a) If $L < 1$, the series converges absolutely.
b) If $L > 1$, the series diverges.
c) If $L = 1$, the series may be converge or diverge.

~~Root Test~~:

Partial Differential Equations:

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

A, B, C are functions of x, y

D is linear function of $x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$

$B^2 - 4AC < 0$ – Elliptic

$B^2 - 4AC$

Assignment-5

1. a) $u_t + x u_x = 0$

$\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = 0$ (linear). $[P(x,t)\,p + Q(x,t)\,q = r(x,t)z + \underbrace{s(x,t)}_{0}$

b) $u_t + a u_x = u^2$

$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = u^2$ (semilinear)

$[\because a(x,y)\,u_x + b(x,y)\,u_y = c(x,y,u)]$

c) $x u_t + u_x = u$

(linear)

d) $u_t + u u_x = 0$ (quasilinear).

e) $-\Delta u(x) = f(x, u(x))$.

$-\frac{\partial u}{\partial x} = f(x, u(x))$ (semilinear).

linear.

f) $\Delta^2 u(x,y)=0$ $\quad \frac{d^2 u(x,y)}{\cancel{dy}\,\cancel{dx}\, d(y,x)^2}=0$

g) $u_t = u^3 u_{xxx}$ (Non-linear)

Ans 2. a) $z(x,y) = xy + f(x^2+y^2)$

$p = y + f'(x^2+y^2)\,2x$

$q = x + f'(x^2+y^2)\,2y$

$\frac{p-y}{q-x} = \frac{x}{y} \Rightarrow py - y^2 = qx - x^2$

$\boxed{py - qx = y^2 - x^2}$

b) $z(x,y) = f(x/y)$

$p = f'\left(\frac{x}{y}\right)\left(\frac{1}{y}\right)$

$q = f'(x/y)\left(\frac{-1}{y^2}\right)(x)$

$\frac{p}{q} = -\frac{y}{x} \Rightarrow \boxed{xp + yq = 0}$

c) $f(x-z, y-z) = 0$

$\cancel{f'(x-z)\ldots}$ $\quad \cancel{f'(x-z)(-q) = 1-q}$

$\cancel{f'(x-z)(-p) = -p}$

$x - z = f(y-z)$

$1 - p = f'(y-z)(-p)$

$-q = f'(y-z)(1-q)$

$\frac{1-p}{-q} = \frac{-p}{1-q}$

$(1-p)(1-q) = pq$

Ans 3. a) $z(x,y) = (x+a)(y+b)$

$p = y + b$

$q = x + a$

$\boxed{z(x,y) = pq}$

b) $z(x,y) = ax + by$

$p = a$

$q = b$

$\boxed{z(x,y) = px + qy}$

c) $z^2(1+a^3) = 8(x+ay+b)^3$

$= 2zp(1+a^3) = 8(x+ay+b)^2 \cdot 3 \ (1)$

$\Rightarrow 2zq(1+a^3) = 8 \cdot 3(x+ay+b)^2 (a)$ — ①

$\frac{p}{q} = \frac{1}{a} \Rightarrow a = \frac{q}{p}$

$\cancel{2zp(1+a^3)} = \cancel{8\times3} \ \cancel{(2zq(1+a^3))}$

$\frac{z}{2q} = \frac{(x+ay+b)}{3}$

$\frac{3z}{2q} = x + \frac{q}{p}y = b$

$z^2\left(1+\frac{q}{p}\right) = 8\left(x+\frac{q}{p}y + \frac{3z}{2q} - x - \frac{q}{p}y\right)$

$z^2\left(1+\frac{q}{p}\right) = 4\left(\frac{3z}{2q}\right)$

Ans 4. a) $\frac{x}{p} + z = c$

$\therefore \frac{x}{p} = \frac{y}{q}$ $\boxed{xq = yp}$ — ①

b) $(x^2+y)\cos^2\theta - (z-c^2)\sin^2\theta = 0$

$2x\cos^2\theta = p\sin^2\theta$

$2y\cos^2\theta = q\sin^2\theta$

$\boxed{\frac{x}{y} = \frac{p}{q}}$ — ② $\therefore$ ① = ②

hence proved.

$z(x,y) = f(x^2+y^2)$

$p = f'(x^2+y^2)(2x)$

$q = f'(x^2+y^2)(2y)$

$\boxed{\frac{p}{q} = \frac{x}{y}}$ hence always holds

Ans 5. a) $xp + yq = z$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

$$xy = c_1 \quad — (1)$$

$$y\cancel{x}z = c_2 \quad — (2).$$

$$\therefore \; f(xy, yz)$$

b) $x^2 p + y^2 q = (x+y)z.$

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{(x+y)z} \quad — (1).$$

$$\frac{dx - dy}{x^2 - y^2} = \frac{dz}{(x+y)z} \quad — (2).$$

c) $yzp + xzq = xy$

$$\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}$$

$$x^2 - y^2 = c_1 \quad — (1).$$

$$z^2 - y^2 = c_2 \quad — (2).$$

d) $$\frac{dx}{x} = -\left(\frac{dy}{y} + \frac{dz}{z}\right)$$

$$x^2 + y^2 + z^2 = c_1.$$

$$\int \frac{d\cancel{x}y}{x(y+z)} = \int \frac{dz}{x(y-z)} = c_2$$

$$c_2 = y^2 - 2yz - z^2.$$

Ans 7. a)