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1. Corner Detection

(a) Basic Principle of corner detection

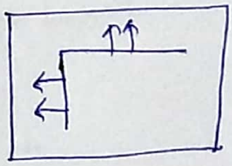
- For a image we define window having neighbour hood point, where corner represent interest point, so goal is to identify those points.

→ The steps to detect corner in local window are

(i) Find correlation matrix in window.

(ii) compute eigenvalues of the matrix

(iii) check if $\lambda_1, \lambda_2 > \tau$



→ The gradient in that window is seen, if there are more than one directions it is said to have a corner.

(b) To find principle direction of gradient orientation in local path.

$$\sum_{i=0}^{n} P_i P^T$$ → as corelation matrix comprising of pi = points in neighborhood.

→ so, we find direction of minimum projection A direction subject to be perpendicular to all previous directions.

→ Direction is eigen vectors of correlation matrix

and projection are proposition to eigen values.

(c) gradient vectors:

$= \{ (0,0), (0,1), (0,2), (0,3), (0,4)$
$(1,0), (1,1), (1,2), (1,3) \}$

→ correlation matrix $= \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i \\ \Sigma y_i x_i & \Sigma y_i^2 \end{bmatrix}$

$$= \begin{bmatrix} 0+0+0+0+0+1+1+1+1 & 0+0+0+0+0+0+1+2+3 \\ 0+0+0+0+0+0+1+2+3 & 0+1+4+9+16+0+1+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 6 & 44 \end{bmatrix}$$

(d) The eigenvalues of the gradient correlation matrix $\lambda_1$ and $\lambda_2$ should be greater than the threshold $\tau$.

$$\lambda_1, \lambda_2 > \tau$$

→ So, we can detect a corner in that neighbourhood if both $\lambda_1$ and $\lambda_2$ are large enough.

(e) Non maximum suppression helps in finding a unique corner for a location when we are using multiple ~~set~~ windows.

→ For point in image multiple windows will find multiple corners.

→ Steps for calculating non-maximum suppression:

(1) compute $\lambda_1, \lambda_2$ for all windows.

(2) select windows with $\lambda_1, \lambda_2 > \tau$ and sort it in decreasing order.

(3) Select the top of the list as corner and delete all other corners in its neighborhood from list.

(4) Stop when detecting x% of the points as corners.

(f) Harris Corner Detection.

$$c(G) = \det(G) - k\, tr^2(G)$$

where, G = gradient correlation matrix
tr(G) = trace of G

→ So, we don't consider eigen values of the gradient corelation matrix directly insted we find determinent and trace.

(g) To determine p we project gradient into edge and choose p minimum projector.

→ Localization of point p

$$p = c^{-1} V$$
$$= c^{-1} \sum_i \Delta \nabla I(P_i) \cdot \nabla I(P_i)^T \cdot P_i$$

where $C = \sum \nabla I(P_i) \cdot \nabla I(P_i)^T$ is a corelation matrix

⇒ condition for solution to exist =
$\lambda_1, \lambda_2 > \tau$ so that C is a non-singular and we can get invers of that.

(h) Feauter points characterization using HOG

(1) Take a window
(2) split each patch into cells which can be overlapping
(3) Compute a histogram of gradient direction for each pixel in each window
(4) Concatenate histograms and we get feature vector.

⇒ For good characterization of feature points
(1) Translation invarient
(2) rotation invarient
(3) scale invarient
(4) illumination invarient.

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(i) SIFT:-

(a) take a large window

(b) split into blocks

(c) compute gradient vector in each block

(d) combine all the gradient vector, into orientation histogram over smaller subregion.

[2] Line Detection

[a] The problem of using the slope and y-intercept of 'a' (slope) because
- The possible value infinite value possible of slop
- How to represent a line. To represent a line you need to take infinite slope.

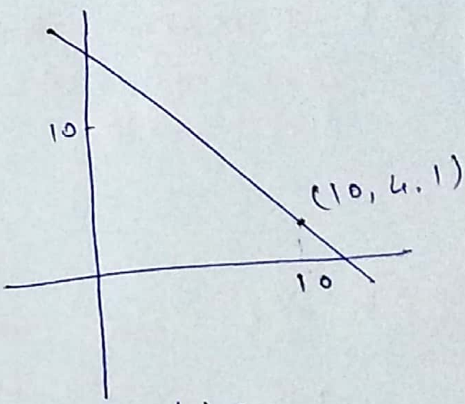
[b] $\theta = 45$, $d = 10$

Equation:

$$x \cos\theta + y \sin\theta - d = 0$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} - 10 = 0$$

$$x + y - 10\sqrt{2} = 0$$



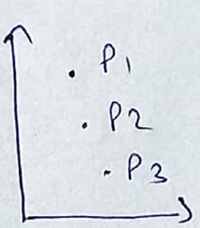

⇒ so detected point is (10, 4.1)

In equation

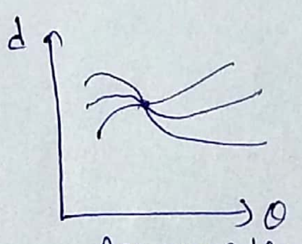
⇒ $10 + 4.1 - 10\sqrt{2} \approx 0$

→ so this satisfy the explicit line equation.

[c] When using the polar representation of lines, the vote of each point in the image looks like sinusoidal curve in the parameter plane.




image




parameter space

(d) Parameter plane will give 2 valve
d and $\theta$

→ d is distance from origin
→ $\theta$ is $\theta$ for normal if line in image plane

So, the equation is

$$x\cos\theta + y\sin\theta - d = 0$$

(e) Larger the bin size more efficient but provide less localization means it is less accurate.

(f) If the normal at each point is known we can find ∇I at voting point and compute $\theta$

So the range will become $(\theta - \Delta \ldots \theta + \Delta)$

→ This is now more accurate.

(g) When using Hough transform for circles, the number of dimensions of the parameter space is 3

## [3] Model fitting

(a) If we use $y = ax + b$ model.

distance of neighbouring point to the line will be increased that will result in non-accurate fitting.

→ Using this equation we have poor fitting near vertical line.

(b) normal (1, 2)
distance $d = 2$

so equation is

$x + 2y - 2 = 0$ where $a = 1$, $b = 2$, $c = -2$

So, $l = [a, b, c]$
$= [1, 2, -2]$

(c) Explicit line equation to minimize geometric distance.

$l^T x = 0$ where all points '$x$' should on the line '$l$'

$$E(l) = \sum_{i=1}^{n} (l^T x_i)^2$$
$$= l^T \left( \sum_{i=1}^{n} (x_i x_i^T) \right) l$$

$E(l) = l^T C l$ $\quad \left[ \because C = \sum_{i=1}^{n} (x_i x_i^T) \right]$

$l^* = \underset{l}{\operatorname{argmin}}\, E(l) \Rightarrow \Delta E(l) = 0$
$Cl = 0$

where
$$C = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i \\ \Sigma x_i & \Sigma y_i & n \end{bmatrix}$$

Scanned by CamScanner

(d) points $[(0,1),(1,3),(2,6)]$

$S = D^T D$

where $D = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 6 & 1 \end{bmatrix}$ $\quad D^T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix}$

$$D^T D = \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$$

$$S = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i \\ \Sigma x_i & \Sigma y_i & n \end{bmatrix} = \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$$

(e) Explicit equation

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 = 1$$

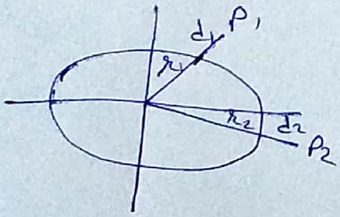
→ Constraint on parameters $a, b, c, d, e, f$. that guarantees the model will be an ellipse is

$$b^2 - 4ac < 0$$

(f) Equation for fitting ellipse:

$\sum_{i=1}^{n} (l^T p_i)^2$ → where $l^T p_i$ is algebric distance

$q_i = l^T p_i$ and $q_i \propto \frac{d_i}{d_i + r_i}$




→ so $q_1 > q_2$

$$\frac{d_1}{d + r_1} > \frac{d}{d + r_2}$$

→ so points close to short - axis will affect fitting more.

(g) $d(P, f) = |P - x| = \frac{|f(P)|}{|\nabla f(x)|}$ ← algebric distance

geometric distance

$x$ is closet point.

→ The problem is what is the closet point $x$ is.

(h)

$$E[\phi(s)] = \int_{\partial s} (\alpha(s)\, E_{continuity} + \beta(s)\, E_{curvature} + \gamma(s)\, E_{image})\, dS$$

- $E_{continuity}$, $E_{curvature}$, $E_{image}$ are energy terms
- $\alpha(s)$, $\beta(s)$, $\gamma(s)$ are coefficients.
- $\alpha(s) \cdot E_{continuity} + \beta(s)\, E_{curvature}$ are internal parameters
- $\gamma(s)\, E_{image}$ is external parameter.

(i) Continuity of discrete curve:

$$E_{continuity} = \left| \frac{d\phi}{ds} \right|^2 \Rightarrow \varepsilon \left| P_i - P_{i-1} \right|^2$$

→ distance between neighbouring points.

→ Curvature of discrete curve:

$$E_{curvature} = \left| \frac{\partial^2 \phi}{\partial s^2} \right|^2 \Rightarrow \varepsilon \left| (P_{i+1} - P_i) - (P_i - P_{i-1}) \right|$$

(j) The continuity of active contours will be $|P_i - P_{i-1}| - d$ to allow for sharp corners.