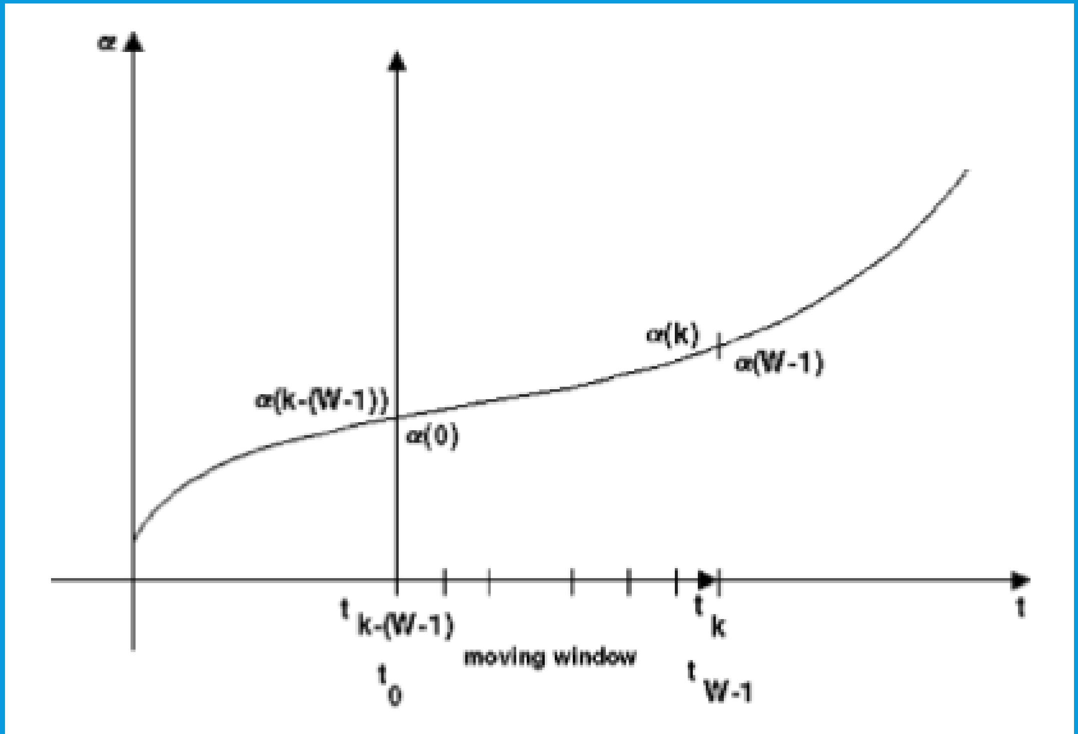
RECURSIVE SPLINE INTERPOLATION METHODS

By Andrew Rozniakowski

BACKGROUND

- Paper written by Alexander Stotsky and Attila Forgo
- Created a new computationally efficient recursive spline interpolation algorithm
- Designed for engine development and real time control of Volvo cars



BACKGROUND CONT.

- $\hat{\alpha} = c_0 + c_1 t + \cdots + c_n t^n,$

- $S = \sum_{i=0}^{w-1} (\alpha_i - (c_0 + c_1 t_i + \cdots + c_n t_i^n))^2$

- $$c_0 w + c_1 \sum_{i=0}^{w-1} t_i + \cdots + c_n \sum_{i=0}^{w-1} t_i^n = \sum_{i=0}^{w-1} \alpha_i,$$
  $$c_0 \sum_{i=0}^{w-1} t_i + c_1 \sum_{i=0}^{w-1} t_i^2 + \cdots + c_n \sum_{i=0}^{w-1} t_i^{n+1} = \sum_{i=0}^{w-1} \alpha_i t_i,$$
  $$\vdots$$
  $$c_0 \sum_{i=0}^{w-1} t_i^n + c_1 \sum_{i=0}^{w-1} t_i^{n+1} + \cdots + c_n \sum_{i=0}^{w-1} t_i^{2n} = \sum_{i=0}^{w-1} \alpha_i t_i^n$$

$\partial S / \partial c_0 = 0,$

$\partial S / \partial c_1 = 0$

$\partial S / \partial c_n = 0.$

RECURSION

$$S_{m_k} = \sum_{i=1}^{w} t_i^m = \Delta t_2^m + (\Delta t_2 + \Delta t_3)^m + \cdots + (\Delta t_2 + \Delta t_3 + \cdots + \Delta t_w)^m$$

$$S_{\alpha m_{k-1}} = \sum_{i=0}^{w-1} \alpha_i t_i^m$$

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$$S_{\alpha m_{k-1}} = \sum_{i=0}^{w-1} \alpha_i t_i^m = \alpha_1 \Delta t_1^m + \alpha_2(\Delta t_1 + \Delta t_2)^m + \cdots + \alpha_{w-1}(\Delta t_1 + \Delta t_2 + \cdots + \Delta t_{w-1})^m$$

$$S_{(m-j)k} = \sum_{i=1}^{w} t_i^{m-j} = \Delta t_2^{m-j} + (\Delta t_2 + \Delta t_3)^{m-j} + \cdots + (\Delta t_2 + \Delta t_3 + \cdots + \Delta t_w)^{m-j},$$

$$S_{\alpha(m-j)k} = \sum_{i=1}^{w} \alpha_i t_i^{m-j} = \alpha_2 \Delta t_2^{m-j} + \alpha_3(\Delta t_2 + \Delta t_3)^{m-j} + \cdots + \alpha_w(\Delta t_2 + \Delta t_3 + \cdots + \Delta t_w)^{m-j}$$

RECURSION CONT.

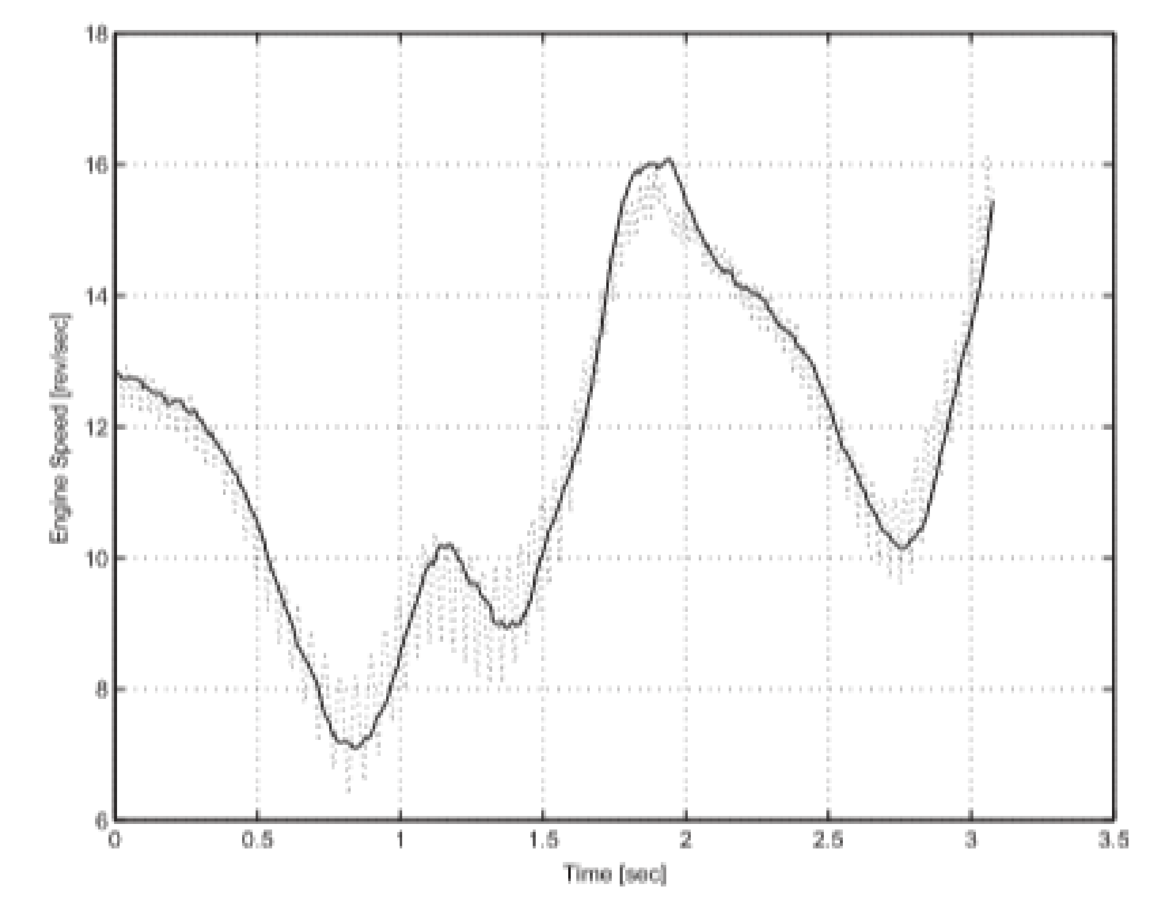
$$S_{m_k} = S_{m_{k-1}} - (w-1)\Delta t_1^m - \sum_{j=1}^{m-1} C_j^m \Delta t^j (S_{(m-j)k}$$
$$- (\Delta t_2 + \cdots + \Delta t_w)^{m-j})$$
$$+ (\Delta t_2 + \Delta t_3 + \cdots + \Delta t_{w-1})^m,$$

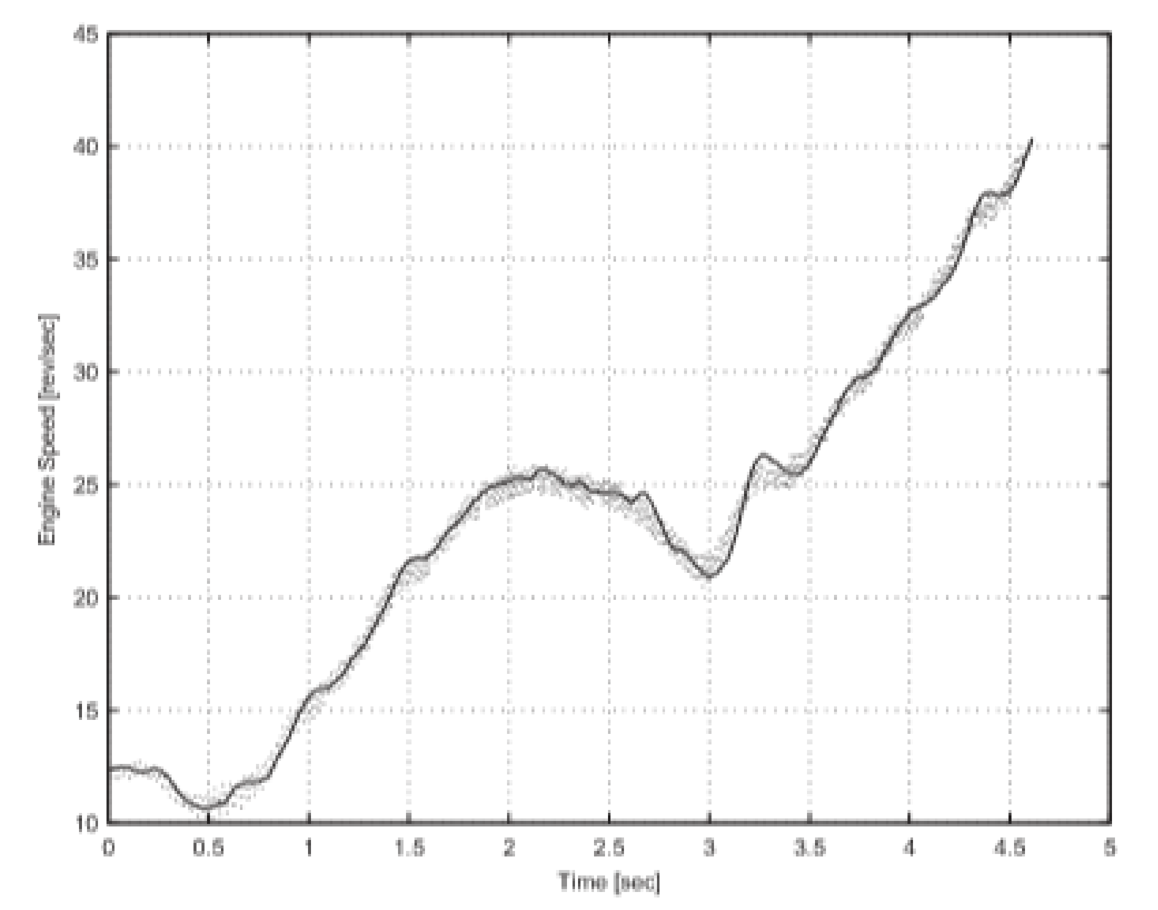
$$S_{\alpha m_k} = S_{\alpha m_{k-1}} - (\alpha_1 + \alpha_2 + \cdots + \alpha_{w-1})\Delta t_1^m$$
$$- \sum_{j=1}^{m-1} C_j^m \Delta t_1^j (S_{\alpha(m-j)k} - \alpha_w$$
$$\times (\Delta t_2 + \Delta t_3 + \cdots + \Delta t_w)^{m-j})$$
$$+ \alpha_w (\Delta t_2 + \Delta t_3 + \cdots + \Delta t_w)^m,$$

SOLVING FOR COEFFICIENTS

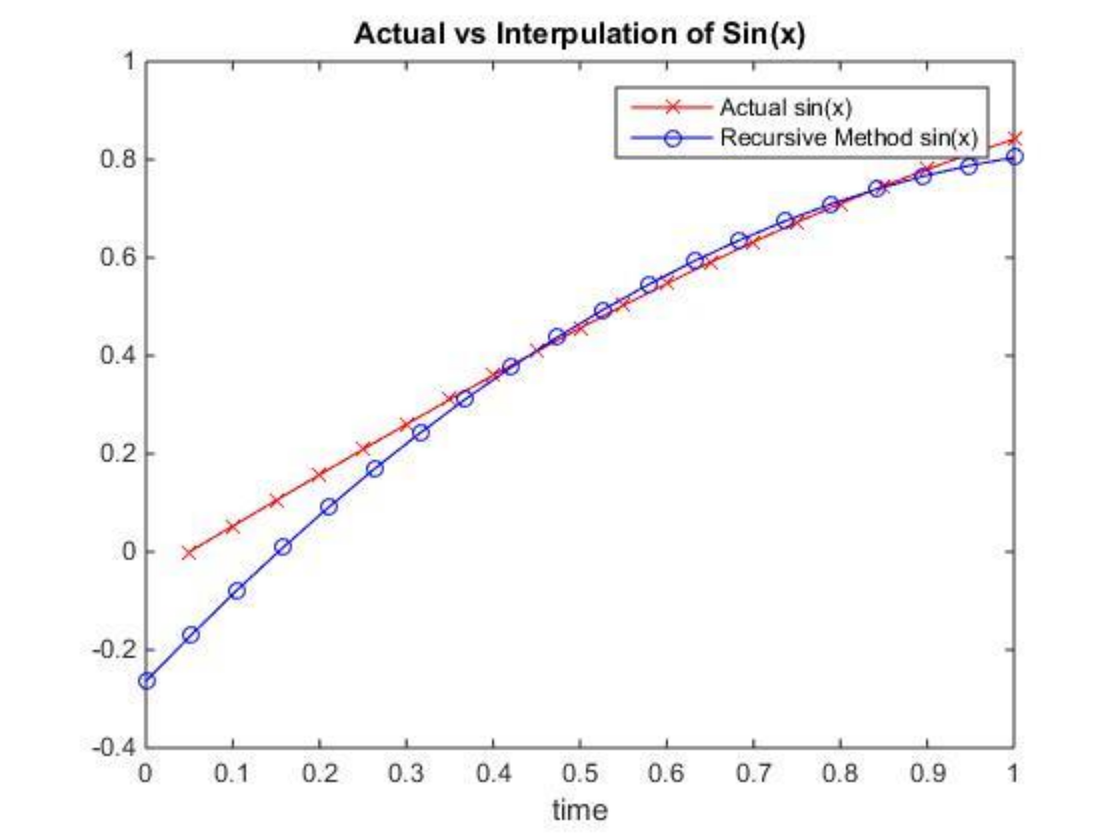
- Ac = b
- $A = \begin{bmatrix} w & S_{1k} & S_{2k} \\ S_{1k} & S_{2k} & S_{3k} \\ S_{2k} & S_{3k} & S_{4k} \end{bmatrix}$
- $b = \begin{bmatrix} S_{\alpha k} & S_{\alpha 1k} & S_{\alpha 2k} \end{bmatrix}$

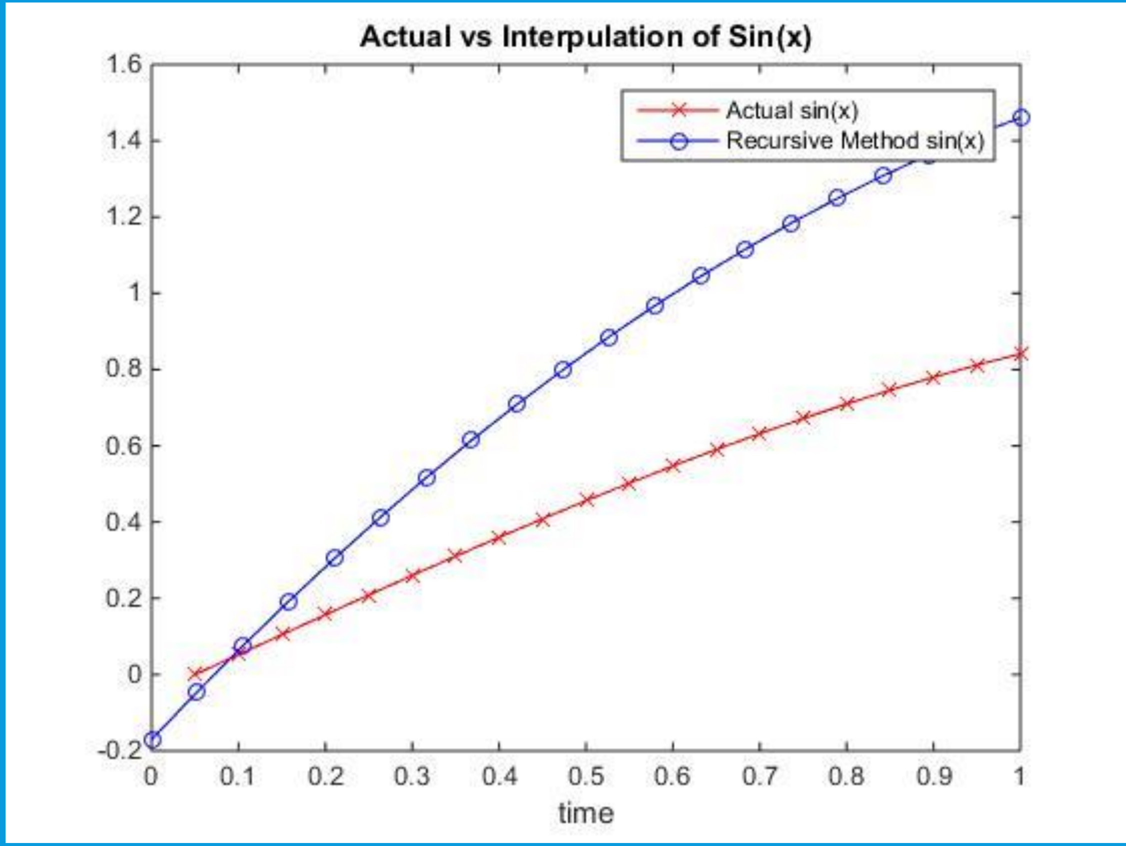
THEIR RESULTS





MY RESULTS


Actual vs Interpulation of Sin(x)
Actual sin(x)
Recursive Method sin(x)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time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time