The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 6

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1. (i) If the differential equation $M(x,y)dx+N(x,y)dy=0$ is homogeneous, then $1/(Mx+Ny)$ is an integrating factor unless $Mx+Ny\equiv 0$, (ii) if the differential equation $M(x,y)dx+N(x,y)dy=0$ is not exact but is of the form $f_1(xy)ydx+f_2(xy)xdy=0$, then $1/(Mx-Ny)$ is an integrating factor unless $Mx-Ny\equiv 0$. Using it, solve the following differential equations:

   (a) $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$ (b) $x^2ydx-(x^3-y^3)dy=0$ (b) $y(1+xy)dx+x(1-xy)dy=0$

2. Solve the following deferential equations:

   (a) $(x+2y^3)\frac{dy}{dx}=y$ (b) $(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$

3. Reduce to linear deferential equations:

   (a) $x\frac{dy}{dx}+y\log y=xye^x$ (b) $\frac{dy}{dx}+\frac{1}{x}\sin 2y=x^2\cos^2 y$ (c) $(xy^2+e^{-\frac{1}{x^3}})dx-x^2ydy=0$

4. Find the orthogonal trajectories of the following families of curves:

   (a) $y=ax^2$ (b) $x^2+y^2=2ax$

5. Find the orthogonal trajectories of the parabolas $r=\frac{2c}{(1+\cos\theta)}$, where c is a parameter.

6. Prove that the orthogonal trajectories of $r^n\cos\ n\theta=c^n$ is $r^n\sin\ n\theta=c^n$.

7. Find the family of oblique trajectories which intersect the family of hyperbola $xy=c$ at an angle of $45°$.
   Note: An oblique trajectory is a curve that intersect each member of a given family of curve at a constant angle $\alpha\neq 90°$.

8. Study the existence of solutions of the initial value problem

$$xy'=\frac{3}{x^3},\qquad y(1)=-1$$

9. Study the existence of solutions of the initial value problem

$$y'=\sqrt{|y|},\qquad y(0)=0$$

10. Show that $xy'=4y,\quad y(0)=1$ has no solution. Does this contradict existence theorem.

11. Find all initial conditions such that the initial value problem $(x^2-2x)y'=2(x-1)y,\ y(x_0)=y_0$ has (a) no solution (b) Infinitely many solutions (c) Unique solution.

12. Find the solution of the initial value problem $y'=2y-x,\quad y(0)=1$, using the Picard's iteration method. Compare with the exact solution.