The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 11

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1. Solve the integral equations:
$(a)\ y(t) + \int_0^t y(\tau)d\tau = u(t-a) + u(t-b)$

$(b)\ e^{-t} = y(t) + 2\int_0^t \cos(t-\tau)y(\tau)d\tau$

$(c)\ 3\sin 2t = y(t) + \int_0^t (t-\tau)y(\tau)d\tau.$

2. Sketch the following functions and find their Laplace transforms:

$(a)\ f(t) = \begin{cases} u(t) - 2u(t-1), & 0 \le t < 2, \\ f(t-2), & t > 2. \end{cases}$ $\quad (b)\ f(t) = \begin{cases} t[u(t) - u(t-1)], & 0 \le t < 2, \\ f(t-2), & t > 2. \end{cases}$

$(c)\ f(t) = \begin{cases} tu(t) - 2(t-1)u(t-1), & 0 \le t < 2, \\ f(t-2), & t > 2. \end{cases}$

3. Find the Fourier series of f (assuming f to be periodic with period 2π):
$(a)\ f(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & \pi < x < 2\pi \end{cases}$ $\quad (b)\ f(x) = \begin{cases} x & -\pi/2 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases}$
$(c)\ f(x) = x^2/4,\ -\pi < x < \pi$ (d) $f(x) = x,\ 0 < x < 2\pi$.
In each case, find the sum of the Fourier series at $x = \frac{101\pi}{2}$.

4. Show that
$(i)\ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$ [Hint: use 3(a)]
$(ii)\ 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12}$ [Hint: use 3(c)]

5. Expand $f(x)$ in a Fourier series on the interval $[-2, 2]$ if $f(x) = 0$ for $-2 \le x < 0$ and $f(x) = 1$ for $0 \le x \le 2$. (Assume f to be periodic with period $p = 2L = 4$).

6. Find the (i) Fourier cosine series of $f(x) = 1 + \sin \pi x$, $0 \le x < 1$, $f(x+2) = f(x)$,
(ii) Fourier sine series for $f(x) = e^x$, $0 \le x < \pi$, $f(x + 2\pi) = f(x)$.

7. Using the Fourier integral representation, show that
(a) $\displaystyle\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2} d\omega = \begin{cases} 0 & x < 0 \\ \frac{\pi}{2} & x = 0 \\ e^{-x} & x > 0 \end{cases}$ $\quad (b)\ \displaystyle\int_0^\infty \frac{\cos x\omega}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, x > 0.$

8. Find $A(\omega)$ such that $\frac{1}{\pi}\displaystyle\int_0^\infty A(\omega)\cos x\omega d\omega = \frac{1}{1+x^2}$.

Supplementary problems from "Advanced Engg. Maths. by E. Kreyszig (8^{th} Edn.)"
(i) Section 4.8, Q. 1,7
(ii) Chapter 4, Review Problems, Q. 32,33
(iii) Section 10.1, Q. 7,10,11,17
(iv) Section 10.2, Q. 1,2,9,10
(v) Section 10.3, Q. 8,11
(vi) Section 10.4, Q. 11, 13,21,25
(vii) Section 10.8, Q. 2,3,9,15