So Lets start with a ~~basis~~ basic statistical problem

We have coin in which probability of head is p, we want to estimate p

To do so, we flip the coin 100 times and get 60 Heads, 40 tails, then likelihood function of the following outcome is

$$f(H=60, T=40 \mid p) = {}^{100}C_{60}\, p^{60} (1-p)^{40}$$

In MLE we try to maximise likelihood func.

$$\text{so} \quad \frac{d\, {}^{100}C_{60}\, p^{60}(1-p)^{40}}{dp} = 60p^{59}(1-p)^{40} - 40p^{60}(1-p)^{39} = 0$$

$$\Rightarrow p = \frac{6}{10} = \frac{3}{5}$$

Now for bayesian we assume prior. , a prior is just a distribution function, we assume from which our parameter that I need to ~~es~~ estimate coming from. for the above example let

prior of p be Uniform(0,1) {we know $0 \le p \le 1$}

~~let~~ so $\pi(p) = 1 \quad | \; 0 \le p \le 1$

now we want to find $\pi(p|X)$ $\left\{ X \Rightarrow \begin{matrix} H = 60 \\ T = 40 \end{matrix} \right\}$

also $f(X|p)\,\pi(p) = \pi(p|x)\, f(x)$

$$\pi(p|x) = \frac{f(x|p)\,\pi(p)}{\int f(x|p)\,\pi(p)\,dp} = \frac{{}^{100}C_{60}\, p^{60}(1-p)^{40}}{{}^{100}C_{60} \int_0^1 p^{60}(1-p)^{40}\,dp}$$

$\pi(p|x) = \text{Beta}(61, 41)$

so $p \sim \text{Beta}(61, 41)$

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let X_is $\sim \exp(\lambda)$ find MLE of λ and posterior distribution, take <u>prior</u> = $\lambda \sim \text{gamma}(a, b)$

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Q let $x_i \sim N(\mu, 1)$ and we know ~~$\mu \in \{ \}$~~ $\mu \in Z$

then MLE of x_is is :-

$$x_i \sim f(x_i \mid \mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2}}$$

Likelihood function = $L(x_1, x_2 \ldots x_n \mid \mu) = \frac{e^{-\sum \frac{(x_i-\mu)^2}{2}}}{(2\pi)^{n/2}}$

$$\log(L(x_is.)) = -\frac{\sum (x_i-\mu)^2}{2} - \frac{n}{2}\log 2\pi$$

$$\frac{d \ln L}{d\mu} = \sum(x_i - \mu) = 0$$

$\left[\mu = \frac{\sum x_i}{n}\right]$ now if $\frac{\sum x_i}{n} \in Z$, $\mu = \frac{\sum x_i}{N}$

otherwise μ = integer closest to $\frac{\sum x_i}{N}$

Q let $X_{is} \sim N(\mu, 1)$, ~~then~~ taking prior of $\mu \sim N(0,1)$

find ~~I want to~~ find posterior distribution of μ

$$f(X_{is}|\mu) = \prod_{i=1}^{n} f(x_i|\mu) = \frac{e^{-\frac{\sum (x_i-\mu)^2}{2}}}{(\sqrt{2\pi})^n}$$

$$\pi(\mu) = \frac{e^{-\frac{\mu^2}{2}}}{\sqrt{2\pi}}$$

$$\pi(\mu|X_{is}) = \frac{\pi(\mu) f(X_{is}|\mu)}{\int_{-\infty}^{\infty} \pi(\mu) f(X_i|\mu) d\mu}$$

$$\pi(\mu) f(x_{is}|\mu) = \frac{1}{(2\pi)^{\frac{n+1}{2}}} e^{-\left(\frac{\sum x_i^2 + n\mu^2 - (\sum x_i) 2\mu + \mu^2}{2}\right)}$$

$$\pi(\mu) f(x_{is}|\mu) = \frac{1}{(2\pi)^{\frac{n+1}{2}}} e^{-\frac{\left(\mu^2 - \frac{2(\sum x_i)\mu}{(n+1)} + \frac{(\sum x_i)^2}{(n+1)^2}\right)(n+1)}{2} + g(x_i)}$$

[Here $g(x_{is})$ is a function of x_{is} only]

then
$$\pi(\mu) f(x_i|\mu) = \frac{1}{(2\pi)^{\frac{n+1}{2}}} e^{-\frac{\left(\mu - \frac{\sum x_i}{n+1}\right)}{2\left(\frac{1}{n+1}\right)} + g(x_i)}$$

$$\pi(\mu \mid x_{is}) = \frac{\cancel{\frac{1}{(2\pi)^{\frac{n+1}{2}}}}\, e^{-\frac{\left(\mu - \frac{\Sigma x_i}{n+1}\right)}{2(1/n+1)}}\, \cancel{e^{g(x_{is})}}}{\cancel{\frac{1}{(2\pi)^{\frac{n+1}{2}}}\, e^{g(x_i)}} \displaystyle\int_{-\infty}^{\infty} e^{-\frac{\left(\mu - \frac{\Sigma x_i}{n+1}\right)}{2(1/n+1)}}\, d\mu}$$

$$\pi(\mu \mid x_{is}) = \frac{1}{\sqrt{2\pi\left(\frac{1}{n+1}\right)}}\, e^{-\frac{\left(\mu - \frac{\Sigma x_i}{n+1}\right)}{2/n+1}}$$

so posterior of $\mu \sim N\left(\frac{\Sigma x_i}{n+1}, \frac{1}{n+1}\right)$

and MLE of $\mu = \frac{\Sigma x_i}{n}$

~~sa~~ so as $n \longrightarrow \infty$ $\quad N\left(\frac{(\Sigma x_i)}{n+1}, \frac{1}{n+1}\right) \longrightarrow \frac{\Sigma x_i}{n}$

Questions

① $(X_i, X_i) \sim f(x,y) = \begin{cases} 3\lambda^2 e^{-\lambda x - 2\lambda y} & 0 < x < y < \infty \\ 3\lambda^2 e^{-\lambda y - 2\lambda x} & 0 < y < x < \infty \end{cases}$

find MLE of λ

② $y_i = a + bt_i + \epsilon_i \quad | \; 1 \le i \le n$

and $\epsilon_i \sim N(0, \sigma^2)$, find MLE of σ^2

if you have data of $Y_1, Y_2 .. \; Y_n$

③ Let $X_1, X_2 .. \; X_n \sim f(x|\lambda, \alpha) = \alpha \lambda x^{\alpha - 1} e^{-\lambda x^{\alpha}}$

let prior of λ be gamma (a,b), find posterior of λ.

④ let $X_1 .. \; X_n \sim \text{Uni}(0, \theta)$, if prior on θ is $\text{Unif}(0,1)$, find the posterior distribution of θ