Assignment - 1

① 1D Steady - State Conduction

General Transport equation is;

$$\frac{\partial \phi}{\partial t} + (U.\nabla)\phi = \nabla(T.\nabla)\phi + S$$

Here $\frac{\partial \phi}{\partial t}$ is time related term,

$(U.\nabla)\phi$ is conduction term,

$\nabla(T.\nabla)\phi$ is Diffusion term,

S is Source term,

For 1-D steady state conduction,

$\nabla(T.\nabla)\phi$ will not be zero, where there is no source term so it will be zero and conduction will also be zero.

Also we are assuming T as constant.

$$\therefore T.\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = 0$$

But as we are taking 1-D only, x term is there and y and z are zero.

$$\therefore \Gamma.\left(\frac{\partial^2 T}{\partial x^2}\right) = 0$$

as T depends only on x, we can rewrite as,

$$\frac{d^2T}{dx^2} = 0$$

Using Finite Difference method,

$$\boxed{\frac{d^2T}{dx^2} = 0} \quad \text{--- [1]}$$

Using Taylor's expansion;

$$T(x+h) = T(x) + \frac{dT}{dx} \cdot h + \frac{d^2T}{dx^2} \cdot \frac{h^2}{2} + \ldots$$

$$T(x-h) = T(x) - \frac{dT}{dx} \cdot h + \frac{d^2T}{dx^2} \cdot \frac{h^2}{2} \mp \ldots$$

$$\therefore T(x+h) + T(x-h) = 2T(x) + \frac{d^2T}{dx^2} \cdot h^2$$

$$\therefore \boxed{\frac{d^2T}{dx^2} = \frac{T(x+h) - 2T(x) + T(x-h)}{h^2}} \quad \text{--- [2]}$$

From eqn ① & ②

$$\therefore \frac{d^2T}{dx^2} = \frac{T(x+h) - 2T(x) + T(x-h)}{h^2} = 0$$

$$\therefore \boxed{T(x) = \frac{T(x+h) + T(x-h)}{2}}$$

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