(1) $D = \begin{bmatrix} 1 & p \\ p & 1 \end{bmatrix}$

$$f(x,y) = \frac{1}{2\pi\sqrt{|D|}} e^{\frac{-[x,y]D^{-1}\begin{bmatrix} x \\ y \end{bmatrix}}{2}}$$

by comparing we get

$\text{cov}(X,Y) = D_{12} = p$

OR

$E(xy) = E(Y E(x|y))$

$E(x|y) \sim N(py, 1-p^2)$

$E(Y^2 p) = p E(y^2) = p \int \frac{x^2 e^{-x^2}}{\sqrt{2\pi}} = 1 \times p$

$\left[E(xy) = p \right]$

$\text{Cov}(X,Y) = E(xy) - E(X)E(Y) = p$

$$\begin{bmatrix} X \sim N(0,1) \\ Y \sim N(0,1) \\ X \mid y = y_1 \sim N(py_1, 1-p^2) \end{bmatrix}$$

(2)

$$\operatorname{Min}(x, y) = \frac{x + y - |x-y|}{2}$$

$$E(\operatorname{Min}(x,y)) = \frac{\cancel{E(x)}^{0} + \cancel{E(y)}^{0} - E(|x-y|)}{2} = \frac{-E(|x-y|)}{2}$$

$$E(|x-y|) = \int_0^\infty \frac{t e^{t^2/4(1-p)}}{\sqrt{\pi(1-p)}}\, dt = \int_0^\infty \frac{2(1-p)}{\sqrt{\pi(1-p)}} e^{-x}\, dx$$

$$= \sqrt{\frac{1-p}{\pi}} \times 2$$

$$E(\operatorname{Min}(x,y)) = -\sqrt{\frac{1-p}{\pi}}$$

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(3) $U = (X+Y)$

$V = X - Y$

$\cdot\ X+Y>0$

$U = X+Y \Rightarrow X = \frac{U+V}{2}$ $\quad |J| = 1/2$

$V = X-Y \quad\quad Y = \frac{U-V}{2}$

$$f(U,V) = \frac{1}{2}\left(\frac{1}{2\pi\sqrt{1-p^2}}\right) e^{-\left(\frac{u^2}{4(1+p)} + \frac{v^2}{4(1-p)}\right)} \Big| U>0$$

when $x+y<0$

$U = -X-Y \Rightarrow X = \frac{V-U}{2}$

$V = X-Y \quad\quad Y = -\frac{(U+V)}{2}$

$$|J| = \begin{bmatrix} -1/2 & 1/2 \\ -1/2 & -1/2 \end{bmatrix} = 1/2$$

$$\cancel{f(U,V) = \frac{1}{2}\left(\frac{1}{2\pi\sqrt{1-p^2}}\right) e^{-\left(\left(\frac{V-U}{2}\right)^2 + \left(\frac{U+V}{2}\right)^2 - 2p\frac{(U-V)}{2}\right)}}$$

$$f(U,V) = \frac{1}{2}\left(\frac{1}{2\pi\sqrt{1-p^2}}\right) e^{-\frac{\left(\left(\frac{U-V}{2}\right)^2 + \left(\frac{U+V}{2}\right)^2 - 2p\frac{(U-V)(V+U)}{4}\right)}{2(1-p^2)}}$$

$$f(U,V) = \frac{1}{2}\left(\frac{1}{2\pi\sqrt{1-p^2}}\right) e^{-\frac{u^2}{4(1+p)} - \frac{v^2}{4(1-p)}} \Big| U>0$$

adding both we get

$$f(u,v) = \frac{1}{2\pi\sqrt{1-p^2}} e^{-\frac{u^2}{4(1+p)}} e^{-\frac{v^2}{4(1-p)}}$$

$$f(u) = \int_{-\infty}^{\infty} e^{-\frac{v^2}{4(1-p)}} dv \times \frac{e^{-\frac{u^2}{4(1+p)}}}{2\pi\sqrt{1-p^2}} \quad | u > 0$$

$$f(u) = \frac{e^{-\frac{u^2}{4(1+p)}}}{\sqrt{4(1+p)}} \quad | u > 0$$