

$$① \quad X_{12} = \{1, 1/2, 3, 7, 1/2\}$$

$$P(X_{12} | \lambda, H_0) = (\lambda)^5 e^{-\lambda(\sum X_i)}$$

$$P(X_{12} | \lambda, H_1) = (2)^5 (\lambda)^5 \left(\prod X_i\right) e^{-\lambda \sum X_i^2}$$

$$\sum X_i = 12$$

$$\prod X_i = 5.25$$

$$\sum X_i^2 = 59.5$$

$$P(X_{12}, \lambda | H_0) = \pi(\lambda) P(X_{12} | H_0, \lambda) = \lambda^5 e^{-\lambda(12)}$$

$$P(X_{12}, \lambda | H_1) = \pi(\lambda) P(X_{12} | H_1, \lambda) = 2^5 \times 5.25 \times \lambda^5 e^{-\lambda(60.5)}$$

$$P(X_{12} | H_0) = \int_0^{\infty} P(X_{12}, \lambda | H_0) d\lambda = \frac{\sqrt{6}}{(12)^5} = 2.48 \times 10^{-5}$$

$$P(X_{12} | H_1) = \int_0^{\infty} 2^5 \times 5.25 \lambda^5 e^{-\lambda(60.5)} d\lambda = \frac{2^5 \times 5.25 \times 5!}{(60.5)^6}$$

$$= 4.1 \times 10^{-7}$$

$$P(H_0) = P(H_1) = 1/2$$

$$P(H_1 | X_{12}) = \frac{P(X_{12} | H_1) P(H_1)}{P(X_{12} | H_1) P(H_1) + P(X_{12} | H_0) P(H_0)}$$

$$P(H_1 | X_{12}) = \frac{4.1 \times 10^{-7}}{2.48 \times 10^{-5} + 4.1 \times 10^{-7}} \approx 1.6\%$$

Q2 $x_{12} = (x_1, y_1)$

$$\pi(\lambda | x_{12}) = \frac{\pi(\lambda) f(x_{12} | \lambda)}{\int \pi(\lambda) f(x_{12} | \lambda) d\lambda}$$

$$\pi(\lambda) = \left(\frac{b^a}{\Gamma a}\right)^2 \lambda_1^{a-1} e^{-\lambda_1 b} \lambda_2^{a-1} e^{-\lambda_2 b}$$

$$f(x_{12} | \lambda) = \lambda_1^4 e^{-10\lambda_1} (\lambda_2)^4 e^{-10\lambda_2}$$

$$\pi(\lambda | x_{12}) \sim \lambda_1^{a+3} \lambda_2^{a+3} e^{-\lambda_1(10+b)} e^{-\lambda_2(10+b)}$$

$$\pi(\lambda | x_{12}) \sim \text{gamma}_{\lambda_1}(a+4, 10+b) \text{gamma}_{\lambda_2}(a+4, 10+b)$$

so posterior of $\lambda_1 \sim \text{gamma}(a+4, 10+b)$
 " " $\lambda_2 \sim \text{gamma}(a+4, 10+b)$

Q3

$$f(x_{12} | \theta) = \left(\frac{1}{\theta}\right)^5 \quad \left[\theta > 2.25 = x_{\max} \right]$$

$$\pi(\theta) = \text{Unif}(2, 3)$$

$$H_0 : \theta < 2.5$$

$$H_1 : \theta > 2.5$$

$$\pi(\theta | x_{12}) = \frac{\left(\frac{1}{\theta^5}\right)}{\int_{2.25}^3 \left(\frac{1}{\theta^5}\right) d\theta} = \frac{\left(\frac{1}{\theta^5}\right) \left(-\frac{1}{4}\right) \left[3^{-4} - 2.25^{-4}\right]}{\left(-\frac{1}{4}\right) \left[3^{-4} - 2.25^{-4}\right]}$$

$$\pi(\theta | x_{12}) = \frac{1}{\theta^5} \left(-\frac{1}{4} [3^{-4} - 2.25^{-4}] \right)^{-1}$$

$$P(H_0 | x_{12}) = \int_{2.25}^{2.5} \left(\frac{1}{\theta^5} \right) \left[-\frac{1}{4} [3^{-4} - 2.25^{-4}] \right]^{-1} d\theta$$

$$P(H_0 | x_{12}) = \frac{-(2.5^{-4} - 2.25^{-4})}{2.25^{-4} - 3^{-4}} = 50.3\%$$

$$P(H_1 | x_{12}) = 49.7\%$$