

Asq 6

Ans 1. a)

$$y \cdot u_{xx} - xu_{yy} = 0$$

$$0 + 4xy > 0$$

hyperbolic (provided x, y should be of same sign)
para (if $y=0$).
elliptical (if x and y of opp sign)

similarly for other questions ($R^2 - 4ST$)

Ans 2. 10

$$u_{xx} - x^2 y u_{yy} = 0$$

$$R = 1, S = 0, T = -x^2 y$$

$$1 + 4x^2 y > 0$$

similarly for other ques:

d) $14u_{xx} + 2xu_{xy} + 5u_{yy} = xu_x$

$$R=1, S=2, T=5$$

$$4 - 4 \times 5 < 0$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$\frac{-4 \pm 4i}{2} = \frac{-2 \pm 2i}{1} = -2 \pm 2i$$

$$\lambda = -2 + 2i$$

$$\lambda_1 = -2 + 2i$$

$$\lambda_2 = -2 - 2i$$

$$y = -2(1-i)x + C_1$$

$$y = -2(1+i)x + C_2$$

$$C_1 = E = y + 2(1-i)x$$

$$C_2 = \eta = y + 2(1+i)x$$

$$u_x = u_E E_x + u_\eta \eta_x$$

$$= u_E (2(1-i)) + u_\eta (2(1+i))$$

$$u_y = u_E E_y + u_\eta \eta_y$$

$$= u_E + u_\eta$$

$$u_{xx} = u_E E_{xx} + E_x (u_{EE} E_x + u_{E\eta} \eta_x) + u_\eta \eta_{xx} + \eta_x (u_{\eta E} E_x + u_{\eta\eta} \eta_x)$$

$$= u_E (0) + 2(1-i) (u_{EE} (2(1-i)) + u_{E\eta} (2(1+i))) + u_\eta (0) + 2(1+i) (u_{\eta E} (2(1-i)) + u_{\eta\eta} (2(1+i)))$$

$$= 4(1-i)^2 u_{EE} + 8(1-i)(1+i) u_{E\eta} + 4(1+i)^2 u_{\eta\eta}$$

Similarly u_{xy} and substitute these values in eqn ①

Ans 3.

same in ques (3) as in (2).

Ans 4.

$$u_{tt} - c^2 u_{xx} = 0.$$

$$\text{B.C.s - } u(0,t) = 0.$$

$$\text{I.C.s - } u(x,0) = g(x), \quad u_t(x,0) = h(x).$$