U-3 Partial diff eq {PDE}

[1] Formulation of P.D.E

[2] linear P.D E of 1^{st} order

[3] non-linear P.D.E of 1^{st} order

[4] Charpit equations

[5] Homogeneous P.D.E with constant coeffients

[6] ~~non homogeneous P.D.E with constant Coeff~~

Formation of PDE

(a) By the elimination of arbitary constants

(b) By the elimination of arbitary functions

Ex $z = ax + by + ab$ eliminate a & b

Ex $z = f_1(y+2x) + f_2(y-x)$ eliminate f_1 & f_2

Note equation of circle is

$$x^2 + y^2 = a^2 \qquad a = \text{constant}$$

diff $2x + 2y\frac{dy}{dx} = 0$

or $\frac{dy}{dx} = -\frac{x}{y}$ {

or $\frac{dy}{dx} + \frac{x}{y} = 0$ { that is a equation of circle in derivative form }

Note If $z = f(x, y)$

$\therefore \frac{\partial z}{\partial x} = p \qquad \frac{\partial z}{\partial y} = q$

$\frac{\partial^2 z}{\partial x^2} = r \qquad \frac{\partial^2 z}{\partial x \partial y} = s \qquad \frac{\partial^2 z}{\partial y^2} = t$

Q(1) eliminate constants a & b from following PDE

$$z = ax + by + a^2 + b^2 \quad \text{— (1)}$$

Sol→ Partialy diff (1) w.r to x & y

$\frac{\partial z}{\partial x} = a \qquad \frac{\partial z}{\partial y} = b$

or $p = a \qquad q = b$

or $\boxed{a = p} \qquad \boxed{b = q}$

the value of a & b putting in (1)

$$z = px + qy + pq \quad \text{Ans}$$

Q(2) eliminate a & b from

$$z = (x+a)(y+b) \quad \text{— (1)}$$

Sol→ P. diff (1) w. r to x & y

$\frac{\partial z}{\partial x} = (1)(y+b) \qquad \frac{\partial z}{\partial y} = (x+a)(1)$

$p = y + b \qquad q = x + a$

these values putting in (1)

$$z = pq \quad \text{Ans}$$

HW Q eliminate a & b from

$$z = (x^2+a)(y^2+b)$$

HW Q eliminate f & g $z = y\,f(x) + x\,g(y)$

Q[3] eliminate f from

$$z = f(x^2-y^2) \quad — (1)$$

Sol→ P. diff w.r to x & y

$$\frac{\partial z}{\partial x} = f'(x^2-y^2)(2x) \qquad \frac{\partial z}{\partial y} = f'(x^2-y^2)(-2y)$$

$$p = 2x\, f'(x^2-y^2) \qquad q = -2y\, f'(x^2-y^2)$$

or $f'(x^2-y^2) = \frac{p}{2x}$ — (II) $\qquad f'(x^2-y^2) = -\frac{q}{2y}$ — (III)

from (II) & (III)

$$-\frac{q}{2y} = \frac{p}{2x}$$

$$-2xq = 2yp$$

or $\quad 2xq + 2yp = 0 \quad$ An

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Q[4] eliminate function f from

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right) \quad — (1)$$

Sol→ P. diff (1) w.r to 'x'

$$\frac{\partial z}{\partial x} = 0 + 2f'\left(\frac{1}{x} + \log y\right)\left(-\frac{1}{x^2}\right)$$

$$p = -\frac{2}{x^2} f'\left(\frac{1}{x} + \log y\right)$$

or $\quad f'\left(\frac{1}{x} + \log y\right) = -\frac{px^2}{2} \quad$ — (2)

Now P. diff (1) w.r to 'y'

$$\frac{\partial z}{\partial y} = 2y + 2f'\left(\frac{1}{x} + \log y\right)\left(\frac{1}{y}\right)$$

or $\quad q = 2y + \frac{2}{y} f'\left(\frac{1}{x} + \log y\right) \quad$ — (3)

the value of (2) putting in (3)

$$q = 2y + \frac{2}{y}\left\{-\frac{px^2}{2}\right\}$$

$$qy = y^2 - 2x^2 p \qquad \text{Am}$$

Q[5] eliminate f & ϕ from following PDE

$$z = f(x+iy) + \phi(x-iy) \quad — (1)$$

→

P. diff (1) w r to x

$$\frac{\partial z}{\partial x} = f'(x+iy) + \phi'(x-iy)$$

$$\frac{\partial^2 z}{\partial x^2} = f''(x+iy) + \phi''(x-iy) \quad — (2)$$

P. diff (1) w.r to 'y'

$$\frac{\partial z}{\partial y} = f'(x+iy)(i) + \phi'(x-iy)(-i)$$

$$\frac{\partial^2 z}{\partial y^2} = f''(x+iy)(i)^2 + \phi''(x-iy)(-i)^2$$

$$\frac{\partial^2 z}{\partial y^2} = -f''(x+iy) - \phi''(x-iy) \quad — (3)$$

adding (2) & (3)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \qquad \text{Ans}$$

Linear P.D.E of 1^{st} order

If $z = f(x, y)$

then $\frac{\partial z}{\partial x} = p$ $\quad \frac{\partial z}{\partial y} = q$ {1^{st} order}

1^{st} order
- → linear P.D.E
- → non-linear P.D.E {charpit equation}

If power of p and q are 1 is called linear otherwise called non-linear

Ex (1) $x^2 p + y^2 q = z$ {linear

(2) $x^2 p^2 + y^2 q^2 = z^2$ {non-linear}

(3) $(x^2 - y^2) p + y^3 q = z^2 + x$ {linear}

(4) $\sqrt{pq}$ {non-linear}

Note – Variable separable

Ex (1) $x\,dy + y\,dx = 0$

$\rightarrow x\,dy = -y\,dx$

$\frac{1}{y} dy = -\frac{1}{x} dx$ {variable sep}

integration

$\int \frac{1}{y} dy = -\int \frac{1}{x} dx$

$\log y = -\log x + \log c$

$\log y = \log \frac{c}{x}$ $\qquad \log a - \log b = \log \frac{a}{b}$

$y = \frac{c}{x}$

Ex $x^2 dx = y^2 dy$ {variable sep}

integ $\int x^2 dx = \int y^2 dy$

$\frac{x^3}{3} = \frac{y^3}{3} + \frac{c}{3}$

$x^3 = y^3 + c$

Ex (3) $(x^2 - y) dx - y\,dy = 0$

$\rightarrow (x^2 - y) dx = y\,dy$ {not variable sep}

Ex (ii) $x\,dy + y\,dx = 0$

$\rightarrow \quad d(\underset{I}{x}\cdot\underset{II}{y}) = 0 \quad$ {varible sep}

intes $\int d(xy) = 0$

$xy = c$

Ex (iii) $y\,dz + z\,dy = 0$

$\rightarrow \quad d(yz) = 0 \quad$ {v. s}

inte $\int d(yz) = \int 0$

$yz = c$

Ex $\dfrac{dy+dz}{y+z}$ put $y+z = t$
$\therefore dy+dz = dt$

$$\frac{dy+dz}{y+z} = \frac{dt}{t}$$

$$= \int \frac{1}{t}\,dt$$

$$= \log t$$

$$\frac{dy+dz}{y+z} = \log(y+z)$$

Ex $\int \dfrac{dy-dz}{y-z} = \log(y-z)$

Ex $\int \dfrac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2} = \log(x^2+y^2+z^2)$

Linear P.DE of 1^{st} order

Langrange method

Consider

$$Pp + Qq = R$$

its AE is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

(i) Solve it by the method of grouping or by the method of multiplier or by both

(ii) its complete solution
$\phi(c_1, c_2) = 0$

Q[I] Solve $y^2 z p + x^2 z q = x y^2$

Sol → Here $P = y^2 z \quad Q = x^2 z \quad R = x y^2$

its AE is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{y^2 z} = \frac{dy}{x^2 z} = \frac{dz}{x y^2}$$

(I) taking 1st & 2nd member

$$\frac{dx}{y^2 \cancel{z}} = \frac{dy}{x^2 \cancel{z}}$$

$$x^2 dx = y^2 dy \quad \{\text{variable sep}\}$$

integration

$$\int x^2 dx = \int y^2 dy$$

$$\frac{x^3}{3} = \frac{y^3}{3} + \frac{c_1}{3}$$

$$x^3 = y^3 + c_1$$

$$\boxed{c_1 = x^3 - y^3}$$

(II) taking 1st & 3rd member

$$\frac{dx}{y^2 z} = \frac{dz}{x y^2}$$

$$\frac{dx}{z} = \frac{dz}{x}$$

$$x\,dx = z\,dz \quad \{\text{variable sep}\}$$

integs

$$\int x\,dx = \int z\,dz$$

$$\frac{x^2}{2} = \frac{z^2}{2} + \frac{c_2}{2}$$

$$x^2 = z^2 + c_2$$

$$\boxed{c_2 = x^2 - z^2}$$

its complete solu

$$\phi(c_1, c_2) = 0$$

or

$$\phi(x^3 - y^3,\ x^2 - z^2)$$

Q 2 Solve

$$xp + yq = 3z$$

Sol → Here $P = x \quad Q = y \quad R = 3z$

its A.E is

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{3z}$$

1 taking 1st & 2nd member

$$\frac{dx}{x} = \frac{dy}{y} \quad \{\text{variable sep}\}$$

integs

$$\int \frac{1}{x}\,dx = \int \frac{1}{y}\,dy$$

$$\log x = \log y + \log c_1$$

$$\log x = \log c_1 y$$

$$x = c_1 y$$

$$\boxed{c_1 = x/y}$$

11 taking 2nd & 3rd member

$$\frac{dy}{y} = \frac{dz}{3z} \quad \{\text{v.s}\} \text{ that is}$$

int

$$\int \frac{1}{y}\,dy = \frac{1}{3}\int \frac{1}{z}\,dz$$

$$3\int \frac{1}{y}\,dy = \int \frac{1}{z}\,dz$$

$$3\log y = \log z + \log c_2$$

$$\log y^3 = \log c_2 z$$

$$y^3 = c_2 z$$

$$\boxed{c_2 = y^3/z}$$

its complete solut

$$\phi(x/y,\ y^3/z) = 0$$

HW

Q Solve $yzp + zxq = xy$

Ans $\phi(x^2 - y^2,\ y^2 - z^2) = 0$

Note → Multiplier
(i) $1/x^2 : 1/y^2 : 1/z^2$
(ii) $1/x^2$
(iii) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$
(iv) x, y, z

Q 3 Solve

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

Sol → Here $P = x^2(y-z)$ $Q = y^2(z-x)$

$R = z^2(x-y)$

its AE is

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$$

(I) taking multiplier $1/x^2, 1/y^2, 1/z^2$

$$\text{each fraction} = \frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{y-z+z-x+x-y}$$

$$\text{each fraction} = \frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0}$$

$$0 = \frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz \quad \{v.s\}$$

inte

$$\int 0 = \int x^{-2}dx + \int y^{-2}dy + \int z^{-2}dz$$

$$\frac{c_1}{-1} = \frac{x^{-2+1}}{-2+1} + \frac{y^{-1}}{-1} + \frac{z^{-1}}{-1}$$

$$\frac{c_1}{-1} = \frac{x^{-1}}{(-1)} + \frac{y^{-1}}{(-1)} + \frac{z^{-1}}{(-1)}$$

$$c_1 = x^{-1} + y^{-1} + z^{-1}$$

$$\boxed{c_1 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

II taking multiplier $1/x, 1/y, 1/z$

$$\text{each fraction} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{xy - xz + yz - xy + xz - yz}$$

$$\text{each fraction} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

$$0 = \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz \quad \{v.s\}$$

inte

$$\int 0 = \int \frac{1}{x}dx + \int \frac{1}{y}dy + \int \frac{1}{z}dz$$

$$\log c_2 = \log x + \log y + \log z$$

$$\log c_2 = \log xyz$$

$$\boxed{c_2 = xyz}$$ its complete s

$$\phi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$$

Q[9] Solve

$$x(y-z)p + y(z-x)q = z(x-y)$$

Sol^n: its AE is

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

[1] taking multiplier $1/x, 1/y, 1/z$

$$\text{each fraction} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{y-z+z-x+x-y}$$

$$\text{each fraction} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

$$0 = \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz \quad \{v.s\}$$

integ $$\int 0 = \int \frac{1}{x}dx + \int \frac{1}{y}dy + \int \frac{1}{z}dz$$

$$\log c_1 = \log x + \log y + \log z$$

$$\log c_1 = \log xyz$$

$$\boxed{c_1 = xyz}$$

[6]

[2] taking multiplier $1, 1, 1$

$$\text{each fraction} = \frac{1dx + 1dy + 1dz}{xy - xz + yz - xy + xz - yz}$$

$$\text{each fraction} = \frac{1dx + 1dy + 1dz}{0}$$

$$0 = 1dx + 1dy + 1dz \text{ fu}$$

$$\int 0 = \int 1dx + \int 1dy + \int 1dz$$

$$\boxed{c_2 = x + y + z}$$

its C.S

$$\phi(xyz, x+y+z) = 0$$

HW

Q Solve

$$x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$$

Q $\boxed{5}$ solve $(x^2-y^2-z^2)p + 2xy\,q = 2xz$

Sol→ It AE is

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

$\boxed{I}$ taking 1st & 2nd member

$$\frac{dy}{2xy} = \frac{dz}{2xz}$$

$$\frac{dy}{y} = \frac{dz}{z} \quad \{\text{variable sep}\}$$

integ $\int \frac{1}{y}dy = \int \frac{1}{z}dz$

$$\log y = \log z + \log c_1$$

$$\log y = \log c_1 z$$

or $y = c_1 z$

or $\boxed{c_1 = y/z}$

$\boxed{II}$ taking multiplier x, y, z

$$\text{each fraction} = \frac{x\,dx + y\,dy + z\,dz}{x^3 - xy^2 - xz^2 + 2xy^2 + 2xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + xy^2 + xz^2}$$

each fraction = (I, II & III rd mem) $\dfrac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)}$

taking III rd & IV th member

$$\frac{dz}{2\cancel{x}z} = \frac{x\,dx + y\,dy + z\,dz}{\cancel{x}(x^2+y^2+z^2)}$$

$$\frac{dz}{z} = \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2} \quad \{v.s\}$$

int $\int \frac{1}{z}dz = \int \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2}$

$$\log z = \log(x^2+y^2+z^2) + \log c_2$$

$$\log z = \log c_2(x^2+y^2+z^2)$$

$$z = c_2(x^2+y^2+z^2)$$

$\boxed{c_2 = z/(x^2+y^2+z^2)}$

Non-linear P.D.E of 1^{st} order
(charpit equation)

charpit method → Consider $\phi(x, y, z, p, q)$

[I] charpit A.E is

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dx}{-\partial f/\partial p} = \frac{dy}{-\partial f/\partial q} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}}$$

[II] let $f = \phi(x, y, z, p, q)$ and we find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial p}$ & $\frac{\partial f}{\partial q}$

these values putting in charpit equation

[III] Solve any two member to find the value of p and q

[IV] Its complete solution to be convert in {varible sep form}

$$dz = p\,dx + q\,dy$$

integration

$$\int dz = \int p\,dx + \int q\,dy$$

Q[1] Solve by charpit method

$$z = px + qy + p^2 + q^2$$

Sol→ ① let $f = px + qy + p^2 + q^2 - z$

∴ $\frac{\partial f}{\partial x} = p \quad \frac{\partial f}{\partial y} = q \quad \frac{\partial f}{\partial z} = -1 \quad \frac{\partial f}{\partial p} = x + 2p$

$\frac{\partial f}{\partial q} = y + 2q$

② these values putting in charpit equation

$$\frac{dp}{p + p(-1)} = \frac{dq}{q + (-1)q} = \frac{dx}{-(x+2p)} = \frac{dy}{-(y+2q)} = \frac{dz}{-p(x+2p) - q(y+2q)}$$

$$\frac{dp}{0} = \frac{dq}{0} = \frac{dx}{-(x+2p)} = \frac{dy}{-(y+2q)} = \frac{dz}{-px - qy - 4pq}$$

[III] taking 1^{st} & 3^{rd} member

$$\frac{dp}{0} = \frac{dx}{-(x+2p)}$$

or $dp = 0$

integ $\int dp = \int 0$

$$\boxed{p = a}$$

taking 2^{nd} & 4^{th} mem

$$\frac{dq}{0} = \frac{dy}{-(y+2q)}$$

or $dq = 0$

integ

$\int dq = \int 0$

$$\boxed{q = b}$$

[IV] Its complete solution is

$$dz = p\,dx + q\,dy$$

$$dz = a\,dx + b\,dy \quad \{\text{that is variable sep}\}$$

integrati. $\int dz = a\int dx + b\int 1\,dy$

$$z = ax + by + c \quad \text{Ans}$$

Q [2] Solve by charpit method $z = pq$

(i) Let $f = pq - z$

$\therefore \frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0 \quad \frac{\partial f}{\partial z} = -1 \quad \frac{\partial f}{\partial p} = q \quad \frac{\partial f}{\partial q} = p$

(ii) these value putting in charpit AE

$$\frac{dp}{-p} = \frac{dq}{-q} = \frac{dx}{-q} = \frac{dy}{-p} = \frac{dz}{-pq - pq}$$

taking 1st & 4th member

$$\frac{dp}{-p} = \frac{dy}{-p}$$

or $dp = dy$ {v.s}

integs $\int dp = \int dy$

$$\boxed{p = y + a}$$

taking 2nd and 3rd member

$$\frac{dq}{-q} = \frac{dx}{-q}$$

or $dq = dx$ {variable sep}

integration

$\int dq = \int dx$

$$\boxed{q = x + b}$$

(iii) its complete solution is

$dz = p\,dx + q\,dy$

$dz = (y+a)dx + (x+b)dy$

$= y\,dx + a\,dx + x\,dy + b\,dy$

$= \{y\,dx + x\,dy\} + a\,dx + b\,dy$

$dz = d(xy) + a\,dx + b\,dy$ {that variable sep}

integs

$\int dz = \int d(xy) + a\int dx + b\int dy$

$z = xy + ax + by + c$ Ans

Q 3 Solve by charpit eq $(p^2+q^2)y = qz$ — (1)

Sol → (i) let $f = (p^2+q^2)y - qz$

or $f = p^2y + q^2y - qz$

$\therefore \frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = p^2+q^2 \quad \frac{\partial f}{\partial p} = 2py.$

$\frac{\partial f}{\partial z} = -q \qquad \frac{\partial f}{\partial q} = 2qy - z$

(ii) these values putting in charpit eq

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dx}{-\partial f/\partial p} = \frac{dy}{-\partial f/\partial q} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}}$$

$$\frac{dp}{-pq} = \frac{dq}{p^2} = \frac{dx}{-2py} = \frac{dy}{-2qy - z} = \frac{dz}{-2p^2y - 2q^2y + qz}$$

(i) taking 1st & 2nd member

$$\frac{dp}{-pq} = \frac{dq}{p^2}$$

or $$\frac{dp}{-q} = \frac{dq}{p}$$

or $p\,dp = -q\,dq$ {variable sep}

integrat $\int p\,dp = -\int q\,dq$

$$\frac{p^2}{2} = -\frac{q^2}{2} + \frac{a^2}{2}$$

$$p^2 = -q^2 + a^2$$

$$p^2 + q^2 = a^2 \quad — \quad (2)$$

these value putting in (1)

$$a^2y = qz$$

$$\therefore \boxed{q = \frac{a^2y}{z}}$$

the value of q putting in (2)

$$p^2 + \left\{\frac{a^2y}{z}\right\}^2 = a^2$$

$$p^2 + \frac{a^4y^2}{z^2} = a^2$$

$$p^2 = a^2 - \frac{a^4y^2}{z^2}$$

$$p^2 = \frac{a^2z^2 - a^4y^2}{z^2}$$

$$\therefore p = \sqrt{\frac{a^2z^2 - a^4y^2}{z^2}}$$

$$\boxed{p = \frac{a}{z}\sqrt{z^2 - a^2y^2}}$$

(3) its complete solution

$$dz = p\,dx + q\,dy$$

$$dz = \frac{a}{z}\sqrt{z^2-a^2y^2}\,dx + \frac{a^2y}{z}\,dy$$

{Now we convert it in variable sep form}

$$z\,dz = a\sqrt{z^2-a^2y^2}\,dx + a^2y\,dy$$

$$z\,dz - a^2y\,dy = a\sqrt{z^2-a^2y^2}\,dx$$

$$\frac{z\,dz - a^2y\,dy}{\sqrt{z^2-a^2y^2}} = a\,dx \qquad \{\text{variable sep}\}$$

integ $$\int \frac{z\,dz - a^2y\,dy}{\sqrt{z^2-a^2y^2}} = a\int dx$$

put $z^2 - a^2y^2 = t^2$

$\therefore\ 2z\,dz - 2a^2y\,dy = 2t\,dt$

or $z\,dz - a^2y\,dy = t\,dt$

$$\therefore \int \frac{t\,dt}{t} = ax + b$$

$$t = ax + b$$

$$\sqrt{z^2-a^2y^2} = ax + b \quad \text{Ans}$$

HW

Q Solve by charpit method $2xz - px^2 - 2qxy + pq = 0$

Hint charpit eq are

$$\frac{dp}{2z - 4px - 2qy} = \frac{dq}{0} = \frac{dx}{x^2-q} = \frac{dy}{2xy-p} = \frac{dz}{px^2 - 2pq + 2x}$$

taking 2^{nd} & 3^{rd} member

$\boxed{q = a}$ putting in eq (1)

$$\therefore \boxed{p = \frac{2x(z-ay)}{x^2-a}}$$

these value {p & q} putting in

$$dz = p\,dx + q\,dy$$

$$dz = \frac{2x(z-ay)}{x^2-a}\,dx + a\,dy$$

{convert in variable separable form}

$$\int \frac{dz - a\,dy}{z - ay} = \int \frac{2x}{x^2-a}\,dx \qquad \{\text{varible sep}\}$$

put $z - ay = t$ $\qquad x^2 - a = v$

Ans $\quad z = ay + c(x^2 - a)$