The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-II ■ Assignment#2

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1. Which of the following are vector space:
(a) R(Q) (b) C(Q) (c) R(C) (d) R(R)
Where, Q: set of all rational numbers; R: set of all real numbers; C: set of all complex numbers.

2. Prove that the set $C[a,b]$ of all real valued continuous functions defined on the closed interval $[a,b]$ forms a real vector space if (i) addition is defined by $(f+g)(x) = f(x)+g(x)$, $f, g \in C[a,b]$, (ii) Multiplication by a real number r is defined by $(rf)(x) = rf(x)$, $f \in C[a,b]$. Prove that the subset $D[a,b]$ of all real valued differentiable functions defined on $[a,b]$ is a subspace of $C[a,b]$.

3. Which of the following are the subspaces of $\mathbb{R}^3$:
(a) $\{(x,y,z)|x \geq 0\}$; (b) $\{(x,y,z)|x+y=z\}$; (c) $\{(x,y,z)|x=y^2\}$; (d) $\{(x,y,z)|xy=0\}$.

4. Express the polynomial $v = x^2+4x-3$ in $P(x)$ as a linear combination of the polynomials $p_1 = x^2-2x+5,\ p_2 = 2x^2-3x,\ p_3 = x-1$.

5. Determine whether the following sets of vectors are linearly independent or not
(a) $S = \{(1,0,2,1),(1,3,2,1),(4,1,2,2)\}$ of $\mathbb{R}^4$,
(b) $S = \{(1,2,6),(-1,3,4),(-1,-4,-2)\}$ of $\mathbb{R}^3$,
(c) $S = \{u+v, v+w, w+u\}$ in a vector space V given that $\{u,v,w\}$ is linearly independent.
(d) $S = \{(1,2,0),(3,-1,1),(4,1,1)\}$

6. Let V be the vector space of functions from R into R. Show that the functions $f(x) = \sin(x),\ g(x) = e^x,\ h(x) = x^2$ are linearly independent.

7. If the set of the vectors $\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ in a vector space V over a field F be linearly dependent, then at least one of them is a linear combination of the remaining others.

8. Check whether the following four vectors in R^4 form a basis of R^4: (1,1,1,1), (0,1,1,1), (0,0,1,1), (0,0,0,1).

9. Check which of the following polynomials in $P_2(x)$ form a basis of $P_2(x)$:
(a) $\{1,\ x,\ x^2\}$ (b) $\{1+x+x^2\}$ (c) $\{1,\ x,\ x^2, 1+x+x^2\}$ (d) $\{1, 1+x, 1+x+x^2\}$.

10. Find the dimension of the following vector spaces
(a) $\{A : A$ is 2×3 real matrices$\}$.
(b) $\{A : A$ is 3×3 real upper - triangular matrices$\}$.
(c) $\{A : A$ is 3×3 real symmetric matrices$\}$
(d) $\{A : A$ is 2×2 real skey-symmetric matrices$\}$

11. Find the dimension and basis of the subspace W of $M_{2,3}$ spanned by

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 & 3 \\ 7 & 5 & 6 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 6 \end{bmatrix}$$

12. For what value of k, the matrix A has rank 2 if

$$\text{(I)} \quad A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 2 & 2 & 0 & 1 \\ 5 & 4 & 3 & k \end{bmatrix} \qquad \text{(II)}\ B = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 3 & 0 & 9 & 3 \\ 2 & 2 & 0 & 1 \\ 4 & 2 & 6 & k \end{bmatrix}.$$