The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 7

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1. Solve the following IBVPs:

   (a)

$$\begin{aligned} u_{tt} - c^2 u_{xx} = 0, \quad & 0 < x < 2\pi,\ t > 0 \\ u(x,0) = \cos x - 1, u_t(x,0) = 0, \quad & 0 \le x \le 2\pi, \\ u(0,t) = u(2\pi,t) = 0, \quad & t \ge 0. \end{aligned}$$

   (b)

$$\begin{aligned} u_{tt} = c^2 u_{xx} - u, \quad & 0 < x < L,\ t > 0 \\ u(x,0) = f(x), u_t(x,0) = 0, \quad & 0 \le x \le L, \\ u(0,t) = u(L,t) = 0, \quad & t \ge 0. \end{aligned}$$

2. Using the method of separation of variables, solve the following heat equations:

   (a)

$$u_t = u_{xx} + u, \qquad 0 < x < 10, t > 0,$$
$$u(x,0) = 3\sin 2\pi x - 7\sin 4\pi x, \quad 0 \le x \le 10 \text{ and } u(0,t) = u(10,t) = 0.$$

   (b)

$$u_t = u_{xx}, \qquad 0 < x < 2, t > 0,$$
$$u(x,0) = \begin{cases} x & \text{if} \quad 0 \le x < 1 \\ 2 - x & \text{if} \quad 1 \le x \le 2 \end{cases} \quad \text{and } u_x(0,t) = u_x(2,t) = 0.$$

   (c)

$$u_t = u_{xx}, \qquad 0 < x < 2, t > 0,$$
$$u(x,0) = \cos^2 \pi x, \quad 0 \le x \le 2 \text{ and } u_x(0,t) = u_x(2,t) = 0.$$

3. Consider the homogeneous Dirichlet diffusion problem:

$$u_t - k u_{xx} = 0 \qquad 0 < x < L, 0 < t \le T,$$
$$u(x,0) = f(x), \quad 0 \le x \le L, \qquad u(0,t) = u(L,t) = 0, \quad 0 \le t \le T.$$

   Define $E(t) = \int_0^L u^2 dx$. Show that $E'(t) \le 0$. If $f(x) = x(2-x)$ and $L = 2$ in the above then show that $\int_0^2 u^2 dx \le \frac{16}{15}$ for $t \ge 0$.

4. Solve the heat equation

$$u_t = u_{xx}, \qquad 0 < x < L, t > 0,$$
$$u(x,0) = f(x), \quad 0 \le x \le L \text{ and } u(0,t) = T_1, \quad u(L,t) = T_2$$

   where T_1 and T_2 are constants.