## Recall

- No. of ways of choosing $r$ objects from $n$ objects

$$^nC_r$$

- No. of ways of $n$ identical things to $r$ people :

$$^{n+r-1}C_{r-1}$$

$$^{n-1}C_{r-1} \quad \text{(at least one to each)}$$

- No. of ways of arranging $n$ distinct objects in $r$ distinct group of size $m_1, m_2 \ldots m_r$

$$\frac{n!}{m_1!\, m_2! \ldots m_r!}$$

- Dearrangement formula = $n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots \frac{(-1)^n}{n!}\right]$

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## N coin tosses

a coin is tossed $n$ times. $\left[\begin{matrix} H=1 \\ T=0 \end{matrix}\right]$ $[P[H] = p]$

$$\Omega = \{0, 1\}^n$$

any outcome $\omega = (\omega_1, \omega_2 \ldots \omega_n)$ $[\omega_i \in \{0,1\}]$

Event A $\Rightarrow$ No. of heads $= n/2$ $[n = \text{even}]$

$$p_\omega = p^{\Sigma \omega_i}(1-p)^{n - \Sigma \omega_i} \quad [\forall\ \omega \in \Omega]$$

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Now to calculate $P(A)$ we need to find $|A|$ as every element of $A$ has same probability

$|A| = {}^nC_{n/2}$

$P(A) = \sum_{\omega \in A} p_\omega = {}^nC_{n/2}\, p^{n/2} (1-p)^{n/2}$

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## Shuffling of Cards

$\Omega = S_{52}$ : [all permutations of 52]

$p_\omega = \frac{1}{52!}$ [ $\omega$ is any ~~permutation~~ permutation $\in S_{52}$ ]

$A$ = getting all Aces together

$P(A) = \sum_{\omega_i \in A} p_{\omega_i} = |A|\, p_\omega$

$|A| = \left({}^{52}C_4\right) 4!\ 48!$

$P(A) = {}^{52}C_4 \frac{4!\ 48!}{52!}$

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# Birthday Paradox

In a room of 25 people, what is probability that two people have same birthday
(atleast)

⇒ lets call this event A then

$$P(A) = 1 - P(A^c)$$

$$P(A^c) = \frac{(365)(364)\ldots(365-24)}{(365)^{25}}$$

$$P(A) = 1 - P(A^c) = 0.569 \approx 57\%$$

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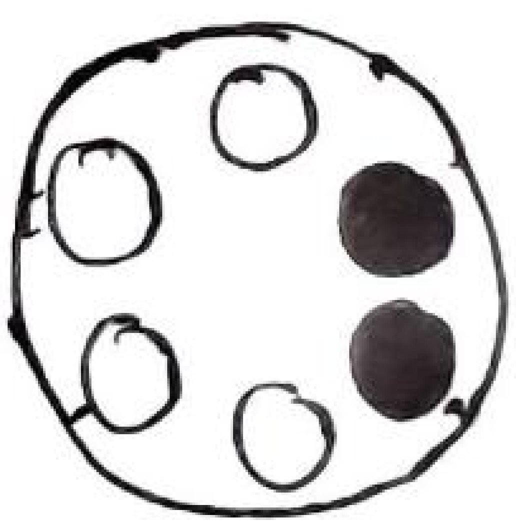
## Questions

① 

You have given a revolver with following arrangement. You rolled the barrel and shot the gun but the bullet ~~doesnt~~ not fired. Now You want to take another shoot, ~~should~~ does rolling the barrel again increase your chances of firing the bullet.

(2) Probability of Accidents on a road is $3/4$ in One hour. What is the probability of accidents in $1/2$ hour.

(3) [Murphy's Law] A fair coin is tossed repeatedly $n$ times. Let $S$ be any sequence of H and T of length $r$.
[ie. $\underbrace{HTTH\ldots H}_{r}$ ]

What is the probability that $S$ will eventually appear in $n$ tosses of the coin [$n \to \infty$]

4) $A$ be countable collection of events then prove

(i) $P[E_i] = 0 \;\; \forall E_i \in A \iff P\left(\bigcup_{E_i \in A} E_i\right) = 0$

(ii) $P(E_i) = 1 \;\; \forall E_i \in A \iff P\left(\bigcap_{E_i \in A} E_i\right) = 1$

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