The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-III
Practice Problems Set #1

---

1. Express the following complex numbers in the form $x + iy$, where x, y are real numbers

   $(i)\ (1+i)^{-1}$ $(ii)\ \dfrac{i}{1+i}$ $(iii)\ \dfrac{1}{-1+i}$ $(iv)\ \dfrac{5+5i}{3-4i}$ $(v)\ \dfrac{3i^{30}-i^{19}}{2i-1}$

2. Perform the indicated operations both analytically and graphically
   (i) (3+4i) + (5+2i) $(ii)\ (6-2i)-(2-5i)$

3. Prove the following relations for any two complex numbers z, w and interpret them geometrically:

   $(i)\ |z+w| \le |z|+|w|$ $(ii)\ |z+w| \ge ||z|-|w||$ $(iii)\ |z+w|^2+|z-w|^2 = 2(|z|^2+|w|^2)$

4. Find the principal argument $Arg\ z$, when
   $(i)\ z = \dfrac{-2}{1+\sqrt{3}i}$ $(ii)\ z=-2-2i$ $(iii)\ z=2-2i$ $(iv)\ z=-2+2i$ $(v)\ z=2+2i.$
   Find, sketch or graph in the complex plane (i) $|Arg\ z| \le \frac{\pi}{4}$ (ii) $Re(\frac{1}{z}) < 1$

Ans (1) $\frac{2\pi}{3}$, (ii) $\frac{-3\pi}{4}$, (iii) $\frac{-\pi}{4}$, (iv) $\frac{3\pi}{4}$, (v) $\frac{\pi}{4}$.

5. Express each of the following complex number in polar form and graph them

   (i) -5-5i (ii) $-\sqrt{6}-\sqrt{2}i$ (iii) 2-2$\sqrt{3}i$

Ans. $(i)\ 5\sqrt{2}(\cos\frac{-3\pi}{4}+i\sin\frac{-3\pi}{4})$ $(ii)\ 2\sqrt{2}(\cos\frac{-5\pi}{6}+i\sin\frac{-5\pi}{6})$ $(iii)\ 4(\cos\frac{-\pi}{3}+i\sin\frac{-\pi}{3})$

6. If $z_1 = r_1(\cos\theta_1 + isin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then evaluate z_1z_2 and $\frac{z_1}{z_2}$. Compute the following:

   $(i)\ [3(\cos 40^\circ + i\sin 40^\circ)][4(\cos 80^\circ + i\sin 80^\circ)]$ $(ii)\ \frac{2(\cos 15^\circ + i\sin 15^\circ)^7}{(\cos 45^\circ + i\sin 45^\circ)^3}$ $(iii)\ \left(\frac{2e^{\frac{\pi i}{3}}}{2e^{-\frac{\pi i}{3}}}\right)^{10}$

Ans. $z_1z_2 = r_1r_2\{\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\}$ and $\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\}$
$(i)\ -6+i6\sqrt{3}$ $(ii)\ \sqrt{3}-i$ $(iii)\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

7. By writing the individual factors on the left in exponential form, performing the needed operations, and finally changing back to rectangular coordinates, show that

   $(i)\ \dfrac{5i}{2+i} = 1+2i$ $(ii)\ (-1+i)^7 = -8(1+i)$ $(iii)\ \left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10} = -\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

8. Find each of the indicated roots and locate them graphically $(i)\ (-1+i)^{1/3}$ $(ii)\ (-2\sqrt{3}-2i)^{1/4}$
   Show that $\sqrt{z} = \pm\left\{\sqrt{\frac{1}{2}(|z|+x)} + sgn(y)i\sqrt{\frac{1}{2}(|z|-x)}\right\}$, where $sgn(y)$ represents signum function.

Ans. $(i)\ \sqrt[6]{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$, $\sqrt[6]{2}(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12})$, $\sqrt[6]{2}(\cos\frac{-5\pi}{12}+i\sin\frac{-5\pi}{12})$
$(ii)\sqrt{2}(\cos\frac{7\pi}{24}+i\sin\frac{7\pi}{24})$, $\sqrt{2}(\cos\frac{19\pi}{24}+i\sin\frac{19\pi}{24})$, $\sqrt{2}(\cos\frac{-5\pi}{24}+i\sin\frac{-5\pi}{24})$, $\sqrt{2}(\cos\frac{-17\pi}{24}+i\sin\frac{-17\pi}{24})$

9. Let $z = \frac{i}{-2-2i}$
   (a) Express z in polar form.
   (b) Express z^5 in polar and cartesian form.
   (c) Express $z^{1/5}$ in Cartesian form.

10. Solve the equation $Z^6+1 = \sqrt{3}i$.

Ans. $\sqrt[6]{2}e^{i40^\circ}$, $\sqrt[6]{2}e^{i100^\circ}$, $\sqrt[6]{2}e^{i160^\circ}$, $\sqrt[6]{2}e^{-i20^\circ}$, $\sqrt[6]{2}e^{-i80^\circ}$, $\sqrt[6]{2}e^{-i140^\circ}$

11. Prove the identity:
    (a) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$
    (b) $\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$, if $\theta \ne 0, \pm1, \pm2, \cdots$ (b) $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$
    (c) $\cos^4\theta = \frac{1}{3}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$