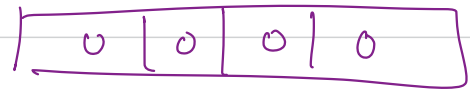
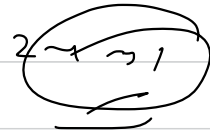
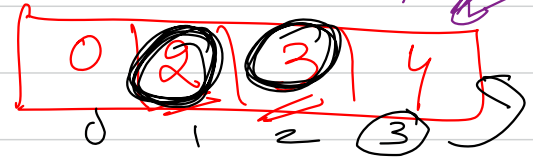


$$\text{freq}[\text{arr}[0]] + 1$$

$$\text{freq}[\text{arr}[1]] + 1$$

$$\text{freq}[\text{arr}[2]] + 1$$



$$\text{output}[\text{freq}[\text{arr}[3]] - 1] = \text{arr}[3]$$

$$\text{output}[\text{freq}[3] - 1] = \text{arr}[3]$$

$$\text{output}[4 - 1] = 3$$

```

1 def counting_sort(arr):
2     # in this implementation we will assume that we get only positive
3     # Space complexity O(n+k)
4     # Time complexity O(n + k)
5     max_element = max(arr)
6     frequency = [0]*(max_element+1)
7
8     # this loop will help to create the frequency list
9     for i in range(0, len(arr)):
10         frequency[arr[i]] += 1
11
12     # now we need to prepare prefix sum
13     for i in range(1, len(frequency)):
14         frequency[i] = frequency[i] + frequency[i-1]
15
16     # frequency[arr[i]] -> the position of the current value arr[i]
17
18     output = [0]*len(arr)
19     for i in range(len(arr)-1, -1, -1): # going from previous
20         output[frequency[arr[i]] - 1] = arr[i]
21         frequency[arr[i]] -= 1
22
23     return output
24

```

$-3, 1, 2, 1$
max element = 3

frequency = [0, 0, 2, 3]

$i \rightarrow [0, 3]$

$i = 0 \times 2 \times 3$

$i \rightarrow [1, 3]$

$i = 1 \times 2 \times 3$

output = [1, 1, 2, 3] final sorted list

$i = [3, 0]$

$i = 3 \times 2 \times 0$
→

disadvantage \rightarrow if range of elements is very high
then it performs poorly

Merge Sort

→ Comparison based sort

First of all forget about sorting.

Q[⇒] Given two sorted lists of size n and m ,
Merge these 2 sorted lists into one new sorted
list & return the final list:-

→ $[1, 5, 7, 9]$
 $[2, 3, 10]$

→ $n, m \leq 10^5$

ans → $[1, 2, 3, 5, 7, 9, 10]$ ← new list

reduced list

$$a \rightarrow [1, 3, 7, 9] \rightarrow n = 4$$

$$b \rightarrow [2, 5, 10] \rightarrow m = 3$$

reduced

output $[1, 2, 3, 5, 7, 9, 10]$
 $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$ $\xrightarrow{(n+m)}$

$$op[k] = \min(a[i], b[j])$$

$k \neq 0$ $i \neq 1$

$j \neq 1$

$O(n)$ $\rightarrow [1, 3, 7, 9]$

$[2, 3, 10]$

\uparrow
 \downarrow

O $[0, 0, 0, 0, 0, 0, 0]_{n+m}$

Merge Sort

5	4	1	7	6	3	0	2
---	---	---	---	---	---	---	---

→ unsorted list

recursively
sorted

5	4	1	7
---	---	---	---

1 4 5 7

6	3	0	2
---	---	---	---

0 2 3 6

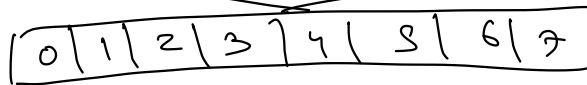
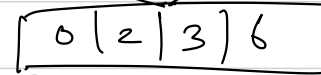
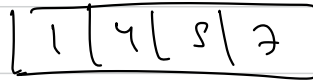
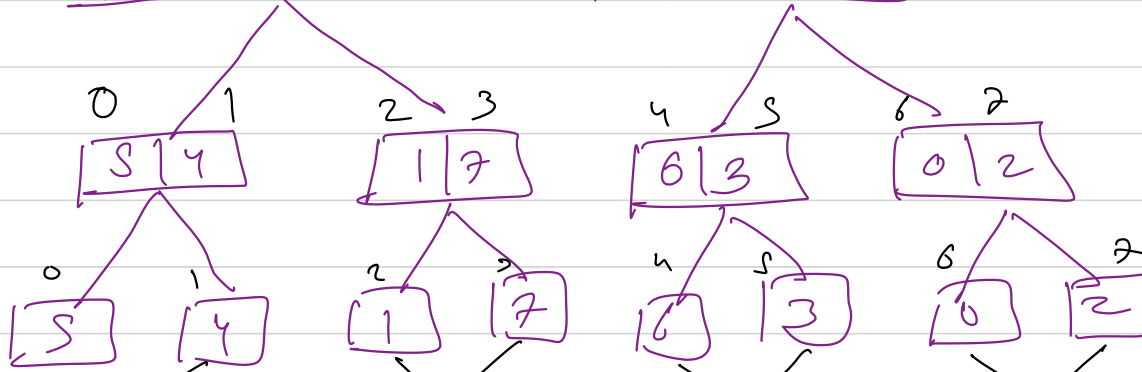
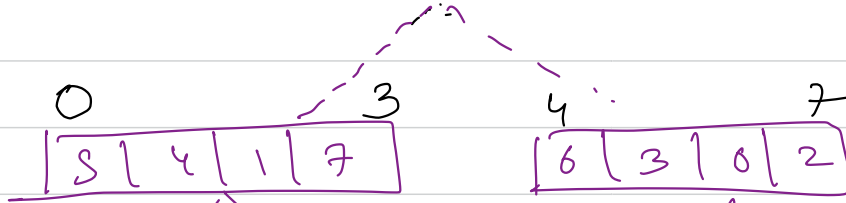
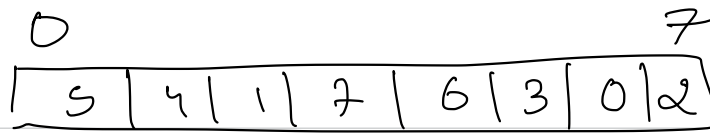
→ Divide Your List Into 2 equal halves

→ Sort the left half recursively.

→ Sort the right half recursively

→ selfwork → merge 2 sorted halves

Py run (game) / Reeb...
 garbage collector



→ final sorted list

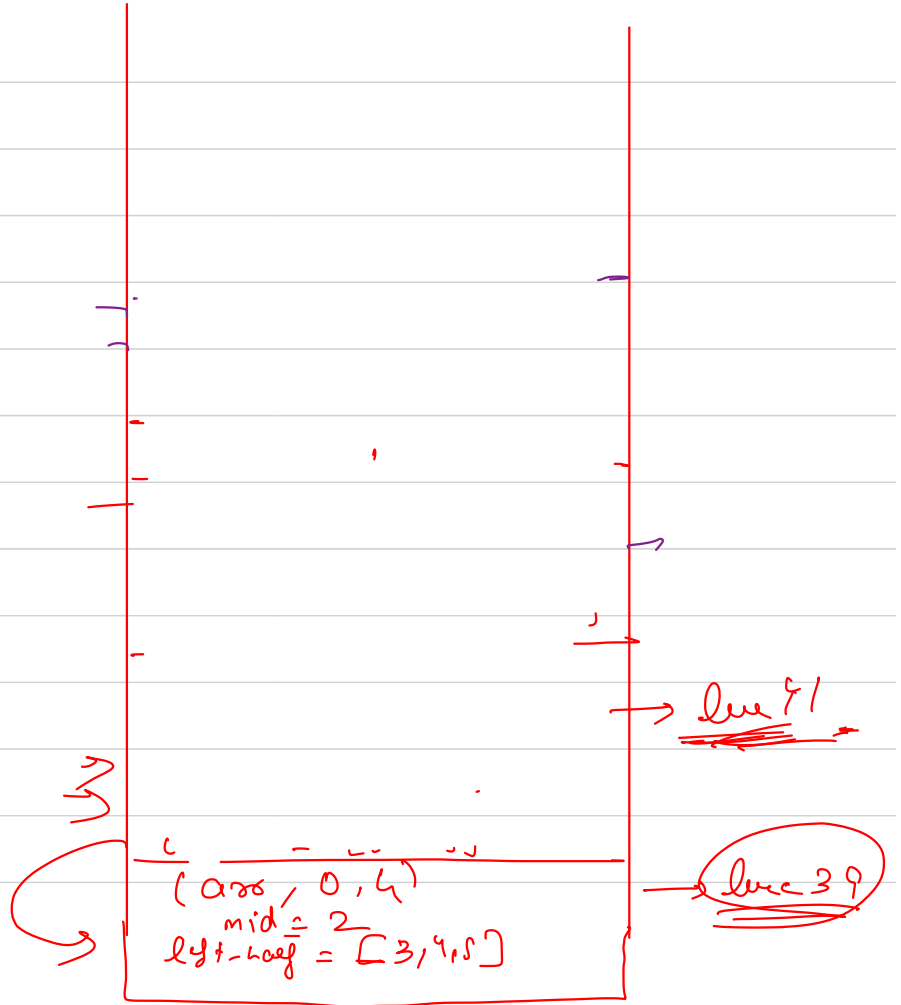
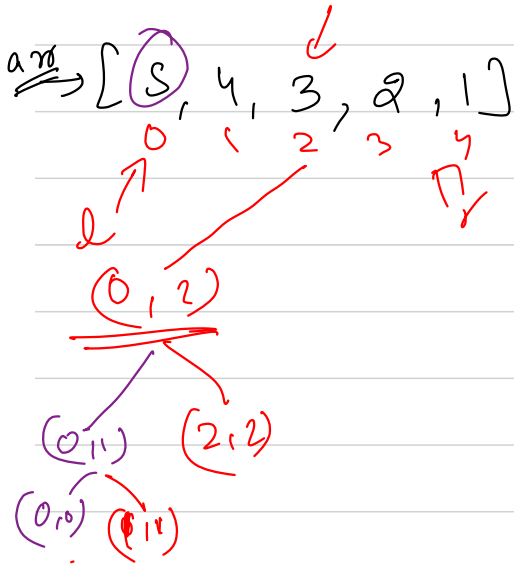
value size of list is 1

Base Case


```

33 def merge_sort(arr, left, right):
34     → if(left == right):
35         # base case
36         return [arr[left]] # or return [arr[right]]
37
38     mid = (left + right) // 2
39     → left_half = merge_sort(arr, left, mid) # recursive
40     right_half = merge_sort(arr, mid+1, right) # recursive
41     output = merge(left_half, right_half) # merged both
42     return output
43

```



$$T(n) = \overset{\text{left}}{T\left(\frac{n}{2}\right)} + \overset{\text{right}}{T\left(\frac{n}{2}\right)} + \overset{\text{merge}}{O(n)}$$

$T(n)$ is a function
 which calculates
 no. of operations
 to apply merge sort
 on a list of
Size n

no of operation
 reqd to sort
 a list of
 $n/2$ size

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

2°

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

→ recurrence relation

$$- 2 \times T\left(\frac{n}{2}\right) = 4T\left(\frac{n}{4}\right) + 2O\left(\frac{n}{2}\right)$$

$$- 4 \times T\left(\frac{n}{4}\right) = 8T\left(\frac{n}{8}\right) + 4O\left(\frac{n}{4}\right)$$

K times

$$2^K = 2^{\log n}$$

$$2^{K-1} T\left(\frac{n}{2^{K-1}}\right) = 2^K T(1) + 2O\left(\frac{n}{2}\right)$$

$$T(n) = O(n) + O(n) + O(n) \dots \dots O(n)$$

K times

$$T(n) = \boxed{K} \times O(n)$$

$$\boxed{K = \log_2 n}$$

$$\underline{\underline{T(n) = O(n \log_2 n)}}$$

$$n \rightarrow \frac{n}{2} \rightarrow \frac{n}{4} \dots \dots \frac{n}{2^k}$$

$$\frac{n}{2^k} = 1$$

$$K = \log_2 n$$

Space Complexity

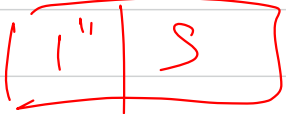
$$O(n + \log_2 n) \rightarrow \underline{O(n)}$$

Inplace \rightarrow No
 Swaps \rightarrow Not available

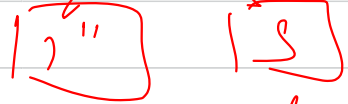
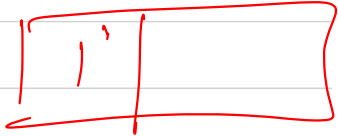
[1 | 1 | 2 | 2]

Stable \rightarrow Yes

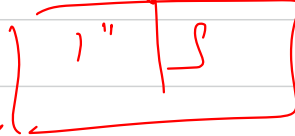
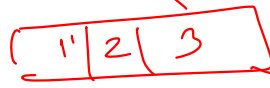
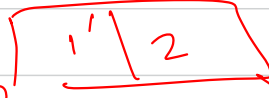
$i^0 = 6$



$i^1 = 2$



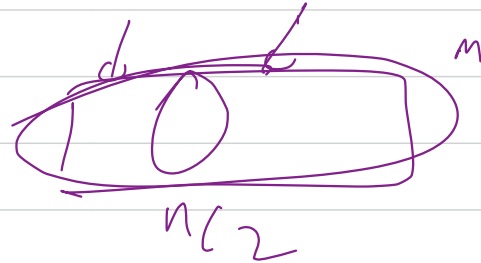
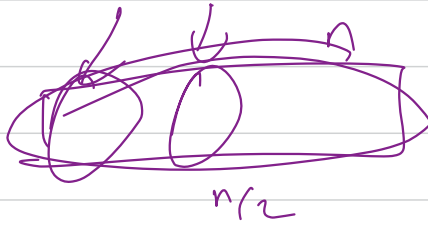
if $a[i] < b[j]$



else

swap $\leftrightarrow b[j]$

No. of Comparison of merge Sort



7 Gen



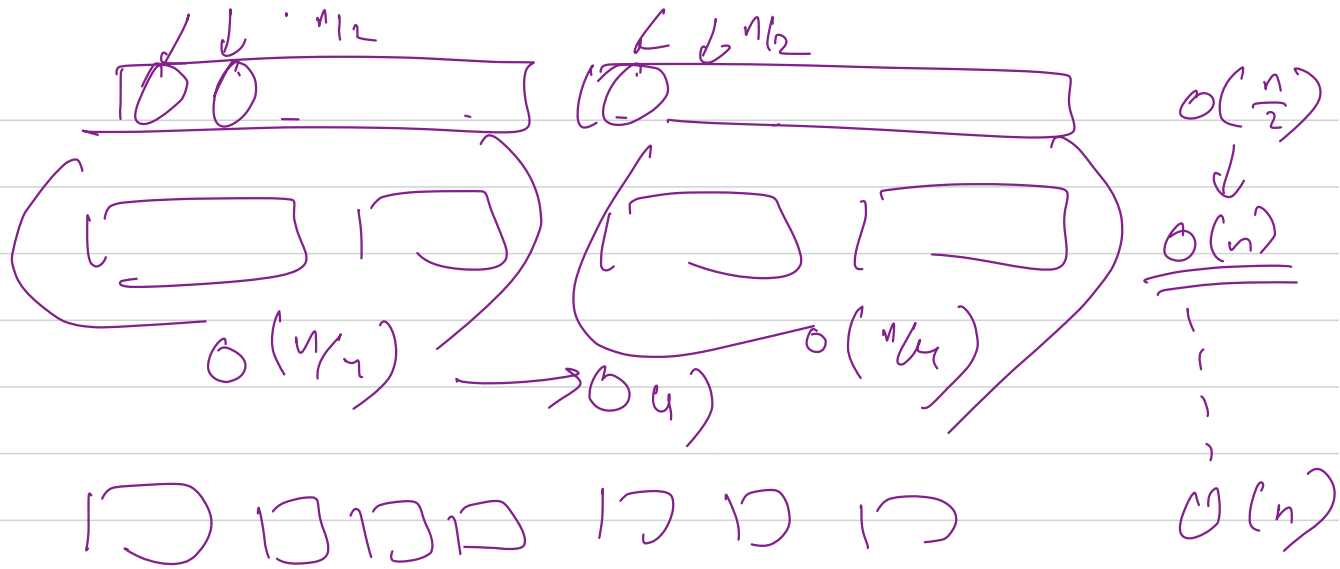
$n = m$

$$O(n/2) \rightarrow O(n)$$



↓

$$O(n \log n)$$



$$\underline{\underline{O(n \log n)}}$$