① $(X_{(i)}, Y_{(i)}) \rightarrow (\text{Min}(X_i, Y_i), \text{Max}(X_i, Y_i)) = (m_i, M_i)$

now $(m_i, M_i) \sim \cancel{f(x,y)}\ f(m, M) = 3\lambda^2 e^{-\lambda m - 2\lambda M}$.

$0 < m < M < \infty$

Likelihood function $= L((m_i, M_i) \mid \lambda) = 3^n \lambda^{2n} e^{-\lambda(\Sigma m_i) - 2\lambda(\Sigma M_i)}$

for MLE, $\frac{dL}{d\lambda} = 0 \Rightarrow \cancel{2n\lambda^{2n-1} e^{-\lambda(\Sigma m_i + 2\Sigma M_i)} \frac{2n}{\lambda}}$,

$\Downarrow$

$2n(\lambda)^{2n-1} e^{-\lambda(\Sigma m_i + 2\Sigma M_i)} - (\Sigma m_i + 2\Sigma M_i)\lambda^{2n} e^{-\lambda(\Sigma m_i + 2\Sigma M_i)} = 0$

$\Rightarrow 2n - \lambda(\Sigma m_i + 2\Sigma M_i) = 0$

$$\left[\lambda = \frac{2n}{\Sigma m_i + 2\Sigma M_i} \right]$$

② $\epsilon_i = y_i - a - bi$

Now $L(\epsilon_i \mid \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^n e^{-\sum \frac{\epsilon_i^2}{2\sigma^2}}$.

$L(\epsilon_i \mid \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{\sum_{i=1}^{n}(y_i - a - bi)^2}{2\sigma^2}}$

$\Rightarrow \sum_{i=1}^{n}(y_i - a - bi)^2 = \Sigma y_i^2 + na^2 + \frac{n(n+1)(2n+1)}{6} b^2 - 2a\Sigma y_i - 2b(\Sigma i y_i) + ab(n)(n+1)$

$\Downarrow$

K

$L(e_i \mid \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{K}{2\sigma^2}}$

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$$\frac{dL}{d\sigma} = \left(\frac{-n}{(\sigma)^{n+1}} + \frac{K}{(\sigma)^{n+3}} \right) e^{-\frac{K}{\sigma^2}} \times \left(\frac{1}{2\pi}\right)^{n/2} = 0$$

$$K = n\sigma^2$$

$$\left[\sigma = \sqrt{\frac{K}{n}} \right]$$

(3) $x_1, x_2 \; . \; x_n \sim f(x \mid \lambda, \alpha) = \alpha \lambda \cancel{x} e^{\alpha - 1} e^{-\lambda x^\alpha}$

$$f(X_{is} \mid \lambda, \alpha) = (\alpha\lambda)^n \left(\prod_{i=1}^{N} x_i^{\alpha-1} \right) e^{-\lambda \Sigma x_i^\alpha}$$

$$f(\lambda \mid a, b) = \frac{b^a}{\Gamma a} \lambda^{a-1} e^{-b\lambda}$$

$$f(\lambda \mid X_{is}, a, b) \propto \cancel{(\alpha^n \lambda^n) \frac{b^a}{\Gamma a}} \lambda^{n+a-1} e^{-\lambda(\Sigma x_i^\alpha + b)}$$

$$f(\lambda \mid X_{is}, a, b] \sim \text{gamma}(n+a, \Sigma x_i^\alpha + b)$$

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(4) $X_1 \ldots x_n \sim \text{uni}(0, \theta)$

$X_m = \text{Max}(X_{is})$

now $f(X_{is} \mid \theta) = \left(\frac{1}{\theta}\right)^n \qquad [\theta \geq X_m]$

$$f(\theta) = \begin{cases} 1 & 0 < \theta < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f(\theta \mid X_{is}) = \frac{f(\theta)\, f(X_{is} \mid \theta)}{\int_{X_m}^{1} \left(\frac{1}{\theta}\right)^n d\theta} = \frac{(n-1)\left(\frac{1}{\theta}\right)^n}{\left(\left(\frac{1}{X_m}\right)^{n-1} - 1\right)}$$

$$\cancel{f(\theta \mid X_{is}) = (n-1)\left(\frac{1}{\theta}\right)^n \left(\frac{1}{X_m}\right.}$$

$$f(\theta \mid X_{is}) = \frac{(n-1)\left(\frac{1}{\theta}\right)^n}{\left(\frac{1}{X_m}\right)^{n-1} - 1} \qquad \left(x_m < \theta < 1\right)$$

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