8696733642

Q (1) $\lambda = \frac{h}{p}$

$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$ = [illegible]

$E = 40\,eV \times 10^3$ (Given)

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 40 \times 1.6 \times 10^{-19} \times 10^3}}$$

$= 3.$[illegible] nm

Q 2. $v_g = \frac{d\omega}{dk}$ (Group velocity)

$$\omega = 2\pi\nu = 2\pi\left(\frac{\gamma mc^2}{h}\right) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$k = \frac{2\pi}{\lambda} = 2\pi\left(\frac{p}{h}\right) = 2\pi\left(\frac{(\gamma m)v}{h}\right)$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = \frac{-\frac{1}{2h} \cdot \frac{2\pi m \cdot (-2v/c^2)}{(1 - v^2/c^2)^{3/2}}}{\frac{2\pi m \cdot 2v}{2hc^2(1 - v^2/c^2)^{3/2}}}$$

$$\boxed{v_g = v}$$ → particle velocity

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Q3.

$$\boxed{n = \frac{c}{v_p} = \frac{ck}{\omega}} \quad \left(v_p = \frac{\omega}{k}\right) \quad \lambda = 2\pi/k$$

$$\frac{dn}{d\lambda} = \frac{dn/dk}{d\lambda/dk}$$

$$\frac{dn}{dk} = \frac{c}{\omega} - \frac{ck}{\omega^2}\frac{d\omega}{dk}$$

$$\frac{d\lambda}{dk} = -\frac{2\pi}{k^2} = \cancel{-\frac{k}{\lambda}} - \frac{\lambda}{k} \quad \left(\lambda = \frac{2\pi}{k}\right)$$

$$\left(v_g = \frac{d\omega}{dk}\right)$$

$$\therefore \frac{dn}{d\lambda} = -\frac{k}{\lambda}\left(\frac{c}{\omega} - \frac{ck}{\omega^2}v_g\right)$$

$$-\frac{\lambda\, dn}{k\, d\lambda} = \frac{c}{\omega} - \frac{ck}{\omega^2}v_g$$

$$\Rightarrow v_g\frac{kc}{\omega^2} = \frac{c}{\omega} + \frac{\lambda}{k}\cdot\frac{dn}{d\lambda}$$

$$v_g\frac{kc}{\omega^2} = \left(\frac{c}{\omega} + \frac{\lambda \cdot c}{\omega n}\frac{dn}{d\lambda}\right)$$

$$\frac{v_g}{v_p} = \left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$$

$$\therefore \boxed{v_g = v_p\left(1 + \frac{\lambda}{n}\cdot\frac{dn}{d\lambda}\right)}$$

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Q4. $\Delta x \times \Delta p \geq \frac{\hbar}{2}$ $\left(\hbar = \frac{h}{2\pi}\right)$

~~$\Delta x = 1.00$ nm~~

For e^- $\Delta x \times m_e \times \Delta V_{e^-} = \frac{h}{4\pi}$ — (1)

For p^+, $\Delta x \times m_p \times \Delta V_p = \frac{h}{4\pi}$ — (2)

$\therefore$ (1) / (2).

$$\frac{\Delta V_{e^-}}{\Delta V_p} = \frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}\ Kg}{9.1 \times 10^{-31}\ Kg}$$

$$\boxed{\frac{\Delta V_{e^-}}{\Delta V_p} = 1835}$$

Q5. Notes

1) ψ must be continuous & single valued.

2) $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous & single valued.

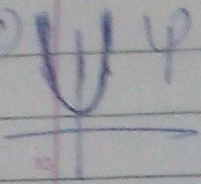
3) ψ must be normalizable which means that ψ must go to 0 as $x \to \pm\infty$. In order to that $\int |\psi|^2 dV$ over all the space be in finite constant.

~~4) One can multiply $\psi(x,t)$ by a constant say N, so~~

Q6

(a). $\psi = A\sec x$
Not a wave f^n (because it is discontinuous)

(b): $\psi = A\tan x$
Not a wave f^n (discont.)

(c)  $\psi = Ae^{x^2}$
It is not a wave f^n (does not satisfies the conditions that at $x \to \infty$ $\psi \to 0$)

(d) $\psi = Ae^{-x^2}$

It is a wave f^n (it satisfies condition that at $x \to \infty$, $\psi = 0$)

(e) $\psi = A\sin x$
It is a wave f^n. (satisfies cont. & diff.)

Q7. TDSE — For $\psi = Ae^{-\frac{i}{\hbar}(Et - px)}$ function only

$$i\hbar \frac{\partial \psi(x)}{\partial t} = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x)$$

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$$E\psi(x) = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x)$$

Q10 (a). As $x \to \infty$, the defined $\psi(x)$ is 0. (given)

also $\int |\psi|^2 dv = \int_0^L N^2 x^2 (L-x)^2 dx$ would be a finite constant.

$\frac{x}{12 \cdot 24 x 5}$ $z \to A + iB$ $\frac{29}{5}$ $\bar{z} \to A - iB$ $\frac{h}{i}$ $\hat{p} = -i\hbar \frac{d}{dx}$

(b) $\langle x \rangle = \int_0^L x \cdot |\psi|^2 dx$

$= \int_0^L x \cdot N^2 x^2 (L-x)^2 dx$

$= N^2 \int_0^L x^3 (L^2 + x^2 - 2Lx) dx$

$= N^2 \left[\frac{L^2 x^4}{4} + \frac{x^6}{6} - \frac{2Lx^5}{5} \right]_0^L$

$= \frac{N^2}{60}$

$\langle x^2 \rangle = \int_0^L x^2 |\psi|^2 dx$

$=$ Solve it.

$\hat{E}\psi(x,t) = \hat{H}\psi(x,t)$

$+i\hbar \frac{\partial}{\partial t}$ $\quad -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$

$\Delta x = \sqrt{(\langle x^2 \rangle - \langle x \rangle^2)}$

(c) $p_x = -i\hbar \frac{\partial \psi}{\partial x}$

~~$\langle p_x \rangle =$~~

$\langle p_x \rangle = \int_0^L \psi^* p_x \psi \, dx$ $\quad |\psi|^2 \to \psi^* \times p_x \psi$

$\int_a^b p_x |\psi|^2 dp_x$

$\int_a^b \psi^* p_x \psi \, dp_x$

$= \int_0^L \left[Nx(L-x) \cdot \left(-i\hbar \frac{d(Nx(L-x))}{dx} \right) \right] dx$

$= \frac{-i}{\hbar} \int_0^L Nx(L-x)(NL - 2Nx) dx$

$= \frac{-iN^2}{\hbar} \int_0^L (L^2 x - 2x^2 L - x^2 L + 2x^3) dx$

$$= \frac{-iN^2}{\hbar}\left\{ \frac{L^2x^2}{2} - \frac{2x^3L}{3} + \frac{x^3}{3} + 2\frac{x^4}{4} \right\}_0^L$$

$$= \frac{-iN^2L^4}{\hbar}(1-1) = 0$$

$$\langle p_x^2 \rangle = \int_0^L \psi^* p^2 \psi \, dx$$

$$\Delta p = \sqrt{\langle p_x^2 \rangle - \langle p_x \rangle^2}$$

(d) $$\langle E \rangle = \int_0^L \psi^* \left(\frac{-\hbar^2}{2m} \hat{p}^2 \right) (\psi(x)) \, dx$$

$$= \int_0^L \psi^* \left(\frac{-\hbar^2}{2m} \right) \left(\frac{\partial^2 (\psi x)}{\partial x^2} \right) dx$$

$$= \int_0^L Nx(L-x) \left(\frac{-\hbar^2}{2m} \right) \left[-2N \right] dx$$

$$= \frac{+\hbar^2 N^2}{m} \int_0^L x(L-x) \, dx$$

$$= \frac{\hbar^2 N^2}{m} \left[\frac{x^2 L}{2} - \frac{x^3}{3} \right]_0^L$$

$$= \frac{\hbar^2}{m} N^2 \frac{L^3}{6}$$

or simply by using $\langle E \rangle = \frac{\langle p^2 \rangle}{2m}$

Q.(11)

$$\psi(x) = A e^{(iax - ibt)}$$

$$\langle p_x \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\hbar}{i} \left(\frac{\partial}{\partial x} \psi(x,t) \right) dx.$$

$$\langle p_x^2 \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\hbar}{i} \left(\frac{\partial^2}{\partial x^2} \psi(x,t) \right) dx$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \underline{\qquad\qquad}$$

(Lazy enough to solve another ques with the same method)

$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}.$$

Q(12.)

$x = 0.1\,nm$

$$E = \frac{2\pi^2 m k^2 z^2 e^4}{n^2 h^2} = \frac{h^2 n^2}{8 m r^2}$$

$$v = \frac{2\pi k z e^2}{n h}, \quad \underline{\underline{r}} = \frac{n^2 h^2}{4\pi^2 m z e^2 k}, \quad \underline{r = x = 0.1\,nm}$$

(a) $\underset{(\Delta E)}{E_2 - E_1} = \frac{h^2}{8 m r^2}\left((2)^2 - (1)^2\right) = \frac{3h^2}{8 m r^2}$

(b). $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h}$$

$$\& \quad \lambda = \frac{hc}{\Delta E}$$

$\Delta p \quad (\langle p^2 \rangle)$

$\int_{-\infty}^{\infty} \psi \frac{\hbar}{i} \left(\frac{\partial}{\partial x} \ldots \right.$