Moment Generating function.
(Mgf is unique for a given pdf)

(i) Uniform $(a,b) \sim X$

$$M_X(t) = \int_a^b \frac{e^{xt}}{b-a}\,dx = \frac{e^{bt} - e^{at}}{t(b-a)}$$

(ii) Normal $(\mu, \sigma^2) \sim X$

$$M_X(t) = \int_{-\infty}^{\infty} \frac{e^{xt}}{\sqrt{2\pi}\sigma} \cdot e^{\frac{(x-\mu)^2}{\sigma^2 \times 2}}\,dx = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

(iii) Now $M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

$$E(X) = \left(\frac{dM_X(t)}{dt}\right)\Big|_{t=0} = \left|\left(e^{\mu t + \frac{\sigma^2 t^2}{2}}\right)(\mu + \sigma^2 t)\right|_{t=0}$$

$$= \mu$$

$$Var(X) = \left|\frac{d^2 \ln(M_X(t))}{dt^2}\right|_{t=0} = \sigma^2$$

~~Ques X & Y are Random variable on same sample space~~

let

Ques $X \sim Pois(\lambda)$ & $f_X(k) = \left\{ \frac{e^{-\lambda}\lambda^k}{k!}, \ k \in \mathbb{N}_0 \right\}$

find Mgf of X.

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* The Mgf of a Random Variable X ~~uniquely~~ uniquely determines the probability distribution of X. , or if X and Y has same Mgf then they have same pdf

for (i) let Mgf of X = $\dfrac{e^{-3t} - e^{-7t}}{4t} \Rightarrow X \sim Uni(-7,-3)$

(ii) Mgf of X :- $e^{-7t + 4t^2} \Rightarrow X \sim N(-7, 8)$

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Change of Variable

Example : $X \sim N(0,1)$, find distribution of X^2

then X^2 is not a one-one function. on $(-\infty,\infty)$

but we can break the support into $(-\infty,0) \cup (0,\infty)$

$$N(0,1) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} & x<0 \\ \frac{1}{\sqrt{2\pi}} e^{-x^2/2} & x>0 \end{cases}$$

then X^2 is one-one on S_1 $(-\infty,0)$ and on S_2 $(0,\infty)$

so $g(x) = x^2$, $\quad g^{-1}(y) = \sqrt{y}$ in S_2

$g^{-1}(y) = -\sqrt{y}$ in S_1

$$f_Y(y) = \underbrace{\frac{1}{\sqrt{2\pi}} e^{-y/2} \left|\frac{1}{2\sqrt{y}}\right|}_{\text{on } S_2} + \underbrace{\frac{1}{\sqrt{2\pi}} e^{-y/2} \left|\frac{1}{-2\sqrt{y}}\right|}_{\text{on } S_1}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \frac{e^{-y/2}}{\sqrt{y}}$$

Ques let X has pdf. $\begin{cases} 6x(1-x) & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

find distribution of $Y = X^2+1$

Joint Pdf

let X_i, $1 \le i \le n$ be independent random variable with pdf, f_{X_i} then $Y = (X_1, X_2 \ldots X_n)$

$$\left[f_Y(x_1, \ldots x_n) = \prod_{i=1}^{n} f_{X_i}(x_i) \right]$$

for example => let $X \sim N(0,1)$, $Y \sim Exp(1)$

$Z = (X, Y)$

$$f_Z(x,y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cdot e^{-y} \left[\begin{array}{l} -\infty < x < \infty \\ 0 < y < \infty \end{array} \right]$$

$\boxed{\text{Now } \cancel{\text{let}}}$ ~~X~~ . X and Y are independent with pdf f_X and f_Y, then $E(XY) = E(X)E(Y)$

<u>proof</u> define $Z = (X, Y)$, $g(x,y) = xy$

$$E(g(x,y)) = \iint g(x,y) f_Z(x,y)\, dx\, dy$$

$$E(XY) = \iint xy\, f_X(x) f_Y(y)\, dx\, dy = \int x f_X(x)\, dx \times \int y f_Y(y)\, dx$$

$$E(XY) = E(X)E(Y)$$

Ques let X, Y be independent random variable on same sample space then prove

$$M_{X+Y}(t) = M_X(t)\, M_y(t)$$

Ques $X_i \sim N(\mu, \sigma^2)$ are independent Random variable ; prove that $Y = \sum_{i=1}^{n} \frac{X_i}{n} \sim N(\mu, \frac{\sigma^2}{n})$

Ques $X_i \sim Exp(1)$ are independen Random variable , prove $Y = \sum_{i=1}^{n} X_i \sim Gamma(n, 1)$

(just google the Mgfs of gamma and Exp)

Ques

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