Properties of inner products

Quiz, 5 questions

4.5/5 points (90%)

Congratulations! You passed!

Next Item

1 / 1
point

1.
The function

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] not symmetric

Un-selected is correct

- [ ] symmetric

Correct
Yes: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

- [ ] bilinear

Correct
Yes:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- For any $\lambda \in \mathbb{R}$ it holds that $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

- [ ] positive definite

Correct

Yes, the matrix has only positive eigenvalues and $\beta(\mathbf{x},\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{0}$ and $\beta(\mathbf{x},\mathbf{x}) = 0 \iff \mathbf{x} = \mathbf{0}$

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- [ ] not positive definite

Un-selected is correct

- [ ] not bilinear

Un-selected is correct

- [ ] not an inner product

Un-selected is correct

- [ ] an inner product

Correct
It's symmetric, bilinear and positive definite. Therefore, it is a valid inner product.

0.50 / 1
point

2.
The function

$$\beta(\mathbf{x},\mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] positive definite

This should not be selected
Try to compute $\beta(\mathbf{x},\mathbf{x})$ with $\mathbf{x} = [1, 1]^T$.

- [ ] not an inner product

This should be selected

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not bilinear

Un-selected is correct

not positive definite

This should be selected

bilinear

Correct

Correct:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

an inner product

This should not be selected

No, it is not positive definite.

symmetric

Correct

Correct: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

not symmetric

Un-selected is correct

---

1 / 1 point

3.

The function

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$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] symmetric

Un-selected is correct

- [ ] not symmetric

Correct
Correct: If we take $\mathbf{x} = [1, 1]^T$ and $\mathbf{y} = [2, -1]^T$ then $\beta(\mathbf{x}, \mathbf{y}) = 0$ but $\beta(\mathbf{y}, \mathbf{x}) = 6$. Therefore, β is not symmetric.

- [ ] bilinear

Correct
Correct.

- [ ] not bilinear

Un-selected is correct

- [ ] an inner product

Un-selected is correct

- [ ] not an inner product

Correct
Correct: Symmetry is violated.

---

1 / 1
point

4.

The function

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$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] not bilinear

Un-selected is correct

- [ ] not positive definite

Un-selected is correct

- [ ] bilinear

Correct
It is the dot product, which we know already. Therefore, it is positive bilinear.

- [ ] positive definite

Correct
It is the dot product, which we know already. Therefore, it is positive definite.

- [ ] not an inner product

Un-selected is correct

- [ ] symmetric

Correct
It is the dot product, which we know already. Therefore, it is symmetric.

- [ ] not symmetric

Un-selected is correct

- [ ] an inner product

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Correct

It is the dot product, which we know already. Therefore, it is also an inner product.

---

1 / 1
point

5.

For any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ write a short piece of code that defines a valid inner product.

```
import numpy as np

def dot(a, b):
  """Compute dot product between a and b.
  Args:
    a, b: (2,) ndarray as R^2 vectors

  Returns:
    a number which is the dot product between a, b
  """

  dot_product = a.T @ np.eye(a.shape[0]) @ b

  return dot_product

# Test your code before you submit.
a = np.array([1,0])
b = np.array([0,1])
print(dot(a,b))
```

Run

Reset

Correct Response

Good job!
