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Assignment - 4

Ans 1 a)

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1 - 2R_2]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix}$$

↳ no zero row

∴ dimention = 3

& basis = {(1,1,2), (2,3,3), (1,3,5)}

b)

$$\begin{bmatrix} 1 & 2 & 2 & -1 & 3 \\ 1 & 2 & 3 & 1 & 1 \\ 3 & 6 & 8 & 1 & 5 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1 - 2R_2]{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 2 & 2 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

↳ one zero row

∴ dimension = 2

basis = { (1,2,2,-1,3) , (1,2,3,1,1) }

Ans 2 i) $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 3R_1 - 2R_2]{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

∴ Row Space : { (1,2,0,-1) , (2,6,-3,-3) }

ii) $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix} \xrightarrow[C_4 \to C_4 + C_1 + \frac{1}{2}C_2]{C_2 \to C_2 - 2C_1 + \frac{3}{2}C_2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \end{pmatrix}$

∴ Column Space : { (1,2,3) , (2,6,10) }

iii) $Ax = 0$

$\Rightarrow$ $x_1 + 2x_2 - x_4 = 0$

$2x_2 - 3x_3 + x_4 = 0$

↓

so 2 free variables

$x_3 = 1, x_4 = 0$ $\quad (-3, 3/2, 1, 0)$

$x_3 = 0, \quad x_4 = 1$ $\quad (0, 1/2, 0, 1)$

$\therefore$ Null space: $\{(-3, 3/2, 1, 0), (0, 1/2, 0, 1)\}$

iv) Rank = no. of non-zero rows in row-echelon form
$= 2$

v) Nullity + Rank = Columns
$\Rightarrow$ Nullity $= 4 - 2 = 2$

vi) $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 2 & -3 & -1 \end{bmatrix}$

Clearly, no zero rows
hence, row space forms sub space of R^4

Ans 4 a) $M_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - R_1 \\ R_4 \to R_4 - 3R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Rank = 1

$M_2 = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 2 \\ 3 & 6 \end{bmatrix} \xrightarrow{C_2 \to C_2 - 2C_1} \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 0 \\ 3 & 0 \end{bmatrix}$ Rank = 1

Similarly, find rest

b) $\begin{pmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{pmatrix} \longrightarrow$

0
-1
-3

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 3 & 1 & 2 \\ 1 & 0 & 1 & 3 & 2 & 5 \\ 4 & 0 & 3 & 9 & 5 & 12 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 4 & 3 & 2 & 0 & 0 & 0 \end{pmatrix}$$

2
6

$\therefore$ Column space : $\{(1,2,1,4), (1,3,2,6), (3,7,5,14)\}$

c) $C_4 = aC_1 + bC_3 + cC_5$

$a+b+3c = 2$ —— ① [from I^{st} terms compare]

$2a+3b+7c = 7$ —— ② [—— II ——]

$a+2b+5c = 5$ —— ③ [—— III ——]

by solving, we get $a=3$, $b=-1$, $c=0$

$\therefore C_4 = 3C_1 - C_3$

d) Rank = 3

Ans4 Max. Dimension of row space = 3 min-

null space = Column - dimen of row

$= 4 - 1$

$= 3$

Ans5 $d_{NS} + d_{RS}$ = No. of columns

$\therefore d_{RS} = 6 - 4 = 2$

~~Ans6 $(A-\lambda I)x = 0$~~

~~$(A^t - \lambda I) = 0$~~

Ans6 Let $A = (a_{ij})$, then we know, $A^T = (a_{ji})$

Assume $n=2$

For eigen values

$$|A-\lambda I| = 0$$

$$\begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda^2 - (a_{11}+a_{22})\lambda + a_{11}a_{22} - a_{21}a_{12} = 0 \quad \text{——— (1)}$$

Also, $|A^T - \lambda I| = 0$

$$\begin{bmatrix} a_{11}-\lambda & a_{21} \\ a_{12} & a_{22}-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda^2 - (a_{11}+a_{22})\lambda + a_{11}a_{22} - a_{21}a_{12} = 0 \quad \text{——— (2)}$$

Clearly, Eqn (1) & (2) are same, hence we get same eigen values for A & A^T.

For eigen vetor,

$$(A-\lambda I)X = 0$$

$$\Rightarrow (a_{11}-\lambda)x_1 + a_{12}y_1 = 0$$

$$a_{12}x_1 + (a_{22}-\lambda)y_1 = 0$$

$y_1 =$ free variable since $|A-\lambda I| = 0$

$= 1$

$$x_1 = \frac{-a_{12}t}{a_{11}-\lambda}$$

$$\therefore X = t\begin{bmatrix} \frac{-a_{12}t}{a_{11}-\lambda} \\ 1 \end{bmatrix} \quad \text{or} \quad t\begin{bmatrix} \frac{\lambda - a_{22}}{a_{21}} \\ 1 \end{bmatrix}$$

$(A^T - \lambda I) Y = 0$

$\Rightarrow \quad x_2 (a_{11} - \lambda) + a_{21} y_2 = 0$

$x_2 a_{12} + y_2 (a_{22} - \lambda) = 0$

y_2 = free variable $= K$

$x_2 = \dfrac{-a_{21} K}{a_{11} - \lambda}$

$Y = K \begin{bmatrix} a_{21}/\lambda - a_{11} \\ 1 \end{bmatrix}$ or $K \begin{bmatrix} \frac{\lambda - a_{22}}{a_{12}} \\ 1 \end{bmatrix}$

Clearly $X \neq Y$, hence they do not have same eigenvectors, until $a_{12} = a_{22}$, i.e., symmetric matrix

Ans 7 a) $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ $\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 2\lambda + 1 - 4 = 0$

$\Rightarrow \lambda = 3, -1$

$(1-\lambda) x_1 + x_2 = 0$ — ①

$4x_1 + (1-\lambda) x_2 = 0$ — ②

$[A - \lambda I] X = 0$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

for $\lambda = 3$,

$-2x_1 + x_2 = 0$

$2x_1 - x_2 = 0$

x_2 = free variable since $|A - \lambda I| = t$

$2x_1 = t \Rightarrow x_1 = t/2$

$X = t \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$

↳ eigen vector

for, $\lambda = -1$

$2x_1 + x_2 = 0$

$2x_1 + x_2 = 0$

} x_2 is free variable $= t$

$x_1 = -t/2$

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$x = t \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$.

↳ eigen vector

Similarly, do (b) & (c)

~~Ans 8~~ $\cancel{\begin{bmatrix} 4-\lambda & -1 & 6 \\ 2 & 1-\lambda & 6 \\ 2 & -1 & 8-\lambda \end{bmatrix} = 0}$

$\Rightarrow$

$$\begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\Rightarrow \quad 2x_1 - x_2 + 6x_3 = 0$

$\therefore$ two free variable,

$x_2 = 1, \; x_3 = 0 \quad \Rightarrow \quad (1/2, 1, 0)$

$x_2 = 0, \; x_3 = 1 \quad \Rightarrow \quad (-3, 0, 1)$