

# Solving 1-D BVP

Praveen. C

`praveen@math.tifrbng.res.in`



Tata Institute of Fundamental Research  
Center for Applicable Mathematics  
Bangalore 560065  
<http://math.tifrbng.res.in>

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# 1-D BVP

$$-u'' = f \quad x \in (0, 1)$$

$$u(0) = u_a, \quad u(1) = u_b$$

Grid with  $n$  points,  $h = \frac{1}{n-1}$ ,  $x_j = jh$

$$0 = x_0 < x_1 < \dots < x_{n-2} < x_{n-1} = 1$$

Finite difference

$$-\frac{u_{j-1} - 2u_j + u_{j+1}}{h^2} = f_j, \quad 1 \leq j \leq n-1$$

or with  $a = \frac{2}{h^2}$ ,  $b = -\frac{1}{h^2}$

$$u_0 = u_a$$

$$bu_{j-1} + au_j + bu_{j+1} = f_j, \quad 1 \leq j \leq n-1$$

$$u_{n-1} = u_b$$

# 1-D BVP

Matrix is non-symmetric. Make it symmetric by eliminating  $u_0, u_{n-1}$  from all other equations.

$$\begin{aligned} au_0 &= au_a \\ au_1 + bu_2 &= f_1 - bu_a \\ bu_{j-1} + au_j + bu_{j+1} &= f_j, & 2 \leq j \leq n-2 \\ bu_{n-3} + au_{n-2} &= f_{n-1} - bu_b \\ au_{n-1} &= au_b \end{aligned}$$