Fitting a non-linear function

Practice Quiz, 5 questions

2.6/5 points (52%)

Try again once you are ready.

Required to pass: 80% or higher

You can retake this as many times as you'd like.

Back to Week 6

Retake

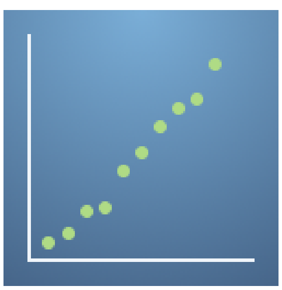
0.60 / 1 point

1.

The previous quiz tested our knowledge of linear regression, and how we can begin to model sets of data. In the last video, we developed on this idea further, looking at the case for data that cannot be effectively modelled by linear approximations. As such, we were introduced to the nonlinear least squares method, as a way of fitting nonlinear curves to data.

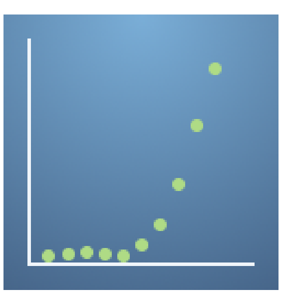
In this question, we have a set of graphs highlighting different distributions of data. Select the appropriate graphs where the nonlinear least squares method can be adapted to provide an effective fit to this data.

- [ ] Option A



This should be selected

- [ ] Option B



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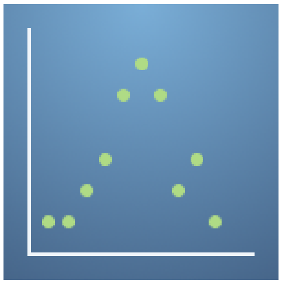
Correct

This data looks similar to an exponential function and should be able to be fitted through the nonlinear regression technique.

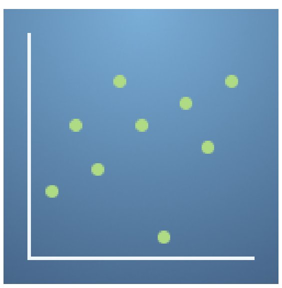
- [ ] Option C



Correct

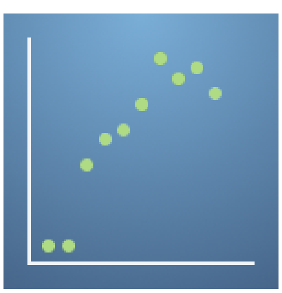
This data looks similar to a Gaussian and should be able to be fitted through the nonlinear regression technique.

- [ ] Option D



Un-selected is correct

- [ ] Option E



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This should be selected

2.6/5 points (52%)

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1 / 1
point

2.

In the previous lecture, you were taken through the example of χ^2 and how it is important in utilising the sum of the differences and the least squares method. We were also introduced to the expression $\chi^2 = \sum_{i=1}^{n} \frac{[y_i - y(x_i; a_k)]^2}{\sigma^2}$. For the parameter χ^2, select all the statements below that are true.

- [ ] When χ^2 is large, we have a good fit to our data.

Un-selected is correct

- [ ] Taking the gradient of χ^2 and setting this to zero allows us to determine effective fitting parameters.

Correct

By finding the gradient, we are finding the minimum of χ^2. This should allow us to build a set of simultaneous equations which can then be analysed to effectively fit the parameters through the steepest descent.

- [ ] The parameter χ is squared so that the effect of bad uncertainties are minimised.

Correct

When calculating χ, we divide by the uncertainty value σ. As a result, χ is squared in order to minimise the effect of dividing by a highly uncertain result.

- [ ] The parameter χ^2 is the uncertainty value of our variables.

Un-selected is correct

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0 / 1
point

3.

In the previous lecture, we took the derivative of χ^2 with respect to our fitting parameters, the form of which is shown below.

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$$\frac{\partial \chi^2}{\partial a_j} = -2 \sum_{i=1}^{n} \frac{y_i - f(x_i, \boldsymbol{a})}{\sigma_i^2} \frac{\partial f(x_i, \boldsymbol{a})}{\partial a_j} for\, j = 1......n$$

Here we will define the matrix $[Z_j] = \frac{\partial f(x_i, \mathbf{a})}{\partial a_j}$

Assuming $f(x_i, \boldsymbol{a}) = a_1x^3 - a_2x^2 + e^{-a_3x}$, select the option that correctly shows the partial differentiation for this function.

- $$\frac{\partial f}{\partial a_1} = x^3, \frac{\partial f}{\partial a_2} = -x^2, \frac{\partial f}{\partial a_3} = -xe^{-a_3x}$$

- $$\frac{\partial f}{\partial a_1} = x^3, \frac{\partial f}{\partial a_2} = 2x^2, \frac{\partial f}{\partial a_3} = xe^{a_3x}$$

- $$\frac{\partial f}{\partial a_1} = x^3 - a_2x^2 + e^{-a_3x}, \frac{\partial f}{\partial a_2} = a_1x^3 - x^2 + e^{-a_3x}, \frac{\partial f}{\partial a_3} = a_1x^3 - a_2x^2 - xe^{-a_3x}$$

- $$\frac{\partial f}{\partial a_1} = 3a_0x^2, \frac{\partial f}{\partial a_2} = -2a_2x, \frac{\partial f}{\partial a_3} = -a_3e^{-a_3x}$$

This should not be selected

We are differentiating our function with respect to the fitting parameters, not with respect to x.

---

0 / 1 point

4.

In this question, we want to put our working knowledge of the partial differentiation of our functions and arrange this into the Jacobian. As a reminder, the Jacobian for the nonlinear least squares method will take the form: $\boldsymbol{J} = \left[\frac{\partial(\chi^2)}{\partial a_k} \right] = \left[\frac{\partial(\chi^2)}{\partial a_1}, \quad \frac{\partial(\chi^2)}{\partial a_2} \right]$.

For the equation $y(x_i; \boldsymbol{a}) = a_1(1 - e^{-a_2x_i^2})$ and assuming $\sigma^2 = 1$, select the correct Jacobian that should be evaluated for our fit function.

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$$\frac{\partial(\chi^2)}{\partial a_1} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](1 - e^{-a_2 x_i^2})$$
$$\frac{\partial(\chi^2)}{\partial a_2} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](a_1 x_i^2 e^{-a_2 x_i^2})$$

$$\frac{\partial(\chi^2)}{\partial a_1} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](1 - e^{-a_2 x_i^2})$$
$$\frac{\partial(\chi^2)}{\partial a_2} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](-x_i^2 e^{-a_2 x_i^2})$$

This should not be selected

Consider the differentiation again, there may be signs and powers that may have been changed incorrectly.

$$\frac{\partial(\chi^2)}{\partial a_1} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](1 + e^{-a_2 x_i^2})$$
$$\frac{\partial(\chi^2)}{\partial a_2} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](a_1 x_i e^{-a_2 x_i^2})$$

$$\frac{\partial(\chi^2)}{\partial a_1} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](e^{-a_2 x_i^2})$$
$$\frac{\partial(\chi^2)}{\partial a_2} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](a_1 x_i^2 e^{-a_2 x_i^2})$$

1 / 1 point

5.

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In this question, we will develop our idea of building the Jacobian further by looking at a function with more fitting parameters. The function below is of a Gaussian distribution with 4 fitting parameters (σ, x_p, I, b) which are to be used in the nonlinear least squares method.

$$y(x; \sigma, x_p, I, b) = b + \frac{I}{\sigma\sqrt{2\pi}} \exp\left\{ \frac{-(x - x_p)^2}{2\sigma^2} \right\}$$

In the lectures, we also showed how to find chi^2 and how this forms the Jacobian shown below:
$\boldsymbol{J} = \left[\frac{\partial(\chi^2)}{\partial a_k} \right] = \left[\frac{\partial(\chi^2)}{\partial \sigma}, \quad \frac{\partial(\chi^2)}{\partial x_p}, \quad \frac{\partial(\chi^2)}{\partial I}, \quad \frac{\partial(\chi^2)}{\partial b} \right].$

where

$$\frac{\partial \chi^2}{\partial a_j} = -2 \sum_{i=1}^{n} \frac{\boldsymbol{y_i} - y(x_i; \boldsymbol{a})}{\sigma_i^2} \frac{\partial y(x_i; \boldsymbol{a})}{\partial a_j} for\, j = 1......n$$

For the Gaussian function above, determine the partial differential

$$\frac{\partial y}{\partial x_p}$$

- ( ) $$\frac{\partial y}{\partial x_p} = \frac{I}{\sqrt{2\pi}} \frac{2(x - x_p)}{\sigma} \exp\left\{ \frac{-(x - x_p)^2}{2\sigma^2} \right\}$$

- (●) $$\frac{\partial y}{\partial x_p} = \frac{I}{\sqrt{2\pi}} \frac{(x - x_p)}{\sigma^3} \exp\left\{ \frac{-(x - x_p)^2}{2\sigma^2} \right\}$$

Correct
Here we are only evaluating one partial derivative that forms part of the Jacobian. In order to correctly fit the Gaussian to a specific set of data, we will need to evaluate all the partial derivatives mentioned previously.

- ( ) $$\frac{\partial y}{\partial x_p} = -\frac{I}{\sqrt{2\pi}} \frac{(x - x_p)}{2\sigma^3} \exp\left\{ \frac{-(x - x_p)^2}{2\sigma^2} \right\}$$

- ( ) $$\frac{\partial y}{\partial x_p} = \frac{I}{\sqrt{2\pi}} \frac{2x}{\sigma} \exp\left\{ \frac{-(x - x_p)^2}{2\sigma^2} \right\}$$

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