Assignment :

1. $\cancel{y'' - \frac{4}{x}y' + \left(\frac{x^2+6}{x^2}\right)y = 0}$ $\qquad y(0) = y'(0) = 0$

$x^2y'' - 4xy' + (x^2+6)\,y = 0 \qquad \text{——} (1)$

$y = x^2 \sin x \qquad y(0) = 0$

$y' = 2x\sin x + x^2\cos x \qquad y'(0) = 0$

$y'' = 2\sin x + 2x\cos x + 2x\cos x - x^2\sin x$

Put in (1)

$2x^2\sin x + \cancel{4x^3\cos x} - x^4\sin x - 8x^2\sin x - \cancel{4x^3\cos x}$
$+ x^4\sin x + 6x^2\sin x$

$= 0$ ✓

$y = 0 \qquad y(0) = 0 ; \; y'(0) = 0$

$0 - 0 + 0 = 0$ ✓

But (1) is not of the type
$y'' + Py' + Qy = 0$
so uniqueness thm. is inapplicable.
No it does not contradict

2. $y'' - y' = 0 \qquad y(0) = 0$

$y'' + Py' + Qy = 0 \qquad y'(0) = 1$

P & Q are cont. $\therefore$ a unique sol^n exists.

$y' = y + c$

$\ln(y+c) = x + c_1$

$\boxed{y = c_1e^x + c_2}$

$y(0) = 0 = c_1 + c_2$

$y'(0) = c_1 = 1 \qquad c_2 = -1$

$\Rightarrow \boxed{y = e^x - 1}$

3\. (i) $y = \cos(ax+b)$

$\cancel{ax+b = t}$
$\cancel{a\,dx = dt}$

$y' = -a\sin(ax+b)$

$y'' = -a^2\cos(ax+b)$

$y'' = -a^2 y$

$\left(\frac{y'}{a}\right)^2 + y^2 = 1$

$a^2 = \frac{(y')^2}{1-y^2}$

$y'' = \left(\frac{(y')^2}{y^2-1}\right)y$

$(y^2-1)y_2 = y_1^2 y$

(ii) $y = ax + \frac{b}{x}$

$y_1 = a - \frac{b}{x^2}$

$y_2 = +\frac{2b}{x^3} \Rightarrow \quad b = \frac{y_2 x^3}{2}$

$a = y_1 + \frac{y_2 x^3}{2x^2}$

$y = \left(y_1 + \frac{y_2 x}{2}\right)x + \frac{y_2 x^2}{2}$

$y = y_1 x + \frac{y_2 x^2}{2}$

(iii) $y = ae^x + bxe^x$

$y_1 = ae^x + be^x + bxe^x = y + be^x$

$y_2 = y_1 + be^x$

$y_1 = y + y_2 - y_1$

$y_2 + y = 2y_1$

4. (a) (i) $m^2+3m+2=0$

$(m+2)(m+1)=0$

$m=-1,-2$

(ii) $m^2-4m+4=0$

$m=2$

(iii) $m^3-2m^2-m+2=0$

$(m-2)(m^2-1)=0$

$(m-2)(m-1)(m+1)=0$

$m=-1,1,2$

(b) (i) $m^2-4m+4=0$

$m=2$

(ii) $m^2-3m-5=0$

$m=\frac{3\pm\sqrt{9+20}}{2}=\frac{3\pm\sqrt{29}}{2}$

5. $z=ay_1+by_2$

$a(y_1''+py_1'+qy_1)+b(y_2''+py_2'+qy_2)$

$=(a+b)r(x)=r(x)$ For z to be a solution

$\boxed{a+b=1}$

~~6. (a) eq^n (1) can have a unique solution~~
~~$\therefore$ either $y=x$ is a sol^n~~
~~or $y=\sin x$.~~
~~$\therefore$ (1) can't have $y=x$ & $y=\sin x$ as its solutions.~~

$y = x$

$0 + p(x) + q(x) \cdot x = 0 \quad -①\quad x \in I.$

$y = \sin x$

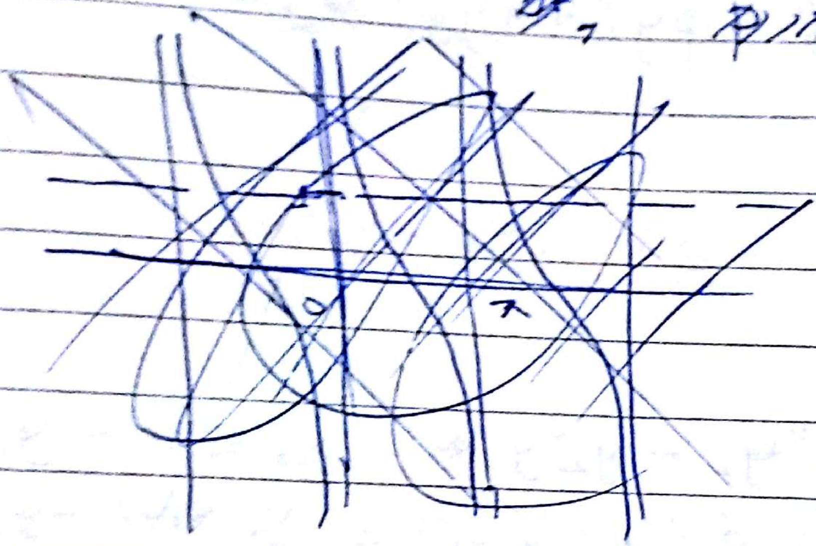
$-\sin x + p \cdot \cos x + q \cdot \sin x = 0 \quad -② \quad x \in I$

$-\sin x \cancel{+} - q \cdot x\cos x + q \cdot \sin x = 0$

$q(\sin x \mp x\cos x) = \sin x$

$q = \dfrac{\sin x}{\sin x \mp x\cos x}.$

$\cancel{q}, \quad q/\!\!/ = \dfrac{1}{1 - x\cot x}$



Let $y = c_1 x + c_2 \sin x$

$y' = c_1 + c_2 \cos x$

$y'' = -c_2 \sin x.$

$-c_2 \sin x + p(c_1 + c_2 \cos x) + q(c_1 x + c_2 \sin x) = 0$

$c_2(-\sin x + p\cos x + q\sin x) + c_1(p + x \cdot q) = 0$

$x\cos x - \sin x$

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1. (a) $y'' + py' + q.y = 0$ $x \in I$.

y_1 & y_2 are linearly independent.
& $y_1 y_2' - y_2 y_1' \neq 0$ — ① $x \in I$.

~~Let y_1 & y_2 be fundamental sol^n~~

~~Let $y_2 \times (y_1'' + py_1' + qy_1 = g_1(x)$ — ②~~
~~$y_1 \times (y_2'' + py_2' + qy_2 = g_2(x)$ — ③~~

~~① - ②~~

~~$-y_1$~~

~~① $y_1'' y_2 - y_2'' y_1 + = y_2 g_1(x) - y_1 g_2(x)$~~
~~$d(y_1 y_2' - y_2 y_1') = y_2 g_1(x) - y_1 g_2(x)$~~
~~$y_1' y_2' + y_1 y_2'' - y_2 y_1' - y_2 y_1''$~~

Let y_1 & y_2 be fundamental sol^ns.

$y_1' = \frac{-qy_1 - y_1''}{p}$

$y_2' = \frac{-qy_2 - y_2''}{p}$

$py_1' + qy_1 = -y_1''$
$py_2' + qy_2 = -y_2''$

unique sol^n exists

$\begin{vmatrix} y_1' & y_1 \\ y_2' & y_2 \end{vmatrix} \neq 0$

(12) (i) e^{-x}, xe^{-x}

$y = c_1 e^{-x} + c_2 x e^{-x}$

$y_1 = -c_1 e^{-x} + c_2 e^{-x} - c_2 x \cdot e^{-x} = -y + c_2 e^{-x}$

$y_2 = -y_1 - c_2 e^{-x} \Rightarrow c_2 = -(y_2 + y_1) e^{x}$

$\cancel{y = c_1 e^{-x} + -(y_1 + y_2) x}$

$\cancel{\Rightarrow c_1 = (y + (y_1 + y_2) x) e^{x}}$

$y_1 = -y - (y_2 + y_1)$

$2y_1 + y + y_2 = 0$

(ii) $e^{-x} \sin 2x, \ e^{-x} \cos 2x$

$y = c_1 e^{-x} \sin x + c_2 e^{-x} \cos x$

$y_1 = -y + 2c_1 e^{-x} \cos x - 2c_2 e^{-x} \sin 2x$

$y_2 = -y_1 - (y_1 + y) - 4c_1 e^{-x} \sin 2x - 4c_2 e^{-x} \cos 2x$

$y_2 = -y_1 - (y_1 + y) - 4y$

$$\boxed{y_2 + 2y_1 + 5y = 0}$$

8.

$y'(x_0) = 0$

$y(x_0) = 0$

$y'' + p(x)\cdot y' + q(x)\cdot y = 0$

$\because p(x)$ & $q(x)$ are cont

By uniqueness thm, it has a unique sol^n

And $y=0$ satisfies it

So $y=0$ is the only sol^n

9. (i)

$y_1(x_1) = 0$

$y_1(x_2) = 0$

$y_1 y_2' - y_2 y_1' \neq 0$

$y_1(x_1)\, y_2'(x_1) - y_2(x_1)\, y_1'(x_1) \neq 0$

$y_1(x_2)\, y_2'(x_2) - y_2(x_2)\cdot y_1'(x_2) \neq 0$

$y_2(x_1)\cdot y_2(x_2) \cdot y_1'(x_1)\cdot y_1'(x_2) \neq 0$

$y_2(x_1)\cdot y_1'(x_1) = 0$

$y_2(x_2)\cdot y_2'(x_2) = 0$

(ii)

$(\alpha y_1 + \beta y_2)(\gamma y_1' + \delta y_2') - (\gamma y_1 + \delta y_2)(\alpha y_1' + \beta y_2')$

$= \cancel{\alpha\gamma y_1 y_1'} + \gamma\beta y_1' y_2 + \alpha\delta y_1 y_2' - \cancel{\gamma\alpha y_1 y_1'} - \gamma\beta y_1 y_2' - \delta\alpha y_2 y_1' - \delta\beta\ldots$

$= (\alpha\delta - \gamma\beta)\, y_1 y_2' \neq (\gamma\beta - \delta\alpha)\, y_2 y_1' \neq 0$

$\alpha\delta \neq \beta\gamma$

10 $y_1(x_0) = y_2(x_0) = 0$

$y_1' y_2 - y_2' y_1$

at $x = x_0$ $0 =$

Q6 Suppose $y_2(x) > 0$ for $x \in [a, b]$

$y_1(a) > 0$ & $y_1(b) < 0$

$y_1(a)$ $y_1(a) \cdot y_2(x) > 0$

$y_1(b) \cdot y_2(b) < 0$

$y_1'(x) \cdot y_2(x) -$

$y_1(x)\, y_2(x) = 0$ $W(y_1, y_2) = 0$

$y_1(x) = 0$. Not possible.

$y_2(x) = 0$ $\therefore$ $y_2(x) = 0$ for some $x \in (a, b)$

Let $\exists$ 2 zeroes of y_2

then $\exists$ 2 zeros for y_1 b/w them

but x_1 & x_2 are consecutive