Change of Variable

$$f_{XY}(x,y) = \lambda_2 \lambda_1 e^{-\lambda_1 x} e^{-\lambda_2 y} \qquad \begin{matrix}(\lambda_1, \lambda_2 > 0) \\ (x, y > 0)\end{matrix}$$

so let find distribution of $U = X+Y$
and $V = X-Y$

then $X = \frac{U+V}{2}$

$$Y = \frac{U-V}{2}$$

$\left[\text{here } \begin{matrix} T_1(X,Y) = X+Y \\ T_2(X,Y) = X-Y \end{matrix} \right] \quad \left[\begin{matrix} T_1^{-1}(U,V) = \frac{U+V}{2} \\ T_2^{-1}(U,V) = \frac{U-V}{2} \end{matrix} \right]$

$$f_{UV}(u,v) = f_{XY}\left(T_1^{-1}(u,v), T_2^{-1}(u,v)\right) J\left(T^{-1}(u,v)\right)$$

$$J(T^{-1}(u,v)) = \begin{bmatrix} \frac{\partial T_1^{-1}}{\partial u} & \frac{\partial T_1^{-1}}{\partial v} \\ \frac{\partial T_2^{-1}}{\partial u} & \frac{\partial T_2^{-1}}{\partial v} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$$

$$|J(T^{-1}(u,v))| = 1/2$$

$$f(u,v) = \lambda_1 \lambda_2 e^{-\frac{(\lambda_1+\lambda_2)}{2}u - \frac{(\lambda_1-\lambda_2)}{2}v}$$

$$\begin{bmatrix} u+v > 0 \\ u-v > 0 \end{bmatrix}$$

Conditional Probability

$$f(x,y) = \frac{1}{2\pi\sqrt{1-p^2}} e^{-\frac{(x^2+y^2-2pxy)}{2(1-p^2)}}$$

$$f(y) = \int_{-\infty}^{\infty} \frac{1}{2\pi\sqrt{1-p^2}} e^{-\frac{(x^2+y^2-2pxy)}{2(1-p^2)}} dx$$

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$$

$$f(x \mid y = y_i) = \frac{f(x, y_i)}{f(y = y_i)} = \frac{1}{\sqrt{2\pi}\sqrt{1-p^2}} e^{-\frac{(x-py_i)^2}{2(1-p^2)}}$$

$$= N(py_i, 1-p^2)$$

Multivariate Normal

$D = [a_{ij}]$ | $a_{ij} = \text{Cov}(x_i, x_j)$ |

[D is a matrix]

for $n=3$

$$D = \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{12} & \sigma_2^2 & \rho_{23} \\ \rho_{13} & \rho_{23} & \sigma_3^2 \end{bmatrix}$$

$$f(x_1, x_2, x_3) = \frac{1}{(2\pi)^{3/2}\sqrt{|D|}} e^{-X^T D^{-1} X} \sim N\left(\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix}, D\right)$$

$$X = \begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \\ x_3 - \mu_3 \end{bmatrix}$$

Q let $f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{(x^2+y^2-2xy\rho)}{2(1-\rho^2)}}$

[show $\mathrm{cov}(X,Y) = \rho$]

Q

Q find $E(\mathrm{Min}(X,Y))$, $f(x,y) = \frac{e^{-\frac{(x^2+y^2-2xy\rho)}{2(1-\rho^2)}}}{2\pi\sqrt{1-\rho^2}}$

{Hint : $\mathrm{Min}(x,y) = \frac{X+Y-|X-Y|}{2}$ }

Q find Distribution of $|X+Y|$ for the above pdf.

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Q find distribution of $|X-Y|$

$$f(x,y) = \frac{1}{2\pi\sqrt{1-p^2}} e^{-\frac{(x^2+y^2-2pxy)}{2(1-p^2)}}$$

Ans

$U = |X-Y|$

$V = X+Y$

$\boxed{\text{for } X>Y}$

$U = X-Y$ $\Rightarrow$ $X = \frac{U+V}{2}$

$V = X+Y$ $\quad$ $Y = \frac{V-U}{2}$

for $(Y<X)$

$U = Y-X$ $\Rightarrow$ $X = \frac{V-U}{2}$

$V = X+Y$ $\quad$ $Y = \frac{U+V}{2}$

now for $(X>Y)$

$$f(U,V) = \left(\frac{1}{2}\right)\frac{1}{2\pi\sqrt{1-p^2}} e^{-\frac{\left(\left(\frac{U+V}{2}\right)^2 + \frac{(V-U)^2}{2} - \frac{2p(V^2-U^2)}{4}\right)}{2(1-p^2)}}$$

$$\left[|J(T^{-1})| = 1/2\right]$$

$$f(U,V) = \frac{1}{4\pi\sqrt{1-p^2}} e^{-\frac{\left(\frac{U^2(1+p)}{2} + \frac{V^2(1-p)}{2}\right)}{2(1-p^2)}}$$

$$f(u,v) = \frac{1}{4\pi\sqrt{1-p^2}} e^{-\frac{u^2}{4(1-p)}} \times e^{-\frac{v^2}{4(1+p)}} \quad [u>0]$$

for $(X<Y)$ we will get same.

$$f(u,v) = \frac{1}{4\pi\sqrt{1-p^2}} e^{-\frac{u^2}{4(1-p)}} \cdot e^{-\frac{v^2}{4(1+p)}} \quad [u>0$$

$$[-\infty < v < \infty]$$

~~da~~ adding both

$$f(u,v) = \frac{1}{2\pi\sqrt{1-p^2}} e^{-\frac{u^2}{4(1-p)}} e^{-\frac{v^2}{4(1+p)}}$$

Now $f(u) = \int_{-\infty}^{\infty} e^{-\frac{v^2}{4(1+p)}} dv \cdot \frac{e^{-\frac{u^2}{4(1-p)}}}{2\pi\sqrt{1-p^2}}$

$$f(u) = \frac{\sqrt{2\pi \, 2(1+p)}}{2\pi\sqrt{1-p^2}} \; e^{-\frac{u^2}{4(1-p)}} \quad u>0$$

$$f(u) = \frac{\sqrt{2}}{\sqrt{2\pi(1-p)}} e^{-\frac{u^2}{4(1-p)}} = \frac{e^{-\frac{u^2}{4(1-p)}}}{\sqrt{\pi(1-p)}}$$

$u>0$