Name: ____________________________

CS354 Midterm Exam, Spring 2014

Useful Formulas

Baye's Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Total Probability Theorem:

$$P(Z) = \sum_{i}^{N} P(X = x_i)P(Z|X = x_i)$$

Recursive State Estimation Formula:

$$Bel(X_k) = \mu P(Z_k|X_k) \sum_{x_{k-1} \in X} P(X_k|x_{k-1})Bel(x_{k-1})$$

Summary of Transform Matrices:

If to convert the k coordinate frame into the j coordinate frame you have to: F_k^j	Then to convert a point in j coordinates into a point in k coordinates, premultiply that point by: T_j^k	The nickname for this transformation is:
Translate along k's x axis by a, along k's y axis by b, along k's z axis by c	$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Trans (a, b, c)
Rotate about k's x axis by θ	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Rot x (θ)
Rotate about k's y axis by θ	$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Rot y (θ)
Rotate about k's z axis by θ	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Rot z (θ)

1. **ROS**

   Write ROS terminal commands for accomplishing each of the following: (4pts each)

   - List all currently active topics.
   - List all currently active nodes.
   - Change directory into the root of the `turtlebot_bringup` package.
   - Display the format of the `LaserScan` message type, which is defined in the `sensor_msgs` package.
   - Display messages published to the `/scan` topic.

   Consider the following partial implementation of a Python constructor for a class representing a ROS node.

```
1   def __init__(self):
2       # Other setup code not shown...
3       rospy.init_node('some_node')
4       rospy.Subscriber(val1, val2, val3)
5       self.pub = rospy.Publisher(val4, val5)
6       rospy.spin()
```

   - What are the three required arguments to the `Subscriber` constructor on line 4? (4pts)
   - What are the two required arguments to the `Publisher` constructor on line 5? (4pts)
   - What is the purpose of the call to `rospy.spin` on line 6? (4pts)
   - Stamped message types (`PointStamped`, `LaserScan`, etc.) store a coordinate frame identifier as well as a time stamp inside of each message. What is the purpose of including this information? How is it used? (4pts)

2. **PID Controllers**

Recall that the purpose of a PID controller is to minimize system error, expressed here as a function of time:

$$e(t) = target(t) - state(t)$$

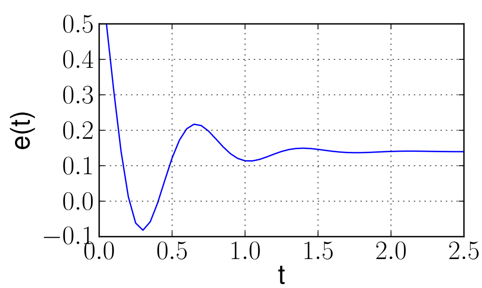
The PID Control signal can be expressed as follows: (Note that the three terms on the right side of this equation are not presented in the standard order.)

$$control(t) = K_? \int_0^t e(\tau)d\tau + K_? \frac{d}{dt}e(t) + K_? e(t)$$

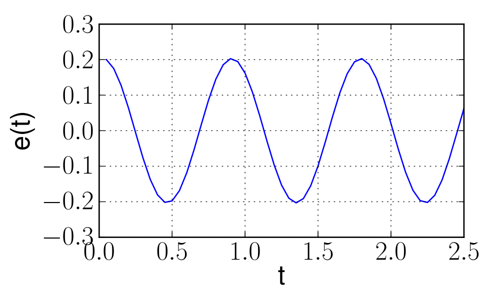
- What do P, I and D stand for? (3pts)

- Label the P, I and D components of the control equation above. (3pts)

- Consider the following graph of error as a function of time. Assuming that this system is being controlled by a PID controller how would you suggest what the gain terms be modified? Justify your answer. (4pts)

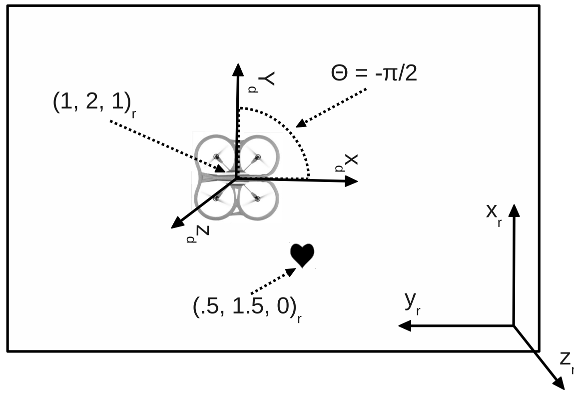


- Consider the following graph of error as a function of time. Assuming that this system is being controlled by a PID controller how would you suggest what the gain terms be modified? Justify your answer. (Note that the system dynamics and gain terms used to generate this figure may not be the same as those from the previous figure.) (4pts)



3. **Coordinate Frames and Forward Kinematics**

Consider the following overhead view of an Ar.Drone in a small room:



The pose of the drone can be expressed as an (x, y, z) position in the room's coordinate frame, along with a Θ value indicating the drone's rotation around its z-axis. The rotation is zero when the drone is pointing in the direction of the room's x-axis and increases when the drone rotates in a counter-clockwise direction.

In the figure above, subscripts are used to label the two coordinate frames: r for the room and d for the drone. Points are subscripted with their coordinate frame. The two points labeled above indicate that the drone is at location (1, 2, 1) in the room coordinate frame, and the heart is at position (.5, 1.5, 0) in the room coordinate frame. The picture is not drawn to scale.

- What is the location of the heart in the drone's coordinate frame? (4pts)

- Assuming that the drone's camera is located at $(.06, 0, 0)_d$, where is the camera in the room coordinate frame? (4pts)

- When expressed in homogeneous coordinates, the location of the heart can be written $[.5\ 1.5\ 0\ 1.0]_r^T$. Write a matrix product that describes the position of the heart in the drone coordinate frame given an arbitrary drone pose. You may use the matrix nicknames in the summary of transform matrices. The following solution is not correct, but it illustrates the format I am expecting: (6pts)

  $[1\ 1\ 0\ 1]_t^T \times \text{Roty}(-2\Theta + x) \times \text{Rotz}(\Theta) \times \text{Trans}(z + x, z, y)$

4. **Grid-Based Localization and Tracking**

We've discussed the application of grid-based localization to the problem of tracking a robot moving in a circular four-room maze. For this question we will track the same robot. The robot may choose to move left or right, and we know that the actions succeed 50% of the time. When an action does not succeed, the robot remains in the same location. Our sensor model tells us that there is an 80% chance that his room sensor will output the true location, and a 20% that it will indicate one of the rooms to the immediate left or right of the true location.

Initially, the robot is 75% likely to be in room "a" and 25% likely to be in room "b":

a	b	c	d
.75	.25	0	0

The robot's first action is "right" and the first sensor output is "b".

- What will the belief distribution be after one step of prediction (before the sensor update)? Show your work. (6pts)

a	b	c	d

- What will the belief distribution be after the sensor update? Show your work. (6pts)

a	b	c	d

5. **Multivariate Normal Distributions and the Kalman Filter**

Recall that the Kalman filter requires both a linear system model and a linear measurement model. The system model (without control) can be expressed as

$\mathbf{x}_{t+1} = \Phi \mathbf{x}_t + \mathbf{v}_t$,

where $\mathbf{x}_t$ represents the system state, Φ expresses the state dynamics, and $\mathbf{v}_t$ is a noise term. The measurement model can be expressed as

$\mathbf{z}_t = \Lambda \mathbf{x}_t + \mathbf{w}_t$,

where $\mathbf{z}_t$ is a measurement value, Λ expresses how sensor values are related to the system state, and $\mathbf{w}_t$ is sensor noise.

For this question assume that we want to use a Kalman filter to track an object moving in one dimension with a fixed acceleration.

The following difference equations describe the system dynamics:

$$
\begin{aligned}
x_{t+1} &= x_t + \dot{x}_t \Delta t \\
\dot{x}_{t+1} &= \dot{x}_t + \ddot{x}_t \Delta t \\
\ddot{x}_{t+1} &= \ddot{x}_t
\end{aligned}
$$

Where x_t is the object position, $\dot{x}_t$ is the velocity and $\ddot{x}_t$ is acceleration and Δt is the size of the time step.

- Assuming that the state of the system is encoded as: $\mathbf{x}_t = \begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}$, what Φ matrix corresponds to the difference equations above? (4pts)

- What should the Λ matrix be to represent the fact that we have a one-dimensional sensor that provides an estimate of the object position, but no information about velocity or acceleration? (4pts)

- Recall that the multivariate normal distribution is parametrized by a mean vector μ and a covariance matrix Σ. Cross out any of the following parameter values that do *not* correspond to a valid normal distribution. (3pts)

$\mu_A = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mu_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \Sigma_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mu_C = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_C = \begin{bmatrix} 2 & -.5 \\ .5 & 1 \end{bmatrix}$
$\mu_D = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_D = \begin{bmatrix} 1 & 0 \\ 0 & .2 \end{bmatrix}$	$\mu_E = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_E = \begin{bmatrix} 1 & -.3 \\ -.3 & 1 \end{bmatrix}$	$\mu_F = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_F = \begin{bmatrix} 0 & .8 \\ .8 & 0 \end{bmatrix}$

- Each of the following figures illustrates one of the probability density functions parametrized above. Label each figure with the matching parametrization (A-F). (3pts)