Probability and Statistics: From Classical to Bayesian

Week 1

1st April-7th April 2021

1 Introduction

Hello and welcome to the project on Probability and Statistics - From Classical to Bayesian. For the first week, we would mostly be covering basics of Discrete and Continuous Probability Spaces, mostly for conceptual bases that shall be used later. We would also revisit counting principles and you might feel back to JEE days questions (just once in a while :)).

Let's start with some basic definitions :

$$P(E) = \frac{Favourable\ Possibilities}{Total\ Possibilities}$$

Now lets extend it to a generalized form using sets and functions. Let the set of all outcomes of any event be denoted by Ω (commonly referred as sample space).

Consider a function p : Ω $\rightarrow$[0,1] which maps probability of each outcome to its probability such that

$$\sum_{\omega \in \Omega} p_\omega = 1$$

Any subset A $\subseteq$ Ω is called an event. For an event A, we define its probability as

$$P(A) = \sum_{\omega \in A} p_\omega$$

2 Laws of Probability

Let (Ω, p) be a discrete probability space.

- P(ϕ) = 0, P(Ω) = 1 and 0 $\leq$ P(A) $\leq$ 1 for any event A.
- P($\bigcup\limits_k A_k$) $\leq \sum_k P(A_k)$ for any countable collection of events A_k.
- P($\bigcup\limits_k A_k$) $= \sum_k P(A_k)$ if A_k is a countable collection of pairwise disjoint events.

3 Independence and Conditional Probability

- For any two events A and B in the same probability space. P(A $\cup$ B) + P(A $\cap$ B) = P(A) + P(B).

  This can be generalised for more events.

- If $P(B) \geq 0$, we define the conditional probability of A given B as

$$P(A|B) := \frac{P(A \cap B)}{P(B)}$$

- We say that A and B are independent if $P(A \cap B) = P(A)P(B)$. If $P(B) \neq 0$, then A and B are independent if and only if $P(A|B) = P(A)$ (and similarly with the roles of A and B reversed). If P(B) = 0, then A and B are necessarily independent since $P(A \cap B)$ must also be 0.

Exercise. If A and B are independent events, show that the following pairs of events are also independent.
(1) A and B^c.
(2) A^c and B.
(3) A^c and B^c.

For the Y20s, Best of the luck for the grades!