# AS-1

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## 1. Geometric Image Formation

(a) $P = (3, 2, 1)$
$f = 10$

$$x = \frac{fx}{x+z} \qquad y = \frac{fy}{y+z}$$

$$= \frac{10 \times 3}{10 + 1} \qquad = \frac{10 \times 2}{10 + 1}$$

$$= \frac{30}{11} \qquad = \frac{20}{11}$$

So, the coordinate of the point P when projecting it onto the image is $\left(\frac{30}{11}, \frac{20}{11}\right)$.

(b)
- The model which is behind the center of projection is better because it gives more accurate image.

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(c)

- When the focal length gets bigger projection gets bigger

- When the distance gets bigger projection gets smaller.

(d)

- 2D point (1, 1)
- its coordinates 2DH : (1, 1, 1)

- one 2DH point is (3, 3, 3).
- Anything we multiply we get another 2DH point that coresponds to a 2D point.

(e) 2DH point (1, 1, 2)

⇒ 2D point will be $\left(\frac{x}{2}, \frac{y}{2}\right)$

so 2D point is $\left(\frac{1}{2}, \frac{1}{2}\right)$

(f) 2DH point (1, 1, 0)
→ It is a point at infinity
- It is a vector and it represent direction.
- It could be difference between two 2DH point.

(g) $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

which is a linear equation :

$u = fx \qquad u = u/w = \frac{fx}{w}$

$v = fy$

$\qquad V = v/w = \frac{fy}{w}$

(h) ~~M =~~ [

$M = K [I|0]$

$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

dimension : $M = 3 \times 4$

$K = 3 \times 3$

$I = 3 \times 3$

$0 = 3 \times 1$

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(i) Projection matrix $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$

3D point $P = (1, 2, 3)$

$$\begin{bmatrix} a \\ b \\ w \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ w \end{bmatrix} = \begin{bmatrix} 1 + 4 + 9 + 4 \\ 5 + 12 + 21 + 8 \\ 1 + 4 + 3 + 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

So, 2D point is : $\left(\frac{18}{10}, \frac{46}{10}\right)$

$= (1.8, 4.6)$

## 2. Modeling transformations:

(a)

(1, 1) find it coordinates after translating it by (2, 3)

coordinates after translation

$= (1+2, 1+3)$
$= (3, 4)$

(b) (1, 1) scale by (2, 2).

coordinates after scaling $= (2\times1, 2\times1)$
$= (2, 2)$

(c) $\theta = 45'$
$(x, y) = (1, 1)$

- Suppose the coordinates after rotating is $(x', y')$

$x' = x\cos\theta - y\sin\theta$
$= \cos(45) - \sin(45)$
$= 0$

$y' = x\sin\theta + y\cos\theta$
$= \sin(45) + \cos(45)$
$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

So, coordinates after rotating is $(0, \sqrt{2})$

(d)

Coordinates $(x, y) = (1+2,\ 1+2)$
$= (3, 3)$

$$x' = \cancel{3}\, x\cos(45) - y\sin(45)$$
$$= 3\cos(45) - 3\sin(45)$$
$$= 0$$

$$y' = x\sin(45) + y\cos(45)$$
$$= 3\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right)$$
$$= 3\sqrt{2}$$

(e) suppose the object is $P$

- we want to rotate first and then translation

so. $P' = TRP$

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(f)

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

when we compare it with

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so, P will be scaled by (3, 2)

(g)

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

when we compare with $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

so P will be translate by (1, 2)

(h)

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Revers effect is

$$\begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i)

$$M = R(45)\,T(1,2)$$

$$M^{-1} = T^{-1}(1,2)\,R^{-1}(45)$$

$$M^{-1} = T(-1,-2)\,R^{T}(45)$$

(j) vector which is perpendicular to (1, 3)

$$(1,3).(x,y) = 0$$
$$x + 3y = 0$$

→ so one vector is $(-3, 1)$

(k) $a = (1, 3)$
$b = (2, 5)$

$$\text{Proj}_b\, a = \left(\frac{a.b}{|b|^2}\right) b$$

$$= \frac{(2+15)}{4+25}\,(2,5)$$

$$= \frac{17}{29}\,(2,5)$$

$$= \left(\frac{34}{29}, \frac{85}{29}\right)$$

3.

(b) Camera is rotated by R and translated by T

- P is a point in world coordinate system and $\hat{x}_c, \hat{y}_c, \hat{z}_c$ is unit vector in camera coordinate.

$$x' = (P-T) \cdot \hat{x}_c = \hat{x}_c^T (P-T)$$
$$y' = (P-T) \cdot \hat{y}_c = \hat{y}_c^T (P-T)$$
$$z = (P-T) \cdot \hat{z}_c = \hat{z}_c^T (P-T)$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \hat{x}_c^T \\ \hat{y}_c^T \\ \hat{z}_c^T \end{bmatrix} (P-T)$$

$$= R^T (P-T)$$

(c) $\hat{x}, \hat{y}, \hat{z}$ unit vectors

Rotation matrix = $\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$

(a)

General Projection matrix is used to relate 3D point in world coordinate system to 2D point in image coordinate.

**(d)** Transformation matrix between world and camera coordinates

$$M = \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix}$$

In this $R^*$ is a rotation of world with respect to camera and $T^*$ is translation with respect to camera.

**(e)**

$$M_{i \leftarrow c} = \begin{bmatrix} K_y & 0 & y_0 \\ 0 & K_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{i \leftarrow c} = \begin{bmatrix} K_y & 0 & 512 \\ 0 & K_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$$

**(f)**

~~M=E~~

$$M = K^* \left[ R^* \mid T^* \right]$$

→ $R^*$ and $T^*$ is rotation and translation of world with respect to camera.

→ $K^k$ is intrinsic parameter.

- This parameters can not change when you change the position of camera.

→ $R^k | T^k$ is a extrinsic parameter.

- This parameters change when you change the position of camera.

(9)

2D skew parameter.

→ It increases the accuracy of model.

- If you want youre model to be accurate then 2D skew parameter is necessary and sometime in some model we can ignore it.

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(h)

→ It complicates the projection.

→ $\lambda$ depends on the distance from the center.
- center is zero so no scale.
when you move from center it makes projection of image more difficult.

(i)

Weak perspective camera.

→ Perspective means distance object looks smaller.

→ A weak perspective is correct when depth variation is smaller compared to distance from camera.
- In weak perspective images look as it is because there is not much perspective.

→ Affine camera is used to make the projection model more easy.

4.

(a)

difference between surface radiance and image irradiance

- Surface radiance is a light that is reflected on surface and image irradiance is a light ~~by~~ that is received at the image.

(b)

$$E(P) = L(P) \frac{\pi}{4} \left(\frac{d}{f}\right)^2 (\cos\alpha)^4$$

where

$E(P)$ is light at image means image irradiance

$L(P)$ is light at surface means surface radiance

$d$ is diameter

$f$ is focal length

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(c) albedo of surface

- It's a reflection coefficient
- It says how good the surface reflection is
- It is between 0 and 1
- If it is 0 then surface did not reflect light and if it 1 then surface reflect excellent light.

(d)
RGB color model is used because our human eye work on this model. It detects the light and send it to brain.

(e) In RGB color cube the line that connect (0, 0, 0) and (1, 1, 1) contains all the grayscale colors.

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(f)

We can map the RGB color to real-world colors by experiments.

- We change the different value of R,G,B and compare it to real-world color and select the value.

(g)

Y: luminance is used to convert the colord image to black & white (grayscale) image.

(h) LAB color space

- It is close resemblance to human perception. It has wide color gamut. It treats Black and white as their own opponent channel.

- We can boost the color in LAB color space.