# ~~Assignment-1~~ Assignment-1 by Yatharth Shah



Q1.) Given $A = (a_{ij})$ define $A^T = (b_{ij})$ [where $b_{ij} = a_{ji}$], then Show that $(AB)^T = B^T A^T$ if $AB$ is defined.

Ans1.) Given $(A^T)^T = B^T$

i.e. $A = B^T$

$A = a_{ij}$

~~A = A~~ $(AB)^T = (A \cdot A^T)^T =$

Q2.) Let A & B be invertible matrices with same dimension, then show that $(AB)^{-1} = B^{-1}A^{-1}$

Ans2.)

~~$A = (a_{ij})_{n \times n}$~~

~~$B = (b_{ij})_{n \times n}$~~

~~$AB = BA = I$~~

~~$B = A^{-1}$~~

$A = (a_{ij})_{n \times n}$

$B = (b_{ij})_{n \times n}$

we need to prove that if A & B are invertible square Matrix then $B^{-1}A^{-1}$ is the inverse of AB. Let us denote $B^{-1}A^{-1}$ by C

Then by defination we need to show that $(AB)C = C(AB) = I$

Substituting $B^{-1}A^{-1}$ for C we get

$(AB)B^{-1}A^{-1} = ABB^{-1}A^{-1} = AIA^{-1} = AA^{-1} = I$

Similarly we can show for $C(AB) = I$.

Q37 Show that every square Matrix can be written as the sum a symmetric and a skew - Symmetric matrices. Further, show that if A & B are symmetric, then AB is also symmetric if and only if AB = BA.

Ans 37 # Symetric Matrix →

$$\boxed{-A = A^T}$$

only possible for

Square Matrix

# Skew-Symmetric Matrix →

$$\boxed{A = A^T}$$

only possible for

square Matrix

Let $A = P + Q$

$$A = \frac{1}{2}(A - A^T) + \frac{1}{2}(A + A^T)$$

$$(A - A^T)^T = A^T - A = -(A - A^T)$$

↓

Skew symmetric

&

$$(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$$

↓

Symmetric Matrix

Given $A = A^T$, $B = B^T$ } Both A & B are symmetric Matrix

$(AB)^T = B^T A^T = BA$

This is possible iff $AB = BA$.

Q4) If A is real orthogonal matrix (i.e., $AA^T = I$), then show that $|A| = \pm 1$

Ans 4)

$AA^T = I$

taking determinant both side.

$|A||A^T| = |I|$

$|A|^2 = 1$

$\underline{|A| = \pm 1}$

Q5) Let A be nilpotent ($A^m = 0$, for some $m \geq 1$) matrix. Show that $I + A$ is invertible.

Ans 5)

$A^m = 0$

$|A + I| \neq 0$

Let's ~~see~~ ~~the~~ assume that $a_0 I + a_1 A + a_2 A^2 + \cdots a_m A^m$ be the inverse of $A + I$ then we need to prove

$(A+I)(a_0 I + a_1 A + \cdots + a_m A^m) = I$ for

some $a_0, a_1, a_2, \cdots, a_m \in I$

$(A+I)(a_0 I + a_1 A + \cdots + a_m A^m)$

$(A+I)(a_0 I + a_1 A + \cdots + a_m A^m$

$\Rightarrow a_0 A + a_1 A^2 + \cdots + a_m A^{m+1} + a_0 I + a_1 A + \cdots + a_m A^m$

$\Rightarrow$ ~~$a_0(A+I) + (a_0 + a_1) A$~~ $\cancel{a_0 I + (a_0 + a_1) A + (a_1 + a_2) A^2 + \cdots (a_m + a_{m-1}) A^m + a_m A^{m+1}}$

$a_0(I+A) + a_1(A+A^2) + a_2(A^2+A^3) + \cdots + a_m(A^m)$

~~I+A~~

if $a_m = (-1)^m$

then

$(I+A) - (A+A^2) + (A^2+A^3) - (A^3+A^4) \cdots + (-1)^m (A^m + A^{m+1})$

for $\underline{m = \text{odd}}$.

$I + \cancel{A} - \cancel{A} - \cancel{A^2} + \cancel{A^2} + \cancel{A^3} - \cancel{A^3} - A^4 \cdots + \cancel{A^m} - \cancel{A^m} - A^{m+1}$

$I + A^{m+1}$ for $m \geq 1$ $A^m = 0$

$A^{m+1} = 0$

thus $\Rightarrow I + A^{m+1} = 0$

$\underline{\text{for} \quad m = \text{even}}$.

$I + \cancel{A} - \cancel{A^2} - \cancel{A^2} + \cancel{A^2} + \cancel{A^3} - \cancel{A^3} - \cancel{A^4} \cdots + \cancel{A^m} + \cancel{A^m} + A^{m+1}$

$\Rightarrow I + A^{m+1}$

Again $A^{m+1} = 0$

thus

$I + A^{m+1} = I$

thus Inverse of $(A+I)$ exist which is equal to $\left[ I_0 + \sum_{n=1}^{n} (-1)^n A^n \right]$ for $\forall n \in \mathbb{N}$

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Use row operations to find the row echelon form and row reduce echelon form of the following matrices.

(a) $\begin{bmatrix} 4 & 3 \\ -8 & -6 \\ 16 & 12 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 & 8 \\ 13 & 2 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$

Ans 6)

(a) $\begin{bmatrix} 4 & 3 \\ -8 & -6 \\ 16 & 12 \end{bmatrix}$

$[R_2 \to R_2 + 2R_1]$

$\begin{bmatrix} 4 & 3 \\ 0 & 0 \\ 16 & 12 \end{bmatrix}$

$[R_3 \to R_3 - 4R_1]$

$\begin{bmatrix} 4 & 3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow$ (Row-echelon form)

$[R_1 \to R_1/4]$

$\begin{bmatrix} 1 & 3/4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow$ (Row-Reduced echelon form)

(b)

$$\begin{bmatrix} 1 & 5 & 8 \\ 3 & 2 & 9 \end{bmatrix}$$

$[R_2 \rightarrow R_2 - 3R_1$

(RE) $\leftarrow \begin{bmatrix} 1 & 5 & 8 \\ 0 & -13 & -15 \end{bmatrix}$ $\left[R_2 \rightarrow \frac{R_2}{13}\right]$

(RRE) $\leftarrow \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & \frac{15}{13} \end{bmatrix}$

(c)

$$\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 0 \end{bmatrix}$$

$\left[R_2 \rightarrow \frac{R_2}{2}\right]$

$$\begin{bmatrix} 0 & 2 & -3 \\ 1 & 0 & 5/2 \\ -3 & 5 & 0 \end{bmatrix}$$

$[R_1 \rightarrow R_1 + R_2]$

$\begin{bmatrix} 0 & 2 & -3 \\ 1 & 0 & 5/2 \\ -1 & \frac{5}{3} & 0 \end{bmatrix} \leftarrow \left(R_3 \rightarrow \frac{R_3}{3}\right)$

$$\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 1 & 0 & 5/2 \\ -3 & 5 & 0 \end{bmatrix}$$

$\left[R_2 \rightarrow R_2 + \frac{R_3}{3}\right]$

$$\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & 5/3 & 5/2 \\ -1 & \frac{5}{3} & 0 \end{bmatrix}$$

$[R_3 \rightarrow R_3 + R_1]$

$$\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & 5/3 & 5/2 \\ 0 & \frac{11}{3} & -\frac{1}{2} \end{bmatrix}$$

$\left[R_3 \rightarrow \frac{3R_3}{11}\right]$

$$\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & 5/3 & \frac{5}{2} \\ 0 & 1 & -\frac{11}{6} \end{bmatrix}$$

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(c) $\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_2 \to \frac{R_2}{2} \right]$

~~Long~~

$\begin{bmatrix} 0 & 2 & -3 \\ 1 & 0 & \frac{5}{2} \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_1 \to R_1 + R_2 \right]$

$\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 1 & 0 & \frac{5}{2} \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_2 \to R_2 - R_1 \right]$

$\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & -2 & 3 \\ & & \end{bmatrix}$

$\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_1 \longleftrightarrow R_2 \right]$

$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 2 & -3 \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_1 \to \frac{R_1}{2} \right]$

$\begin{bmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 2 & -3 \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_2 \to \frac{R_2}{2} \right]$

$\begin{bmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -3/2 \\ -3 & 5 & 0 \end{bmatrix}$ $\left[ R_3 \to R_3 + 3R_1 \right]$

$\begin{bmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -3/2 \\ 0 & 5 & \frac{15}{2} \end{bmatrix}$ $\left[ R_3 \to \frac{R_3}{5} \right]$

$\begin{bmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -\frac{3}{2} \\ 0 & 1 & \frac{3}{2} \end{bmatrix}$ $\left[ R_3 \to R_3 - R_2 \right]$

$\begin{bmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 3 \end{bmatrix}$ [RRE

(d)

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

$[R_2 \to R_2 - 2R_1]$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

$[R_3 \Longleftrightarrow R_2]$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 3 & 6 & -6 & 9 & 13 \\ 0 & 0 & 2 & 4 & 6 \end{bmatrix}$$

$[R_2 \to R_2 - 3R_1]$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 3 & 6 & 7 \\ 0 & 0 & 2 & 4 & 6 \end{bmatrix}$$

$[R_3 \to R_3/2]$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 3 & 6 & 7 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

Apply Gauss Elimination method to solve the following system :- (5)

(I) $x - y + z = 0$

$-x + y - z = 0$

$10y + 25z = 90$

$20x + 10y = 80$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right] \quad [R_2 \leftarrow R_1 + R_2]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right] \quad [R_4 \leftarrow R_4 - 20R_1]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & +30 & -20 & 80 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad [R_3 \leftarrow R_3 + 30R_1]$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & -95 & -190 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left( \begin{aligned} x &= 0 \\ y &= 4 \\ z &= 2 \end{aligned} \right)$$

$x - y + z = 0$

$10z = 80$

$z = 8$

$10y + 25y = 90$

$10y + 25 \times 8 = 90$

$y = -11$

$x + 11 + 8 = 0$

$x = -19$

(II)

$3x + 2y + z = 3$

$2x + y + z = 0$

$3x + y + 2z = 3$

$$\left[\begin{array}{ccc|c} \boxed{3} & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 3 \end{array}\right] = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 3 \end{array}\right] R_2 \leftarrow R_2 - R_1$$

$R_1 \leftarrow \frac{R_1}{3}$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 1 & 1 & 0 & -3 \\ 1 & 0 & 1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 1 & 1 & 0 & -3 \\ 1 & 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 1 & 1 & 0 & -3 \\ 1 & 0 & 1 & 3 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$ $\quad$ $R_3 \leftarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 0 & 1/3 & -1/3 & -4 \\ 0 & -2/3 & 2/3 & 2 \end{array}\right]$$

$R_3 \leftarrow 2R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 0 & 1/3 & -1/3 & -4 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

[No Solution]

(III)

$$3x + 2y + z = 0$$
$$2x + y + z = 0$$
$$6x + 2y + 4z = -6$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & -6 \end{array}\right]$$

$R_3 \leftarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right] \quad [R_2 \leftrightarrow R_1]$$

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & 2 & 1 & 3 \\ 0 & -2 & 2 & 0 \end{bmatrix} \quad \left[R_1 \to \frac{R_1}{2}\right]$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 3 & 2 & 1 & 3 \\ 0 & -2 & 2 & 0 \end{bmatrix} \quad [R_2 \to R_3 + R_2]$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 3 & 0 & 3 & 3 \\ 0 & -2 & 2 & 0 \end{bmatrix} \quad \left[R_2 \to \frac{R_2}{3}\right]$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 0 & 1 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix} \quad [R_2 \to R_2 - R_1]$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix} \quad \left[R_3 \to \frac{R_3}{4}\right]$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

It can be easily solved ahead

But the Result will be no-Solution as we will be getting

$0(x) + 0(y) + 0(z) = K$ where $K \neq 0$

that's it

7 Consider the following linear non-homogeneous system:

$$x + y + z = 5$$
$$2x + 3y + 5z = 8$$
$$4x + 5z = 2$$

(a) find the coefficient matrix A and augmented matrix [A|b], where b is the right hand side vector. What can we say about the existance of the solution for the given system.

(b) Apply Gauss Jordan elimination method to find the solution.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 0 & 4 & 5 & 2 \end{array}\right]$$

$\cancel{R_2 \leftarrow R_2 - 2R_1}$

$\cancel{R_3 \leftarrow R}$

$[R_2 \to R_2 - 2R_1]$

$[\cancel{R_3 \to R_3 - 4R_1}]$

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 4 & 5 & 2 \end{bmatrix}$$

$[R_3 \to R_3 - 4R_1]$

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & +1\cancel{7} & \cancel{10}\ -26 \end{bmatrix}$$

$[R_1 \to R_1 - R_2]$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & +1 & 2 \end{array}\right]$$

$[R_1 \to R_1 + 2R_3]$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{bmatrix} x = 11 \\ y = -4 \\ z = 2 \end{bmatrix}$$

Q. Choose h & k such that the system has (a) no solution (b) a unique solution (c) infinitely many solution

(I) $x + hy = 2$
$4x + 8y = k$

(II) $x + 3y = 2$
$3x + hy = k$

**Ans 9.7**

$$\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{bmatrix}$$

$8 - 4h = 0$ , $k \neq 8$

$h = 2$ , $k \neq 8$ (unique)

$h \neq 2$ , $k = 8$ (no sol) [$k \neq 8$, $h = 2$]

$h = 2$ $k = 8$ → infinite

(II)
$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & h-9 & k-6 \end{bmatrix}$$

no sol

$h = 9$, $k \neq 6$

$h \neq 9$, $k$ [illegible] (unique)

$h = 9$, $k = 6$ [illegible]

Q10) find the inverse of the following matrices by using Gauss-Jordan Elimination

(a) $\cancel{\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}}$

(a) $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$

$[R_2 \rightarrow R_2 - 2R_1]$

$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$ $[R_1 \rightarrow R_1 - 3R_2]$

$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$

thus the Inverse of Matrice is

$\hookrightarrow \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right] \quad [R_3 \to R_3 - 4R_1]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ \cancel{2} & \cancel{-1} & \cancel{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right] \quad [R_3 \leftrightarrow R_2]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 2 & -1 & 3 & 0 & 1 & 0 \end{array}\right] \quad [R_3 \to R_2 + R_3]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 2 & 0 & 3 & -4 & 1 & 1 \end{array}\right] \quad [R_3 \to R_3 - 2R_1]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right] \quad [R_1 \to 2R_3 + R_1]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 13 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right] \quad [R_3 \to -R_3]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 13 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array}\right]$$

**Inverse** $\to \begin{bmatrix} 13 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{bmatrix}$$

$B$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 1 & 5 & -1 & 0 & 1 & 0 \\ 3 & 13 & -6 & 0 & 0 & 1 \end{array}\right]$$

$[R_2 \to R_2 - R_1$
$R_3 \to R_3 - 3R_1]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -2 & -5 & -1 & 1 & 0 \\ 0 & 4 & -18 & -3 & 0 & 1 \end{array}\right]$$

$[R_3 \to R_3 + 2R_2]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -2 & -5 & -1 & 1 & 0 \\ 0 & 0 & -8 & -5 & 2 & 1 \end{array}\right]$$

$[R_3 \to \frac{R_3}{2}]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -2 & -5 & -1 & 1 & 0 \\ 0 & 0 & -4 & \frac{5}{2} & 1 & \frac{1}{2} \end{array}\right]$$

$[R_1 \to R_1 + R_3]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{7}{2} & 1 & \frac{1}{2} \\ 0 & -2 & -5 & -1 & 1 & 0 \\ 0 & 0 & -4 & 5/2 & 1 & \frac{1}{2} \end{array}\right]$$

$[R_2 \to \frac{R_2}{2}]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{7}{2} & 1 & \frac{1}{2} \\ 0 & -1 & -\frac{5}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -4 & \frac{5}{2} & 1 & \frac{1}{2} \end{array}\right]$$

$[R_3 \to \frac{R_3}{[illegible]} \times \frac{5}{[illegible]}]$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{7}{2} & 1 & \frac{1}{2} \\ 0 & -1 & -\frac{5}{2} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{5}{2} & \frac{25}{16} & \frac{5}{8} & \frac{5}{16} \end{array}\right] \quad [R_2 \to R_2 - R_3]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{7}{2} & 1 & \frac{1}{2} \\ 0 & -1 & 0 & -\frac{33}{16} & -\frac{1}{8} & -\frac{5}{16} \\ 0 & 0 & -\frac{5}{2} & \frac{25}{16} & \frac{5}{8} & \frac{5}{16} \end{array}\right] \quad \left[\begin{array}{l} R_3 \to \frac{2R_3}{5} \\ R_2 \to R_1 + 3R_2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{11}{4} & \frac{21}{8} & -\frac{7}{16} \\ 0 & -1 & 0 & -\frac{33}{16} & -\frac{1}{8} & -\frac{5}{16} \\ 0 & 0 & -1 & \frac{5}{8} & \frac{1}{4} & \frac{1}{8} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{11}{4} & \frac{21}{8} & -\frac{7}{16} \\ 0 & 1 & 0 & \frac{33}{16} & \frac{1}{8} & \frac{5}{16} \\ 0 & 0 & 1 & -\frac{5}{8} & -\frac{1}{4} & -\frac{1}{8} \end{array}\right]$$

Inverse $\begin{bmatrix} -\frac{11}{4} & \frac{21}{8} & -\frac{7}{16} \\ \frac{33}{16} & \frac{1}{8} & \frac{5}{16} \\ -\frac{5}{8} & -\frac{1}{4} & -\frac{1}{8} \end{bmatrix} \Rightarrow \begin{bmatrix} -44 & 42 & -7 \\ 33 & 2 & 5 \\ -10 & -4 & -2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 1 & 0 & 0 \\ 1 & 5 & -1 & 0 & 1 & 0 \\ 3 & 13 & -6 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$ $\quad$ $R_3 \leftarrow R_3 - 3R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 1 & 0 & 0 \\ 0 & 2 & 3 & -1 & 1 & 0 \\ 0 & 4 & 6 & -3 & 0 & 1 \end{array}\right]$$

$1[-30+13] - 3[-6+3] - 4[13-15]$ Non-invertible

$-17 + 3\times 3 - 4\times 2$

$-17 + 8 + 9 = 0$

If any row becomes 0 then non-invertible matrix