Q:) You are given a number n, check if a number is prime or not ??

Ex → 7 → Yes $n \le 10^9$

→ 8 → NO

Q:- Given 2 number a, b. Calculate a^b without using (**) exponent operator or any inbuilt function

⇒ loop → a^b → $\underbrace{a * a * a * \cdots\cdots a}_{b\text{ times}}$

1 a a a a

a) O(n²)
b) O(n)
c) O(1)
d) None

n = b

temp = 1

$\boxed{\text{temp} = \text{temp} * a}$ → b times

Recursively → Base Case
→ Self work
→ Recursion

$a^1 = a$

a^b → Base Cases ← { $b == 1$ → a ; $b = 0$ → 1 }

$a^b = \boxed{a \times a^{b-1}}$

to go and calculate a^{b-1} → recursive intuition

multiply the result by a → selfwork

$f(a, b) = a \times f(a, b-1)$

a^b

works perfect

$b == 0 \rightarrow 1$

$b == 1 \rightarrow a$

$\rightarrow$ base case

a) $O(b)$ ✓

b) $O(1)$

c) $O(b^2)$

d) None

space complexity

call stack $\rightarrow$ space

2^4

4

2, 1

2, 2

2, 3

2, 4

n

calls

2^3

2^2

2^1 → base case

b stack frame

$2 \times b \approx b$

$O(b)$

$f(2, n)$

$\to f(2, n-1)$

$\to f(2, n-2) \to \boxed{\text{Base case}}$

$\vdots$

$f(2, 1)$

$3 \times (n-1)$

$3(n-1) \approx n$

$O(n)$

Can we optimise a^b calculation

$2^4 = 2^2 * 2^2$

$7/2 \to 3$

$2^7 = 2^3 * 2^3 * 2$

$$f(a,b) = a * f(a, b-1)$$

$\Downarrow$

$$f(a,b) = f(a, b/2) * f(a, b/2)$$

$b \to$ even

$$f(a,b) = f(a, b/2) * f(a, b/2) * a$$

$b \to$ odd

a) $O(b)$
b) $O(\log b)$ ✓
c) $O(1)$
d) None

$n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8$ - - - - - $\frac{n}{2^k}$ last term

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{K calls}}$

$\frac{1}{2^k} \neq 0$

$n \neq 0$

$\frac{n}{2^k} = 1$

$10/2 / 2 / 2 2 h \rightarrow 0/1$

$n = 2^k$

$K = \log_2 n$

$100/2$

$50/2$

$25/2$

$\vdots$

1

$O(\log n)$

K operation

T is a fun

$$T(b) = \underbrace{T(b/2)}_{\text{no of operations for } b/2} + \underbrace{1}_{\text{one comparison for subproblem}}$$

Recurrence relation

How many operation for size b

$T(b/2) = T(b/4) + 1$

$T(b/4) = T(b/8) + 1$

$\vdots$

$T(2) = T(1) + 1$

k terms

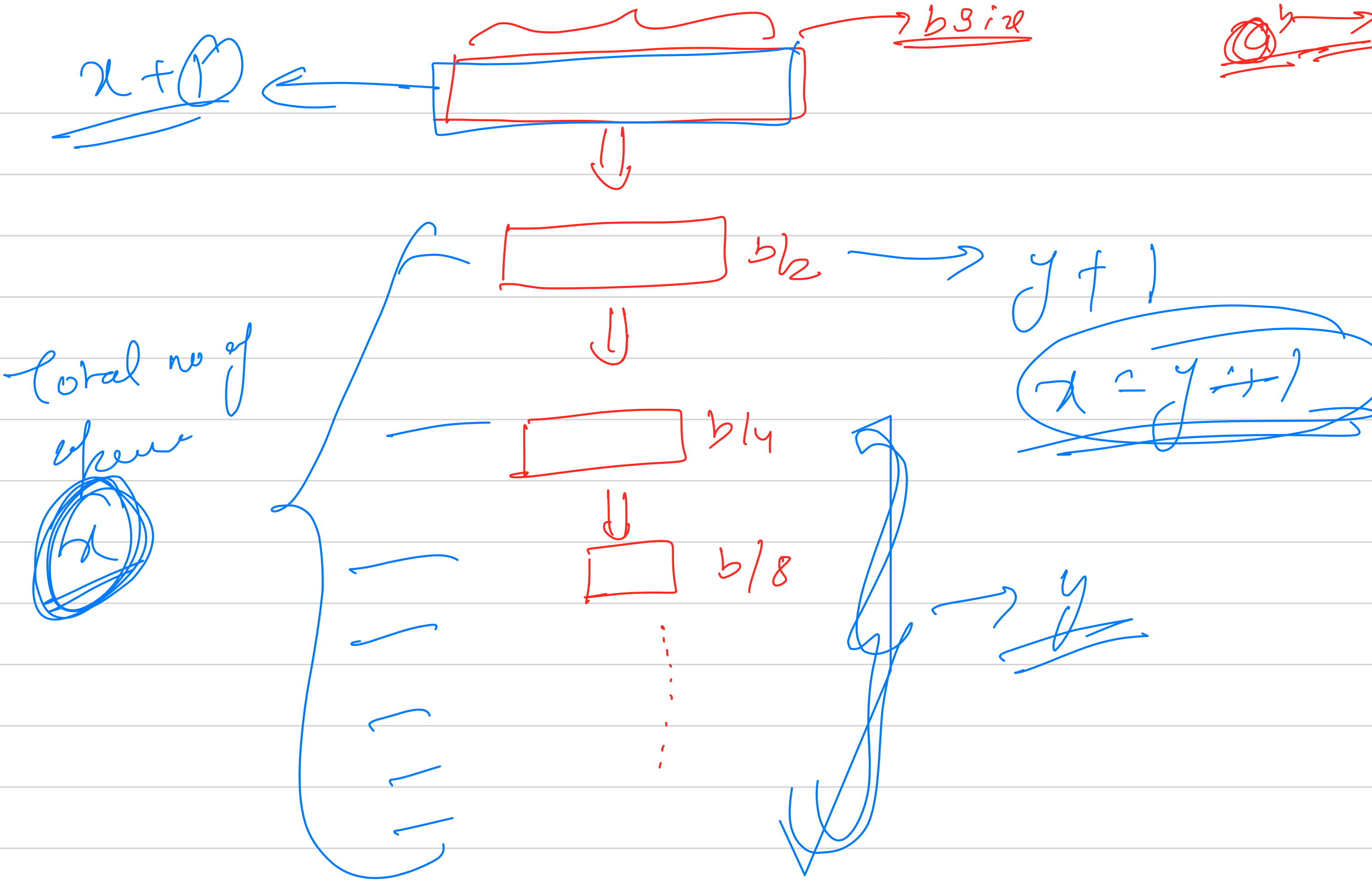
$K = \log b$

$T(b) = T(1) + k \times 1$

Total actual comparisons

$1 \times \log_2 b$

$O(\log b)$



b size
x + 1
b/2
y + 1
x = y + 1
b/4
b/8
y
total no of elements
x

$$\boxed{b}$$

$$\downarrow$$

$$\boxed{b/4}$$

$$\downarrow$$

$$\boxed{b/16}$$

$$\vdots$$

$a/1 = b$

$$b \rightarrow \frac{b}{4} \rightarrow \frac{b}{16} \text{ - - - - - } \frac{b}{4^k}$$

$$k = \log_4 b$$

$$b \rightarrow \frac{b}{3} \rightarrow \frac{b}{3^2} \rightarrow \frac{b}{3^3} \text{ - - - - } \frac{b}{3^k}$$

$$k = \log_3 b$$

$f(a, b) = f(a, b/3) * f(a, b/3) * f(a, b/3) \rightarrow$

if ( ) ←

elif ( ) ←

if ($b \% 3 == 0$) $b\%3==0$

elif $b \% 3 == 1$

else

$2 \rightarrow 2^2 \times 2^2 \times 2^2$

$2^2 \cdot 2 = 2^2 \times 2^2 \times 2^2 \times 2$

$\rightarrow 2^8 = 2^2 \times 2^2 \times 2^2 \times 2 \times 2$

$2^9 \rightarrow 2^3 \times 2^3 \times 2^3$

$2^{10} \rightarrow 2^3 \times 2^3 \times 2^3 \times 2$

$2^{11} \rightarrow 2^3 \times 2^3 \times 2^3 \times 2 \times 2$

9

8/3

9

Merge Sort

Binary Search

Heap Sort

$$T(b) = \cancel{T(b/3)} + 2$$

$$\cancel{T(b/3)} = \cancel{T(b/3^2)} + 2$$

$$T(b/3^2) = T(b/3^3) + 2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\cancel{T(3)} = T(1) + 2$$

$$T(b) = T(1) + \underline{2K}$$

$K = \log_3 b$

k few

$2\log_3 b$

x $\qquad$ y

$b \to b/1$

$2\log_3 b$ $\qquad$ $\log_2 b$ $\qquad$ $b/3$ $\qquad$ we change the base

a) $x > y$

b) $x < y$

c) $x == y$

d) None

$2\log_3 b = 2 \times \log_2 b$

LHS > RHS

$\log_2 3$

$\frac{2}{\log_2 3} \log_2 b \longleftrightarrow 2 \log_2 b$

$\log_2 3 > 1$

coefficient > coeffn

$\frac{2}{1.12}$ $\qquad$ $2/1.12$

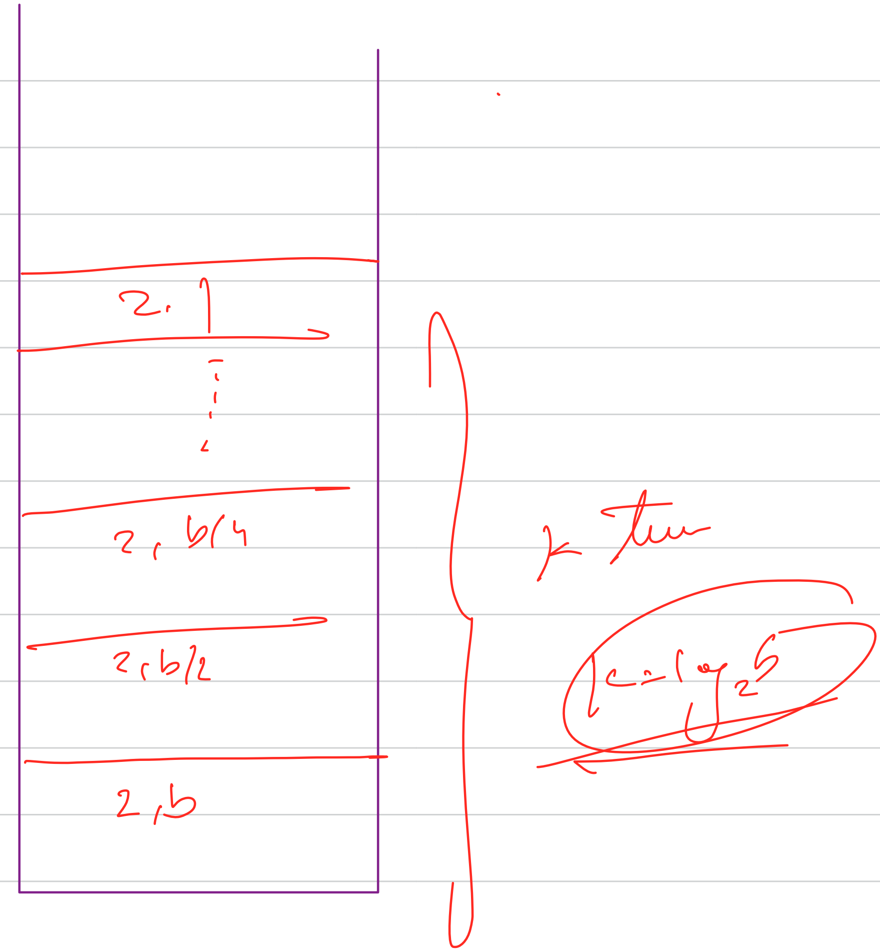
lesser bit

a) $O(b)$

b) $O(\log b)$ ✓

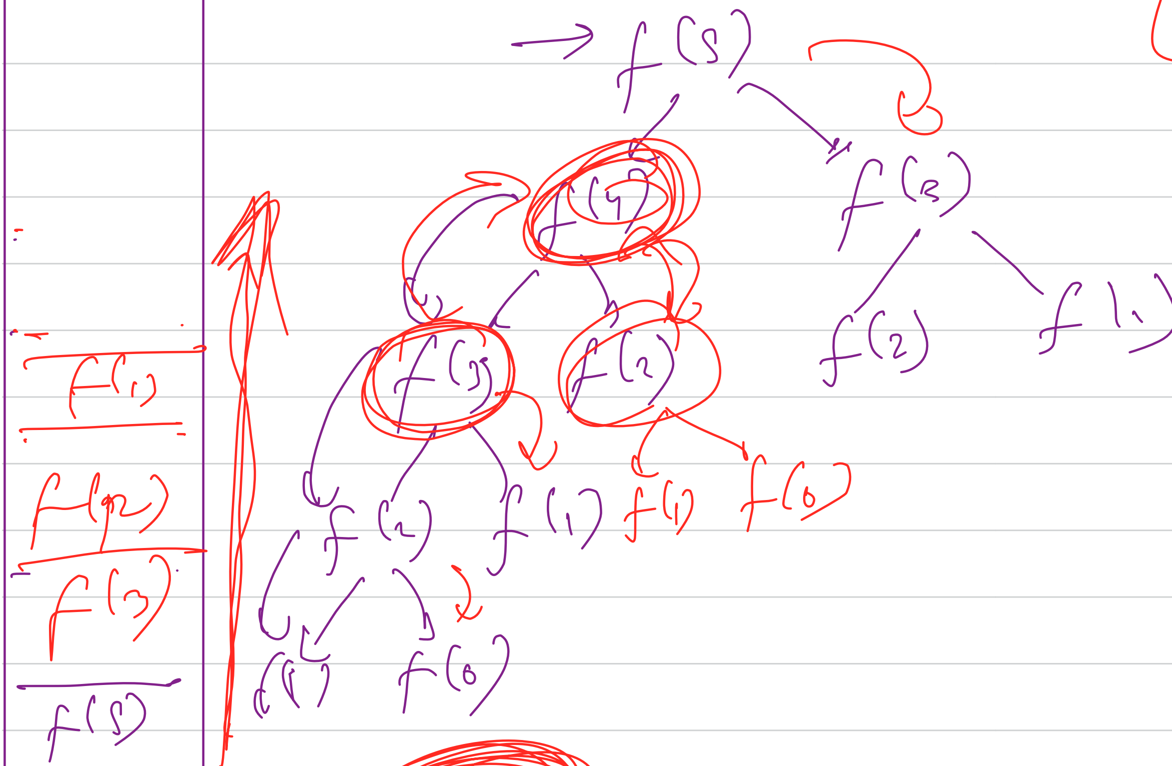
c) $O(1)$

d) None

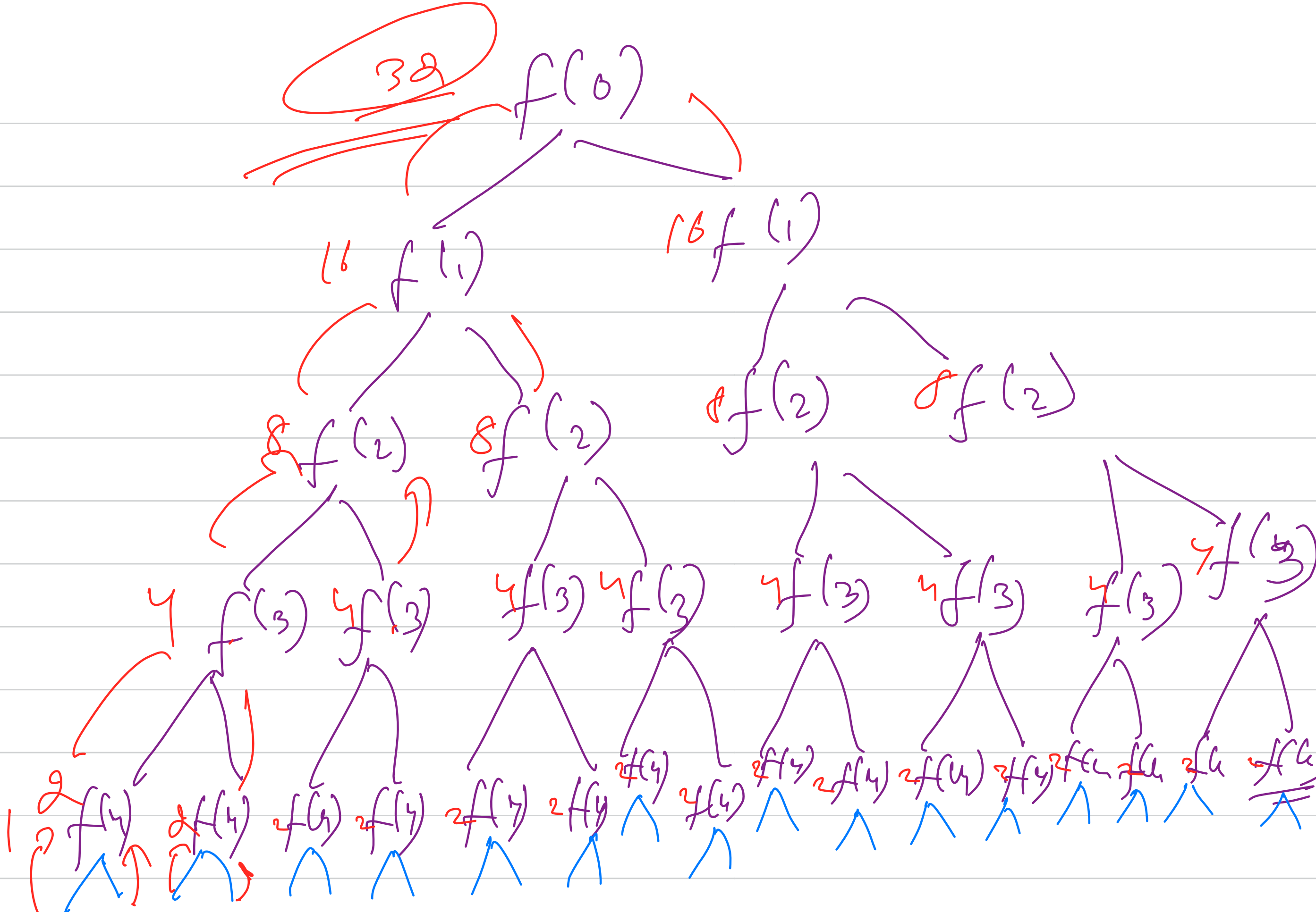


$f(n) = f(n-1) + f(n-2) \rightarrow O(2^n)$

$SC \rightarrow O(n)$



A pos



38
f(0)
16 f(1)
16 f(1)
8 f(2)
8 f(2)
8 f(2)
8 f(2)
4 f(3)
f(4)