Recall

- No. of ways of choosing r objects from n objects

$${}^{n}C_{r}$$

- No. of ways of n identical things to r people :

$${}^{n+r-1}C_{r-1}$$

$${}^{n-1}C_{r-1} \quad \text{(at least one to each)}$$

- No. of ways of ~~dist~~ arranging n distinct objects in r distinct group of size $m_1, m_2 \dots m_r$

$$\frac{n!}{m_1!\, m_2! \dots m_r!}$$

- Dearrangement formula $= n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots \frac{(-1)^n}{n!}\right]$

N coin tosses

a coin is tossed n times. $\left[\begin{matrix} H = 1 \\ T = 0 \end{matrix}\right]$ $[P[H] = p]$

$$\Omega = \{0, 1\}^n$$

any outcome $\omega = (\omega_1, \omega_2 \quad \dots \quad \omega_n)$ $[\omega_i \in \{0, 1\}]$

Event A $\Rightarrow$ No. of heads $= n/2$ $[n = \text{even}]$

$$p_\omega = p^{\Sigma \omega_i} (1-p)^{n - \Sigma \omega_i} \quad [\forall \ \omega \in \Omega]$$

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Now to calculate P(A) we need to find $|A|$ as every element of A has same probability

$$|A| = {}^nC_{n/2}$$

$$P(A) = \sum_{\omega \in A} p_\omega = {}^nC_{n/2}\, p^{n/2} (1-p)^{n/2}$$

Shuffling of Cards

$\Omega = S_{52}$: [all permutations of 52]

$p_\omega = \frac{1}{52!}$ [ω is any ~~permutation~~ $\in S_{52}$]
permutation

A = getting all Aces together

$P(A) = \sum_{\omega_i \in A} p_{\omega_i} = |A| p_\omega$

$|A| = \cancel{({}^{52}C_4)\ 4!\ 48!}$ $\cancel{49!\,48!}$ $49!\ 4!$

$P(A) = \cancel{\frac{{}^{52}C_4\ 4!\ 48!}{52!}}$ $\frac{49!\ 4!}{58!}$

Birthday Paradox

In a room of 25 people, what is probability that two people have same birthday
(atleast)

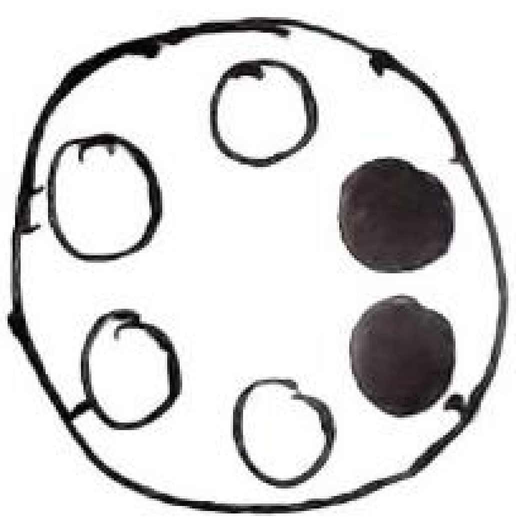
⇒ lets call this event A then

$$P(A) = 1 - P(A^c)$$

$$P(A^c) = \frac{(365)(364)\ \ldots\ (365-24)}{(365)^{25}}$$

$$P(A) = 1 - P(A^c) = 0.569 \approx 57\ \%$$

Questions

(1) 

You have given a revolver with following arrangement. You rolled the barrel and shot the gun but the bullet ~~doesn't~~ not fired. Now You want to take another shoot, ~~should~~ does rolling the barrel again increase your chances of firing the bullet.

(2) Probability of Accidents on a road is $3/4$ in One hour. What is the probability of accidents in $1/2$ hour.

(3) [Murphy's Law] A fair coin is tossed repeately n times. Let S be any sequence of H and T of length r.

[ie. $\underbrace{HTTH\ldots H}_{r}$]

What is the probability that S will eventually appear [redacted] in n tosses of the coin [$n \to \infty$]

4) A be countable collection of events then prove

(i) $P[E_i] = 0 \ \forall E_i \in A \iff P\left(\bigcup_{E_i \in A} E_i\right) = 0$

(ii) $P(E_i) = 1 \ \forall E_i \in A \iff P\left(\bigcap_{E_i \in A} E_i\right) = 1$

1) $A =$. Next ~~bullet~~ chamber contain bullets
$B =$ chamber is empty

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/6}{4/6} = 1/4$$

Probability of firing a bullet after rolling the ~~barrel~~ barrel $= 1/3$.

2) $p =$ probability of accident in half ~~hour~~ hour

No. accident in hour $=$ (no-accident in half$)^2$

$$1/4 = (1-p)^2$$

$$\left[p = 1/2\right]$$

3) we can break sequence of n tosses in $\lfloor n/r \rfloor$ parts each of length r

than probability of even S occuring in $k^{th}+1$ part is

$$\left[\left[1 - 1/2^r\right]^{ka} \left[1/2^r\right]\right]$$

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now $0 \leq K \leq \lfloor n/\gamma \rfloor$

so as $n \to \infty, K \to \infty$

and as all K events are disjoint so probability will be sum of everyone

$$P(S) = \sum_{K=0}^{\infty} \left[1 - \tfrac{1}{2\gamma}\right]^{K} \left[\tfrac{1}{2\gamma}\right] = 1$$

$$= \cancel{\left[\tfrac{1}{2\gamma}\right]\left[\quad\right]} \quad 1 - \tfrac{1}{2\gamma}$$

(4) $\underline{P[E_i] = 0} \quad \forall E_i \in A$.

$$P(\cup E_i) \leq \sum P(E_i)$$

$$P(\cup E_i) \leq 0$$

$$P(\cup E_i) = 0$$

$\boxed{\underline{P(\cup E_i) = 0}}$.

$$P(E_i) \leq P(\cup E_i)$$

$$P(E_i) \leq 0$$

$$P(E_i) = 0 \quad \forall E_i \in A$$

now $i \Rightarrow ii$, take $\left[A_i = E_i^c\right]$

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