The LNM Institute of Information Technology
Jaipur, Rajasthan

Ist Mid Semester Exam MATH-I, August 21, 2014

Part-B Time: 45 minutes, Maximum Marks: 30

Instructions: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be penalized by the deduction of **2 marks**.

Question No.				
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1. (a) Let $x > 1$, then show that the $S = \{x^n : n \in \mathbb{N}\}$ is unbounded above. In particular, the set $\{10^n : n \in \mathbb{N}\}$ is unbounded set and hence show that there exists $n \in \mathbb{N}$ such that $\frac{1}{10^n} < b$ for $b \in \mathbb{R}^+$. **[5 marks]**

   (b) Show that the limit of a convergent sequence is unique i.e. if $\lim_{n\to\infty} x_n = l$, $\lim_{n\to\infty} x_n = m$ then $l = m$. **[5 marks]**

2. (a) Consider the sequence (x_n) defined by

$$x_1 = \sqrt{3}, \quad x_n = \sqrt{3 + x_{n-1}}, n > 1.$$

   Show that (x_n) converges and find its limit. **[3+2 marks]**

   (b) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ x, & x \notin \mathbb{Q}. \end{cases}$$

   is continuous only at $x_0 = 0$ and discontinouos everywehre else. **[5 marks]**

3. (a) Let a, b, and c be real numbers. Show that the equation

$$4ax^3 + 3bx^2 + 2cx = a + b + c$$

   always has a root between 0 and 1. **[4 marks]**
   (*Hint: You can think of applying MVT to a suitable function*).

   (b) Consider $f : [0, 2] \to [0, 2]$ defined by $f(x) = (1 + x)^{1/5}$. Apply fixed point iteration method to generate a sequence of approximate solutions of the equation $x^5 - x - 1 = 0$. Show that the sequence converges to a root of this equation and find first three approximations to this root in the interval $[0, 2]$. **[6 marks]**