Assignment-3

Ans 1 first let us find the n-bit words containing no two consecutive 1's corressponding to $n = 1, 2, 3, 4$:

$n=1$	$n=2$	$n=3$	$n=4$
0	00	000	0000
1	01	001	0001
	10	010	0010
		100	0100
		101	0101
			1010
			1001
			1000

$a_1 = 2$

$a_2 = 3$

$a_3 = 5$

$a_4 = 8$

Now consider arbitrary n-bit word. It may end in 0 or 1.

case 1 suppose n bit word ends in 0. Then the (n-1)st bit can be a 0 or a 1, so there are no restrictions on the (n-1)st bit.

$$\overbrace{\quad 0}^{n \text{ bit}}$$

$$\underbrace{\quad}_{\text{no restriction}} 1\ 0 \quad \text{— (n-1)st bit}$$

~~(n-1) bit~~

Therefore, a_{n-1} n bit words end in 0 and contain no consecutive ones. 1's.

case 2: Suppose n-bit word end in 1. Then the (n-1)st bit must be a zero. Further, there are no restrictions on the (n-1)nd bit

$$\overbrace{- - - 0}^{n \text{ bit}}$$

$$\underbrace{- - -}_{\text{no restriction}} 1\ 0\ 1 \leftarrow \text{(n-2)st bit}, \quad (n-1)$$

Thus, a_{n-2} n bit words end in 1 and contain no two consecutive 1's.

Since, the two cases are mutually exculsive, by the addition principle, we have,

$a_1 = 2 \quad a_2 = 3$ Initial conditions.

$a_n = a_{n-1} + a_{n-2} \quad n \geq 3$ recurrence relation.

recurrence relation is same as the fibonacci recurrence relation, but with different initial conditions.

Qu 6

(c)

for i=1 to n do
  for j=1 to $\lfloor i/2 \rfloor$ do
    $x \leftarrow x+1$

Case 1 Let $n=1$ then the statement $x \leftarrow x+1$ is executed zero times. So $a_1 = 0$

Case 2 Let $n>1$ and even when n is even $\lfloor n/2 \rfloor = n/2$

$a_n = a_{n-1} + n/2$

Case 3 Let $n>1$ and odd. when $\lfloor n/2 \rfloor = (n-1)/2$.

$a_n = a_{n-1} + (n-1)/2$

Sol^n (2) $L_1 = 1 \quad L_2 = 3$

$L_n = L_{n-1} + L_{n-2} \quad n \geq 3$

$L_3 = L_{3-1} + L_{3-2} = L_2 + L_1$

$= 3+1 = 4$

$L_4 = L_3 + L_2 = 4+3 = 7$

$L_5 = L_4 + L_3 = 7+4 = 11$

$L_6 = L_5 + L_4 = 11+7 = 18$

first 6 Lucas numbers 1, 3, 4, 7, 11, 18.

Sol^n (3)

$$gcd(x,y) = \begin{cases} gcd(y,x) & \text{if } y>x \\ x & \text{if } y \leq x \text{ and } y=0 \\ gcd\{y, x \bmod y\} & \text{if } y \leq x \text{ and } y>0 \end{cases}$$

$gcd\{\underset{x}{28}, \underset{y}{18}\}$ Here $y \leq x$ and $y>0$ Cond^n 3

$= gcd\{18, 28 \bmod 18\}$

$= gcd\{18, 10\}$ Here $x=18$ $y=10$ $y \leq x$

$= gcd\{10, 18 \bmod 10\} = gcd\{10, 8\}$

$= \gcd\{10, 8\}$ $\quad x = 10 \quad y = 8 \quad y \le 8$ cond③

$= \gcd\{8, 10 \bmod 8\}$

$= \gcd\{8, 2\}$

$= \gcd\{2, 8 \bmod 2\} = \gcd\{2, 0\}$ $\quad x = 2 \quad y = 0$

$= 2 \quad \Rightarrow$ {condition 2nd satisfy}.

Another Example.

$\gcd(24, 75) = \gcd(y, x)$ $\quad y \ge x$ condⁿ①

$= \gcd(75, 24)$

$= \gcd(24, 75 \bmod 24)$

$= \gcd(24, 3)$

$= \gcd(3, 24 \bmod 3)$

$= \gcd(3, 0)$

$= 3.$

Solⁿ ④ ⓐ $1, 4, 7, 10, 13$

$a_1 = 1$

$a_2 = 4 = 1 + 3 = a_1 + 3$

$a_3 = a_2 + 3 = 4 + 3 = 7$

$a_4 = a_3 + 3 = 7 + 3 = 10$

$a_5 = a_4 + 3 = 10 + 3 = 13$

Recursive ~~dt~~ definition

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + 3 & n > 1 \end{cases}$$

ⓑ $1, 2, 5, 26, 677$

$a_1 = 1$

$a_2 = 1 + 1 = 1^2 + 1 = 2$ $\quad [a_1^2 + 1]$

$a_3 = a_2^2 + 1 = 2^2 + 1 = 4 + 1 = 5$

$a_4 = a_3^2 + 1 = 5^2 + 1 = 25 + 1 = 26$

$a_5 = a_4^2 + 1 = 26^2 + 1 = 676 + 1 = 677$

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1}^2 + 1 & n \ge 1 \end{cases}$$

Q5 $f(x) = \begin{cases} x-10 & \text{if } x > 100 \\ f(f(x+11)) & \text{if } 0 \le x \le 100 \end{cases}$

a) $f(99) = f(f(99+11))$
$= f(f(110))$
$= f(110-10) = f(100)$
$= f(f(100+11))$
$= f(f(111)) = f(111-10)$
$= f(101) = 101-10$
$= 91$

b) $f(\underline{f(99)}) = f(91)$
$= f(f(91+11))$
$= f(f(102))$
$= f(102-10) = f(92)$
$= f(f(92+11)) = f(f(103)) = f(103-10)$
$= f(93) \ldots\ldots$
$\vdots$
$f(99)$
$\vdots$
$\Rightarrow 91$ Ans.

Sol 6 (a) $x = 0$

for $i = 1$ to n do
 for $j = 1$ to i do
  $x \leftarrow x+1$

first the initial condition satisfied by a_n when $n = 1$ $i = j = 1$ so the assignment stmt executed exactly once. Thus

$a_1 = 1$

to find recurrence relation satisfied by a_n

let $n \geq 2$ As i runs from 1 to $n-1$ by definition, the statement is executed a_{n-1} times.

when $i = n$, the inner loop becomes.

for j=1 to i do
x ← x+1

for each value of j, where $1 \leq j \leq i$ the innermost loop execute i times

$$a_n = \begin{bmatrix} \text{no of times stmt} \\ \text{executed as } i \text{ runs from} \\ 1 \text{ through } n-1 \end{bmatrix} + \begin{bmatrix} \text{no of times stmt} \\ \text{executed as } j \text{ runs} \\ \text{from } 1 \text{ to } i \end{bmatrix}$$

$$= a_{n-1} + n$$

$$\boxed{\begin{array}{l} a_1 = 1 \\ a_n = a_{n-1} + n \quad , \quad n \geq 2 \end{array}}$$

(b) for i=1 to n do
for j=1 to i do
for k=1 to i do
x ← x+1

Initial condition $n=1$ $i=j=k=1$ $a_1 = 1$.

recurrence relation,

let $n \geq 2$ i runs from 1 to $n-1$ by defn

stmt executed a_{n-1} times.

$i=n$ the inner loop becomes.

for j=1 to i do
for k=1 to i do
x ← x+1

inner loop executed 2 times of i for every value of n

$$a_n = \begin{bmatrix} \text{no of times the stmt} \\ \text{executed as } i \text{ runs} \\ \text{from 1 to } n-1 \end{bmatrix} + \begin{bmatrix} \text{no of times stmt} \\ \text{executed when} \\ i = n \end{bmatrix}$$

$$= a_{n-1} + n^2 \qquad n \geq 2$$

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + n^2, & n \geq 2 \end{cases}$$

Que 7 $S(2,2)$ = condition 1 satisfy

Here $r = n$

$= 1$

solve for $S(5,2)$

exercise

Q8 a) $\underset{m\ n}{A(0,7)} = n+1 = 7+1 = 8$

b) $\underset{m\ n}{A(4,0)} = A(3,1)$ if $n=0$

$= A(2, A(3,0))$

$= A(2, A(2,1))$

$= A(2, A(1, \underline{A(2,0)}))$

$= A(2, A(1, \underline{A(1,1)}))$

$= A(2, A(1, A(0, \underline{A(1,0)})))$

$= A(2, A(1, A(0, \underline{A(0,1)})))$

$= A(2, A(1, A(0,2)))$

$= A(2, A(1, 3))$

$= A(2, A(0, A(1,2)))$

$= A(2, A(0, A(0, A(1,1))))$

$= A(2, A(0, A(0, A(0, A(1,0)))))$

$= A(2, A(0, A(0, A(0, A(0,1)))))$

$= A(2, A(0, A(0, A(0,2))))$

9 (a) $a_0 = 0$

$a_n = a_{n-1} + 4n \quad n \geq 1$

$a_n = [a_{n-2} + 4(n-1)] + 4n$

$= [a_{n-3} + 4(n-2)] + 4(n-1) + 4n$

$= [a_{n-4} + 4(n-3)] + 4(n-2) + 4(n-1) + 4n$

$=$ ⋮

$= a_0 + 4[n + (n-1) + (n-2) + \cdots 1]$

$= a_0 + \cancel{4}^{2}\left[\frac{n(n+1)}{\cancel{2}}\right] = 2n(n+1)$

$= 0 + 2n(n+1)$

(b) $S_1 = 1$

$S_n = S_{n-1} + n^3 \quad n \geq 2$

$= S_{n-2} + (n-1)^3 + n^3$

$= S_{n-3} + (n-2)^3 + (n-1)^3 + n^3$

$= S_{n-4} + (n-3)^3 + (n-2)^3 + (n-1)^3 + n^3$

⋮

$= S_1 + [\cancel{(n-n)^3 + (n} \; n^3 + (n-1)^3 + (n-2)^3 + \cdots 2]$

$\left[\frac{n(n+1)}{2}\right]^2$

$= 1 + [n^3 + (n-1)^3 + (n-2)^3 + \cdots + 2]$

$= n^3 + (n-1)^3 + (n-2)^3 + \cdots 2 + 1$

$= 1^3 + 2^3 + 3^3 + \cdots (n-2)^3 + (n-1)^3 + n^3$

$= \left(\frac{n(n+1)}{2}\right)^2$

(c) $a_1 = 1$

$a_n = 2a_{n-1} + (2^n - 1) \quad n \geq 2$

$a_n = 2[2a_{n-2} + 2^{n-1} - 1] + 2^n - 1$

$= 2[2[2a_{n-3} + 2^{n-2} - 1] + 2^{n-1} - 1] + 2^n - 1$

$= 2[2 \cdot 2\, a_{n-3} + 2 \cdot 2^{n-2} - 2 \cdot 1 + 2^{n-1} - 1] + 2^n - 1$

$= 2^3 a_{n-3} + 2^2 2^{n-2} - 2^2 \cdot 1 + 2 \cdot 2^{n-1} - 2 \cdot 1 + 2^n - 1$

$= 2^3 a_{n-3} + 2^2 2^{n-2} + 2^1 2^{n-1} + 2^0 2^n - 2^2 \cdot 1 - 2^1 \cdot 1 - 2^0 \cdot 1$

$= 2^3 a_{n-3} + \underbrace{2^n + 2^n + 2^n} - \underbrace{2^2 1 - 2^1 1 - 2^0 \cdot 1}$

$= 2^3 a_{n-3} + 3 \cdot 2^n - [2^2 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1]$

G.P

a, r^{n-1}

⋮

$= 2^n a_1 + n 2^n - [1 \times 2^{n-1}]$

$= 2^n a_1 + n 2^n - 2^{n-1}$

$= 2^n + n 2^n - 2^{n-1}$

$= 2^n (n+1) - 2^{n-1}$