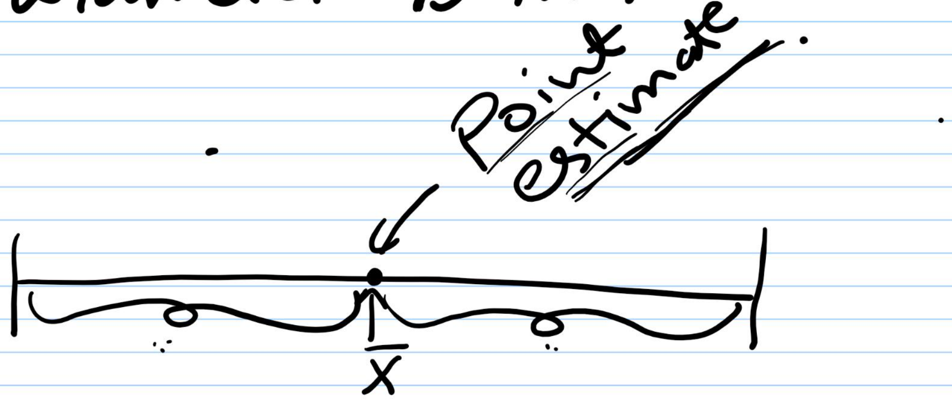
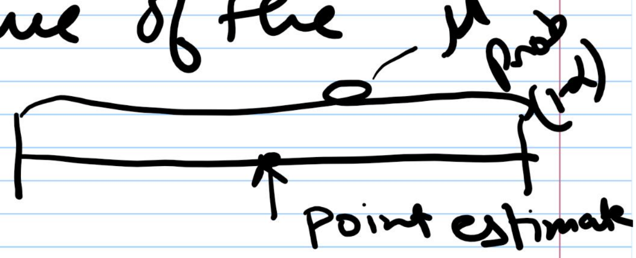
Confidence Interval.

* In Statistics, a confidence interval (CI) is a estimate computed from the Statistists of the observed data
* This Proposes a range of possible values for an unknown parameter.
* The interval has an associated Confidence level that the true parameter is in the CI range



* Given observation $x_1, \dots, x_n$ and a confidence level $(1-\alpha)$, a valid CI has probability $(1-\alpha)$ containing the true parameter.

→ Point estimate of a parameter is value of a statistic that estimate the value of the parameter.



→ Sample mean $\bar{x}$, is the best point estimate of the population mean μ.

The confidence interval for the population mean depends upon three factors

1. The point estimate of the population $\bar{X}$

2. The level of confidence: $1-\alpha$

3 The Standard deviation of the Sample mean

$$\boxed{\sigma_{\bar{x}} = \sigma/\sqrt{n}}$$

σ: Population SD.

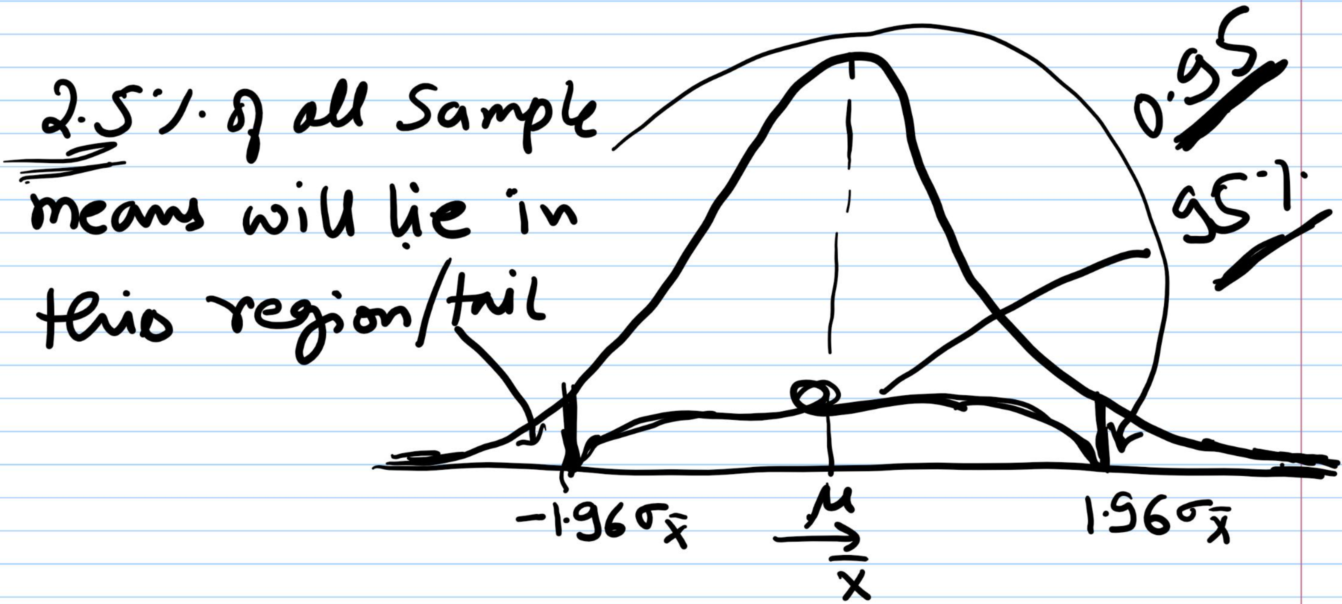
n: Sample size.

Suppose we obtain a simple random sample from a population. Provided that the population is normally distributed or the Sample size is large (> 30) the distribution of the sample mean will be normal with

mean $= \mu$

$SD = \sigma/\sqrt{n}$

* Since $\bar{X}$ is normally distributed 95% of all sample mean should lie within 1.96 SD of the population mean μ & 2.5% will lie in each tail



$\Rightarrow$ 95% of all the sample mean are in the interval.

$$\Rightarrow \mu - 1.96\frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + 1.96\frac{\sigma}{\sqrt{n}}$$

$$\boxed{\bar{X} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}}$$

$$\Rightarrow P\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$= 1 - \alpha$$

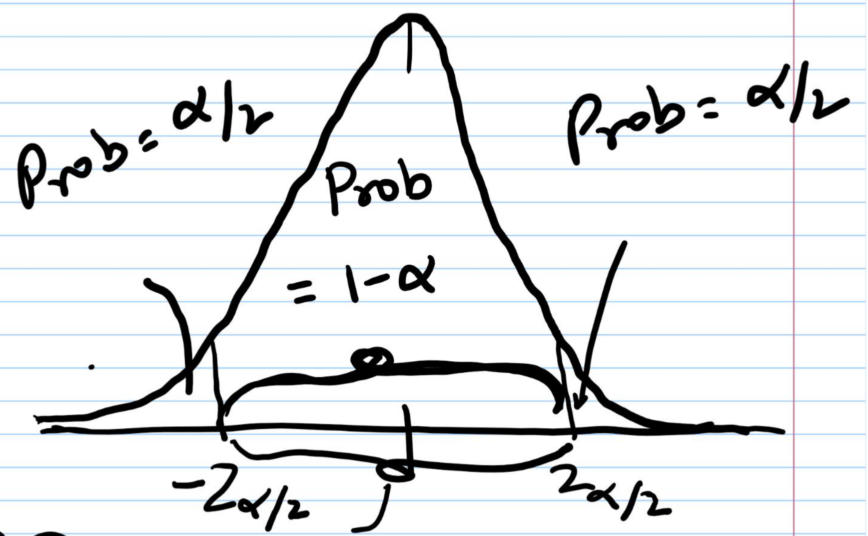
where α is significance level.

So interms of α ←

$$\boxed{\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}$$

Sample size n must be greater than or equal to 30 or population must be normally distributed.

$$CI = \left[\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$



where $n > 30$

The margin of Error, E in $(1-\alpha)*100\%$ Confidence interval in which σ is known is

$$\boxed{E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}}$$

Confidence interval About mean is σ is unknown.

In this case we use T-statistics.

Suppose a Simple random Sample of size n is taken from a Population. & Population is Normally distributed. then

$$t_\alpha = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim \text{Student T-distribution}$$

with $n-1$ degree of freedom, α SL

$$\boxed{t = 1.684 \quad df = 40, \alpha = 0.05}$$

So in the case when population SD is unknown.

then

Sample SD

$$\boxed{\bar{X} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2}\frac{s}{\sqrt{n}}}$$

where S is sample standard deviation.

$$CI = \left[\bar{X} - t_{\alpha/2}\frac{S}{\sqrt{n}},\ \bar{X} + t_{\alpha/2}\frac{S}{\sqrt{n}}\right]$$

$\alpha = 0.05$ $\quad \alpha/2 = 0.025$

df = $\underline{n-1}$.

Hypothesis Testing.

- ↳ Population (entire space)
- ↳ Sample (Subset of the entire space/population)

Hypothesis : Claims.

Claims are generally about the population.

- ↳ Proportion of the population
- ↳ interms of average behavior of the population (mean behavior)
- ↳ interms of standard deviation (Variance)

In Order to test we use different Statistical test / Method.

↳ Z Test → Standard
→ P-Value

Z Statistics

↳ T - Test → Standard
→ P-Value

T - Statistics

↳ χ^2 test (χ^2 statistics)

$F(z=3.34) = 0.9996$

Ex Sample of 465 Choclate piece. The sample mean is 0.8635 gm.
Population SD is 0.0565 gm

Test the claim that mean weight of the Choclate piece is greater than 0.8535 gm.
Significance Level is 0.01

Step 1 Claim $\boxed{\mu > 0.8535}$

Opposite $\mu \leq 0.8535$

Step 2 $H_0: \mu = 0.8535$

$\boxed{H_1: \mu > 0.8535}$

Step 3 $\alpha = \underline{0.01}$

Step 4 Test Statistics

$$\boxed{Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}$$

$\bar{X} = 0.8635$

$\mu = 0.8535$

$\sigma = 0.0565$

$n = 465$

$Z_{stat} = \underline{3.8166}$

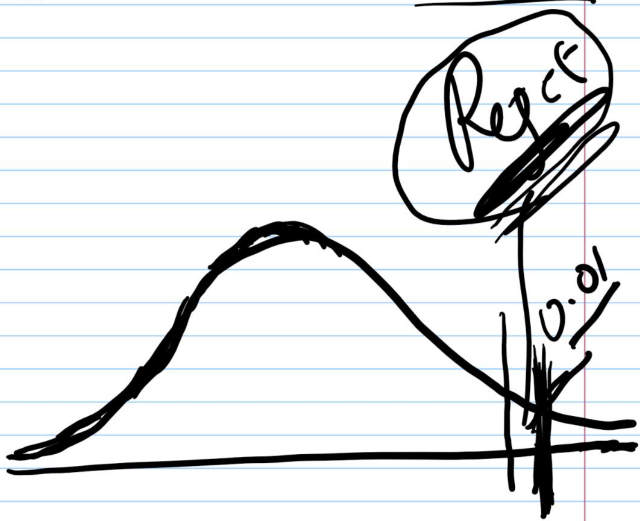
Step 5 test

Right tail Test

H_1: $>$ Go for RTL

$<$ Go for LTF

$\neq$ Two tail test



$Z_{critical} = \underline{2.33}$

H_1 is accepted

Z_{stat}		$Z_{critical}$
3.8166	$>$	2.33

H_0 is Rejected

If hypothesis is given in terms of Proportion

$$Z_{Stat} = \frac{\hat{P} - P}{\sqrt{\frac{P \times q}{n}}}$$

$\hat{p}$: Sample Proportion

P : Population Proportion

$q = 1 - P$

n : Size of Sample

Ex In a Survey of 206 companies it has been found that 61% of CEOs are Male

Claim: Most of the CEOs are Male

Significance Level $\alpha = 0.05$

Sol

Steps to Solve the question

Step 1: State the Claim & its opposite.

Claim: $P > 0.5$

opposite: $P \leq 0.5$

Step 2: formulate the Hypothesis

NULL HYPOTHESIS, H_0 : $p = 0.5$

Alternate Hypothesis

H_1 : $p > 0.5$

Step 3: State the Significance Level : $\alpha = 0.05$

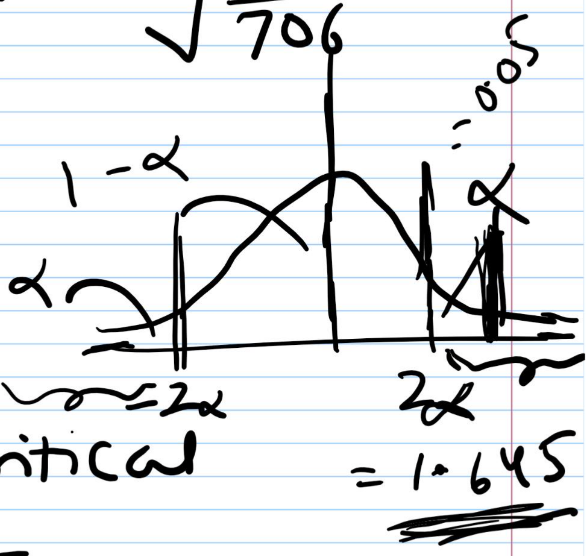
Step 4: Test Statistics

$$Z_{Stat} = \frac{(\hat{p} = 0.61) - (p = 0.5)}{\sqrt{\frac{(p = 0.5)(q = 0.5)}{(n = 706)}}}$$

$$\boxed{\therefore q = 1 - p}$$

$$Z = \frac{0.61 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{706}}} = \frac{0.11}{\sqrt{\frac{0.25}{706}}}$$

$$Z_{stat} = 5.84$$



Step 5: Find $Z_{critical}$

$\therefore 1 - \alpha = 0.95$

$\Rightarrow Z_{critical} = Z_{\alpha} = 1.645$

Step 6: Make a decision.

$$\underbrace{Z_{stat}}_{5.84} > \underbrace{Z_{\alpha}}_{1.645}$$

⇒ NULL hypothesis is Rejected.

Step 7: Interpretation

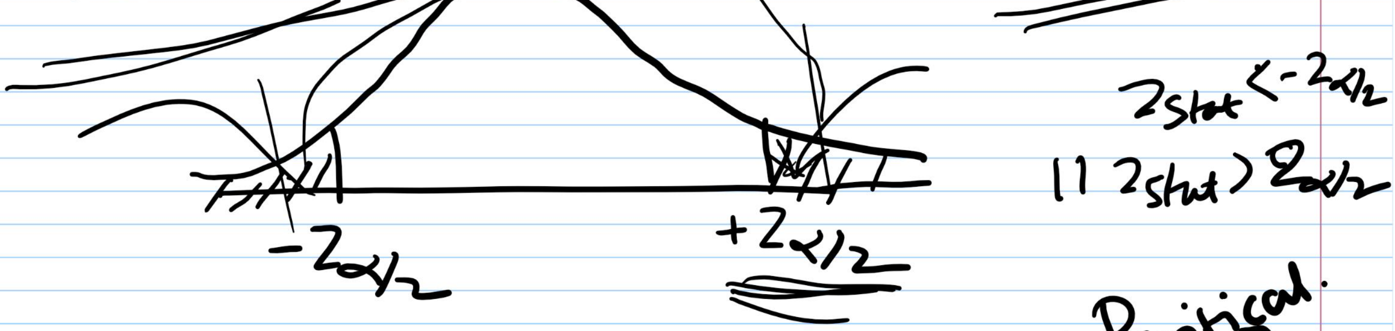
∴ NULL hypothesis is opposite to the claim.

hence alternate hypothesis is Accepted.

∴ "There is enough evidences to support the claim that Most of the CEO's are Male"

P-Value Method: Probability associated with A test Statistics

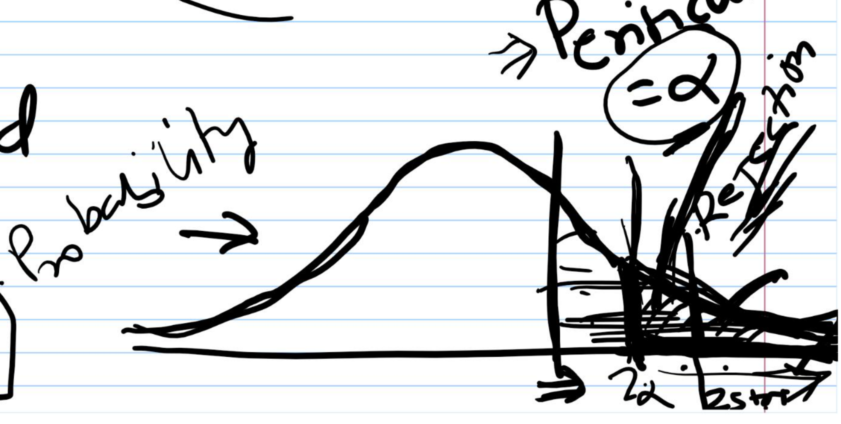
Traditional Method: Reject H_0 if Test Statistics falls in the Rejection Region.



P-Value Method

Reject H_0 if

P-Value < α



$X + PValue = 1$

$\Rightarrow$ P-Value $= 1 - X$

$$Z_{stat} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

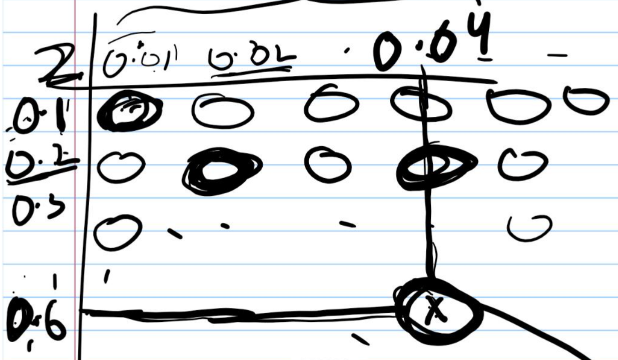
$$\frac{\hat{p} - p}{\sqrt{pq/n}}$$

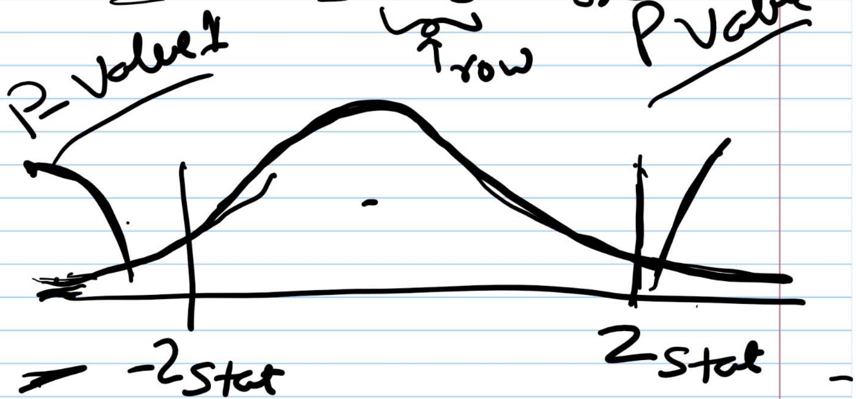
for Two tail test

$Z_{stat} = 0.64$

$= 0.6 + 0.04$

row, col





$$X = \int_{-\infty}^{Z_{stat}} \text{Pdf Normal } dz$$

$$\text{PValue} = \int_{-\infty}^{-Z_{stat}} \text{Pdf Normal } dz = cdf(-Z_{stat})$$

P-Value = P-Value 1 + P-Value 2

If P-Value $\leq P_{critical} = \alpha$

$\boxed{\text{Reject } H_0}$

300 Corp Sample

183 CEOs Male

$\% = \frac{183}{300} \times 100 = \%$ Male

$\alpha = 0.01$

Proportion = $\frac{183}{300}$ =

0 — 1

⇒ Claim → $p > 0.5$

⇒ Opposite → $p \leq 0.5$

$\hat{p} = \frac{183}{300}$

$H_0 : p = 0.5$

$H_1 : p > 0.5$

$q = 0.5$ $n = 300$

$$Z_{stat} = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{\frac{183}{300} - 0.5}{\sqrt{\frac{0.25}{300}}}$$

$Z_{stat} = 3.81$

$$Z_{stat} = \underbrace{3.8}_{row} + \underbrace{0.01}_{col}$$

$$X = \boxed{0.99993}$$

$$P_{value} = 1 - 0.99993$$

$$P_{value} = \underline{0.00007}$$

$$\alpha = 0.01$$

P-Value		α
0.00007	$<$	0.01

$\Rightarrow$ H_0 is Rejected.

$\Rightarrow$ H_1 is accepted $\Rightarrow$ Claim is accepted.

T-Test

↳ When population SD is not given.

↳ if Sample Size is less than 30

$$T_{Stat} = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

✓ $\bar{X}$: is Sample mean

✓ μ: population mean

✓ S: Sample SD

✓ n: Sample Size.

α : Significance level

$T_{\alpha, df}$

⇒ $df = n - 1$

Ex Sample of 39 cans of Softdrink with Sample mean volume of the Softdrink is 121.1 mL & Sample SD is 0.27

Test the Claim that the mean volume is greater than 120 mL with $\alpha = 0.01$

$\Rightarrow \bar{X} = 121.1$ $\quad S = 0.27$

$\mu = 120$ $\quad \alpha = 0.01$

$n = 39$ $\quad df = 39 - 1 = 38$

$\Rightarrow$ Step 1 Claim $\mu > 120$

Opposite $\mu \leq 120$

Step 2

$H_0 : \mu = 120 \leftarrow$

$H_1 : \mu > 120$

→ Step 3 $\alpha = 0.01$

Step 4

$$T_{stat} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{121.1 - 120}{0.27/\sqrt{39}}$$

$$= \frac{1.1 \times \sqrt{39}}{0.27}$$

$$= 25.44258$$

$$T_{stat} \simeq 25.4425$$

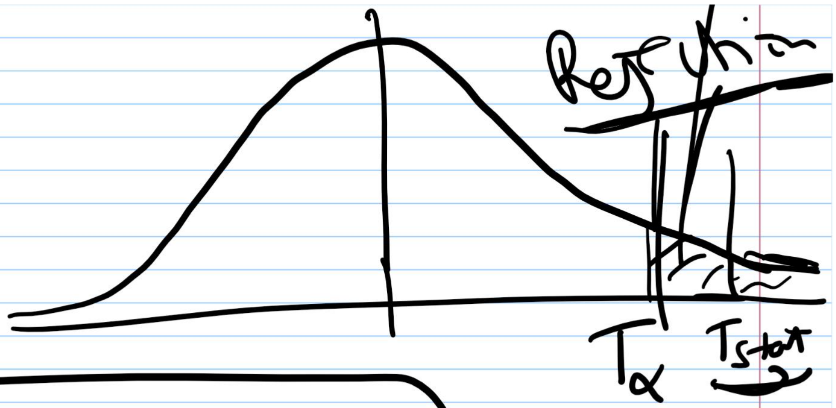
$$T_{critical} = T_{\alpha, df} =$$

degree of freedom $= n - 1$

$= 39 - 1 = 38$

$$T_{critical} = T_{\alpha} = 2.4286$$

T_{stat}		T_{α}
25.4425	$>$	2.4286



$\Rightarrow$ $\boxed{H_0 \text{ is Rejected.}}$

Interpretation.

"There are enough evidences to support our claim that "the soft drink can have mean vol greater than 12oz"

Sample of 35 Chocolate piece

mean Sample wt 0.86 gm

Sample SD 0.0565 gm

Test the Claim mean wt is greater than 0.85 gm

$\alpha = 0.01$

$n = 35$, $\bar{x} = 0.86$

$\mu = 0.85$ $\quad S = 0.0565$

$\alpha = 0.01$

$df = n - 1 = 34$

$$T_{Stat} = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{0.86 - 0.85}{0.0565/\sqrt{35}}$$

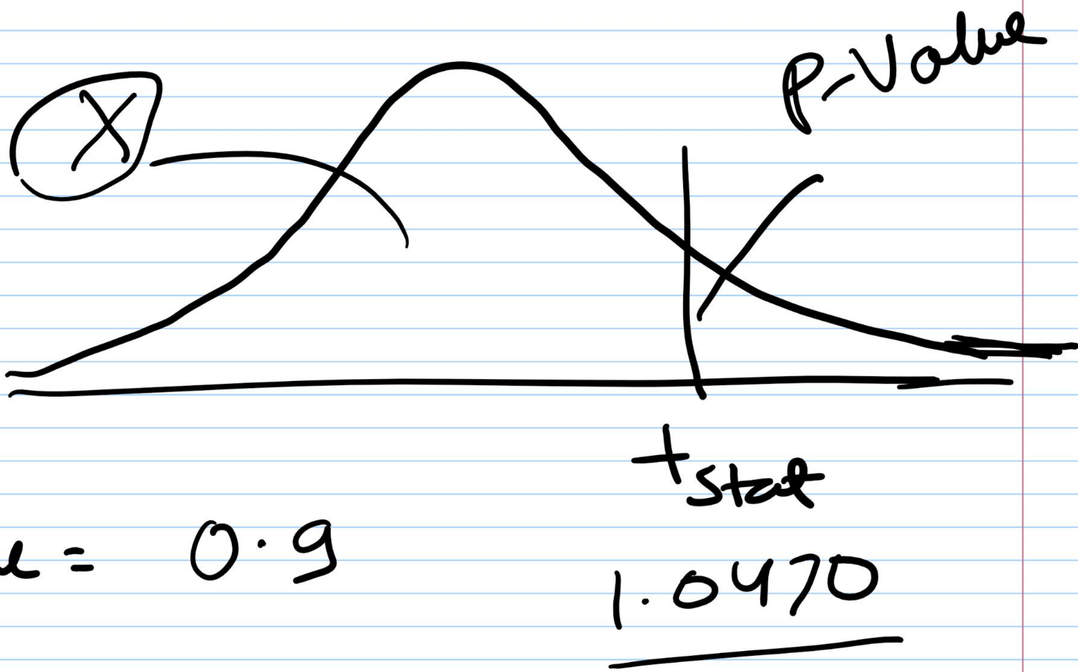
$T_{Stat} = 1.0470$

$T_\alpha = T_{0.01,34} = 2.4411$

T_{stat} T_α

$1.0470 < 2.4411$

$\alpha = 0.1$



P-Value = 0.9

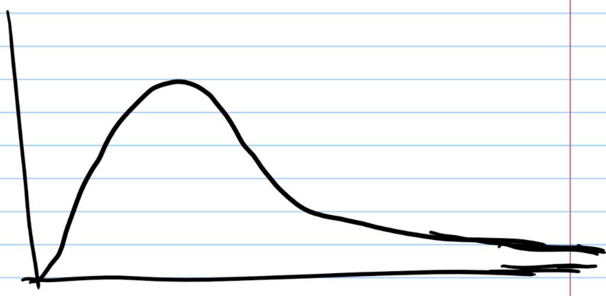
$\alpha = 0.01$

Pvalue > α

H_0 Failed to Reject

Test a Claim about σ

SD



χ^2 Test

$$\chi^2_{stat} = \frac{(n-1)s^2}{\sigma^2}$$

n : Sample Size

s : Sample SD

σ : Population SD with the claim.

Ex In a Sample of 37 coins. mean weight is 2.49910 gm with SD 0.01648 gm.

Test the Claim that population SD is less than 0.023gm at 0.05 Significance level

Sol

Step 1 Claim: $\sigma < 0.023$

Opposite: $\sigma \geq 0.023$

Step 2

$H_0: \sigma = 0.023$

$H_1: \sigma < 0.023$

Step 3 $\alpha = 0.05$

Step 4 Test Statistics: χ^2

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$n = 37$

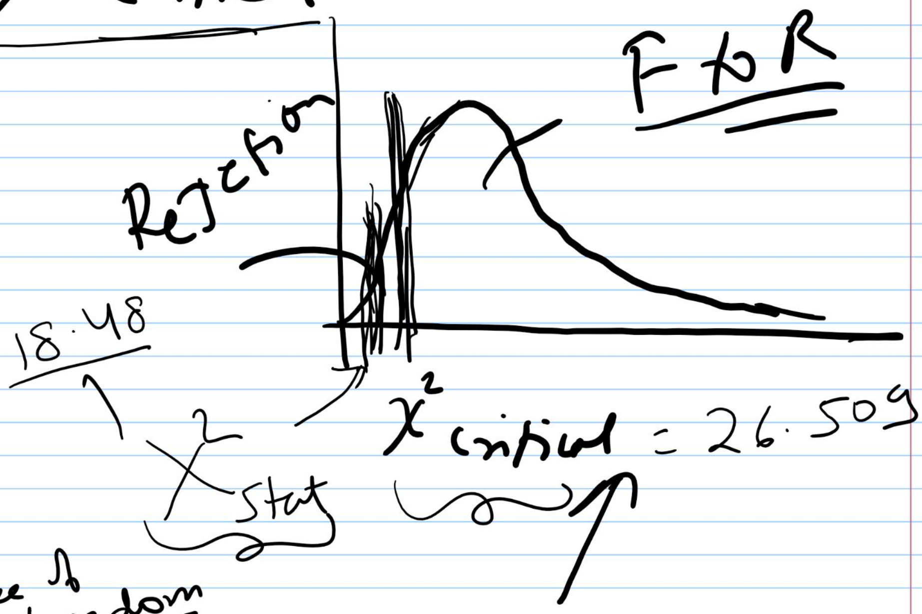
$s = 0.01648$

$\sigma = 0.023$

$$\chi^2 = \frac{36 \times (0.01648)^2}{(0.023)^2}$$

$$\chi^2 = 18.48$$

Step 5 $\chi^2_{critical}$.



$\chi^2_{critical}$

$$= \chi^2_{1-\alpha,\, n-1}$$

degree of freedom

$$= \chi^2_{0.95,36} = 26.509$$

Step 6 Make a decision.

⇒ χ^2_{stat} $\chi^2_{critical}$

$$18.48 < 26.509$$

⇒ Ho is Rejected ⇐

⇒ H_1 is accepted ⇐

Step 7

Interpretation: "There is enough evidences to support the claim that population SD is less than 0.023 gm".

→ [One Sample test]

Two Sample Test

if The Samples are drawn from Two different/ Independent Population

We go for Two

Sample test:

- Z test
- T test
- χ^2 test

Ex: Say there are Two Call centers. Call centers provide solution to Clients. Each call center have its own employees.

If you wish to see that if there is any difference in their average call duration.

Claim: The mean difference of call length between two call center is zero

	Call Center 1	Call Center 2
mean	μ_1	μ_2
σ (Population)	1.2 min	1.5 min

Step 1

→ Claim: $\mu_1 - \mu_2 = 0 \Leftarrow D_0$

→ Opposite: $\mu_1 - \mu_2 \neq 0 \Leftarrow$

Step 2.

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

σ is known we can use Z test.

Step 3

$\alpha = 0.05$

Step 4

One Sample test

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

two Sample test

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

D_0 = the Claim mean difference

	Call Center 1	Call Center 2
Sample mean	11.91	12.02
Pop SD	1.2	1.5
Sample size.	30	30

$$Z_{stat} = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}}$$

$$= \frac{(11.91 - 12.02) - 0}{\sqrt{\frac{1.2^2 + 1.5^2}{30}}}$$

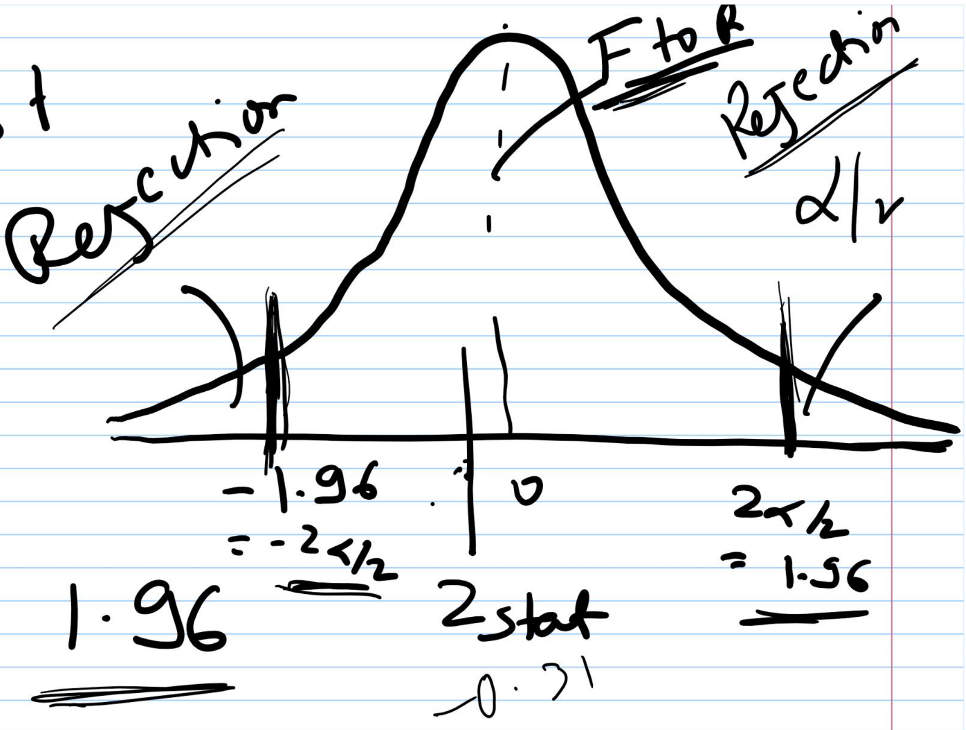
$$Z_{stat} = \frac{-0.11}{0.3507} = -0.31$$

Steps $Z_{critical}$.

$\left.\begin{array}{l} \rightarrow \text{R Tail test} \\ \rightarrow \text{L Tail test} \\ \rightarrow \text{2 Tail test} \end{array}\right\}$ 2 Tail test

2 Tail test

$Z_{\alpha/2}$



$Z_{0.025} = 1.96$

Step 6 Take the Decision.

$$-Z_{\alpha/2} < Z_{stat} < Z_{\alpha/2}$$

$$-1.96 < -0.31 < 1.96$$

H_0: Failed to Reject.

Step 7 Interpretation.

"There is not a enough evidences to Reject the Claim that Call centers differ in mean Call duration"

If the population SD is not given

T test

	Call Center 1	Call Center 2
Sample Size	30	30
Sample Mean	11.91	12.02
Sample SD	1.2	1.5

Claim : $\mu_1 - \mu_2 = 0$

Opposite: $\mu_1 - \mu_2 \neq 0$

H_0 : $\mu_1 - \mu_2 = 0$

H_1 : $\mu_1 - \mu_2 \neq 0$

$\alpha = 0.05$

$$T_{stat} = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{(11.51 - 12.02) - 0}{\sqrt{\frac{1.2^2 + 1.5^2}{30}}}$$

$$T_{stat} = -0.3136$$

$T_{critical}$ ← Two tail test

One Sample test

$T_{\alpha, df}$ → $n-1$

Significance

degree of freedom

Two Sample test

$T_{\alpha, df}$

Sig.

dof

$$\boxed{df_{2ST} = \frac{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}{\frac{1}{(n_1-1)}\left(\frac{s_1^2}{n_1}\right) + \frac{1}{(n_2-1)}\left(\frac{s_2^2}{n_2}\right)}}$$

if you put $s_1 = s_2$ & $n_1 = n_2$

$$= \frac{s_1^2/n + s_1^2/n}{\frac{1}{n-1}\left(\frac{s_1^2}{n}\right) + \frac{1}{n-1}\left(\frac{s_1^2}{n}\right)}$$

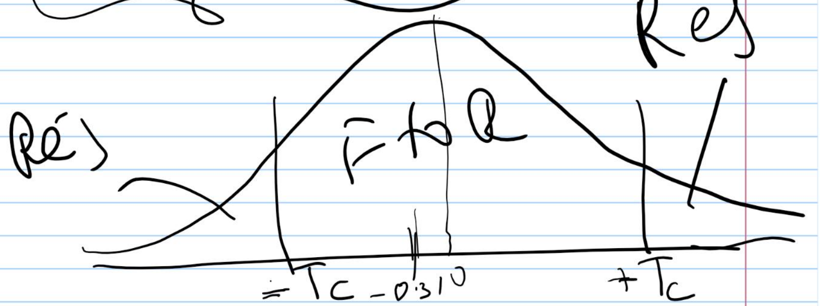
$$df = \frac{2 s_i^2/n}{\frac{1}{n-1}\left(2 s_i^2/n\right)} = n-1$$

$$df = \frac{\frac{1.2^2}{30} + \frac{1.5^2}{30}}{\frac{1}{29}\left[\frac{1.2^2}{30} + \frac{1.5^2}{30}\right]}$$

$$\boxed{df = 29}$$

$$T_{0.05,\,29} = \pm\ 2.0452$$

T_{stat}



$$-T_{\alpha,df} < T_{stat} < +T_{\alpha,df}$$

$$-2.0452 < -0.3136 < 2.0452$$

H_0 : is failed to Reject.

Case 3

Claim is about σ

χ^2 test.

$$df = \frac{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}{\left(\frac{1}{n_1-1}\right)\left(\frac{s_1^2}{n_1}\right) + \left(\frac{1}{n_2-1}\right)\left(\frac{s_2^2}{n_2}\right)}$$

α

$$\chi^2_{\alpha, df} = \;??$$

Ex Ram Grows Tomatoes in two separate fields. When tomates are ready to picked. He is interested to check whether the size of Tomatoes plants are differ in size.

He collect a random sample from each fields & measure the heigh of the plants.

	field 1	field 2
Sample Mean	1.3	1.6
Sample SD	0.5	0.3
Sample Size	22	24

Claim is : $\underline{\mu_1 - \mu_2 = 0}$

Opposite : $\mu_1 - \mu_2 \neq 0$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 \neq 0$$

$$\alpha = 0.05$$

$$T_{stat} = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{(1.3 - 1.6) - 0}{\sqrt{\frac{0.5^2}{22} + \frac{0.3^2}{24}}}$$

$$T_{stat} = \frac{-0.3}{0.1229375} = \underline{\underline{-2.44}}$$

$$T_{critical} = T_{\alpha, df}$$

$$df = \frac{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}{\frac{1}{(n_1-1)}\left(\frac{S_1^2}{n_1}\right) + \frac{1}{(n_2-1)}\left(\frac{S_2^2}{n_2}\right)}$$

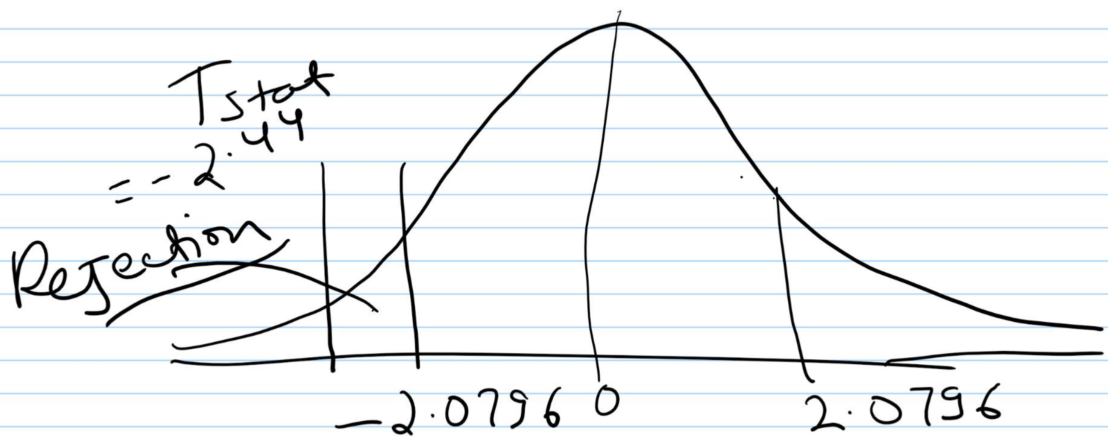
$$= \frac{\left(\frac{0.25}{22} + \frac{0.09}{24}\right)}{\frac{1}{21} \times \frac{0.25}{22} + \frac{1}{23} \times \frac{0.09}{24}}$$

$$= \frac{0.015113}{7.0416 \times 10^{-4}}$$

$$df = 21.4621$$

$$df \cong \underline{21}$$

$$\underline{T_{0.05, 21}} = \pm 2.0796$$



$$T_{stat} < -T_{\alpha, df} = -T_{critical}.$$

H_0 is Rejected.

Interpretation

There is enough evidences to reject the claim.

ANOVA:

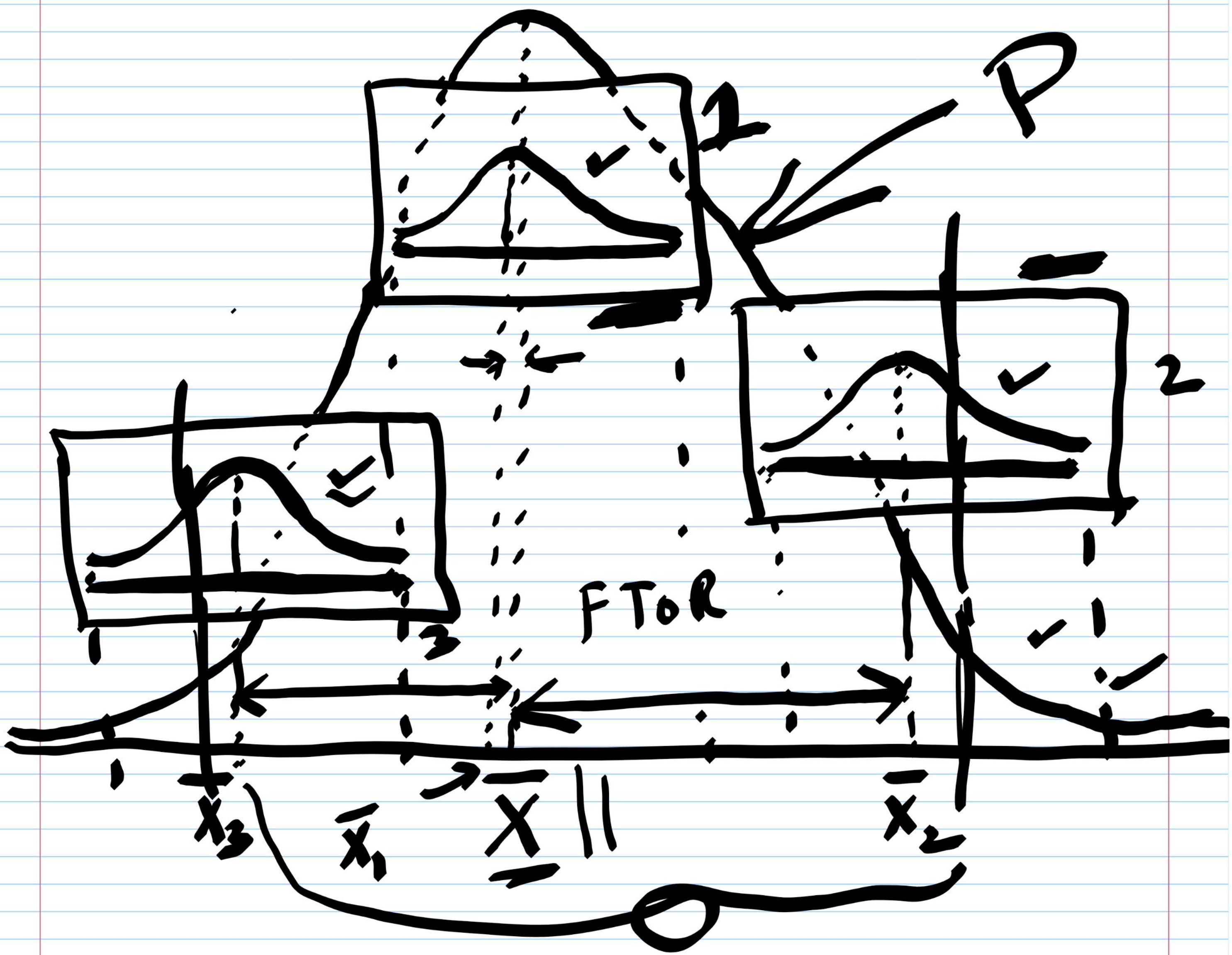
(ANalysis Of VAriance)

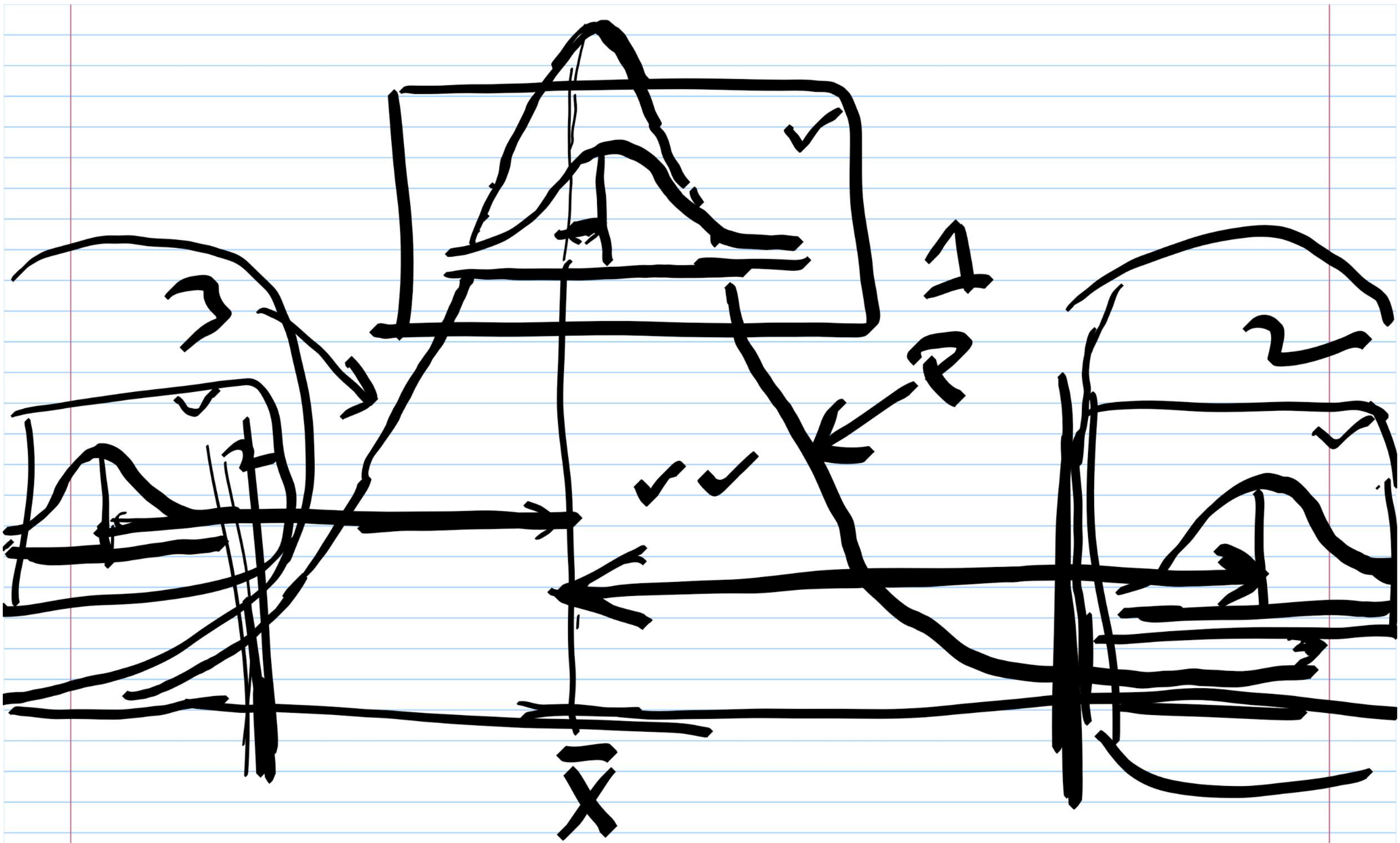
* ONE SAMPLE TEST: 1 Popu.
* Two Sample Test: 2 popu.

If we wish to Compare means of more than two population.

For this we Use ANOVA

Do all these sample means comes from a common population?





NULL HYPOTESIS

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

All three Samples likely come from Same Population.

	$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_3$
$\bar{X}_1$	Pairwise Comparision	Three test	T-Test
$\bar{X}_2$	$H_0: \bar{X}_1 = \bar{X}_2$ $\alpha = 0.05$	95% 5%	Confidence error
$\bar{X}_3$	$H_0: \bar{X}_1 = \bar{X}_3$ $\alpha = 0.05$	$H_0: \bar{X}_2 = \bar{X}_3$ $\alpha = 0.05$	

errors compounds with each test

$$\text{Conf} \doteq (0.95)(0.95)(0.95)$$

$$\doteq 85.7\% = 0.857$$

$$\text{Significance} = 1 - 0.857$$

$= \underline{0.147}$

$\underline{14.7\%}$

If have **more** # of Sample $\rightarrow$

Significance $\alpha \rightarrow \underline{1}$

ANOVA

Test the

: Variability

between the

Samples & within

the sample

$$= \frac{\{\text{Variability Between the means}\}}{\{\text{Variability within the Samples.}\}}$$

$$= \frac{\text{Variance Between}}{\text{Variance within}}$$

$$\sim \text{F - distribution}$$

⇒ Variance Between is Some large value

Variance within is some Small value.

$$\frac{\text{Large}}{\text{Small}} > 1$$

is much greater than One then Reject H_0

$$\frac{\text{Similar}}{\text{Similar}} \approx 1$$

Failed to Reject H_0

$$\frac{\text{Small}}{\text{Large}} < 1$$

Failed to Reject H_0

SSC [Sum of Square between the Sample]

SSE [SUM of Square within the Sample]

$$MSC = \frac{SSC}{df_{Sample\ (column)}}$$

$$MSE = \frac{SSE}{df_{error}}$$

V.B

$$F = \frac{MSC}{MSE} \sim F\ dist.$$

V. within

$$\underline{\underline{SSC}} = \sum_{i=1}^{K=3} \left(\bar{X}_i - \bar{\bar{X}} \right)^2 * \underline{\text{Size}}$$

$$\underline{\text{of Sample}}$$

$$= \sum_{i=1}^{K} \left(\bar{X}_i - \bar{\bar{X}} \right)^2 * n$$

$$\boxed{n: \text{ Sample Size}}$$

$$SSE = \sum_{j=1}^{K} \sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j \right)^2$$

$$df_{\text{Sample(col.)}} = \# \text{ of Sample} - 1$$

$$df_{\text{error}} = N - \# \text{ of Sample}$$

$$= N - K$$

Ex 21 Student at Some University which are Selected for an Study about Students Skills.

$$\underbrace{7\ I^{st}\ yr}\quad \underbrace{7\ II^{nd}\ yr}\quad \underbrace{7\ III^{rd}\ yr}$$

Skill test is conducted total Score 100

Interested to Study that whether or not there is a difference in Skills exist between three differ.

year levels.

	Year 1	Year 2	Year 3
1	82	71	64
2	93	62	73
3	61	85	87
4	74	94	91
5	69	78	56
6	70	66	78
7	53	71	87
	$\bar{X}_1 = 71.71$	$\bar{X}_2 = 75.29$	$\bar{X}_3 = 76.57$

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{3}$$

$H_0: \mu_1 = \mu_2 = \mu_3$

$\alpha = 0.05$

$$= \frac{\sum_{i=1}^{21} x_i}{21} = \underline{74.52}$$

$$SSC = \sum_{i=1}^{3} \left(\bar{X}_i - \bar{\bar{X}}\right)^2 * 7$$

$$= \left[(71.71 - 74.52)^2 + (75.29 - 74.52)^2 + (76.57 - 74.52)^2\right] * 7$$

$$= \underline{88.6667}$$

$$SSE = \sum_{j=1}^{3} \boxed{\sum_{i=1}^{7} \left(x_{ij} - \bar{X}_j\right)^2}$$

$$= \sum_{j=1}^{3} \boxed{Var\left(X_j\right) * (n-1)}$$

$$= \sum_{j=1}^{3} \underline{Var\left(X_j\right) * 6}$$

$$= \underline{2812.571429}$$

$$\underline{MSC} = \frac{SSC}{df_{col.}}$$

$$df_{col} = 3 - 1$$
$$= 2$$

$$MSC = \frac{88.66667}{2}$$

$$= \underline{44.3333}$$

$$\underline{MSE} = \frac{SSE}{df_{with.}}$$

$$df_{with} = \underline{N - C}\,(=K)$$

# of sample

$$= 21 - 3$$
$$= \underline{18}$$

$$MSE = \frac{2812.571429}{18}$$

$$= \underline{156.254}$$

$$F = \frac{MSC}{MSE} = \frac{44.3333}{156.254}$$

$$= 0.283726 \ldots$$

$$\underline{F_{stat}} \simeq \underline{0.2837}$$

$\rightarrow$ F table

$$\underline{F_{critical}} = F_{\alpha, df_{col}, df_{with}}$$

$$= \underline{3.554455715}$$

$$\underline{F_{score}}_{\text{stat}} < \underline{F_{critical}}$$

$\underline{H_0 : \text{Failed to Reject}}$