 20$ GeV^2) on deep-inelastic μp, μn, νN and $\bar{\nu}N$ scattering have been fitted by

$$Q_0 = 0.8 \text{ GeV} \quad , \qquad \Lambda = 0.65 \text{ GeV} \quad , \tag{2.6}$$

$$a = 0.65 \quad , \qquad b = 0.35 \quad . \tag{2.7}$$

Eqs.(2.6) and (2.7) describe two separate aspects of the valons: Q_0^{-1} gives an estimate of the effective size of the valons, while a and b specify the momentum distribution of the valons. We shall use (2.6) for all valons regardless of the type of hadron they are in, but the values of a and b obviously will depend on the particular hadron.

The single-valon inclusive distribution is obtained from (2.4) by integrating out the two unspecified y variables. The result is

$$G_{U/p}(y) = 7.98\, y^{0.65}\, (1 - y)^2 \quad , \tag{2.8a}$$

$$G_{D/p}(y) = 6.01\, y^{0.35}\, (1 - y)^{2.3} \quad . \tag{2.8b}$$

The two-valon inclusive distribution can be even more simply obtained by making one trivial integration. If the valons have no internal motion in the proton, then the G functions in (2.8) should be proportional to $\delta(y - 1/3)$. The fact that they are so widely spread out indicates that the binding effects are not negligible. Yet, the ability to fit all high-Q^2 data implies that (2.1) is sensible; hence, the impulse approsimation contained in (2.1) must be approximately valid at such high Q^2 despite the significant binding effect on the valon distributions.

It should be remarked that (2.8) should not be trusted in the $y \to 1$ limit because theoretically the off-mass-shell effects have not been taken into account and experimentally the data fitted are only for moments with $n \leqslant 10$. But in the bulk of the y range our result should be a reliable description of $|\langle UUD|P\rangle|^2$. Note that, as implied earlier, the three valon state is a nearly complete description of the proton. This is to be contrasted from the behavior in the $x \to 1$ limit where the Fock state containing three quarks $\langle uud|$ becomes more important than the other states with more quarks.[10] Because the kinematical region and the constituents are different from ours, the large-x behavior of $|\langle uud|P\rangle|^2$ based on counting rule should not bear any resemblance to (2.8).

For the pion we can derive the valon distribution from the pion structure function inferred from lepton-pair production data,[11] at least over the region of x for which the higher-twist effects[12,13] are not important. It has been found[14] that the pion structure function can be well fitted by use of a valon distribution that has the simple form

$$G_{v/\pi}(y) = 1 \quad . \tag{2.9}$$

This is a non-trivial phenomenological result even though it seems to be a trivial solution of the sum rules

$$\int_0^1 G_{v/\pi}(y)dy = 1 \quad , \tag{2.10a}$$

$$\int_0^1 G_{v/\pi}(y)y\,dy = 1/2 \quad . \tag{2.10b}$$

It can easily be verified that any expression of the form $\alpha\, y^a(1 - y)^a$ can satisfy both of (2.10).

To summarize what has been discussed so far, we have found a valon representation for the description of the hadron. In terms of quarks and gluons at high Q^2 the wave function $\langle q\ldots,\bar{q}\ldots,g\ldots|P\rangle$ is untractable since the different parton-number states in the entire Fock space are all important. In terms of valons at low Q^2 the wave function $\langle UUD|P\rangle$ is simple. Its relationship to the parton inclusive distribution at high Q^2 is made calculable in QCD by virtue of the definition of the valons, i.e. leading-order evolution in Q^2.

III. HADRONIZATION OF QUARKS

The problem of hadronization of quarks has thus far been studied only in the recombination model.[2,8,15] In the early version[2] the recombination function was discussed in the context of counting rule.[16] But the extent of its validity was uncertain except that it yielded a sensible result. Moreover, the normalization was unspecified and the role of gluons ignored. Clearly, a more thorough study of the hadronization problem is needed.

The availability of a valon representation for the hadron facilitates a quantitative description of the hadronization problem. The knowledge of the proton wave function in terms of the valons $\langle UUD|P\rangle$ obviously implies a knowledge of the recombination function from valons to proton $|\langle P|UUD\rangle|^2$. Similarly, for pion formation we would have $|\langle\pi^+|U\bar{D}\rangle|^2$. Defining R to be the probability for recombination in invariant phase space, we then have[8,15] from (2.9) and (2.4)

$$R^{\pi}(x_1,x_2,x) = \frac{x_1x_2}{x^2}\,\delta\left(\frac{x_1}{x} + \frac{x_2}{x} - 1\right) \quad , \tag{3.1}$$

$$R^N(x_1,x_2,x_3,x) = \frac{105}{2\pi}\left(\frac{x_1x_2x_3}{x^3}\right)^{3/2} \delta\left(\frac{x_1}{x}+\frac{x_2}{x}+\frac{x_3}{x}-1\right) \quad , \qquad (3.2)$$

where in (3.2) we have neglected flavor dependence, i.e. setting a=b=1/2 in (2.4). Note that not only is the dependence on x_1 and x_2 in (3.1) exactly as suggested in Ref.2, the normalization is now fixed.

Because the valons are parton clusters that include gluons, the role of gluons in recombination is therefore not ignored. It then seems that the burden is on the determination of the valon distribution in a process just before hadronization. However, it is only necessary to calculate the quark and antiquark distribution (for the production of pions) because a quark (or antiquark) can in time dress itself up by virtual processes and turn into a valon with the same momentum. Since the convolution integral that describes hadronization in the recombination model involves integration over momenta of the recombining constituents, it does not matter whether we call them quarks or valons: the same momentum distribution is involved.

We have tested the recombination mechanism by calculating the quark fragmentation function in e^+e^- annihilation processes using perturbative QCD to determine the $q\bar{q}$ distribution in a quark jet and then (3.1) for hadronization. The result[15] agrees well with data, and represents a first successful attempt to describe quark fragmentation process all the way to the hadron level. Since no adjustable parameters have been used, it gives a good check of the recombination model for hadronization, certainly necessary if not sufficient.

In the calculation described above the contribution due to resonance decay has been added by hand according to known procedure[17] developed for familiar hadronic reactions. It would be more preferable if resonance production can be independently calculated in the recombination model. However, the corresponding recombination function is unavailable. What is needed is an analysis of the inclusive π distribution in photoproduction and by use of the recombination model determine the structure function of ρ in just the same way that the structure function of π and K have been determined.[18] The recombination function for ρ then follows from the valon distribution in ρ .

IV. LOW-p_T REACTIONS

The stage is now set for treating the low-p_T problem. We cannot delay further the first question asked at the beginning: is the constituent picture reliable when there is no large Q^2 in the problem? We have already made progress toward answering this question by showing the relevance of valons. Before collision an incident hadron can be described by wave function in terms of the valons. After collision hadronization takes place also from the valons. The central question now is how the initial valons turn into a multitude of final valons. In the fragmentation region of one incident particle the other incident particle is not relevant by virtue of short-range correlation in parton rapidity. Thus we ask how three valons (in the case of proton) break up into partons which dress themselves up after a long time to become many valons. Since we confine our attention to the fragmentation region, we need not be specific about what takes place in the central region; color separation may or may not be an important mechanism. We need only recognize that the bags are broken upon collision, and the unknown confinement mechanism that keeps the three valons as distinct entities in the static problem is no longer operative. The partons in each valon which are originally virtual in the static problem become on-shell and take on definite world lines in accordance to their momentum distributions in the valon, which in turn has a momentum distribution in the initial hadron. The partons in the fragmentation region are all very near the light cone at high energies and the time scale involved for hadronization is highly dilated in the c.m. system. What we need is the parton distribution in the valons.

Is it meaningful to discuss parton distribution without being at high Q^2? The question can best be approached first on phenomenological grounds. We know that even for Q^2 in the $1 - 3$ GeV^2 range in the early days of electroporoduction experiments at SLAC, "precocious" scaling was already found for $\nu W_2(x)$, even though theoretically there was no compelling reason for it to be so. At higher Q^2 the modification on $\nu W_2(x)$ is only of order $\log Q^2$ correction which is calculable in perturbative QCD. What the phenomenology reveals for us is that there is a primitive parton distribution which is not calculable at present and which gives rise to the observed $\nu W_2(x)$ at low Q^2 where the parton model itself may be questionable as originally formulated. We venture to use parton model at low Q^2 not on the usual basis that the interaction time is short compared to the lifetime of a parton in a particular state, but rather on the following basis. Each valon has a primitive parton distribution, which spread out over a wide range in the rapidity space because of soft gluon radiation. Hadronization, however, occurs in a limited range in rapidity space for each detected hadron. Thus in calculating inclusive cross section one is essentially focussing on a narrow region in rapidity, ignoring whatever that goes on in the rest of the rapidity space. This is very akin to the impulse approximation which gives justification for ignoring the response of the residual system. Indeed, there are models which give a close relationship between rapidity and time. Short-range parton interaction in rapidity therefore justifies the discussion of partons at low Q^2. The assumption of the existence of an universal primitive parton distribution in every valon then enables us to express the parton distributions in a hadron by convolution integrals.

The unknown in the above description is the primitive parton distribution. We have determined it phenomenologically by fitting the low Q^2 data on $\nu W_2(x)$.[8] Once done, there are no free parameters left in the problem. If we denote by $P(z)$ the primitive single-parton distribution and $P(z_1,z_2)$ the primitive two-parton distribution in a valon, then the $q\bar{q}$ distribution in a hadron is

$$F(x_1,x_2) = F^{(1)}(x_1,x_2) + F^{(2)}(x_1,x_2) \quad , \tag{4.1}$$

$$F^{(1)}(x_1,x_2) = \sum_v \int_{x_1+x_2}^{1} dy\, G_{v/h}(y) P_{q\bar{q}}\left(\frac{x_1}{y}, \frac{x_2}{y}\right) \quad , \tag{4.2}$$

$$F^{(2)}(x_1,x_2) = \sum_{v_1 v_2} \int_{x_1}^{1} dy_1 \int_{x_2}^{1-y_1} dy_2\, G_{v_1 v_2/h}(y_1,y_2) P_q\left(\frac{x_1}{y_1}\right) P_{\bar{q}}\left(\frac{x_2}{y_2}\right). \tag{4.3}$$

Clearly, $F^{(1)}$ and $F^{(2)}$ describe separately the contributions from a single valon and two valons, respectively. As we have discussed in Sec.III, these quark and antiquark can develop their own clusters, with the corresponding valons still having momentum fractions x_1 and x_2. Hence, we can directly use the recombination formula[2]

$$\frac{x}{\sigma}\frac{d\sigma}{dx} = \int F(x_1,x_2) R(x_1,x_2,x) \frac{dx_1}{x_1}\frac{dx_2}{x_2} \quad , \tag{4.4}$$

in the calculation of inclusive pion distribution.

The result of our no-parameter calculation is remarkably good.[8] It is shown in Fig.1. It agrees with data[19] over a variation of three orders of magnitudes. It gives strong indication that the formalism presented has captured the essence of hadron fragmentation at low-p_T.

Application of the formalism to various reactions involving different beam and detected particles can readily be carried out. In particular, attention to reactions involving kaons should lead to a determination of the valon distribution in kaons.[14]

V. DISCUSSION

It is useful to point out that the formalism presented here does not rely on detailed description of color separation in low-p_T reactions. Nor does it make use of any quark fragmentation functions borrowed from high-Q^2 e^+e^- annihilation data. To use phenomenological fragmentation function in low-p_T physics is to abandon any attempt to treat the hadronization problem. The point of view of our approach is to start from the valon-parton basis in describing inclusive reactions and hopefully to derive Regge behavior eventually as an output. In contrast, the fragmemtation model[4] is based on dual-topological-unitarization diagrams intimately related to Regge theory. It may be eminently sensible for describing the central region, but its extention to the fragmentation region does not offer an elucidation of the hadronization process. The two approaches may be complementary. Since they emphasize different regions, unification is a possibility that cannot be ruled out.

It is of interest to point out also that the valon representation of a hadron offers an intriguing possibility of investigating certain problems that have thus far been mainly the concern of those solving bound-state problems. The valon distribution is a statement about the wave function of the hadron, but instead of solving a potential problem or the bag problem, we have obtained it from deep inelastic scattering data. It is our hope that on the basis of our valon distribution we can calculate such quantities as the pion decay constant f_π. Preliminary effort[20] in that direction indicates that a numerical determination of f_π is not only feasible but seems to yield a result that is close to the experimental value.

In the area of low-p_T physics the recombination model has been used to calculate two-particle correlation with remarkable success.[21] Since the two-particle distributions reveal detailed properties of the hadronization process in the fragmentation region, they may well pose crucial tests for the various models of hadronic reactions at low-p_T.

Another area of application for the valon-parton description of hadronic processes is the fragmentation of the so-called "diquark" system. In a hard scattering process in which a quark suffers a large-momentum-transfer collision, the residual parton system of the hadron fragments in a completely calculable way in the framework outlined here. The result of this investigation will soon become available and should provide a stringent test of the theoretical ideas when compared to experimental data which are in the process of being analyzed.

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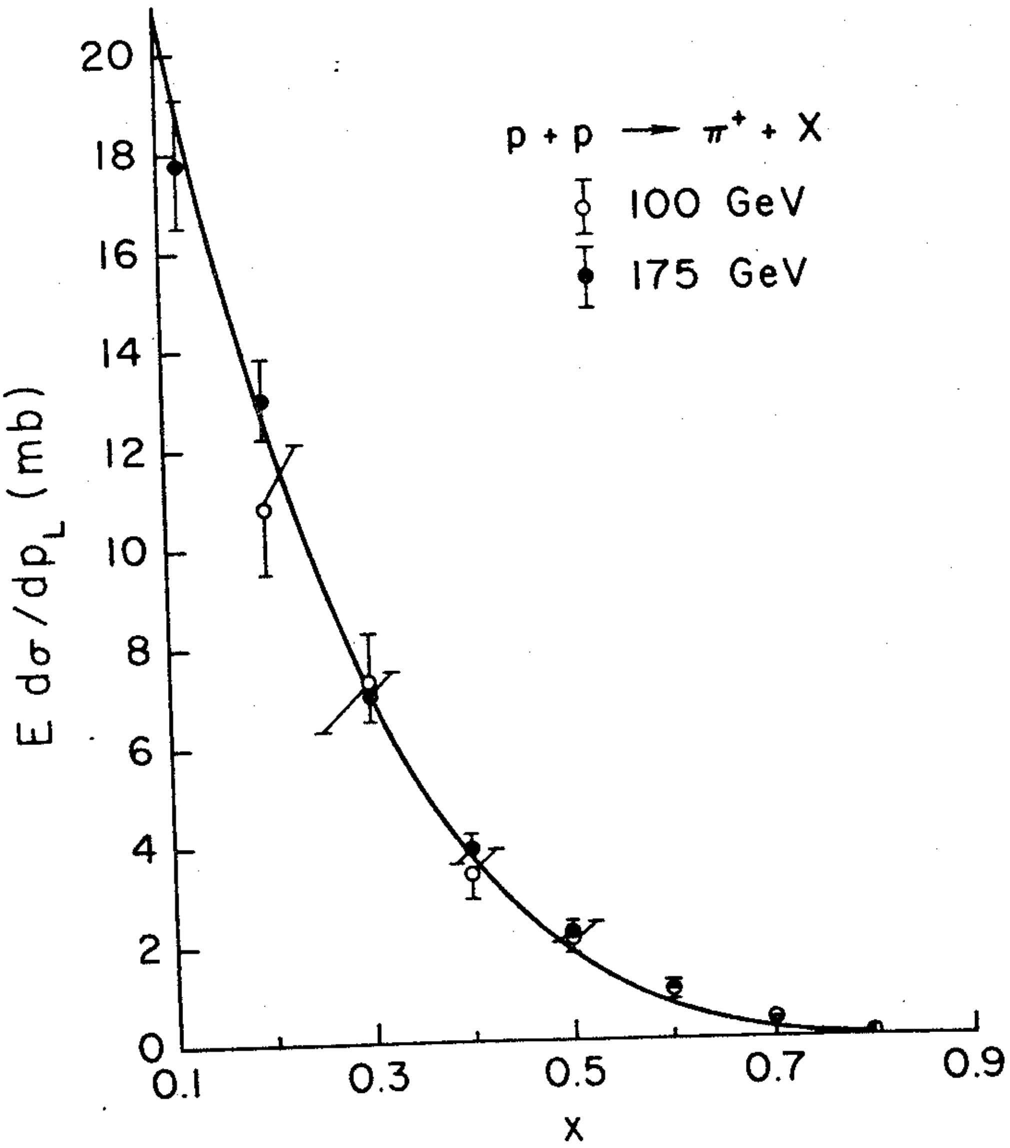

Fig.1 Theoretical prediction (solid line) compared to the data (Ref.9) for the inclusive cross section $pp \to \pi^+X$ at 100 and 175 GeV.