Assignment - 3rd

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Q1)
Ans 1)

(a) $|\psi_{(x)}|^2 = \psi_{(x)} \cdot \psi^*_{(x)}$

$= 4\alpha^3 x^2 e^{-2\alpha x}$

$\frac{\partial |\psi_{(x)}|^2}{\partial x} = 8\alpha^3 x e^{-2\alpha x} + (-2\alpha) 4\alpha^3 (x^2) e^{-2\alpha x}$

$= 0$

$\left[x = \frac{1}{\alpha} \right]$

(b) $\langle x \rangle = \int_0^x x |\psi_{(x)}|^2 dx$

$\Rightarrow \int_0^x 4\alpha^3 x^3 e^{-2\alpha x} dx$

$= \frac{x^3 e^{-2\alpha x}}{-2\alpha} - \int_0^x \frac{x^3 e^{-2\alpha x}}{-2\alpha} dx = \frac{3}{2\alpha}$

$\boxed{\langle x^2 \rangle = \frac{3}{2\alpha}}$

(c) $P = \int_0^{\alpha} |\psi|^2 dx = 1$ & $\Rightarrow \int_0^{1/\alpha} |\psi|^2 dx$

$\Rightarrow \int_0^{1/\alpha} 4\alpha^3 x^2 e^{-2\alpha x} = 0.32$

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Q3
Ans 2

T.I.S.E $\rightarrow$ $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{\partial x^2} + U(\psi) = E(\psi)$$

$V = 0$ in II $\Rightarrow$ $$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE(\psi)}{\hbar^2} = 0$$

$$\frac{\partial^2(\psi_0)}{\partial x^2} + k^2\psi_0 = 0$$

$k^2 = \frac{2mE}{\hbar^2}$ general sol^n $\psi_0(x) = A\sin kx + B\cos kx$

Boundary conditions $\psi_0 = 0$ at $x = -a$

$$0 = A\sin(kx - \omega t) + B\cos((k)(-a))$$

$$A\sin ka = B\cos(ka)$$

$$\sin(ka) = \frac{B}{A}$$

Now at $x = a$ & $\psi(0) = 0$

$$0 = A\sin ka + B\cos ka \quad -②$$

from ③ & ④

$$2B\cos(ka) = 0$$

$B = 0$ and $A = 0$

Case ① $A = 0$ and $B \neq 0$

$B\cos(ka) = 0$

if $B \neq 0$

$0 = B\cos(ka) = 0$

for $ka = \frac{n\pi}{2}$

$n = 1, 3, 5, \ldots$

$E = \frac{n^2\pi^2\hbar^2}{}$

② $A \neq 0$ & $B = 0$

$A\sin ka = 0$

if $A \neq 0 \Rightarrow \sin ka = 0$

for $ka = \frac{\pi n}{2}$ where

$n = 0, 2, 4, 6 \ldots$

$$\psi_{(x)} = \begin{cases} \frac{A \sin n\pi x}{2a} & ; n = 2, 4, 6 \ldots \\ A \cos \frac{n\pi a}{2a} & ; n = 1, 3, 5 \ldots \end{cases}$$

Normalizing $\int |\psi|^2 dx = 1 \Rightarrow A^2 \int \sin^2 \frac{n\pi x}{2} = 1$

by $\rightarrow \left\{ B = \frac{1}{\sqrt{a}} \right\}$ and $\left\{ A = \frac{1}{\sqrt{a}} \right\}$

(3) $\int |\psi|^2 dx \Rightarrow 1$

$$\int_{-1}^{1} |A \sin \pi x e^{-i\omega t}| \cdot A \sin \pi x e^{i\omega t}$$

$$\Rightarrow \int_{-1}^{1} A^2 \sin^2 \pi x \, dx = 1$$

$$\boxed{A = \pm 1}$$

& Normalized wave function is

$$\psi(x,t) = \pm \sin(n\pi x) e^{i\omega t)}$$

Q4) $\langle x \rangle = \frac{L}{2}$

$$\langle p_x \rangle = \int \psi^* \left(-i\hbar \frac{\partial}{\partial x^2} \psi_{(x)} \right) dx$$

$$= \int 2 \left(\frac{i\hbar}{L} \right) \sin\left(\frac{n\pi x}{L} \right) \cos\left(\frac{n\pi x}{L} \right) dx \times \frac{n\pi}{L}$$

$$= \frac{-2\hbar}{L} i n \frac{\pi}{2} \int_0^L \sin\left(\frac{n\pi x}{L} \right) \cos\left(\frac{n\pi x}{2} \right) dx = 0$$

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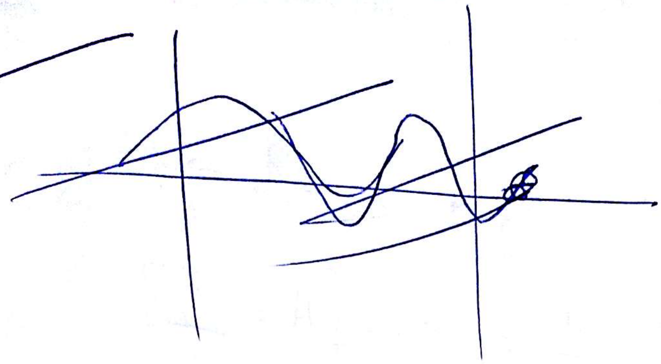
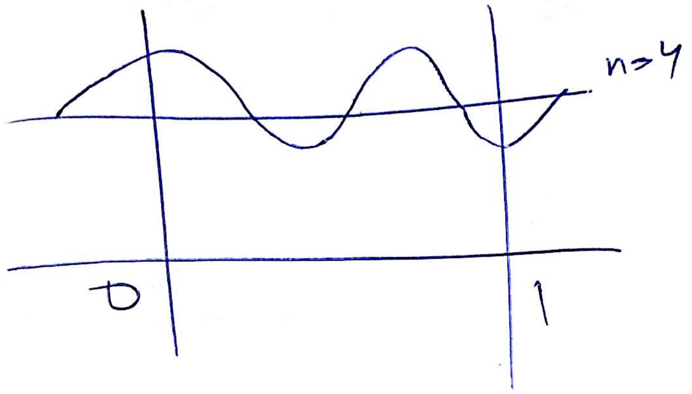
Q5) $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{4\pi x}{L}\right)$

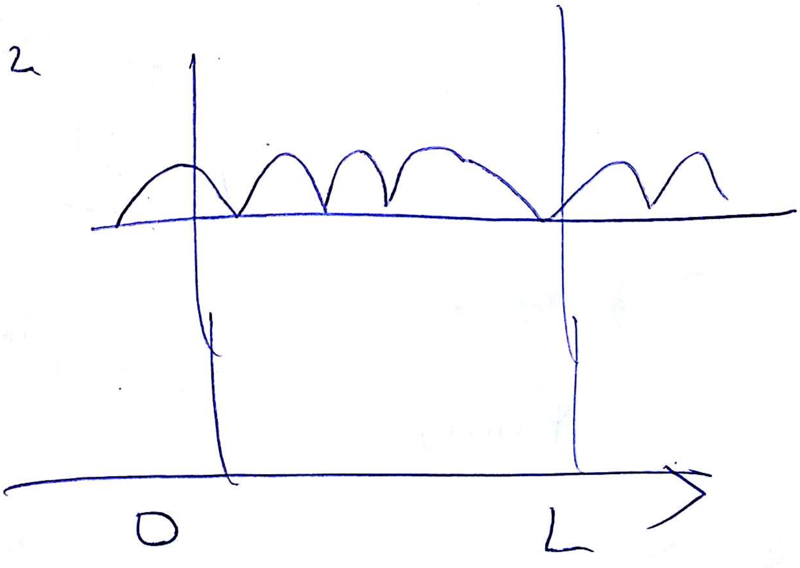
~~$|\psi(x)|$~~



$|\psi(x)|$



Q6-) $|\psi(x)|^2$



Q7) $P = \int_{L/4}^{L/2} (\psi(x))^2 \, dx = \frac{2}{L} \int_{L/4}^{L/2} \sin^2 \frac{\pi x}{L} \, dx - \frac{1}{L} \int_{L/4}^{L/2} 1 - \cos\left(\frac{2\pi x}{L}\right)$

$\Rightarrow \frac{1}{4} - \frac{\sin\left(\frac{2\pi x}{L}\right)}{2\pi} \times L$

$\Rightarrow \frac{1}{4} - \frac{1}{L}\left(\frac{-L}{2\pi}\right) = \frac{1}{4} + \frac{1}{2\pi}$

Q8)
Ans8)

$$E_n = \frac{n^2 h^2 \pi^2}{8ma^2} = E_4 - E_2$$

$$\Rightarrow \frac{[(4)^2 - (2)^2]\, h^2 \pi^2}{8ma^2} \Rightarrow \frac{12 \pi^2 h^2}{8ma^2}$$

$$\Rightarrow \frac{3}{2}\left(\frac{\pi^2 h^2}{ma^2}\right) \Rightarrow 6.67 \times 10^{-34} \times 3.43 \times 10^{6}$$

Q9)
Ans9)

TISE for Region 1

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + 0 = E(\psi)_I(x)$$

$$\frac{\partial^2 \psi}{\partial x^2} = k_1^2 (\psi_I(x)) = 0$$

$$k_1^2 = \frac{2mE}{h^2}$$

Q10)
Ans10)

$$T = 16\frac{E}{V}\left(1 - \frac{E}{V}\right)e^{-2k_2 L}$$

$$\Rightarrow 16 \times \frac{12}{30}\left(1 - \frac{12}{30}\right)e^{-2k_2 5 \times 10^{-11}}$$

$$\Rightarrow \frac{32}{5}\left(\frac{3}{5}\right)e^{-16 \times k_2}\ ;\ k_2 = \sqrt{\frac{V_0 - E}{\hbar^2} \times 2m}$$

$$k_2 = \sqrt{\frac{30-12}{(\hbar)^2}(2 \times 9.1 \times 10^{-31})}$$