1. Define the domain in which the following partial differential equations can be classified as either elliptic or parabolic or hyperbolic:

   (a) $yu_{xx} = xu_{yy}$

   (b) $u_{yy} - xu_{xy} + yu_x + xu_y = 0$

   (c) $y^2 u_{xx} + 2xy u_{xy} + x^2 u_{yy} = 0$

   (d) $u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} = 0$

2. Reduce the following equations to canonical form:

   (a) $u_{xx} - x^2 y u_{yy} = 0, \qquad (y > 0)$

   (b) $e^{2x} u_{xx} + 2e^{x+y} u_{xy} + e^{2y} u_{yy} = 0$

   (c) $x^2 u_{xx} + y^2 u_{yy} = 0, \qquad (x > 0, y > 0)$

   (d) $u_{xx} + 2u_{xy} + 5u_{yy} = xu_x.$

3. Reduce the following equations to canonical forms and hence find solutions:

   (a) $u_{xx} + 2\sqrt{3} u_{xy} + u_{yy} = 0$

   (b) $x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} + xu_x + yu_y = 0$

   (c) $u_{xx} - 2\sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$

   (d) $u_{xx} + 4u_{xy} + 4u_{yy} = 0$

4. Using D'Alembert's formula, solve the Cauchy problem

$$u_{tt} - c^2 u_{xx} = 0, \qquad x > 0, \, t > 0,$$

   with initial conditions

$$u(x, 0) = g(x), \quad u_t(x, 0) = h(x), \quad x > 0,$$

   and boundary conditions

$$u(0, t) = 0, \qquad t \geq 0.$$

5. Solve the Cauchy problem

$$u_{tt} = 16 u_{xx}, \qquad -\infty < x < \infty, \, t > 0,$$

$$u(x, 0) = 6 \sin^2 x, \quad u_t(x, 0) = \cos 6x, \quad -\infty < x < \infty.$$

6. Solve the wave equation (Hint: Duhamel's principle):

$$u_{tt} = u_{xx} + x^2 - t, \qquad -\infty < x < \infty, \, t > 0,$$

$$u(x, 0) = u_t(x, 0) = 0, \quad -\infty < x < \infty.$$