<u>Discrete Mathematics Assignment-2</u>

1. Let x and y be any two words over an alphabet z. Prove that $\|xy\| = \|x\| + \|y\|$.
2. If n + 1 integers are selected from the set {1, 2,…, 2n}, one of them divides another integer that has been selected.
3. A C++ identifier contains 37 alphanumeric characters. Show that at least two characters are the same.
4. The total cost of 13 refrigerators at a department store is $12,305. Show that one refrigerator must cost at least $947.
5. If five points are chosen inside a unit square, then the distance between at least two of them is no more than $\sqrt{2}/2$. (*Use the pigeonhole principle)*
6. Five points are chosen inside an equilateral triangle of unit side. The distance between at least two of them is no more than 1/2. (*Use the pigeonhole principle)*
7. Determine if the given function is invertible. If it is not invertible, explain why.
   $f: ASCII \rightarrow W$ defined by $f(c) =$ ordinal number of the character c.

[The character sets ASCII, multinational 1, box drawing, typographical symbols, math/scientific symbols, and Greek symbols, used by *WordPerfect* are denoted by the character set numbers 0, 1, 2, 3, 4, 6, and 8, respectively. Let A - {0, 1, 2, 3, 4, 6, 8}. Each character in a character set is associated with a unique decimal number, called its ordinal number (or its relative position). For example, the ordinal number of the character '&' in ASCII is 38.Let B - {32, 33, 3 4 , . . . , 60}, the set of ordinal numbers. Then we can define a function f :A x B ~ C defined by *f(i,j) - c,* where c is the character with ordinal number j in the character set i. For example, f(0,36) = '$' and f(8,38) = 'E'.]

8. Determine true or false
$$\sum_{i=m}^{n} i = \sum_{i=m}^{n} (n + m - i)$$
9. Solve the following problem.
   a) Sums of the form $S = \sum_{i=m+1}^{n}(a_i - a_{i-1})$ are telescoping sums. Show that $S = a_n - a_m$.
   b) Using Exercise 9(a) and the identity $\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+!}$, derive a formula for $\sum_{i=1}^{n} \frac{1}{i(i+1)}$
10. Expand this
$$\sum_{1 \le i \le j < 3} (a_i + a_j)$$
11. Let *A* be an *m* x *n* matrix, *B* a *p* x *q* matrix, and C an *r* x *s* matrix. Under what conditions is each defined? Find the size of each when defined. *(Note:* A^2 means AA.)
    a) $A + B$

b) $B - C$
c) BC
d) A^2

12. A summer vacation lodge in sunny California expects four guests: A, B, C,and D. They plan to stay at the lodge for 7, 14, 21, and 28 days, respectively. Each of them has diabetes. Since the nearest drugstore is several miles away, the manager of the lodge decides to store three different types of insulin semi-Lente, Lente, and ultra --needed by these guests. Their daily insulin requirements are summarized in Table 1.

Insulin	**Guests**			
	A	**B**	**C**	**D**
Semi-Lente	**25**	**40**	**35**	**0**
Lente	**20**	**0**	**15**	**15**
ultra	**20**	**0**	**30**	**40**

Each gram of insulin of the three types costs 10, 11, and 12 cents, respectively. Using matrices, compute each:

a) The number of grams of each type of insulin needed.
b) The total cost of the insulin.
c) The insulin requirements if the guests decide to stay an additional 3, 5, 8, and 13 days, respectively.
d) The insulin requirements if the guests decide to stay three times their original time.

13. Apply the euclidean algorithm to find gcd{ 2076, 1024}.
14. **(Twelve Days of** Christmas) Suppose you sent your love 1 gift on the first day of Christmas, 1 + 2 gifts on the second day, 1 + 2 + 3 gifts on the third day and so on.
    a) How many gifts did you send on the 12th day of Christmas?
    b) How many gifts did your love receive in the 12 days of Christmas?
15. Using PMI, prove each for every integer n > 1.
    a) $\sum_{i=1}^{n}(2i-1) = n^2$
    b) $n^4 + 2n^3 + n^2$ is divisible by 4.
16. Find the value of x resulting from executing each algorithm fragment.
    a) $x \leftarrow 0$
       $for\ i = 1\ to\ n\ do$
       $x \leftarrow x + (2i - 1)$
    b) $x \leftarrow 0$
       $for\ i = 1\ to\ n\ do$
       $for\ j = 1\ to\ i\ do$
       $x \leftarrow x + 1$