Discrete Mathematics Assignment-1

1. Testing Whether One Set Is a Subset of Another

   Let A = {1} and B = {1, {1}}.

   a) Is $A \subseteq B$?
   b) If so, is A a proper subset of B?

2. Define sets A and B as follows:

   $A = \{m \in Z \mid m = 2a \text{ for some integer } a\}$

   $B = \{n \in Z \mid n = 2b - 2 \text{ for some integer } b\}$

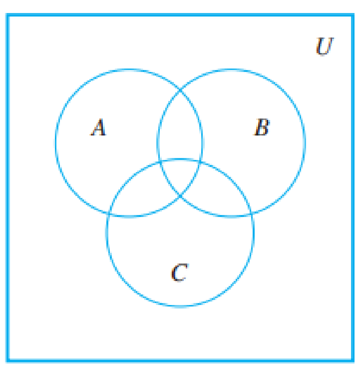
   Is A = B?

3. Let A = {a, b, c}, B = {b, c, d}, and C = {b, c, e}.

   a) Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?
   b) Find $A \cap (B \cup C)$, $(A \cap B) \cup C$, and $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?
   c) Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?

4. Consider the Venn diagram shown below. For each of (a)–(f), copy the diagram and shade the region corresponding to the indicated set.

   a) $A \cap B$ b) $B \cup C$ c) A^c
   d) $A - (B \cup C)$ e) $(A \cup B)^c$ f) $(A^c \cap B^c)$



5. Construct an algebraic proof that for all sets A and B, $A - (A \cap B) = A - B$

6. Using set laws verify that (X – Y) - Z = X – (Y – Z)
7. Simplify the set expression (A ∩ B') U (A' ∩ B) U (A' ∩ B').
8. Use the fuzzy sets A = {Angelo 0.4, Bart 0.7, Cathy 0.6} and B = {Dan 0.3, Elsie 0.8, Frank 0.4} to find each fuzzy set.

   a) A U B   b) A ∩ B   c) A'   d) A U B'

   e) A ∩ B'   f) A ∩ A'

   The **union** of A and B is AUB, where $d_{AUB} = \max\{d_A(x), d_B(x)\}$; their **intersection** is A∩ B, where $d_{A \cap B} = \min\{d_A(x), d_B(x)\}$; and the **complement** of A is A', where

   $d_A = 1 - d_A(x)$;

9. A set with n elements has 2^n subsets, where n >_ 0.
10. Find the cardinality of each set.
    a) The set of letters of the English alphabet.
    b) The set of letters of the word TWEEDLEDEE.
    c) The set of months of the year with 31 days.
    d) The set of identifiers in Java that begin with 3.
11. Find the number of positive integers < 500 and divisible by:
    a) Two or three.
    b) Two, three, or five.
    c) Two or three, but not six.
    d) Neither two, three, nor five.
12. A recent survey by the MAD corporation indicates that of the 700 families interviewed, 220 own a television set but no stereo, 200 own a stereo but no camera, 170 own a camera but no television set, 80 own a television set and a stereo but no camera, 80 own a stereo and a camera but no television set, 70 own a camera and a television set but no stereo, and 50 do not have any of these. Find the number of families with:
    a) Exactly one of the items.
    b) Exactly two of the items.
    c) At least one of the items.
    d) All of the items.
13. Let S be the set defined recursively as follows.

a) $2 \in S$.

b) If $x \in S$, then $x^2 \in S$.

Describe the set by the listing method.

14. Find the domain and range of these functions.

a) the function that assigns to each pair of positive integers the first integer of the pair.

b) function that assigns to each positive integer its largest decimal digit.

c) the function that assigns to a bit string the difference between the number of ones and the number of zeros in the string .

d) the function that assigns to each positive integer the largest integer not exceeding the square root of the integer .

e) the function that assigns to a bit string the longest string of ones in the string.

15. Determine whether $f: Z \times Z \to Z$ is onto if

a) $f(m, n) = 2m - n$

b) $f(m, n) = m^2 - n^2$

c) $f(m, n) = m + n + 1$

d) $f(m, n) = |m| - |n|$

e) $f(m, n) = m^2 - 4$

16. Determine whether each of these functions is a bijection from R to R .

a) $f(x) = -3x + 4$

b) $f(x) = -3x^2 + 7$

c) $f(x) =$(x+1)/(x+2)

d) $f(x) = x^5 + 1$

17. Suppose that g is a function from A to B an f is a function from B to C .

a) Show that if both f and g are one-to-one functions, then $f\ o\ g$ is also one-to-one.

b) Show that if both f and g are onto functions, then $f\ o\ g$ is also onto.

18. Let $f, g, h : Z \to Z$ be defined by $f(x) = x - 1,\ g(x) = 3x$

$$h(x) = \begin{cases} 1, & x\ odd \\ 0 & x\ even \end{cases}$$

Determine

a) $f\ o\ g,\ gof, goh, hog, fo(goh), (fog)oh$

b) $f^2, f^3, g^2, g^3, h^2, h^3, h^{500}$

19. Let f be a function from A to B. Let S and T be subsets of B. Show that

   a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
   b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

20. Let S be a subset of a universal set U . The characteristic function f_s of S is the function from U to the set {0,1} such that $f_s(x) = 1$ if x belongs to S and $f_s(x) = 0$ if x does not belong to S . Let A and B be sets. Show that for all x

   a) $f_{A \cap B}(x) = f_A(x).f_B(x)$
   b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x).f_B(x)$
   c) $f_{\bar{A}}(x) = 1 - f_A(x)$
   d) $f_{A \otimes B}(x) = f_A(x) + f_B(x) - 2f_A(x).f_B(x)$