Asg-3

Ans1. a) $y = x^2 + 1 \Rightarrow dy = 2x\,dx$.

$\int_{(0,1)}^{(2,5)} (3x + x^2 + 1)\,dx + (2(x^2+1) - x)(2x)\,dx$.

b) $\frac{x-0}{2-0} = \frac{y-1}{5-1}$

$(y - 1) = \frac{4}{2}(x - 0) \Rightarrow y - 1 = 2x \Rightarrow dy = 2dx$

c) $(0,5) \to (2,5)$: $dy = 0,\ y = 5$

$(0,1) \to (0,5)$: $x = 0,\ dx = 0$

Ans5. $\int_0^{2\pi} \bar{z}^2\, dz$ around the circle $|z| = 1$. $z = e^{i\theta}$

$dz = i e^{i\theta} d\theta$.

$\bar{z} = e^{-i\theta} d\theta$.

$I = \int_0^{2\pi} ((e^{-i\theta})^2\, i e^{i\theta} d\theta = i\int_0^{2\pi} e^{-i\theta} d\theta$.

b) $|z - 1| = 1$

$z - 1 = e^{i\theta}$

$z = 1 + e^{i\theta}$

$dz = i e^{i\theta} d\theta$.

$\bar{z} = 1 + e^{-i\theta}$

$I = \int_0^{2\pi} (1 + e^{-i\theta})^2\, i e^{i\theta} d\theta$.

e.g. $|z + i| = 3$

$z = i + 3e^{i\theta}$

8. $L = \int_{-1}^{1} z'(t)\, dt$

9. curve is circle of radius R.

$\therefore L = 2\pi R$

$$f(z) = \left|\frac{\log z}{z^2}\right| = \frac{|\ln|z| + i\,\mathrm{Arg}\,z|}{|z|^2} = \frac{\ln R + i\pi}{R^2}$$

max value of Arg

$$\therefore \int_{|z|=R} \frac{\log z}{z^2}\,dz \le 2\pi R\left(\frac{(\ln R + i\pi)}{R^2}\right)$$

$$= 2\pi\left(\frac{\ln R + i\pi}{R}\right) \le$$

Now using traigle inequality:

$$\frac{|\ln R| + |\pi| 2\pi}{R^2} = \frac{(\ln R + \pi)2\pi}{R}$$

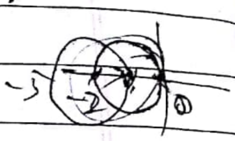
(Hence proved)

Ans 10. $|e^z| = |e^x||e^{iy}| = |e^x|$

$\hookrightarrow x$ max value is 1.

$\therefore$ max value is $\underline{e}$.

$$\therefore \int_C e^z\,dz \le \underline{e \cdot (3)}. \rightarrow \text{(length)}.$$



Ans 11. $|e^{x^2-y^2}|\;|e^{-2xyi}|$ $\hookrightarrow 1$

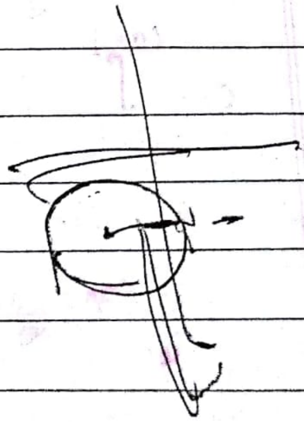
$x^2 - y^2 = \cos^2\theta - \sin^2\theta = \cos 2\theta \rightarrow$ max value $= 1$.

$\therefore \underline{e^1}$

$\therefore$ $ML = \underline{2\pi e}$

e.g. $f(z) = (z - i)^{1/3}$

$z - i = 0 \Rightarrow z = i \Rightarrow$ branch point.



Asg-3

1.a) $\int_{(0,1)}^{(2,5)} (3x+y)\,dx + (2y-x)\,dy =$

$y = x^2+1$ $\quad dy = 2x\,dx.$

$\int_0^2 (3x+x^2+1)\,dx + (2(x^2+1)-x)\,2x\,dx$

$= \frac{3x^2}{2} + \frac{x^3}{3} + x \Big|_0^2 + \frac{4x^4}{4} + \frac{2x^2}{2} - \frac{2x^3}{3}\Big|_0^2$

$= 6 + \frac{8}{3} + 2 + 16 + 8 - \frac{16}{3} = 32 - \frac{8}{3}$

$= \frac{96-8}{3} = \frac{88}{3}$

b) $y - 1 = \frac{4}{2}(x) \Rightarrow y = 1+2x.$

$= \int_0^2 (3x+1+2x)\,dx + (2(1+2x)-x)\,2dx$

$= \int_0^2 (5x+1)\,dx + 2(2+3x)\,dx = \int_0^2 (11x+5)\,dx$

$= \frac{11x^2}{2}\Big|_0^2 + 5x\Big|_0^2 = 11\times 2 + 5\times 2 = 32$

c) $\int_{(0,1)}^{(0,5)}$ $\quad \int_{(4,5)}^{(2,5)}$

5. $\oint_C \bar{z}^2\,dz$ $\quad$ a) $|z|=1$ $\quad$ b) $|z-1|=1.$

$\oint_C (e^{-i\theta})^2 \cdot i e^{i\theta}\,d\theta = i\oint_C e^{-i\theta}\,d\theta = i\, e^{-i\theta} = 0$

b) $|z-1| = e^{i\theta}$

$z = 1+e^{i\theta}$

d $\quad |z|=2$

Ans 8. a) $|1+z^2| > 1 - |z|^2$

$\frac{1}{1+z^2} < \frac{1}{|1+z^2|} < \left|\frac{1}{1-|z|^2}\right|$

$< \frac{\pi}{3}.$

$L = \frac{\pi}{2\times 2\pi} \times 2\pi R$

$\frac{\pi R}{2} = \pi$

8. $\oint$ ... $\int 2\sqrt{2t}\,dt$

8. $z = (1-i)t^2 \quad -1 \le t \le 1.$

$x = t^2 \quad y = -t^2$

$x^2 = t^4 \quad y^2 = t^4$

$\int \sqrt{4t^3 + 4t^3}\,dt = \int \sqrt{8t^3}\,dt = 2\sqrt{2}|t^{3/2}|\,dt$

$= 2t^{5/2}$

$\sqrt{(2t)^2+(2t)^2} = \sqrt{8t^2} \quad = \int_{-1}^{1} 2\sqrt{2}|t|\,dt = 2\sqrt{2}$

$\int_{-1} 2t\,dt = 2\frac{t^2}{2}$

$|\log|z|| + |i\arg z|$

$\le |\log|z|| + |i \arg z|$

$\log R + \pi \cdot 2\pi R$

3. a) $\int \frac{2z+3}{z}\,dz = \int_{\pi}^{0} (2e^{i\theta}+3)\,2ie^{i\theta}\,d\theta$

$z = 2e^{i\theta}$

$\int_{\pi}^{0} (4e^{i\theta}+3)\,i\,d\theta = \frac{4e^{i\theta}i}{i}\Big|_{\pi} + 3i\theta\Big|_{\pi}$

$= 4 + 4 - 3\pi i$

b) $\int_{0}^{\pi} (4e^{i\theta}+3)\,i\,d\theta = -8 + 3\pi i$

c) $\int_{0}^{-\pi} (4e^{i\theta}+3)\,i\,d\theta = 4e^{i\theta}\Big|_{-\pi}^{0} = 8 + 3\pi i$

d) $\int_{-\pi}^{0} (4e^{i\theta}+3)\,i\,d\theta = 8 + 3\pi i$

$\frac{\pi}{2\cdot 2\pi} \times 2\pi R \quad \frac{\pi R}{2}$

5. $\oint_C \bar{z}^2\,dz =$

$\int_C (x^2 - iy^2)\,dz$

$1 + z^2$

$\frac{3z-1}{(\;)(\;)} = \frac{A}{z} + \frac{Bz+C}{z^2+2}$

$A(z^2+2) + B(z^2) + Cz$

$\frac{1}{1+z^2} \quad \frac{1}{1+|z|^2}$

18. $\oint \frac{3z-1}{z(z^2+2)} = \oint \ldots$

$\frac{1}{1+z^2} \le \frac{1}{|1+z^2|}$

$\pi R \times \frac{1}{}$

$2\pi R \cdot \frac{1}{}$

i

$(-1,2) \quad (1,2) \quad i \quad 0$

$2\pi i \tan \pi i$

$1+z^2$

$-2\pi i \tanh \pi$