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M-II

Assignment-1

Ans1 Let dimention of A be $m \times n$ & B be $n \times l$
(∵ AB is defined)

$$A = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & & \vdots \\ a_{1m} & a_{2m} & \cdots & a_{nm} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{21} & \cdots & b_{l1} \\ b_{12} & b_{22} & \cdots & b_{l2} \\ \vdots & \vdots & & \vdots \\ b_{1n} & b_{2n} & \cdots & b_{ln} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \qquad B^T = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{l1} & b_{l2} & \cdots & b_{ln} \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} \sum_1^n a_{i1} b_{1i} & \sum_1^n a_{i1} b_{2i} & \cdots & \sum_1^n a_{i1} b_{li} \\ \sum_1^n a_{i2} b_{1i} & \sum_1^n a_{i2} b_{2i} & \cdots & \sum_1^n a_{i2} b_{li} \\ \vdots & \vdots & & \vdots \\ \sum_1^n a_{im} b_{1i} & \sum_1^n a_{im} b_{2i} & \cdots & \sum_1^n a_{im} b_{li} \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} \\ \\ ,, \\ \\ \\ \end{bmatrix}$$

Hence $(AB)^T = B^T A^T$

Ans 2 $(AB)^{-1} = B^{-1}A^{-1}$ (To Prove)

Multiply AB both side

$(AB)(AB)^{-1} = (AB)B^{-1}A^{-1}$

$\Rightarrow A(BB^{-1})A^{-1} = I$ $[\because AA^{-1} = I$ & $(AB)C = A(BC)]$

$\Rightarrow AIA^{-1} = I$

$\Rightarrow AA^{-1} = I$ $[\because AI = A]$

$\Rightarrow I = I.$

Hence Proved.

Ans3 ① For a matrix A, let A^T be its transpose.

$\therefore$ a) $(A + A^T)^T = A^T + (A^T)^T$ $[\because (A+B)^T = A^T + B^T]$

$= A^T + A$ $[\because (A^T)^T = A]$

b) $(A - A^T)^T = A^T - (A^T)^T$

$= A^T - A = -(A - A^T)$

Clearly, $A+A^T$ is symmetric & $A-A^T$ skew-symmetric

Also, we can write

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

Hence, any matrix can be represented as the sum of symmetric & skey-symmetric matrix.

② $A = A^T$, $B = B^T$

[∵ A&B are sym. matrix]

$\therefore (AB)^T = B^T A^T$

$= BA$

$= AB$ ~~[given]~~ [if $AB = BA$]

Clearly, AB is sym. iff $AB = BA$.

$$\begin{array}{r}104\\ -75\\ \hline 29\end{array}$$

Ans4 We know,

$|A| = |A^T|$, $|AB| = |A||B|$ & $|I| = 1$

given, $AA^T = I$

Taking determinant Both side

$$|AA^T| = |I|$$
$$\Rightarrow |A||A^T| = 1$$
$$\Rightarrow |A|^2 = 1$$
$$\Rightarrow |A| = \pm 1$$

Hence proved.

Ans5 given A is a nilpotent matrix.

$$\Rightarrow A^m = 0 \quad \text{for } m \geq 1$$

To prove: A+I is invertible.

$$(A+I)(A+I)^{-1} = (A+I)\,[I - A + A^2 + ----- + (-1)^n A^m]$$
$$= I - \cancel{A} + \cancel{A^2} + ---- + \cancel{(-1)^m A^m}$$
$$+ \cancel{A} - \cancel{A^2} + ----- + (-1)^m A^{m+1}$$
$$= I + \cancel{(-1)^m A^{m+1}}^{\,0} \quad [\because A^m = 0$$
$$\Rightarrow (A+I)(A+I)^{-1} = I$$
$$\Rightarrow |A+I|\,|(A+I)^{-1}| = 1 \neq 0$$

$\therefore$ $|A+I| \neq 0$, hence A+I is invertible.

Ans6 a)

$$\begin{bmatrix}4 & 3\\ -8 & -6\\ 16 & 12\end{bmatrix} \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 + 2R_1} \begin{bmatrix}4 & 3\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Big\downarrow R_1 \to R_1/4$$

row echelon form & row reduced echelon form $\longleftarrow$ $\begin{bmatrix}1 & 3/4\\ 0 & 0\\ 0 & 0\end{bmatrix}$

b)

$$\begin{bmatrix}1 & 5 & 8\\ 3 & 2 & 9\end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix}1 & 5 & 8\\ 0 & -13 & -15\end{bmatrix}$$

$$\Big\downarrow R_2 \to R_2/-13$$

row reduced echelon form $\begin{bmatrix}1 & 0 & 29/13\\ 0 & 1 & 15/13\end{bmatrix} \xleftarrow{R_1 \to R_1 - 5R_2} \begin{bmatrix}1 & 5 & 8\\ 0 & 1 & 15/13\end{bmatrix} \to$ row echelon form

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c) $$\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} -3 & 5 & 0 \\ 2 & 0 & 5 \\ 0 & 2 & -3 \end{bmatrix}$$

$$\xrightarrow[\text{b) } R_2 \to R_2 - 2R_1]{\text{a) } R_1 \to R_1 + 2R_2} \begin{bmatrix} 1 & 5 & 10 \\ 0 & -10 & -15 \\ 0 & 2 & -3 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_2]{R_2 \to R_2/-5} \begin{bmatrix} 1 & 5 & 10 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{bmatrix}$$

$$\xrightarrow[R_3 \to R_3/-6]{R_2 \to R_2/2} \begin{bmatrix} 1 & 5 & 10 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[\text{b) } R_2 \to R_2 - \frac{3}{2}R_3 \text{ c) } R_1 \to R_1 - \frac{5}{2}R_3]{\text{a) } R_1 \to R_1 - 5R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

row echelon form ↓ (middle matrix)   row reduced E.F. ↓ (last matrix)

d) $$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 3 & 6 & 7 \end{bmatrix}$$

$$\xrightarrow[\text{b) } R_3 \to R_3 - 3R_2]{\text{a) } R_2 \to R_2/2} \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix} \xrightarrow{R_3 \to R_3/-2} \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Row echelon form

$$\xrightarrow[\text{b) } R_2 \to R_2 - 3R_3]{\text{a) } R_1 \to R_1 + 3R_2} \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Row reduced echelon form

Ans 7 $[A \vdots B]$

(I) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ [illegible]0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right]$$

a) [illegible]

~~b) $R_3 \to R_3/10$~~

c) $R_3 \to R_3/5$

d) $R_2 \to R_2 + R_1$

e) $R_2 \leftrightarrow R_4$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 1 & 0 & 8 \\ 0 & 2 & 5 & 18 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 3 & -2 & 8 \\ 0 & 2 & 5 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2/3$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -2/3 & 8/3 \\ 0 & 0 & 19/3 & 38/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to \frac{3R_3}{19}$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -2/3 & 8/3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↳ row echelon form

$R_2 \to R_2 + \frac{2}{3} R_3$

$R_1 \to R_1$ [illegible] R_3

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↳ row reduced echelon form

(II) $$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 3 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$R_1 \to R_1/3$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + 3R_2$

$R_2 \to -3R_2$

$R_3 \to R_3/3$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$$

↳ row echelon

3) 1

2) $R_2 \to R_2 + R_3$

1) $R_1 \to R_1 - \frac{2}{3} R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 4/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$$

↳ row reduced echelon form

(III) $\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 4 & -3 \end{array}\right] \xrightarrow[\text{before}]{\text{same as}}$

Ans 8 a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 4 & 0 & 5 \end{bmatrix}$ $[A|b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{array}\right]$

Since, no. of variables = no. of distinct eq^n(s).
Hence, system has unique solution.

b) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{array}\right] \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & -4 & 1 & -18 \end{array}\right]$

$R_3 \to R_3 + 4R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -10/13 \end{array}\right] \xleftarrow{R_3 \to R_3/13} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -10 \end{array}\right]$

$\therefore$ $z = -10/13$

$y + 3z = -2 \Rightarrow y = 4/13$

$x + y + z = 5 \Rightarrow x = 5 + 6/13 = \frac{71}{13}$

Ans 9 ~~(I)~~ $\left[\begin{array}{cc|c} 1 & h & 2 \\ 4 & 8 & k \end{array}\right]$

(I) a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{4} = \frac{h}{8} \neq \frac{2}{k}$

$\Rightarrow h = 2$ & $k \neq 8$

b) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{4} \neq \frac{h}{8}$

$\Rightarrow h \neq 2$ & $k \in R$

c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{4} = \frac{h}{8} = \frac{2}{k}$

$\Rightarrow h = 2$ & $k = 8$

(II) a) $\frac{1}{3} = \frac{3}{h} \neq \frac{2}{k}$

$\Rightarrow h = 9$ & $k \neq 6$

b) $\frac{1}{3} \neq \frac{3}{h}$

$h \neq 9$ & $k \in R$

c) $\frac{1}{3} = \frac{3}{h} = \frac{2}{k}$

$\Rightarrow h = 9$ & $k = 6$

Ans10 We know that, $AI = A$, so define an augmented matrix $[A : I]$

a) $$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \xrightarrow[R_1 \to R_1 - 3R_2]{R_2 \to R_2 - 2R_1} \left[\begin{array}{cc|cc} 1 & 0 & -5 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -5 & -3 \\ -2 & 1 \end{bmatrix}$$

b) $$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 + R_2$
$R_1 \to R_1 + 2R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & 8 & -2 & -1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array}\right] \xleftarrow[R_2 \to R_3 - R_2]{R_3 \to -R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -11 & 2 & 2 \\ 8 & -2 & -1 \\ 6 & -1 & -1 \end{bmatrix}$$

c) $$\left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ -1 & -1 & 5 & 0 & 1 & 0 \\ 2 & 7 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 = R_3 - 2R_1 - 3R_2]{R_2 = R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & -5 & -3 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_2 + 3R_3$
$R_3 \to R_3/2$
$R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -16 & -11 & 3 \\ 0 & 1 & 0 & 7/2 & 5/2 & -1/2 \\ 0 & 0 & 1 & -5/2 & -3/2 & 1/2 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -16 & -11 & 3 \\ 7/2 & 5/2 & -1/2 \\ -5/2 & -3/2 & 1/2 \end{bmatrix}$$

d) $A = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{bmatrix}$

$|A| = (-30+13) - 3(-6+3) - 4(13-15)$

$= -17 + 9 + 8$

$= 0$

Hence, inverse is not possible.