Question 1

Question 1: A die is rolled. What is the probability of getting:
(a) An even number
(b) A number greater than 4

(a) Probability of getting an even number

- **Favourable outcomes:** The even numbers are {2, 4, 6}.
- **Number of favourable outcomes:** 3

P(Even) = 3 / 6 **P(Even) = 1 / 2**

Answer: The probability of getting an even number is **1/2**.

(b) Probability of getting a number greater than 4

- **Favourable outcomes:** The numbers greater than 4 are {5, 6}.
- **Number of favourable outcomes:** 2

P(Number > 4) = 2 / 6 **P(Number > 4) = 1 / 3**

Answer: The probability of getting a number greater than 4 is **1/3**.

Question 2

Question 2: In a class of 50 students:
- 20 like Mathematics (M)
- 15 like Science (S)
- 5 like both subjects

What is the probability that a student chosen at random likes Mathematics or Science?

To find the probability that a student likes Mathematics **or** Science, we use **Rule 4 - The Addition Rule**. This rule is used when calculating the probability of either event A or event B occurring.

1. Identify the given values:

- Total number of students (Sample Space) = 50
- Number who like Mathematics, n(M) = 20
- Number who like Science, n(S) = 15
- Number who like **both** subjects, n(M and S) = 5

2. Apply the Formula: According to the slides, the formula for the Addition Rule is:

P(A or B) = P(A) + P(B) – P(A and B)

Note: We subtract the intersection (P(A and B)) because if the events overlap, their common part is counted twice, so we must remove it once.

3. Calculate the Probability: First, convert the values into probabilities:

- P(Mathematics) = 20 / 50
- P(Science) = 15 / 50
- P(Both) = 5 / 50

Now, substitute these into the formula: P(M or S) = (20 / 50) + (15 / 50) – (5 / 50) P(M or S) = (20 + 15 – 5) / 50 P(M or S) = 30 / 50

Answer: The probability that a student chosen at random likes Mathematics or Science is **3/5** (or 0.6).

Question 3

Question 3: A bag has 3 red and 2 blue balls. If one ball is drawn randomly and is red, what is the probability that the next ball is also red (without replacement)?

This is a case of **Conditional Probability**, where the first event affects the second.

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- **Initial Count:** 3 Red, 2 Blue (Total = 5).
- **After 1st Draw (Red):** One red ball is removed and not replaced.
  - **Red balls remaining:** 2
  - **Total balls remaining:** 4
- **Calculation:** P(Next is Red) = (Remaining Red) / (Remaining Total) P(Next is Red) = 2 / 4 = **1/2**

Answer: The probability is **1/2**.

Question 4

Question 4: The population of a school is divided into 60% boys and 40% girls. If you want equal representation of both genders in the sample, which method should you use: Simple Random Sampling or Stratified Sampling? Why?

Answer:

Stratified Sampling is the correct method to use.

Explanation:

1. Why Stratified Sampling? According to the slides, **Stratified Sampling** is a method where the population is divided into distinct, non-overlapping groups called "strata" based on specific characteristics. In this scenario, the strata would be "Boys" and "Girls."

- **Mechanism:** Once the strata are defined, a random sample is taken from each stratum.
- **Benefit:** This method allows the researcher to ensure that all groups are represented in the survey results. By using this method, you can deliberately select an equal number of students from each group (e.g., 50 boys and 50 girls) to achieve equal representation, regardless of the original population proportions.

2. Why NOT Simple Random Sampling?

Simple Random Sampling is defined as a method where every individual in the population has an equal chance of being selected.

- **The Issue:** Because selection is based purely on chance, a simple random sample will naturally tend to reflect the characteristics of the larger population. Since the school is 60% boys and 40% girls, a simple random sample would likely result in a similar 60/40 split, failing to achieve the desired equal representation.

Question 5

Question 5: The average height of 1000 students = 160 cm. A sample of 100 students shows an average height = 158 cm.

Find the sampling error.

Solution:

Solution:

1. Identify the Given Values:

- **Population Mean:** 160 cm (Average height of the entire group of 1000 students).
- **Sample Mean:** 158 cm (Average height of the subset of 100 students).

2. Definition and Formula: According to the provided notes, **Sampling Error** is defined as the "difference between a sample statistic (e.g., sample mean) and the corresponding population parameter (e.g., population mean)".

The formula is:

Sampling Error = Sample Mean – Population Mean

3. Calculation:

Substitute the values into the formula:

Sampling Error = 158 cm – 160 cm

Sampling Error = -2 cm

4. Explanation: The sampling error is **-2 cm**. This negative value indicates that the sample mean underestimates the true population mean by 2 cm. This error occurs "due to the randomness of sampling," meaning the specific group of 100 students selected happened to be slightly shorter on average than the total population.

Answer:

The sampling error is **-2 cm**.

Question 6

Question 6: The population mean salary is ₹50,000 with σ = ₹5,000. If we take a sample of 100 employees, what is the standard error of the mean (SEM)?

Given:

• Population Mean (μ) = ₹50,000

• Population Standard Deviation (σ) = ₹5,000

• Sample Size (n) = 100

Formula:

The Standard Error of the Mean (SEM) is calculated using the formula:

$SEM = \sigma / \sqrt{n}$

Calculation:

$SEM = 5000 / \sqrt{100}$

$SEM = 5000 / 10$

$SEM = 500$

Answer:

The standard error of the mean (SEM) is ₹500.

Question 7

Question 7: In a group of 100 students:

40 like Cricket (C)

30 like Football (F)

10 like both Cricket and Football

Find the probability that a student likes at least one sport.

Given:

• Total Students = 100

• Students who like Cricket (C) = 40

• Students who like Football (F) = 30

• Students who like Both $(C \cap F)$ = 10

Objective:

Find the probability that a student likes at least one sport $(P(C \cup F))$.

Formula:

First, find the number of students who like at least one sport using the formula:

$n(C \cup F) = n(C) + n(F) - n(C \cap F)$

Then, calculate the probability:

$P(C \cup F) = \frac{\text{Number of students who like at least one sport}}{\text{Total students}}$

Calculation:

1. Find the number of students:

$n(C \cup F) = 40 + 30 - 10$

$n(C \cup F) = 60$

2. Calculate Probability:

$P(C \cup F) = \frac{60}{100}$

$P(C \cup F) = 0.6$

Answer:

The probability that a student likes at least one sport is 0.6 (or 60%).

Question 8

Question 8: From a deck of 52 cards, two cards are drawn without replacement. What is the probability that both are Aces?

Question 8

Given:

• Total number of cards = 52

• Number of Aces in the deck = 4

• Conditions: Two cards are drawn without replacement.

Logic:

Since the cards are drawn without replacement, the outcome of the second draw depends on the first. This is an example of conditional probability.

1. First Draw: The probability of drawing an Ace is the number of Aces divided by the total cards.

2. Second Draw: Since one Ace has already been removed, there are now 3 Aces left and 51 total cards remaining.

Calculation:

• Probability of 1st Ace: $P(A_1) = \frac{4}{52} = \frac{1}{13}$

• Probability of 2nd Ace (given 1st was Ace): $P(A_2 \mid A_1) = \frac{3}{51} = \frac{1}{17}$

Multiply the probabilities of the dependent events:

$P(\text{Both Aces}) = \frac{1}{13} \times \frac{1}{17}$

$P(\text{Both Aces}) = \frac{1}{221}$

Answer:

The probability that both cards are Aces is $\frac{1}{221}$ (or approximately 0.45%).

Question 9

Question 9: A factory produces bulbs with 2% defective rate. If 5 bulbs are chosen at random, what is the probability that all are non-defective?

Given:

• Defective rate (P_d) = 2% = 0.02

• Non-defective rate (P_{nd}) = 100% - 2% = 98% = 0.98

• Number of bulbs chosen (n) = 5

Logic:

Since the sample size is small relative to total production, we treat the selection of each bulb as an independent event. To find the probability that all 5 bulbs are non-defective, we multiply the probability of a single bulb being non-defective by itself 5 times.

Calculation:

$$P(\text{All 5 Non-defective}) = (P_{nd})^5$$

$$P = 0.98 \times 0.98 \times 0.98 \times 0.98 \times 0.98$$

$$P = (0.98)^5$$

$$P \approx 0.9039$$

Answer:

The probability that all 5 bulbs are non-defective is approximately 0.9039 (or 90.39%).

Question 10

Question 10: Differentiate between discrete and continuous random variables with examples.

1. Definitions

• Discrete Random Variable: A variable that can only take on specific, distinct values. There are "gaps" between the numbers (e.g., you can have 1 or 2, but not 1.5).

• Continuous Random Variable: A variable that can take on any value within a range. It usually represents a precise measurement with decimals.

2. Key Differences

• Nature of Values:

• Discrete: Countable values (finite).

• Continuous: Uncountable values (infinite).

• Method of Measurement:

• Discrete: Obtained by counting.

• Continuous: Obtained by measuring.

• Probability:

• Discrete: We calculate the probability of specific values (e.g., $P(X=5)$).

• Continuous: We calculate the probability over an interval (e.g., $P(X > 5)$), because the probability of one exact point is zero.

3. Examples

• Discrete Examples:

• The number of students in a class (you can't have half a student).

• The outcome of rolling a die (1, 2, 3, 4, 5, or 6).

• The number of cars in a parking lot.

• Continuous Examples:

• The exact height of a person (e.g., 175.4 cm).

• The time taken to run a race (e.g., 12.35 seconds).

• The temperature of a city today.