

# Problem Set 8

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## 1 Model Overview

The effectiveness of fiscal policy has been a debated topic. Several stimulus checks have been issued by the United States federal government since the start of Covid-19 pandemic in order to increase aggregate demand. To understand the effectiveness of fiscal policy under uncertainty, I build a model using a two-state Markov chain approach. In this model, the agent is households, but I model it from the perspective of a social planner.

Social planner's problem:

$$\max_{\{c_t, tr_t, D_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t) \quad (1)$$

subject to

$$\begin{aligned} i) \quad & c_t = w_t y_t + tr_t, t = 0, 1, 2, \dots \quad \forall t \\ ii) \quad & tr_t = D_t(1 + r_t) - D_{t+1}, t = 0, 1, 2, \dots \quad \forall t \\ iii) \quad & c_t \geq 0 \quad \forall t \\ iv) \quad & y_t > 0 \quad \forall t \\ v) \quad & NPG : \lim_{T \rightarrow \infty} \left( \prod_{s=1}^T \left( \frac{1}{1 + r_s} \right) D_{T+1} \right) = 0 \end{aligned}$$

$c_t$ : consumption in period  $t$

$tr_t$ : government transfers in period  $t$

$D_t$ : government debt in period  $t$

$r_t$ : fixed rate of interest = 0.1, so  $r_t = r$

$w$ : fixed wage rate = 1, so  $w_t = w$

$y_t \in \{y_L, y_H\}$ : random shock to the consumer's income.  $y_t$  follow a two-state Markov chain with  $P(y' = y_H \mid y = y_H) = p$  and  $P(y' = y_L \mid y = y_L) = q$ .

Equation (i) is the household budget constraint

Equation (ii) is the government budget constraint

State variable:  $D_{t+1}, y_t$

Control variable:  $c_t, tr_t$

## 2 Definitions

1. Population of agents

Continuum of household uniformly distributed on  $[0,1]$

2. Preferences

log utility:  $u'(c_t) > 0, u''(c_t) < 0$

$0 < \beta < 1$

3. Productive technology

No production technology. Income,  $y_t$ , is exogenous.

4. Information technology

The probability of high state and low state economy is known to all consumers.

## 3 Bellman Equation

Log utility  $\Rightarrow c_t > 0; y_t > 0$  as given. We have binding constraints, so we can substitute all constraints into the objective function.

$$V(D, y) = \max_{\{tr\}_{t=0}^{\infty}} \ln(D(1+r) - \tilde{D} + wy) + \beta E_y[V(\tilde{D}, \tilde{y}) \mid D, y] \quad (2)$$

## 4 Derive First Order Conditions

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \ln(c_t) + E_t \sum_{t=0}^{\infty} \lambda_t [D_t(1+r) - \tilde{D}_{t+1} + wy_t] \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \frac{1}{c_t} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial D_{t+1}} = E_{t+1}[\lambda_{t+1}(1+r)] - \lambda_t = 0$$

## 5 Results

Figure 1 shows the resulting value function, plotted against the value of debt (next period). The figure suggests that as the size of debt increases, the value function increase. For the same level of debt, the value function of the agent who starts with a high wage shock is higher.

Figure 2 is the policy function of consumption plotted against debt. It indicates that for the agent that starts with a high shock, consumption level initially slightly increases with debt, but gradually decreases with debt increases. It is mostly flat, it changes slightly because they have wage shocks. However, for the agent that starts with low wage shock, consumption initially increases dramatically with the size of debt increases and it slowly diminishes. This is because the low agent starts with a very low wage due to the shock, but slowly smoothes their consumption using debts. For the

same level of debt, consumption is higher for the low agent because they start with a much lower wage shock, so it converges to the steady-state level faster using more debts.

Figure 3 is the policy function of transfer plotted against debt. It shows that transfer increases with debt. This is obvious because you need debt to have the transfers. In this analysis, I allow for negative transfer as a tax. For the same level of debt, the transfer is higher for the agent that starts with high wage shock because they have higher initial income and hence need less debt to achieve the same level of consumption.

Overall, the results suggest that in order to maximize consumption, the government needs to provide more transfers for individuals that experienced more negative shocks during an economic recession. This implies that under simplified assumptions, lower-income people should have gotten higher stimulus checks during Covid-19 pandemics.



