M2 Assignment # 5

①

(i) $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} = \sin y$

order: 2
Degree: 1

(ii) $(y')^{3/2} = 2x(y\cancel{a}e^{-x^2} - y - 3x)$

$\cancel{(y')}\left(\frac{dy}{dx}\right)^3 = 4x^2(ye^{-x^2} - y - 3x)^2$

Order: 1
Degree: 3

(iii) $\frac{d^3y}{dx^3} + 2x\frac{dy}{dx} + x^2 = \sqrt{y}\cot x$

order: 3
degree: 1.

②

(i) $\frac{dy}{dx} = \cancel{2}x^{2/3} - 2y \longrightarrow$ Linear

(ii) $yy'' + 3y' = e^{3x^2} \longrightarrow$ NOT linear {because of yy'' term}

(iii) $y'' + 4y' = x\cos y \longrightarrow$ NOT linear {because of $\cos y$ term}

$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{4} ...$

(iv) $x^2y'' + 4\sqrt{x}\,y = \cos x \longrightarrow$ Linear

(v) $y'' + 4y^2 = 3x \longrightarrow$ Non linear, {y^2 term}

(vi) $y'' + 2x^2y' = \sqrt[3]{x}$

$\longrightarrow$ linear

③. i] ~~So~~ $\log y' = 2x+3y$

$y' = e^{2x} \cdot e^{3y}$

$$\int \frac{dy}{e^{3y}} = \int e^{2x}\,dx$$

$$-3e^{-3y} = 2e^{2x} + c$$

$$2e^{2x} + 3e^{3y} + c = 0$$ solved.

ii] $$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$$

$$\int \frac{dy}{e^y} = \int dx\,(e^x + x^2 e^{x^3})$$

$$-1e^{-y} = e^x + \frac{1}{3}e^{x^3} + c$$

$$\Rightarrow e^{x+y} + \frac{1}{3}e^{x^3+y} + ce^{y} = -1$$ Solved.

4.

i) $$\frac{dy}{dx} = (4x+y+1)^2 \quad — ①$$

$$4x+y+1 = v$$

$$4 + \frac{dy}{dx} = \frac{dv}{dx}$$

in ①

$$\frac{dv}{dx} - 4 = v^2$$

$$\frac{dv}{dx} = v^2 + 4$$

$$\int \frac{dv}{v^2+4} = \int dx \Rightarrow \frac{1}{2}\tan^{-1}\frac{v}{2} + c = c$$

where $v = 4x+y+1$

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(ii) $(x+y)^2 \frac{dy}{dx} = a^2$

$$x+y = v$$

$$\frac{dv}{dx} - 1 = \frac{dy}{dx}$$

$$\left(\frac{dv}{dx} - 1\right) = \frac{a^2}{v^2}$$

$$\frac{dv}{dx} = \frac{a^2+v^2}{v^2}$$

$$\int \frac{v^2\,dv}{a^2+v^2} = \int 1\,dx$$

$$\int 1\,dv - \int \frac{a^2\,dv}{a^2+v^2} = \int dx$$

$$v - \frac{a^2}{a}\tan^{-1}\left(\frac{v}{a}\right) = x + c$$

$$\cancel{x+y} \quad y = a\tan^{-1}\left(\frac{x+y}{a}\right) + c$$

(iii) $\frac{dy}{dx} = \sec(x+y)$.

$$x+y = v$$

$$\frac{dv}{dx} - 1 = \frac{dy}{dx}$$

$$\frac{dv}{dx} = \frac{\cos v + 1}{\cos v}$$

$$\int \frac{\cos v\,dv}{1+\cos v} = \int dx \qquad \cancel{x}+y - \int \frac{1}{1+\cos v}\,dv = \int dx$$

$$y = \int \frac{1}{1+\cos v}\,dv + C$$

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$$y = \int \frac{1}{1+\cos v}\,dv + c = \int \frac{1}{\cancel{\sin^2\frac{v}{2}} + \cos^2\frac{v}{2} + \cos^2\frac{v}{2} - \cancel{\sin^2\frac{v}{2}}}\,dv + c$$

$$= +\frac{1}{2}\int \sec^2\frac{v}{2}\,dv + c$$

$$= \frac{\cancel{1}}{\cancel{2}}\frac{\left(\tan\frac{v}{2}\right)}{\cancel{1/2}} + c$$

$$\boxed{y = \tan(x+y) + c}$$

~~Ans~~

5

(1) $(y\,dx + x\,dy)\,x\cos\frac{y}{x} = y\sin\frac{y}{x}(x\,dy - y\,dx)$

$(x\,dy - y\,dx)\,y\sin\left(\frac{y}{x}\right) = (y\,dx + x\,dy)\,x\cos\frac{y}{x}$

$\Rightarrow xy\sin\frac{y}{x}\,dy - y^2\sin\frac{y}{x}\,dx$

$= xy\cos\left(\frac{y}{x}\right)dx + x^2\cos\frac{y}{x}\,dy$

$\left[xy\sin\left(\frac{y}{x}\right) - x^2\cos\frac{y}{x}\right]dy = \left[xy\cos\frac{y}{x} + y^2\sin\frac{y}{x}\right]dx$

$$\frac{dy}{dx} = \frac{xy\cos\left(\frac{y}{x}\right) + y^2\sin\left(\frac{y}{x}\right)}{xy\sin\frac{y}{x} - x^2\cos\left(\frac{y}{x}\right)}$$

Dividing numerator & denominator by x^2.

$$\frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right) + \frac{y^2}{x^2}\cos\frac{y}{x}}{\frac{y}{x}\sin\frac{y}{x} - \cos\frac{y}{x}} \quad \text{--- (1)}$$

Let ~~v = y x~~ $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Putting value of $\frac{dy}{dx}$ and y in (1).

$$v + \frac{x\,dv}{dx} = \frac{\frac{vx}{x}\cos\frac{vx}{x} + \frac{v^2x^2}{x^2}\sin\left(\frac{vx}{x}\right)}{\frac{vx}{x}\sin\left(\frac{vx}{x}\right) - \cos\left(\frac{vx}{x}\right)}$$

~~v + c~~

$$x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v} - v$$

$$\frac{x\,dv}{dx} = \frac{2v\cos v}{v\sin v - \cos v}$$

$$\left(\frac{v\sin v - \cos v}{v\cos v}\right) dv = 2\frac{dx}{x}$$

$$\left(\frac{v\sin v}{v\cos v} - \frac{\cos v}{v\cos v}\right) dv = 2\frac{dx}{x}$$

$$\left(\tan v - \frac{1}{v}\right) dv = 2\frac{dx}{x}$$

Integrating both sides

$$\int\left(\tan v - \frac{1}{v}\right) dv = 2\int\frac{dx}{x}$$

$$\int \tan v\, dv - \int\frac{dv}{v} = 2\int\frac{dx}{x}$$

$$\log|\sec v| - \log|v| = 2\log|x| + \log|c|$$

$$\log\left|\frac{\sec v}{v}\right| = \log|x^2 c|$$

$$\frac{\sec v}{v} = x^2 c$$

$$\sec\left(\frac{y}{x}\right) = cxy \quad \xleftarrow{\text{Putting } v = y/x}$$

(ii) $$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

$$\frac{y}{x} = v \implies \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\cancel{v} + x\frac{dv}{dx} = v\log v \cancel{+v}$$

$$\int\frac{dv}{v\log v} = \int\frac{dx}{x} \Rightarrow \boxed{\log v = xc}$$

or $\boxed{y = x\cdot e^{xc}}$

6.

① $\frac{dy}{dx} = \left(\frac{x+2y-3}{2x+y-3}\right)$

$-h+2k+3=0$

$2h+k-3=0$

$3h-3=0 \Rightarrow h=1$

$\Rightarrow k = 3-2h = 1.$

$X = x+h$

$Y = y+k$

$\frac{dY}{dX} = \frac{X+2Y}{2X+Y}$

$\frac{dY}{dX} = \frac{1+2Y/X}{2+Y/X} = f(Y/X) = g(X,Y)$ (say)

if we replace Y & X by λY & λX then $\boxed{\lambda \neq 0} \in R$

$$\boxed{\frac{dY}{dX} = \frac{1+2Y/X}{2+Y/x}} \Rightarrow g(X,Y) = g(\lambda X, \lambda Y)$$

$\Rightarrow$ Reduced to Homo generous form

② $\frac{dy}{dx} = \frac{(-x+y+2)}{x-2y-3}$.

$X = x+h$

$Y = y+k$

$-h+k+2=0$

$h-2k-3=0$

$-k+2-3=0$

$\Rightarrow \boxed{\begin{matrix} k=-1 \\ h=1 \end{matrix}}$ (1, -1)

$$\boxed{\frac{dY}{dX} = \frac{\frac{Y}{X}-1}{1-2\frac{Y}{X}}} \leftarrow \text{Homogeneous form}$$

Q7.

i) $\left(y\left(1+\frac{1}{x}\right)+\cos y\right)=M$

$x+\log x-x\sin y=N$

$$\frac{\partial M}{\partial y}=1+\frac{1}{x}-\sin y=\frac{\partial N}{\partial x} \Rightarrow \text{exact}$$

Sol.

$$\underbrace{\int M\,dx}_{\{\text{treating } y \text{ as constant}\}} + \int (\text{terms of } N, \text{ not containing } x)\,dy = c$$

$$\int y\left(1+\frac{1}{x}\right)+\cos y\,dx+\int 0\,dy=c$$

$$\Rightarrow yx+y\log x+x\cos y=C$$

ii) $$x\,dx+y\,dy=\frac{y\,dx-x\,dy}{x^2+y^2}$$

LHS: $$\frac{d(x^2+y^2)}{2}=x\,dx+y\,dy$$

RHS: $$d\left(\tan^{-1}(y/x)\right)=\frac{\frac{x\,dy-y\,dx}{\cancel{x^2}}}{\frac{x^2+y^2}{\cancel{x^2}}}=\frac{y\,dx-x\,dy}{x^2+y^2}$$

$$\Rightarrow \int \frac{1}{2}d(x^2+y^2)=\int d\left(\tan^{-1}(y/x)\right)$$

$$x^2+y^2=2\tan^{-1}y/x+C \longleftarrow \text{Solution}$$

8.

$$\frac{dy}{dx} = -\frac{3x^2+\lambda e^y}{2x e^y+3y^2}$$

$$\overbrace{(2xe^y+3y^2)}^{N} dy + \overbrace{(3x^2+\lambda e^y)}^{M} dx = 0.$$

for exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow 2e^y + \overset{0}{\cancel{0}} = 0 + \lambda e^y$

$$\boxed{\lambda = 2}$$

$\cancel{\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = 2e^y} - 2e^y = 0$

Solution:

$$\int (3x^2+2e^y)\,dx + \int 3y^2\,dy = c$$

$$\boxed{x^3 + 2xe^y + y^3 = c}$$

1\. (i) $y(x+y)\,dx + (x+2y-1)\,dy = 0$

$$\frac{\partial M}{\partial y} = x+2y, \quad \frac{\partial N}{\partial x} = 1$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = 1 \Rightarrow I.f = e^{\int 1\,dy} = e^y$$

(ii) $y(x+y+1)\,dx + x(x+3y+2)\,dy = 0.$

$$\frac{\partial M}{\partial y} = x+2y+1, \quad \frac{\partial N}{\partial x} = 2x+3y+2$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y} \Rightarrow I.f = e^{\int \frac{1}{y}dy} = y$$

Solved

K.S.