The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-III
Assignment 5

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1. Classify the following PDE as linear, semi-linear, quasi-linear or fully nonlinear:

$$(a)\ u_t + xu_x = 0, \qquad (b)\ u_t + au_x = u^2, \qquad (c)\ xu_t + u_x = u,$$

$$(d)\ u_t + uu_x = 0, \text{ (Inviscid Burgers' Eqn.)} \quad (e)\ -\Delta u(x) = f(x, u(x)),\ x \in \mathbb{R}^n, \text{ (Poisson Eqn.)}$$

$$(e)\ \Delta^2 u(x,y) = 0,\ (x,y) \in \mathbb{R}^2, \text{ (Biharmonic Equation)} \qquad (f)\ u_t = u^3 u_{xxx}, \text{ (Dym Equation)}$$

2. Find the first order PDE by eliminating the arbitrary function f, satisfied by z:

$$(a)\ z(x,y) = xy + f(x^2 + y^2), \quad (b)\ z(x,y) = f(x/y), \quad (c)\ f(x - z, y - z) = 0.$$

3. Find the first order PDE by eliminating the arbitrary constants a and b, satisfied by z:

$$(a)\ z(x,y) = (x + a)(y + b), \quad (b)\ z(x,y) = ax + by, \quad (c)\ z^2(1 + a^3) = 8(x + ay + b)^3.$$

4. Find the first order PDE whose solution is the given function (or surface) $z(x,y)$:
   (a) The family of all spheres in $\mathbb{R}^3$ whose centres lie on the z-axis with radius a, *i.e.*, $x^2 + y^2 + (z - c)^2 - a^2 = 0$.
   (b) The family of all right circular cones with vertex at $(0,0,c)$, whose axes coincides the z-axis with inclination θ, *i.e.*, $(x^2 + y^2)\cos^2\theta - (z - c)^2 \sin^2\theta = 0$. Compare the PDE obtained with the one obtained in (a). Further, show that any surface $z(x,y) = f(x^2 + y^2)$ satisfies the same PDE as obtained in (a).

   Note: All surfaces of revolution with z-axis as the axis of revolution are governed by the equation $z(x,y) = f(x^2 + y^2)$.

5. (a) Let $v(x,y)$ be a given function of x, y and $f : \mathbb{R} \to \mathbb{R}$ is an arbitrary (or unknown) one variable function. Find the first order PDE whose solution is $u(x,y) = f(v)$.
   (b) Let $f, g : \mathbb{R} \to R$ be two arbitrary (or unknown) one variable functions. Find the second order PDE whose solution is $u(x,y) = f(x - ay) + g(x + ay)$, for some given $a \in \mathbb{R}$.

6. In the following, find general integral *i.e.*, solution containing arbitrary function:

$$(a)\ xp+yq = z, \quad (b)\ x^2p+y^2q = (x+y)z, \quad (c)\ yzp+xzq = xy, \quad (d)\ (z^2-2yz-y^2)p+x(y+z)q = x(y-z).$$

7. Discuss the existence of integral surface of $2p+3q+8z = 0$, which contains the curve: (a) $\Gamma : z = 1-3x$ and the line $y = 0$; (b) $\Gamma : z = x^2$ on the line $2y = 1 + 3x$, (c) $\Gamma : z = e^{-4x}$ on the line $2y = 3x$.

8. Find the integral surface for the following Cauchy's problem: (a) $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$, $\Gamma : x_0(s) = 1, y_0(s) = 0, z_0(s) = s$; (b) $x^3p + y(3x^2 + y)q = z(2x^2 + y)$, $\Gamma : x_0(s) = 1, y_0(s) = s, z_0(s) = s(1 + s)$.