Electrodynamics

(Physics-II, Module-B)

- **Review of Mathematical Tools** 6 lectures
- **Electrostatics** 4 lectures
- **Special techniques** 3 lectures
- **Concepts of Dipole** 2 lectures
- **Electric Field in Materials** 3 lectures
- **Magnetostatics** 4 lectures
- **Magnetic Field in Materials** 2 lectures
- **Electrodynamics** 1 lectures
- **Maxwell’s Equation** 1 lectures

Reference Books

1. Introduction to Electrodynamics by David. J. Griffiths

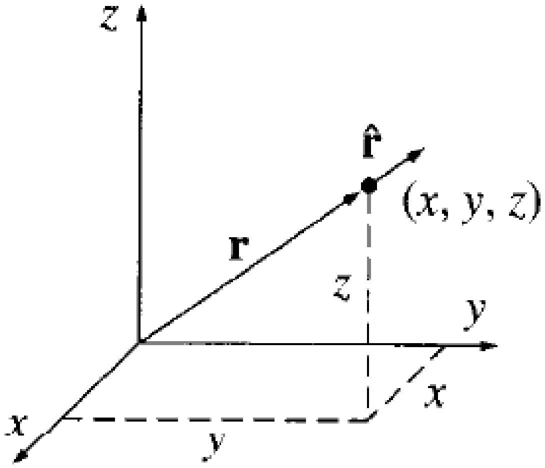
2. Classical Electrodynamics by John David Jackson

3. Electricity and Magnetism by Edward M. Purcell

	%
End Course Examination	40
Surprise quizzes and attendance and daily evaluation	10
Total	50

Vector Calculus

Position Vector, Separation Vector and Infinitesimal Displacement vector



$$\mathbf{r} \equiv x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}.$$

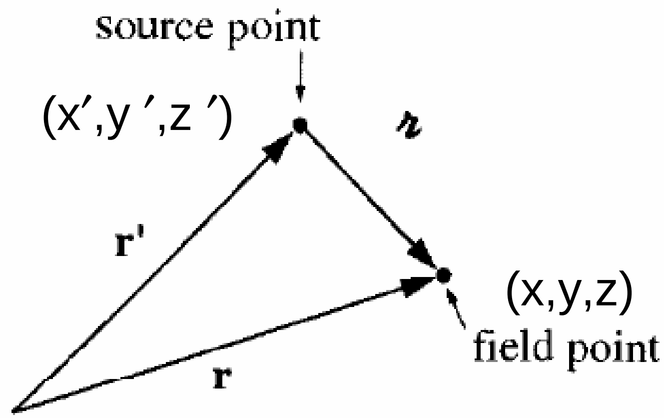
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{\sqrt{x^2 + y^2 + z^2}}$$

$$\boldsymbol{\mathscr{r}} \equiv \mathbf{r} - \mathbf{r}'$$

$$\mathscr{r} = |\mathbf{r} - \mathbf{r}'|$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}} = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$



$$\boldsymbol{\mathscr{r}} = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}},$$

$$\mathscr{r} = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2},$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{(x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$

infinitesimal displacement vector

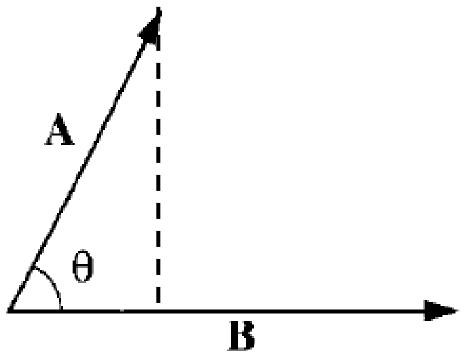
$$(x, y, z) \text{ to } (x+dx, y+dy, z+dz),$$

$$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

Dot product of two vectors

$$\mathbf{A} \cdot \mathbf{B} \equiv AB\cos\theta$$

$$\vec{A} \bullet \widehat{B} = \left|\vec{A}\right| \cos\theta$$

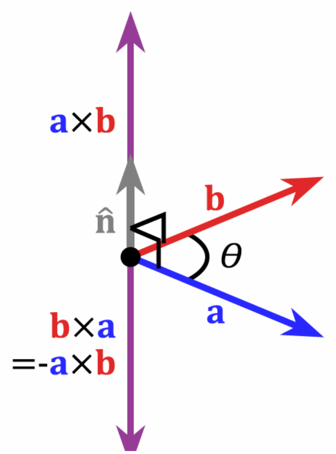


$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}) \cdot (B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}) \\ &= A_xB_x + A_yB_y + A_zB_z. \end{aligned}$$

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

$$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1; \quad \hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = 0$$

Cross product of two vectors



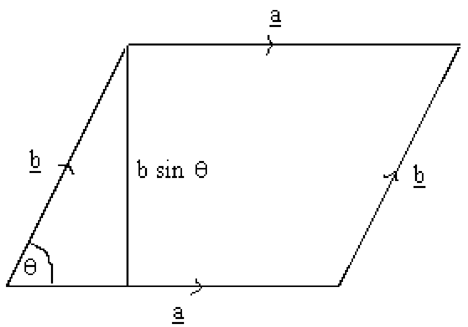
$$\mathbf{A} \times \mathbf{B} \equiv AB \sin\theta \, \hat{\mathbf{n}}$$

$$\mathbf{A} \times \mathbf{B} = (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}) \times (B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}})$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\hat{\mathbf{x}} + (A_zB_x - A_xB_z)\hat{\mathbf{y}} + (A_xB_y - A_yB_x)\hat{\mathbf{z}}$$

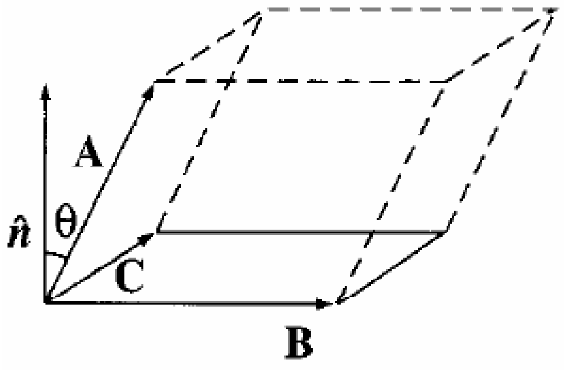
$$(\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})$$



$$\begin{aligned} \hat{\mathbf{x}} \times \hat{\mathbf{x}} &= \hat{\mathbf{y}} \times \hat{\mathbf{y}} &&= \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0, \\ \hat{\mathbf{x}} \times \hat{\mathbf{y}} &= -\hat{\mathbf{y}} \times \hat{\mathbf{x}} &&= \hat{\mathbf{z}}, \\ \hat{\mathbf{y}} \times \hat{\mathbf{z}} &= -\hat{\mathbf{z}} \times \hat{\mathbf{y}} &&= \hat{\mathbf{x}}, \\ \hat{\mathbf{z}} \times \hat{\mathbf{x}} &= -\hat{\mathbf{x}} \times \hat{\mathbf{z}} &&= \hat{\mathbf{y}}. \end{aligned}$$

Triple Products

Scalar triple product: $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$



$|\mathbf{B} \times \mathbf{C}|$ is the area of the base

$|\mathbf{A}\cos\theta|$ is the altitude

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Calculus to study Scalar Function / Field

Scalar Field/Function

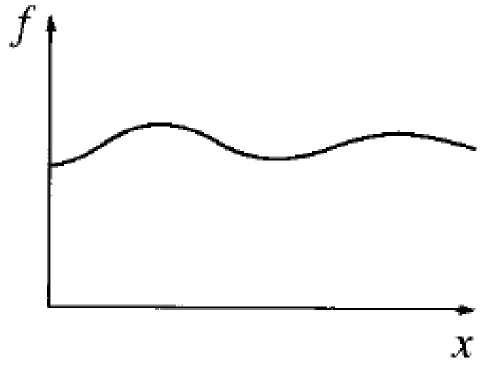
1-D, 2-D,………..n-D

Ordinary derivatives

$T = f(x)$

$$\frac{df}{dx} \qquad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$df = \left(\frac{df}{dx}\right) dx$$



Partial derivatives

$$T = T(x, y, z)$$

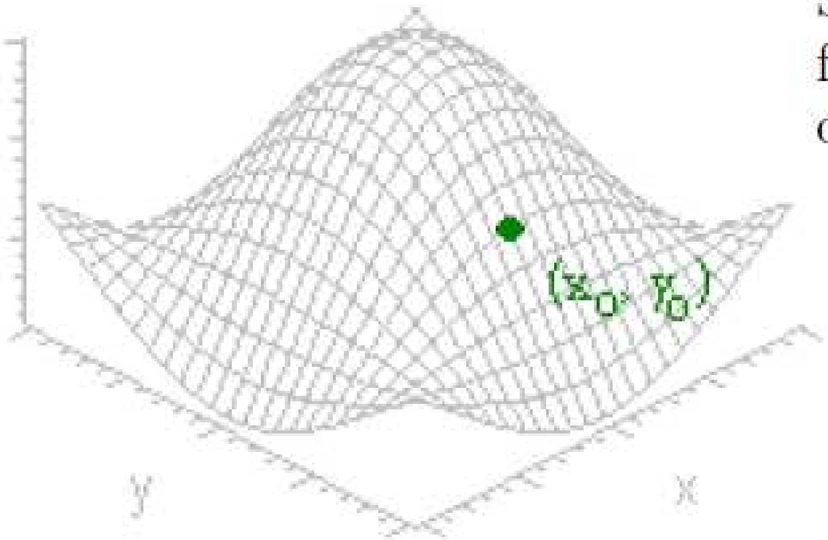
$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz.$$

$$f_x(x, y, z) = \lim_{h \to 0} \frac{f(x+h, y, z) - f(x, y, z)}{h}$$

$$f_y(x, y, z) = \lim_{h \to 0} \frac{f(x, y+h, z) - f(x, y, z)}{h}$$

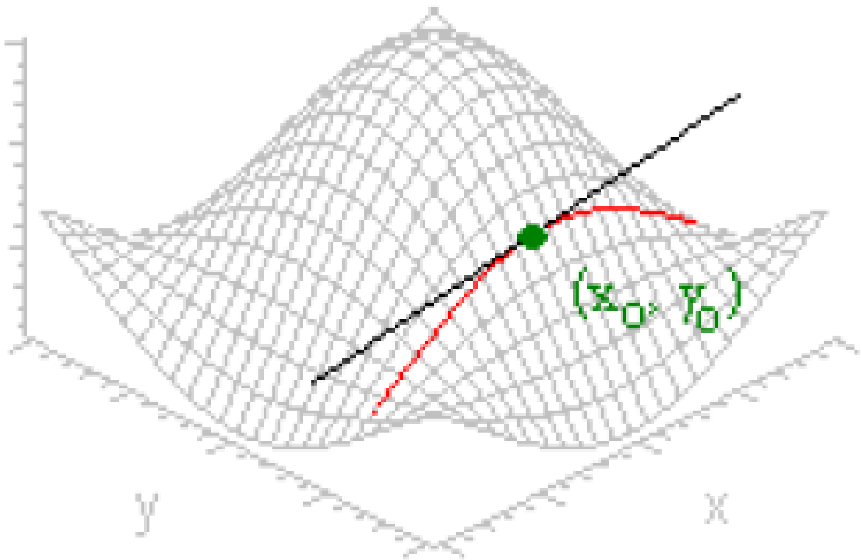
$$f_z(x, y, z) = \lim_{h \to 0} \frac{f(x, y, z+h) - f(x, y, z)}{h}$$

Geometrical Meaning



Suppose the graph of $z = f(x, y)$ is the surface shown. Consider the partial derivative of f with respect to x at a point (x_0, y_0).

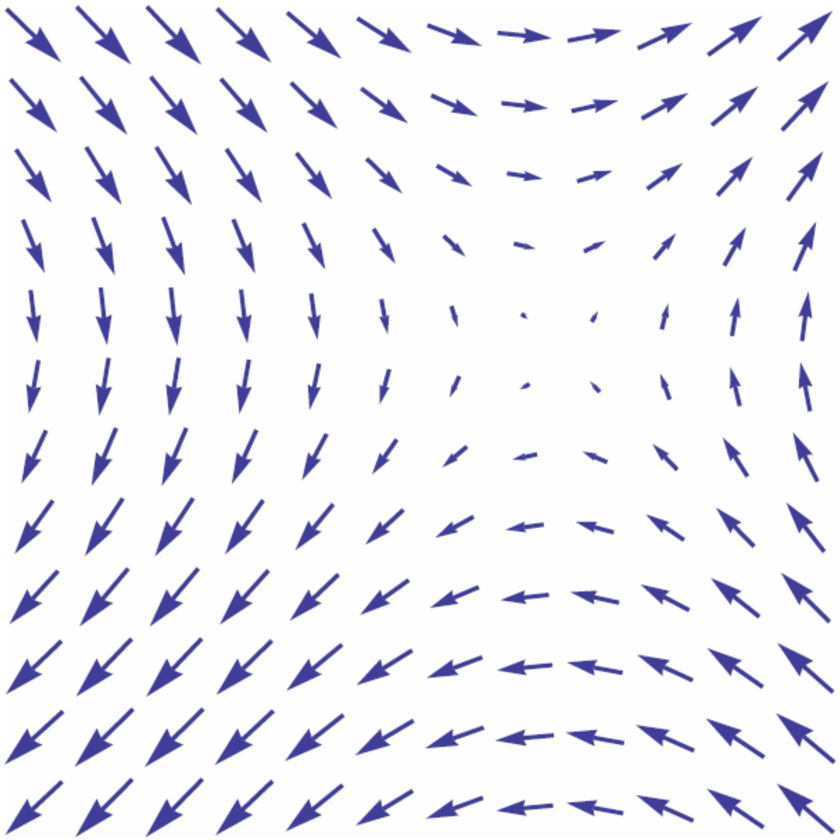
$$\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}$$



Vector calculus

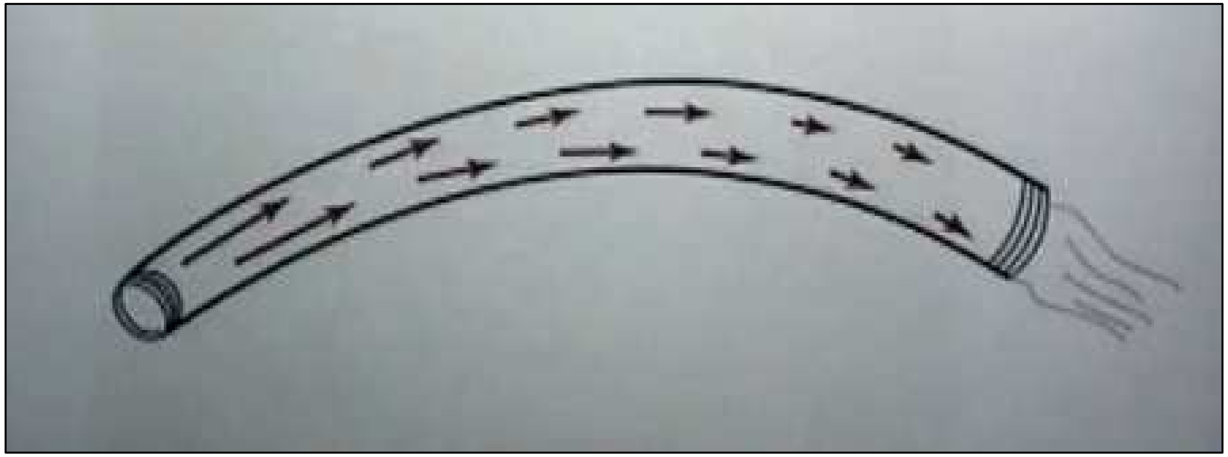
Vector Field / Vector Function

Scalar Field / Scalar Function



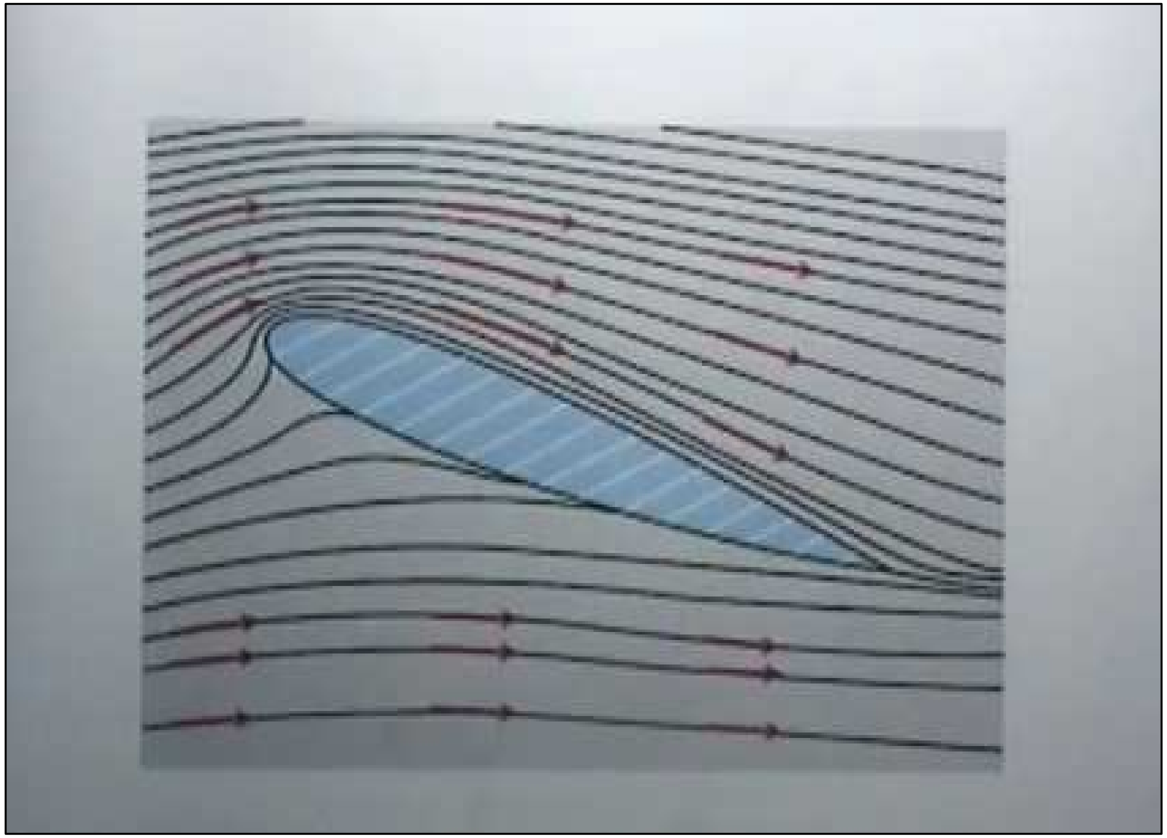
$$\vec{F} = \sin y\hat{i} + \sin x\hat{j}$$

http://en.wikipedia.org/wiki/Vector_field

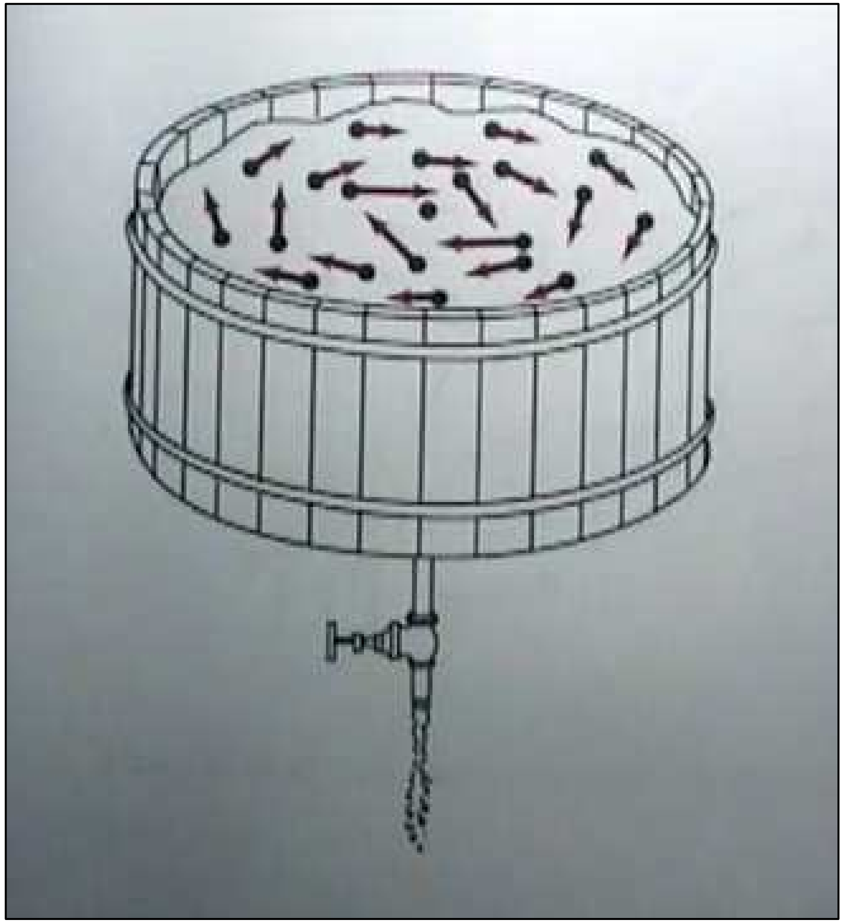


A vector field describing the velocity of a flow in a pipe

Note: All the figures related of vector field has been taken from a lecture series given by Dr. Chris Tisdell, UNSW

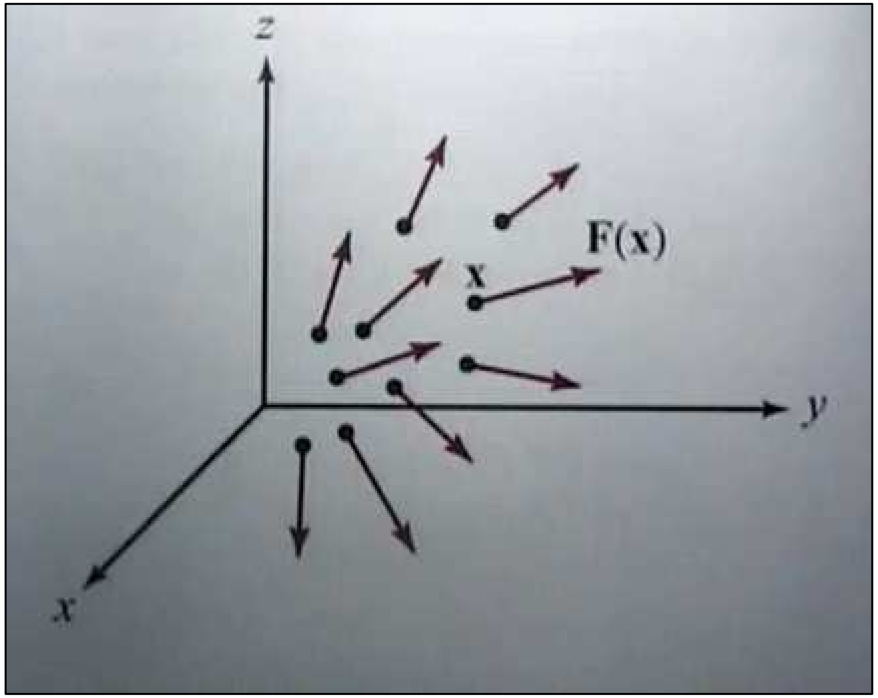


Velocity vector field of a flow around a aircraft wing



Circular flow in a tub

Vector Field or Vector function



$$\vec{F}(x,y,z) = F_1(x,y,z)\hat{i} + F_2(x,y,z)\hat{j} + F_3(x,y,z)\hat{k}$$

$$\vec{F}(x,y,z) = xyz\hat{i} - x^2 z^4\hat{j} + x\hat{k}$$

Sketching of Vector Function/Field

$$\vec{V} = x\hat{i}$$

$$\vec{V} = x^2\hat{i}$$

$$\vec{V} = -y\hat{i} + x\hat{j}$$