classmate

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Ans ① let X_i be a Random variable such that $X_i = 1$ if $f(i) = i$, 0 otherwise.

then the $X = \sum_{i=1}^{N} X_i$ be the random varible ~~that~~ of no. of fixed point.

then $E(X) = E(\sum X_i) = \sum_{i=1}^{N} E(X_i)$

$E(X_i) = 0 * \left(1 - \frac{(n-1)!}{n!}\right) + \left(\frac{(n-1)!}{n!}\right) 1 = 1/n$

[probability that $f(i) = i$ is $\Rightarrow \frac{(n-1)!}{n!}$]

$E(X) = n \times 1/n = 1$

Variance of $(X) = E(X^2) - E(X)^2$

$= E((\sum X_i)^2) - 1$

$= E\left(\sum X_i^2 + \sum\sum_{i \neq j} X_i X_j\right) - 1$

$= \sum_{i=1}^{N} E(X_i^2) - 1 + \sum_{i=1}^{N} \sum_{i \neq j} E(X_i X_j)$

$= \sum_{i=1}^{N} \sum_{i \neq j} E(X_i X_j)$

now [as $E(X_i^2) = E(X_i)$ [$X_i^2 = X_i$]]

now $\operatorname{var}(X) = \sum_{i=1}^{N} \sum_{j \neq 1} E(X_i X_j]$

$$E(X_i X_j) = \frac{(n-2)!}{n!} = \frac{1}{(n)(n-1)}$$

$$\operatorname{Var}(X) = n(n-1) \frac{1}{n(n-1)} = 1$$

④

(a) for N coupons

let X_i be the random variable, representing no. of tries to get i^{th} coupon given that $(i-1)$ coupons are selected.

~~the~~ then X, the random variable representing No. of tries for selecting all coupons.

$$X = \sum_{i=1}^{N} X_i$$

$$E(X) = \sum_{i=1}^{N} E(X_i)$$

Now every X_i has a geometric distribution $\left[p \text{ of success} = \frac{N-i+1}{N} \right]$

as for X_i no. of favourable coupons $= \underline{\underline{N-i+1}}$

Now $E(X_i) = \dfrac{N}{N-i+1}$

[in the case of X_i we are counting no. of tries to get 1st success]

so $E(X) = \sum E(X_i) = N\left[1 + \frac{1}{2} + \dots \frac{1}{N} \right]$