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A. $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

(1) $2A - B$

$$= 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

(2) $||A|| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$$= \sqrt{1+4+9}$$

$$= \cancel{2\sqrt{3}} \quad \sqrt{14}$$

$\Rightarrow$ Angle to positive x-axis

$$\cos\alpha = \cancel{\frac{1}{2\sqrt{3}}} \quad \frac{1}{\sqrt{14}}$$

so, $\alpha \simeq 75^\circ$

(3) unit vector in direction of A

$\|A\| = \sqrt{14}$

So,

unit vector of $A = \dfrac{A}{\|A\|}$

$= \left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} \right)$

(4) the direction of cosines of A

$\cos \alpha = \dfrac{A_x}{\|A\|} = \dfrac{1}{\sqrt{14}}$

$\cos \beta = \dfrac{A_y}{\|A\|} = \dfrac{2}{\sqrt{14}}$

$\cos \gamma = \dfrac{A_z}{\|A\|} = \dfrac{3}{\sqrt{14}}$

(5)

$A . B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} =$

$= 4 + 10 + 18 = 32$

$B . A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 4 + 10 + 18 = 32$

So, $A . B = B . A$

(6)

$$\cos\theta = \frac{A \cdot B}{\|A\|\,\|B\|}$$

$$= \frac{32}{(\sqrt{14})(\sqrt{77})}$$

$$= \frac{32}{32.83}$$

$$= 0.974$$

$$\theta = \cos^{-1}(0.974)$$

$$\theta \simeq 13^\circ$$

(7) A vector which is perpendicular to A

→ Suppose X is a vector which is perpendicular to A

~~x = ([x y z]~~

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

so $A \cdot X = 0$

$$x + 2y + 3z = 0$$

$$\cancel{x}\,1 + 2(-2) + 3(1) = 0$$

so

$$X = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

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(8) $A \times B$ and $B \times A$

$$A \times B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6\times 2 - 5\times 3 \\ 3\times 4 - 6\times 1 \\ 5\times 1 - 4\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times 3 - 6\times 2 \\ 6\times 1 - 4\times 3 \\ 4\times 2 - 5\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

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(9) Suppose a vector which is perpendicular to both A and B is x.

So, $A \times B$ is a vector which is perpendicular to A and B

$$x = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= -3\hat{i} + 6\hat{j} - 3\hat{k}$$

$$= (-3, 6, -3)$$

→ And we can see that $A.x$ and $B.x$ is zero so $x = (-3, 6, -3)$ is perpendicular to both A and B

(10) the linear dependency between A, B, C

$\Rightarrow$ To find a dependency we take constant x, y, z such that

$$xA + yB + zC = 0$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 3 & 0 \end{array}\right]$$

$$\frac{R_3}{3} \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right]$$

$$R_3 - R_1 \Rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$$\frac{R_3}{2} \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 2 & 5 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$C_3 + C_1 \to C_3$$

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$$\left[\begin{array}{ccc|c} 1 & \cancel{6}3 & -1 & 0 \\ 2 & 6 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 - R_3 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x + 3y = 0$
$2x + 6y = 0$
$z = 0$

So, $\cancel{x}$ $y = \frac{-x}{3}$

$0 = xA + yB + zC$
$0 = xA - \frac{x}{3}B$

$0 = 3xA - xB$
$3A = B$

This is not true so A, B, C are not depedent

(iii) $A^T B$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= 4 + 10 + 18$$

$$= 32$$

~~* AB^T~~

$$\cancel{B^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}}$$

$$\cancel{AB^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}}$$

$$\cancel{= 4 + 10 + 18 = 32}$$

$$AB^T = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{3\times 1} \underbrace{\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}}_{1\times 3}$$

$$= \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

[B]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

(1) $2A - B$

$$= 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

(2) $A \cdot B$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+2-6 & 1+-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 10-3 & 5+2 & -20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

*

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 \\ \\ \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4 & 4-2-20 & 6+3+4 \\ 3-8 & 6+4+5 & 9-6-1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

(3)

$(AB)^T$

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

* $B^T A^T$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1+4+9 & 4-4+9 & 10-3 \\ 2+2-6 & 8-2-6 & 5+2 \\ 1-8+3 & 4+8+3 & -20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

(4) $|A|$ and $|C|$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} - 2\begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} + 3\begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix}$$

$= -13 + 8 + 60$

$= 55$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 5 & 6 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ -1 & 3 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ -1 & 1 \end{vmatrix}$$

$= 9 - 36 + 27$

$= 0$

5 orthogonal set

For A

$\Rightarrow (1, 2, 3) \cdot (4, -2, 3)$

$= 4 - 4 + 9$

$= 9$

Dot product is not zero so A is not orthogonal set.

For B

$\Rightarrow (1, 2, 1)\ (2, 1, -4)$

$= 2 + 2 - 4 = 0$

$\Rightarrow (2, 1, -4)\ (3, -2, 1)$

$= 6 - 2 - 4 = 0$

$\Rightarrow (3, -2, 1)\ (1, 2, 1)$

$= 3 - 4 + 1$

$= 0$

so, B has row vectors which form an orthogonal set.

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(6) A^{-1}

$|A| \neq 0$ so A has a inverse

$$A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$\text{Adj}(A) = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$= \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 20/55 & -5/55 & -10/55 \end{bmatrix}$$

$$= \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 4/11 & -1/11 & -2/55 \end{bmatrix}$$

* B^{-1}.

$$\cancel{Adj(B)}, = B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 11 \end{bmatrix}$$

$$|B| = -7 - 28 - 7 = -42$$

$$Adj(B) = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times Adj(B)$$

$$= \begin{bmatrix} 7/42 & 4/42 & 9/42 \\ 14/42 & 2/42 & -6/42 \\ 7/42 & -8/42 & +3/42 \end{bmatrix}$$

$$= \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 1/3 & 1/21 & -1/7 \\ 1/6 & -4/21 & +1/14 \end{bmatrix}$$

[C]

(1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

→ The values of λ for which determint of $(A - \lambda I)$ is zero is the eigenvalues.

$$|A - \lambda I| = \left| \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$\therefore (1-\lambda)(2-\lambda) - 6 = 0$

$\therefore (1-\lambda)(2-\lambda) = 6$

$\therefore \lambda^2 - 4\lambda + \lambda - 4 = 0$

$\therefore (\lambda - 4)(\lambda + 1) = 0$

$\lambda = 4$ or $\lambda = -1$

For $\lambda = 4$

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}$$

$$V_4 = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \left[\begin{array}{cc|c} -3 & 2 & 0 \\ 3 & -2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} -3 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$-3v_1 + 2v_2 = 0$$

$$2v_2 = 3v_1$$

$$v_2 = \frac{3}{2}v_1 \qquad \therefore v_1 = \frac{2}{3}v_2$$

~~for~~ $\cancel{v_1 = 1}$ $\cancel{v_2 = \frac{3}{2}}$ $\quad v_4 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

so, for $\lambda = 4$

$$\cancel{v_4 = \begin{bmatrix} 1 \\ 3/2 \end{bmatrix}} \qquad v_4 = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

for $\lambda = -1$

$$v_{(-1)} = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 3 & 3 & 0 \end{array}\right]$$

so, $v_1 + v_2 = 0$

$\cancel{v_2 = -v_1}$ $\quad v_1 = -v_2$

so, the eigenvector is $\begin{bmatrix} -1 \\ +1 \end{bmatrix}$

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2 $V^{-1}AV$

$$V = \begin{bmatrix} 2/3 & -1 \\ 1 & 1 \end{bmatrix} \cancel{\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}}$$

$$|V| = \frac{2}{3} + 1 = +\frac{5}{3}$$

$$adj(V) = \begin{bmatrix} 1 & 1 \\ -1 & 2/3 \end{bmatrix}$$

$$\cancel{V^{-1} = \frac{1 \times 3}{-1} \begin{bmatrix} 1 & 1 \\ -1 & 2/3 \end{bmatrix}}$$

$$\cancel{= \begin{bmatrix} -3 & -3 \\ 3 & -2 \end{bmatrix}}$$

$$\cancel{V^{-1}AV = \begin{bmatrix} -3 & -3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2/3 & -1 \\ 1 & 1 \end{bmatrix}}$$

$$\cancel{= \begin{bmatrix} 6-12 & -12 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 2/3 & -1 \\ 1 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} -20 & 0 \\ 0 & 5 \end{bmatrix}$$

$$V^{-1} = \frac{5}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 & 5/3 \\ -5/3 & 10/9 \end{bmatrix}$$

$$V^{-1}AV = \begin{bmatrix} 5/3 & 5/3 \\ -5/3 & 10/9 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2/3 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20/3 & 20/3 \\ 5/3 & 10/9 \end{bmatrix}\begin{bmatrix} 2/3 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 100/9 & 0 \\ 20/9 & -5/9 \end{bmatrix}$$

(3) dot product between eigenvectors of A

$$= \begin{bmatrix} \cancel{1} \\ \cancel{3/2} \end{bmatrix} \cdot \begin{bmatrix} \cancel{1} \\ \cancel{-1} \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \cancel{1 - \frac{3}{2}} = \cancel{\frac{-1}{2}} = \frac{-2}{3} + 1 = \frac{1}{3}$$

(4) dot product between eigenvectors of B

$$|B - \lambda I| = \left| \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix} \right| = 0$$

$\therefore (2-\lambda)(5-\lambda) - 4 = 0$

$\therefore \lambda^2 - 7\lambda + 6 = 0$

$\therefore (\lambda - 6)(\lambda - 1) = 0$

$\lambda = 6$ and $\lambda = 1$

for $\lambda = 6$

$$v_{(6)} = \begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \left[\begin{matrix} -4 & -2 \\ -2 & -1 \end{matrix} \right| \left. \begin{matrix} 0 \\ 0 \end{matrix} \right]$$

$\therefore -2v_1 - v_2 = 0$

$\therefore \cancel{v_2 = -2v_1}$ $\quad \therefore v_1 = -\dfrac{v_2}{2}$

$\cancel{\text{for } v_1 = 1}$

$\cancel{v_2 = -2}$

for $\lambda = 1$

$v_{(1)} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore \left[\begin{array}{cc|c} 1 & -2 & 0 \\ -2 & 4 & 0 \end{array}\right]$

$= \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$v_1 - 2v_2 = 0$

$v_1 = 2v_2$

$\cancel{v_2 = \frac{v_1}{2}}$

$\cancel{\text{for } v_1 = 1}$

$\cancel{v_2 = 1/2}$

~~dot product~~

$\cancel{\begin{bmatrix} 1 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}}$ $\qquad \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\cancel{= 1 - 1}$ $\qquad = -1 - 1$

$= 0$ $\qquad = -2$

5 Properties of the eigenvectors of B

$\Rightarrow$ $B = X \lambda X^{-1}$. So the matrix B is diagonalizable

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[D]

$f(x) = x^2 + 3$, $g(x,y) = x^2 + y^2$

(1) first and second derivatives of $f(x)$ with respect to x

$f(x) = x^2 + 3$

$f'(x) = 2x$

$f''(x) = 2$

(2) Partial derivatives

$\Rightarrow \frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2)$

$\frac{\partial g}{\partial x} = 2x$

$\Rightarrow \frac{\partial g}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2)$

$\frac{\partial g}{\partial y} = 2y$

(3) gradiant vector $\nabla g(x,y)$

$\nabla g(x,y) = \begin{bmatrix} \frac{\partial}{\partial x}(x^2+y^2) \\ \frac{\partial}{\partial y}(x^2+y^2) \end{bmatrix}$

$= \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

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4 Univariate Gaussian distribution

$$p(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)}$$

Probability density function of univariate Gaussian distribution is

$$= \frac{1}{\sqrt{2\pi}} \quad \text{where } \mu=0,\ x=0,\ \sigma=1$$