The LNM Institute of Information Technology
Jaipur(Raj)-302031

Optimization Techniques & Applications
Self Practice Problems

Non-Linear Programming

Constrained multi-variable Optimization with Equality constraints(Lagrange multipliers)

1. Optimize $Z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^3 - 4x_3$ subject to the constraints $x_1 + x_2 + x_3 = 7$. and $x_1, x_2, x_3 \geq 0$
   Does the solution maximize or minimize the problem? Justify.
   (Ans $x_1 = 4, x_2 = 2, x_3 = 1$ Min $Z = -35$)

2. Maximize $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$
   subject to $x_1 + 2x_2 = 2$ and $x_1, x_2 \geq 0$.

   (Ans $x_1 = 1/3, x_2 = 5/6$ Max $Z = 4.166$)

3. Min $Z = x_1^2 + x_2^2 + x_3^2$
   subject to $4x_1 + x_2^2 + 2x_3 = 14$, $x_1, x_2, x_3 \geq 0$
   (Ans $x_1 = 81/100, x_2 = 7/20, x_3 = 7/25$ Min $Z = 857/1000$)

Constrained multi-variable Optimization with Inequality constraints(Kuhn-Tucker Condition Method)

1. Maximize $Z = 2x_1^2 + 12x_1x_2 - 7x_2^2$
   subject to $2x_1 + 5x_2 \leq 98, x_1, x_2 \geq 0$
   (Ans $x_1 = 44, x_2 = 2$ Max $Z = 4900$)

2. Maximize $Z = -2x_1^2 + 3x_1 + 4x_2$
   subject to

$$x_1 + 2x_2 \leq 4$$

$$x_1 + x_2 \leq 2,$$
$$x_1, x_2 \geq 0$$

3. A manufacturing farm produces two products A and B. It produces them at the per unit cost of Rs 3/- and Rs 4/- respectively. The cost of production of these two products is given in the following table Because of the limited available resources, the firm has to bear within the

No. of Units produced	Cost of production(Rs)
Product-A(x_1)	$30 + 1.2x_1 + 0.001x_1^2$
Product-B (x_2)	$40 + 2x_2 + 0.005x_2^2$

   restrictions: $2x_1 + 3x_2 \leq 2500$ and $x_1 + 2x_2 \leq 1500$.
   Determine optimal level of production of product A and B by the firm.

Integer Linear Programming

1. Maximize $Z = 4x_1 + 3x_2$ subject to

$$x_1 + 2x_2 \leq 4$$
$$2x_1 + x_2 \leq 6$$

   $x_1, x_2 \geq 0$ and are integers.

   (Ans $x_1 = 3, x_2 = 0$ Max $Z = 12$)

2. $Z = 7x_1 + 9x_2$ subject to

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

   $x_1, x_2 \geq 0$ and are integers.
   (Ans $x_1 = 4, x_2 = 3$ Max $Z = 55$)

Quadratic Programming(Wolfe's Method)

1. Max $Z = 2x_1 + 3x_2 - 2x_1^2$
   subject to

$$x_1 + 4x_2 \leq 4$$
$$x_1 + x_2 \leq 2$$

   $x_1, x_2 \geq 0$
   (Ans $x_1 = 0, x_2 = 1$ Max $Z = 3$)

2. Min $Z = x_1^2 + x_2^2 + x_3^3$ subject to

$$x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$

   $x_1, x_2, x_3 \geq 0$
   (Ans $x_1 = 81/100, x_2 = 7/20$ Max $Z = 17/20$)

3. Max $Z = 2x_1x_2 - 5x_1 - 13x_2 + 3x_2^2 - 10$ subject to

$$x_1 + x_2 \leq 1$$
$$4x_1 + x_2 \geq 2$$

   $x_1, x_2 \geq 0$
   (Ans $x_1 = 1/2, x_2 = 0$, Max $Z = -15$)

End