The LNM Institute of Information Technology
Jaipur, Rajasthan

Mid Semester Exam

MATH-I, September 8, 2015

Part-II

Time: 90 minutes, Maximum Marks: 20

Instructions: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. **Start a new question on a new page and answer all its parts in the same place.** Please make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be penalized by the deduction of **2 marks**.

Question No.				
Page No.				

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1. Consider the sequence (a_n) defined by

$$a_1 := 1 \text{ and } a_{n+1} := 1 + \frac{1}{a_n} \text{ for } n \in \mathbb{N}.$$

Show that (a_n) converges and find its limit. **[5 marks]**

2. Show that function $f : \mathbb{Z} \to \mathbb{R}$ defined by

$$f(m) = (-1)^m + 2^m.$$

is continuous everywhere on $\mathbb{Z}$. **[5 marks]**

3. Consider $f : [0,1] \to \mathbb{R}$ defined by $f(x) = \dfrac{1}{1+x^2}$. Use the Picard Convergence Theorem to show that f has a unique fixed point in $[0,1]$ and any Picard sequence with its initial point $x_0 \in [0,1]$ will converge to this fixed point. Compute the first three values of the Picard sequence for f when $x_0 = 0$. **[5 marks]**
(Hint: Compute f'' to estimate $|f'(x)|$)

4. Let $n \geq 2$, $r > 0, a \in \mathbb{R}$. Let $f^{(n)}$ be continuous on $[a-r, a+r]$. Assume that $f^{(k)}(a) = 0$ for $1 \leq k \leq n-1$, but $f^{(n)}(a) < 0$. If n is even, then show that a is point of local maximum for f. **[5 marks]**