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[1]

a) outliers are points that are different from all other points  
- outliers do not fit the right model that is why they give wrong solution.

(b) objective function used for robust estimation is:

$$E(\theta) = \sum_{i=1}^n \rho_{\sigma}(d(x_i; \theta))$$

→ The standard least error equation is more sensitive to outliers

(c) German-MacLure function for robust estimation:

$$\rho_{\sigma}(x) = \frac{x^2}{x^2 + \sigma^2}$$

- Its advantage is that it lower the influence of outliers.

- If  $\sigma$  is bigger the open range will bigger, if  $\sigma$  is smaller, the model become more selective of outliers.

(d) The main principle of RANSAC algorithm is to find use minimum number of points to fit the model and try this process several times and choose the best model after many trials. Number of points drawn at each attempt should be small because there are less chance of getting outlier and get a better model.



(e) The parameters for RANSAC algorithm

$n$  = number of points drawn at each evaluation

$d$  = minimum number of points needed to estimate model

$K$  = number of trials

$t$  = distance to determine inliers

=> Formula for estimating the number of trials  $K$ :

$$K = \frac{\log(1-P)}{\log(1-w^n)}$$

$P$  = Probability that atleast one of the trials will succeed

$w$  = probability that a point is inlier

$n$  = number of points drawn at each trial.

(f) Objective of image segmentation is to separate foreground from the background.

Merge approach: start with each pixel in separate cluster iteratively merge cluster.

Split approach: start with all pixels in one cluster iteratively split cluster.

(g) K means -

- select  $K$

- select initial guess of  $K$ -means:  $m_1, \dots$

- Assign  $l_i = \arg \min_{j \in [1, K]} \|x_i - m_j\|^2$  for each

pixels & assign correspondent cluster.

- recompute mean:  $m_j = \frac{\sum_{i \in S_j} x_i}{\# S_j}$

- stop when  $m_j$  doesn't change.

Mixture of Gaussian segmentation :

- It is like k-means with additional parameters.
- Replacing  $d = \|f_i - m_j\|^2$  with  $d = (f_i - m_j)^T$
- Replacing  $d = \|f_i - m_j\|^2$  with  $d = (f_i - m_j)^T \Sigma_j^{-1} (f_i - m_j)$

$$m_j = \frac{\sum_{i \in S_j} f_i}{\# S_j}$$

$$\Sigma_j = \frac{\sum (f_i - m_j)(f_i - m_j)^T}{\# S_j}$$

$$d = (f_i - m_j)^T \Sigma_j^{-1} (f_i - m_j)$$

distance                      mean of cluster                      covariance of cluster

b) mean shift is similar to k-means.

- Instead of  $m_j$  in k-means use weighted from sample to the mean.

$$m_j = \frac{\sum_{i \in S_j} w(f_i - m_j) f_i}{\sum_{i \in S_j} w(f_i - m_j)}$$

- The closest a sample is to the mean, more weighted or effect to the mean.



## (2) Camera Calibration:

(a) Forward projection: given a 3D world point  $P$  project into the image using the projection matrix  $M$

\* Camera Calibration: given the image coordinate and the world coordinate of the object, find the camera parameters (internal & external) used in the projection.

\* Reconstruction: given image point  $P$ , compute world point  $P$ .

→ Forward projection is easiest and Reconstruction is most difficult.

(b) For camera calibration we need a set of 3D world points and its 2D corresponding points.

(c) Step 1: estimate projection matrix  $M$   
Step 2: find parameter  $(K^*, R^*, T^*)$  given matrix  $M$ .

(d)

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad P_i = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$
$$P_i = M P_i = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ 14 \\ 7 \end{bmatrix}$$

2DH

In 2D  $P_i = \begin{bmatrix} 18/7 \\ 2 \end{bmatrix}$

(f) we need at least 6 points to get matrix  $M$  because we need to solve 11 unknown in order to solve it we need 6 points to get 12 equations.

(e) World point = (1, 2, 3)  
image point = (100, 200)

so, for matrix:

$$\Rightarrow \begin{bmatrix} p_1^T & 0^T & -x_1 p_1^T \\ 0^T & p_1^T & -y_1 p_1^T \end{bmatrix} \begin{bmatrix} [M_1] \\ [M_2] \\ [M_3] \end{bmatrix} = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & -100 & -200 & -300 & -100 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 1 & -200 & -400 & -600 & -200 \end{bmatrix}$$

(g) The principle that is used to extract the unknown parameters from the projection matrix  $M$  is:

$$m = R^* [K^* | T^*]$$

- We use the fact that the rotation matrix has the orthogonal vectors along the rows.
- we take dot product and cross product of the rows in 'M' and this will cancel out some unknown parameters.



(h) We need to compute the error

$$E(K^*, R^*, T^*) = \sum_{i=1}^n \left( x_i - \frac{M_1^T P_i}{M_3^T P_i} \right) + \left( y_i - \frac{M_2^T P_i}{M_3^T P_i} \right)$$

(i) Planar Calibration Steps:

- 1) Estimate 2D homography between calibration plane and image
  - 2) Estimate intrinsic parameters.
  - 3) Compute extrinsic parameters for view of interest.
- Planar solve 2DH points. Non-planar solve 3DH points.

$$(j) \quad \overset{2DH}{P_i} = K^* [R^* | T^*] \overset{3DH}{P_i}$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = K^* \underbrace{\begin{bmatrix} r_1 & r_2 & r^* \end{bmatrix}}_{K^* \text{ Homography}} \overset{2DH:}{\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}}$$

$$P_i = \begin{bmatrix} x_i \\ y_i \\ 0 \\ 1 \end{bmatrix}$$

assume z coordinate is zero