The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-II ■ Assignment 3

---

1. $C[a,b]$ denotes the vector space of all continuous functions on the closed interval $[a,b]$, then for $f, g \in C[a,b]$ define the inner product as: $< f,\ g >= \int_a^b f(t)g(t)dt$................$(I)$ Show that postulates of inner product hold in (I).

   (a) For $f(t) = t+2$, $g(t) = t^2 - 3t + 4$, $a = -1$ and $b = 1$, compute $< f, g >$, $||f||$, and $||g||$.

   (b) For $f(t) = 3t - 5$, $g(t) = t^2$, $a = 0$ and $b = 1$, compute $< f, g >$, $||f||$, and $||g||$.

   (c) Verify the Cauchy -Schwarz inequality for the vector f and g in (a) and (b).

2. Verify that the following is an inner product on R^2, where $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$:

   (a) $< \alpha, \beta >= x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$

   (b) $< \alpha, \beta >= 5x_1y_1 - x_2y_2$

   (c) $< \alpha, \beta >= x_1y_1 + x_2y_2 + 5$

   (d) $< \alpha, \beta >= x_1y_1 + x_1y_2 + x_2y_1 + 3x_2y_2$

   (e) $< \alpha, \beta >= x_1x_2 + 5y_1y_2$

3. Find the value of a so that the following is an inner product on R^2, where $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$:
$$< \alpha, \beta >= x_1y_1 - 3x_1y_2 - 3x_2y_1 + ax_2y_2$$

4. Show that the norm of a vector in a vector space V has the following three properties
(a) $\|v\| \geq 0$ and $\|v\| = 0$ if and only if $v = 0$.
(b) $\|\lambda v\| = |\lambda|\|v\|$ for all $\lambda \in \mathbb{R}$ and $v \in V$.
(c) $\|v + w\| \leq \|v\| + \|w\|$ for all $v, w \in V$.
Note that $\| \cdot \| =< \cdot, \cdot >^{1/2}$, where $< \cdot, \cdot >$ is an inner product defined on V.

5. Let U be the subspace of R^4 spanned by $\{(1,1,1,1), (1,-1,2,2), (1,2,-3,-4)\}$. Then by using Gram-Schmidt process find the orthonormal basis and orthonormal basis under the usual inner product on R^4.

6. Use Gram-Schmidt process to transform each of the following into an orthonormal basis;

   (a) $\{(1,0,1), (1,0,-1), (0,3,4)\}$ for $\mathbb{R}^3$ with the standard inner product.

   (b) $\{(1,0,1), (1,0,-1), (0,3,4)\}$ for $\mathbb{R}^3$ using the inner product defined by $< (x,y,z), (x',y',z') >= xx' + 2yy' + 3zz'$.

7. Consider the vector space $P(t) = \{a_0 + a_1t + a_2t^2 : a_0, a_1, a_2 \in R\}$ with inner product $< f,\ g >= \int_0^1 f(t)g(t)dt$ Apply the Gram-Schmidt algorithm to the set $\{1, t, t^2\}$ to obtain an orthogonal set $\{f_0, f_1, f_2\}$ with integer coefficients.