CODING THEORY
UNIT-III
Cyclic Redundancy Code
CSE 2052

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Cyclic Redundancy Code

This is an one of the common error detecting codes. For a k-bit block of bits the (n, k) CRC encoder generates $(n-k)$ bit long `frame check sequence`(FCS). Let use the following notations:

- $T =$ n-bit frame to be transmitted.
- $D =$ k-bit message block (information bits).
- $F =$ Predetermined divisor, a pattern of $(n-k+1)$ bis.

Note that predetermined divisor P should be able to devide the codeword T. Thus $\frac{T}{P}$ has no reminder.

Cont..

- D is the k-bit message block. Therefore, $2^{n-k}D$ amounts to shifting the k bits to the left by $(n-k)$ bits.
- The resulting string will be padded with zeros. The codeword T can be represented as

$$T = 2^{n-k}D + F$$

Cont..

- Divide $2^{n-k}D$ by P, we obtain
$$\frac{2^{n-k}D}{P} = Q + \frac{R}{P}$$
where Q is the quotient and $\frac{R}{P}$ is the reminder.
  - Let we use R as FCS, then $T = 2^{n-k}D + R$.
    - Divide both side by P, we obtain
$$\frac{T}{P} = \frac{2^{n-k}D+R}{P}$$
$$= \frac{2^{n-k}D}{P} + \frac{R}{P}$$
$$= Q + \frac{R}{P} + \frac{R}{P} = Q + \frac{R+R}{P} = Q$$

Cont..

Thus there is no reminder $i.e$, T is exactly divisible by P. To generate such FCS, we simply divide $2^{n-k}D$ by P and use $(n-k)$-bit reminder as FCS.

Error detection

- Let an error E occur, when T is transmitted over noise channel. The received word is given by

$$V = \ T + E$$

- The scheme will fail to detect the error only if V is completely divisible by P. This shows that E is completely divisible by P since T is divisible by P.

Example

let the message $D = 1010001101$, $i.\ k = 10$. Predetermine devisor $P = 110101$. The number of FCS bits $= 5$. Therfore $n = 15$. Determine FCS.

Solution: Multiply 2^5 with message D (left shift by 5 and pad with 5 zeros). This yields

$$2^5 D = 101000110100000$$

Divide by $P = 110101$. By long division, we obtain $Q = 1101010110$ and $R = 01110$. The reminder is added to $2^5 D$. This yields $T = 101000110101110$.
T is the transmitted codeword.
Note If no error occurs in the channel, the received word, when devided by P will yield 0 as reminder.