


We will start at 7:35

Aman's Doubt class → 6:30pm Zoom
↳ from → fell your doubts

Agenda → Complexity analysis of algorithms

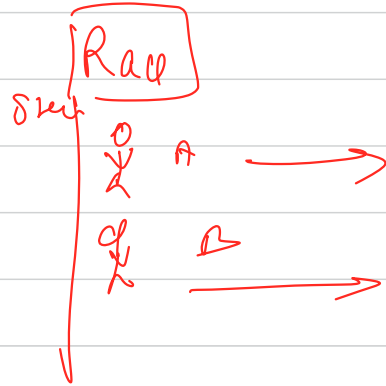
1 hr → theory
↳ practice

⇒ Process consumes some memory

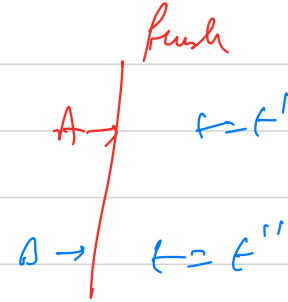
To execute a task process takes some time

Algo → memory efficient
→ Consume less time to execute the task

⇒ How we can calculate how much time an algo takes to execute??



$t = t$



time consumed by A $\rightarrow (t' - t)$
" " " " $\rightarrow (t'' - t)$

Experimental analysis

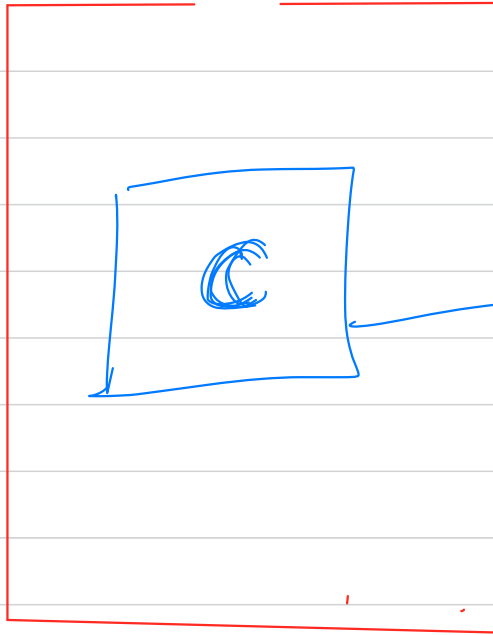
→ point at which code starts to execute → t_1

statements of algo

→ point when the code ends → t_2

$t_2 - t_1$

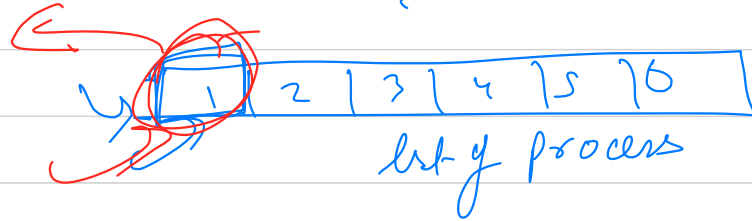
Because → your computer doesn't actually execute
everything parallelly



→ multiprocessing

→ multitasking

$t=t' \rightarrow$ one process



↳ microprocessor (i9, i7, i5...)

$t = 1 \text{ ms}$ ← unit

very very fast

1 sec = 10^8 cycles

time
consumed

algo1

$$y = 10x^2 - 3x + 1$$

$$y = x \log x$$

algo2

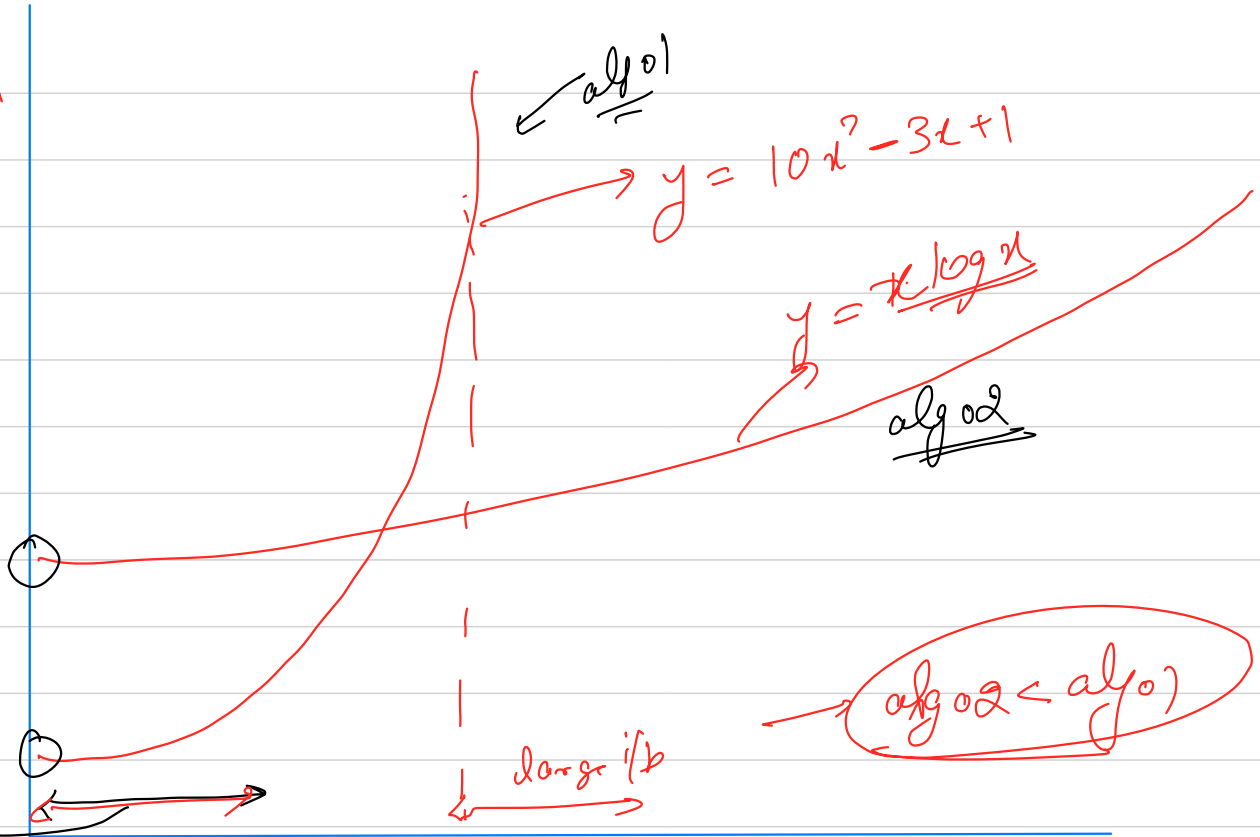
algo1 < algo2

small
i/p

algo2 < algo1

large i/p

input size \rightarrow



The change in execution time based on change in input is a better metric to measure.

→ Rate of growth of algorithm execution time

algorithm having lesser rate of growth will
be better.

$$f(x) = ax^n + bx^{n-1} + cx^{n-2} - \dots + \text{const}$$

$$g(x) = a'x^n + b'x^{n-1} + c'x^{n-2} - \dots + \text{const}$$

degree $\rightarrow \underline{\underline{n}}$

for 2 polynomials having same degree, the rate of growth will be same for large values of x

$$\begin{array}{|c|} \hline 10 \\ \hline 5 \\ \hline \end{array} \begin{array}{c} \textcircled{2} \\ \textcircled{2} \end{array} + \begin{array}{|c|} \hline 99x \\ \hline 13x \\ \hline \end{array} \rightarrow \underline{\underline{\text{use}}}$$

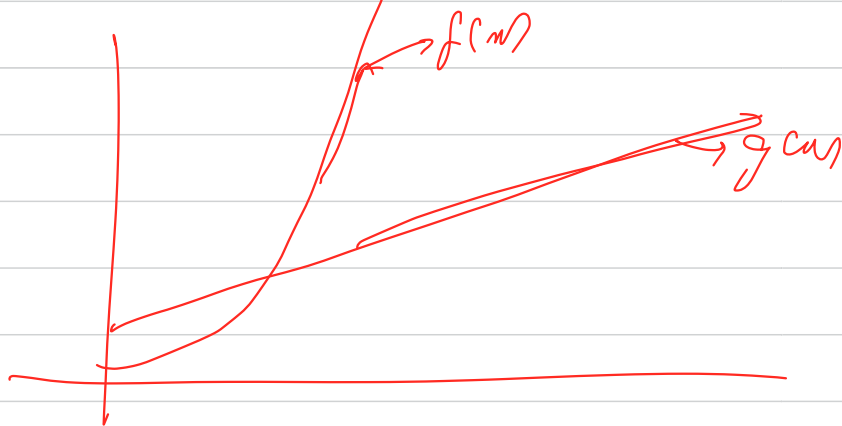
$x \geq 10^9$

$$\underline{\underline{f(n) = 3n^2 + 2n + 1}} \quad \checkmark$$

$$\underline{\underline{g(n) = 5n + 3}} \quad \checkmark$$

$g(n)$

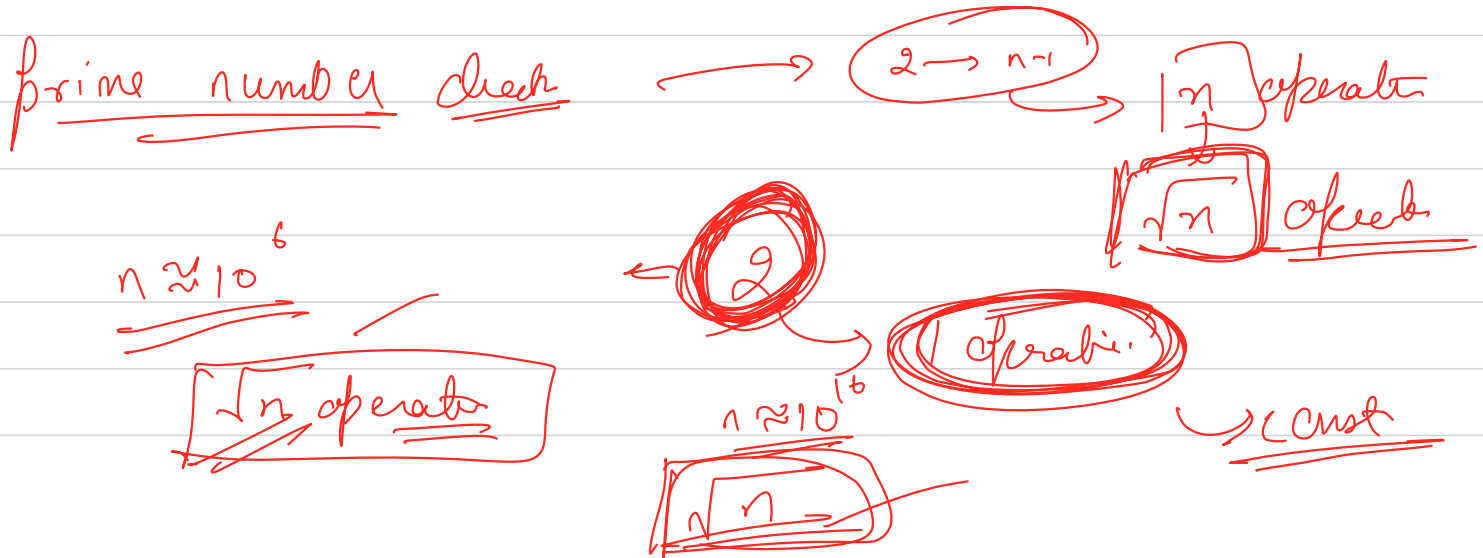
rate of growth is less



→ If we can approximately visualize the rate of growth in terms of the highest degree polynomial, we can compare algo.

→ asymptotic analysis

- ↳
- 1) Best Case \rightarrow Big omega notation $\sim \underline{\underline{\Omega(1)}}$
 - 2) Avg Case \rightarrow Big theta notation $\rightarrow \Theta(\sqrt{n})$
 - 3) Worst Case \rightarrow Big O notation $\rightarrow O(\sqrt{n})$
- ↓

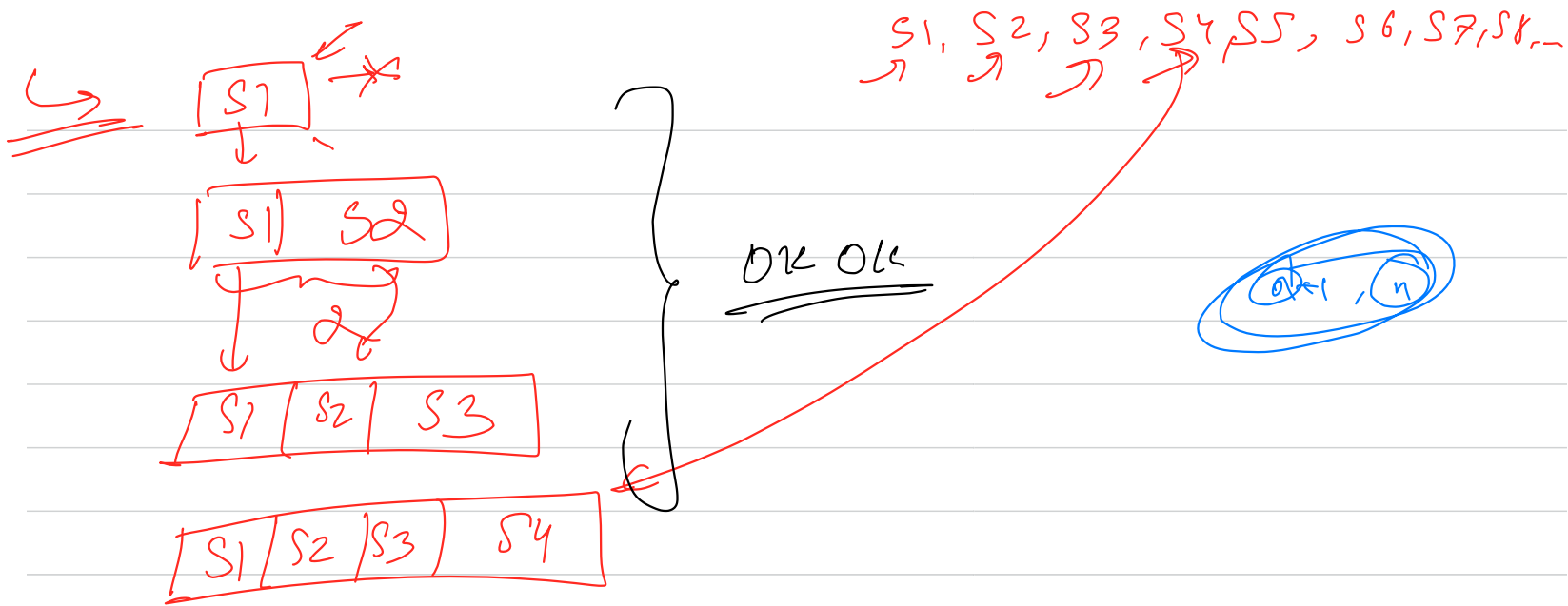


→ implement your own insertion algorithm that inserts at the last of a list, where you can only create list of constant size.

→ You can only create a list of const size. ^{→ can never grow}

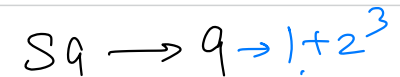
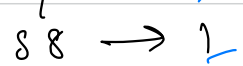
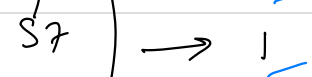
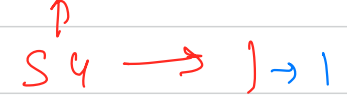
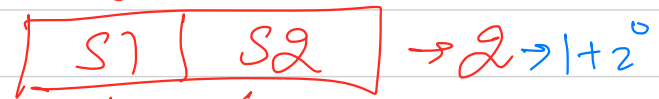
→ You cannot use append funcⁿ

→ Instead write your own funcⁿ called as add at last, that adds element at last.



If you have done n insertions till now,
 how many operations you will do for $(n+1)^{\text{th}}$
insertion

$(n+1)$



S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, ...

Best Case $\rightarrow \Omega(1)$, const

Worst Case $\rightarrow \underline{O(n)}$

Avg case \rightarrow

Total no. of operation
no. of insertion

$(1 + 2 + 3 + 1 + 5 + 1 + 1 + 9 + \dots)$

n

$1+2^k \geq \underline{\underline{\text{val}}}$ n ra mit

$$n \rightarrow 1 + 2^k$$

$$2^k \rightarrow n-1$$

$$k = \log_2(n-1)$$

$$k = \log_2 n$$

$$1 + (1+2^0) + (1+2^1) + 1 + (1+2^2) + 1+1+1 + (1+2^3) + \dots$$

$$\underbrace{(1+1+1+1+\dots)}_{n \text{ times}} + \underbrace{(2^0 + 2^1 + 2^2 + \dots + 2^k)}_{\text{CRP}}$$

$$\log_2 n = n$$

$$n + \frac{2^0 \times (2^{\log_2 n + 1} - 1)}{2 - 1}$$

$$k+1$$

$$2 \times 2^{\log_2 n}$$

$$\Rightarrow n + 1 \times (n-1)$$

$$\Rightarrow \underline{2n-1} \rightarrow \text{const} \rightarrow \underline{\underline{O(1)}}$$

$O \rightarrow$ w cost

$$\frac{2^{n-1}}{n} \rightarrow \underline{\underline{\text{cost}}}$$

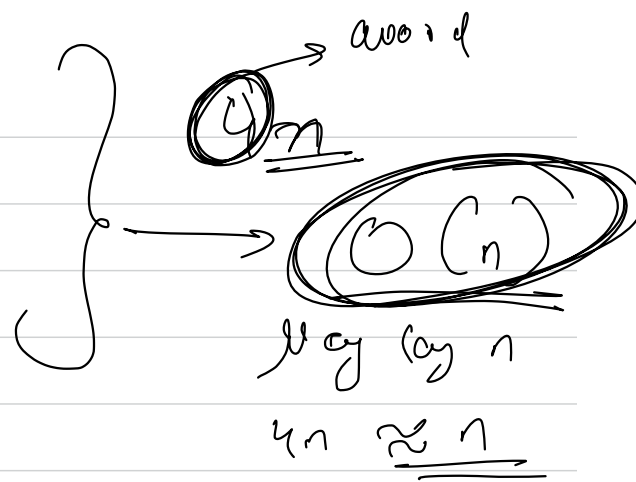
$\Theta \rightarrow$ avg

$\Omega \rightarrow \underline{\underline{\text{best}}}$

$i=0$
 while ($i \leq n$) :

$n+1 \approx n$

4 statements



no of operation \approx no of lines executed \times no. of lines
 they are exec

$i = 0$
while ($i \leq n$):

$\equiv \equiv \equiv$ 4 states $\rightarrow 4n$

$j = 0$
while ($j \leq m$):

$\equiv \equiv \equiv$ 4 states $\rightarrow 4m$

$4n + 4m$

$\approx \underline{\underline{O(n + m)}}$

nested
loop

time complexity

$i = 0$
while ($i \leq n$): $\leftarrow n$

$j = 0$
while ($j \leq n$):
print("Hello") $\leftarrow n$

$k = 0$
while ($k \leq m$):
print("World") $\leftarrow m$

$$O(n \times (n + m))$$

$$\Omega(n \times (n + m))$$

$$\Theta(n \times (n + m))$$

We are more concerned about any & waste case &

mainly waste case

if (_____) :

===== 4 statements

else if (_____) :

===== 3 statements

else :

=====

2 statements

↓
rate of growth

const
p

1000 statements?

O(1)

rate of growth

$$f(n) = 10000 \times n^0$$

10 const

$$\underline{\underline{O(n^2)}}$$

$$\underline{\underline{i=1}}$$

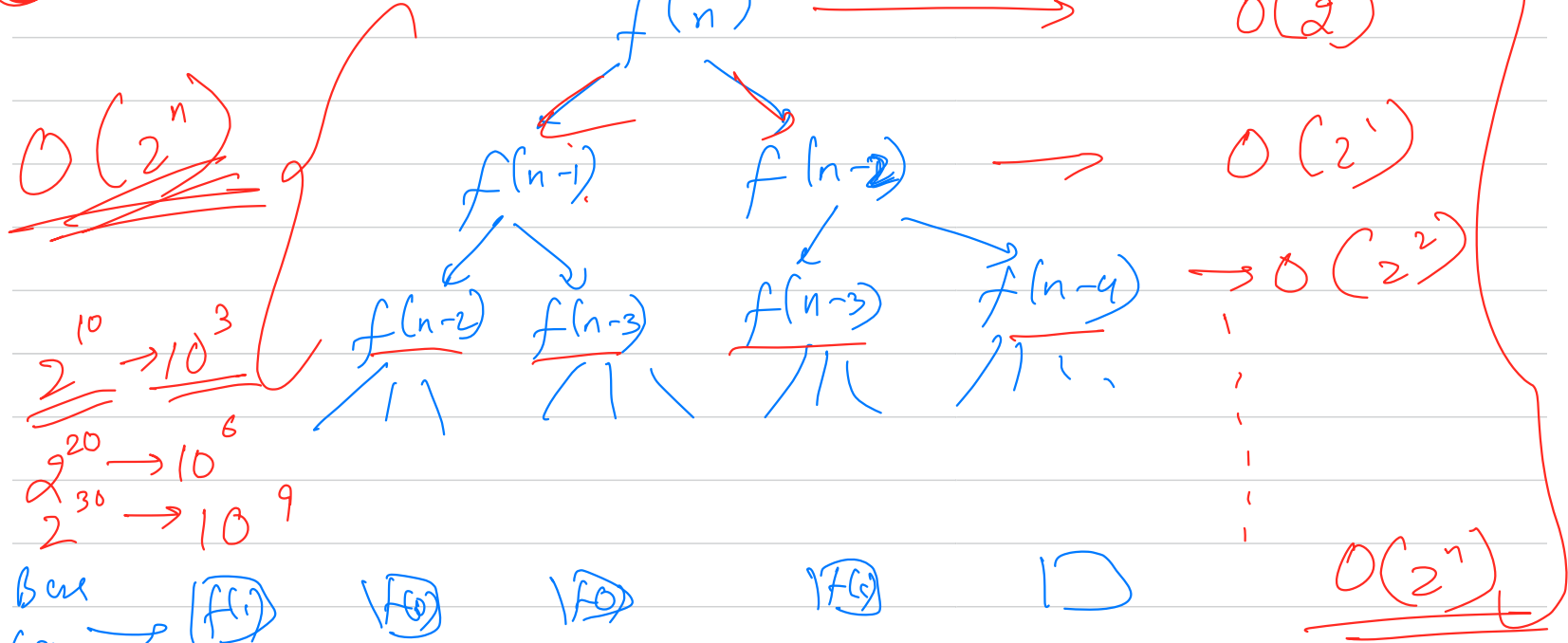
$$i=2$$

$$i=3$$

$$1 + 2 + 3 + \dots$$

2^n
 $2^{n-1} + 2^{n-2} + \dots + 1$

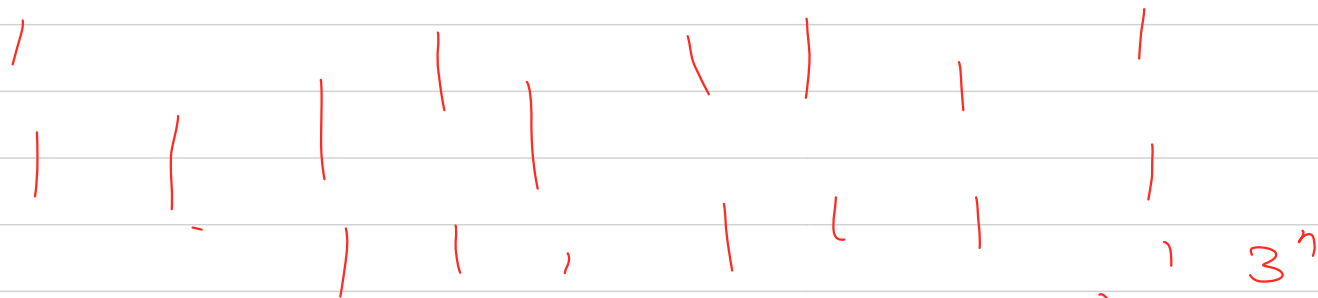
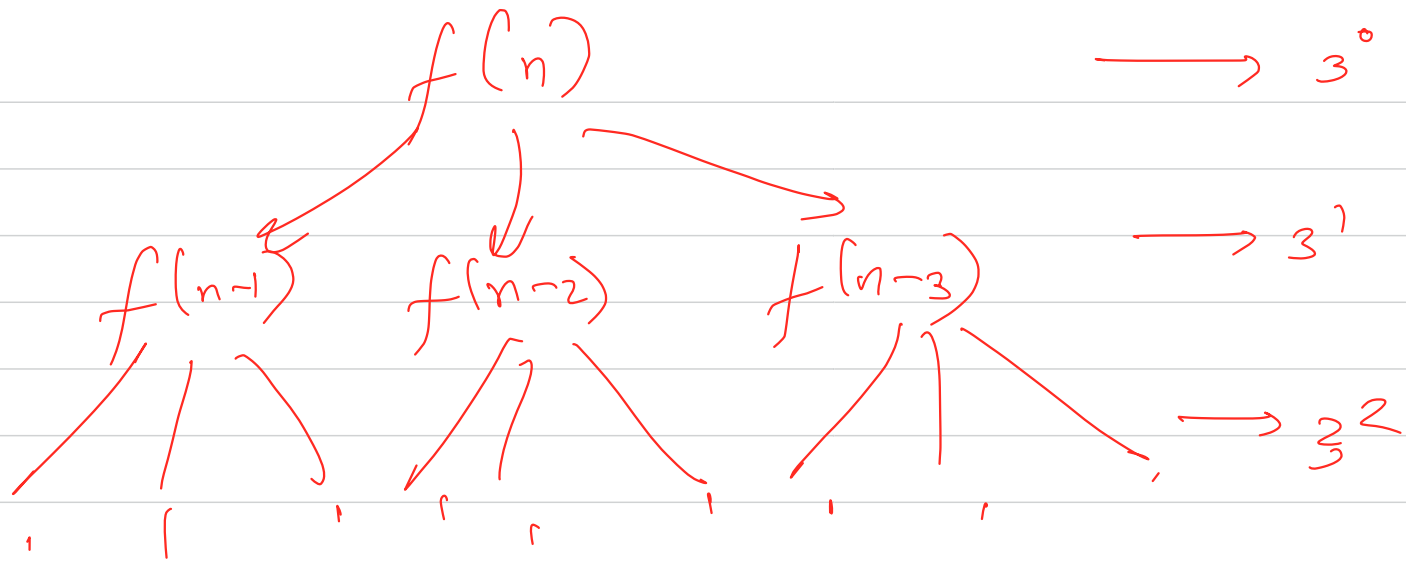
$$f(n) = f(n-1) + f(n-2)$$



$2^{10} \rightarrow 10^3$
 $2^{20} \rightarrow 10^6$
 $2^{30} \rightarrow 10^9$

Base case: $f(0)$, $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$

n is small



$\hookrightarrow \underline{\underline{O(3^n)}}$