Que 13 euclidean algo to find gcd {2076, 1024} ①

Apply division algo. with 2076 (largest of 2 numbers) as the dividend and 1024 as the divisor.

$$2076 = 2 \cdot 1024 + 28$$

Apply division algo again with 1024 and 28 using 1024 as the dividend and 28 as the divisor.

$$1024 = 36 \cdot 28 + 16$$

Continue this procedure untill zero remainder is ~~obtained~~ obtained.

$$28 = 1 \cdot 16 + 12$$

$$16 = 1 \cdot 12 + 4$$ ← last non zero remainder

$$12 = 3 \cdot 4 + 0$$

Since the last nonzero remainder is 4.

$$\gcd(2076, 1024) = 4$$

Que 14 a) gift receive on 12th day $\frac{n(n+1)}{2}$

$$1+2+3+ \cdots 12 = \sum_{i=1}^{12} i = \frac{12 \times 13}{2} = 78$$

b) on the first day you get 1

2nd day ---- 1+2

3rd day ---- 1+2+3

⋮

n th day ---- 1+2+3 --- n

S_n = Sum of n days in 1...n (n is number of days)

$$= S\left(\frac{n(n+1)}{2}\right) = S\left(\frac{n^2}{2} + \frac{n}{2}\right)$$

$$= S\left(\frac{n^2}{2}\right) + S\left(\frac{n}{2}\right)$$

Sum of $\frac{n^2}{[illegible]}$ for n in 1....n days is $\frac{n(n+1)(2n+1)}{6}$

Sum of $\frac{n}{[illegible]}$ for n in 1....n days is $\frac{n(n+1)}{2}$

$$S(n) = \frac{n(n+1)(2n+1)}{6\times 2} + \frac{n(n+1)}{2\times 2}$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \cancel{\frac{n(n+1)}{4}} \frac{(n^2+n)(2n+1)}{12} + \frac{n^2+n}{4}$$

$$= \frac{(2n^3+n^2+2n^2+n)+(3n^2+3n)}{12}$$

$$= \frac{2n^3+6n^2+4n}{12} = \frac{2n(n^2+3n+2)}{12}$$

$$\boxed{S(n) = \frac{n(n^2+3n+2)}{6}}$$

Here value of n is 12

$$S(12) = \frac{12(12^2+3\times 12+2)}{6} = 364.$$

<u>Qu 15</u> Using PMI

<u>a)</u> $\sum_{i=1}^{n}(2i-1) = n^2$

Basis step : when $n=1$

LHS $1^2 = 1$

RHS $\sum_{i=1}^{1}(2\times 1-1) = 1$

LHS = RHS. P(1) is true.

<u>Induction step</u>. Assume P(K) is true.

$$\sum_{i=1}^{K}(2i-1) = K^2$$

then $\sum_{i=1}^{k} (2i-1)+(2k+1) = k^2 + (2k+1)$

$= (k^2 + 2k + 1)$

$= (k+1)^2$

$\therefore$ P(K+1) is true

Thus the given result follows by PMI.

(b) $n^4 + 2n^3 + n^2$ is divisible by 4.

Basis step for n=1 $1^4 + 2 \times 1^3 + 1^2 = 1+2+1$

$= 4$ is div by 4.

So condition is true for n=1

Induction step Let the condition be true for n=k.

$k^4 + 2k^3 + k^2$ is divisible by 4

$k^4 + 2k^3 + k^2 = 4m$

Now we have to show the condition is true for n=k+1

$(k+1)^4 + 2(k+1)^3 + (k+1)^2$

$= k^4 + 6k^2 + 4k^3 + 4k + 1 + 2(k^2 + 3k^2 + 3k + 1) +$

$k^2 + 2k + 1$

$= k^4 + 6k^3 + 13k^2 + 12k + 4$

$= (k^4 + 2k^3 + k^2) + 4k^3 + 12k^2 + 12k + 4$

$= \cancel{k^4+2k^3} \; 4m + 4(k^3 + 3k^2 + 4k + 1)$

$= 4(m + k^3 + 3k^2 + 4k + 1)$ is divisible by 4.

Que 10 $\sum_{1 \le i \le j < 3} (a_i + a_j)$

$= (a_1 + a_1) + (a_1 + a_2) + (a_2 + a_1) + (a_2 + a_2)$

$= 4a_1 + 4a_2$

$= 4(a_1 + a_2)$

Q4 m= 12305 piegonhole $= \lfloor (m-1)/n \rfloor + 1 = \lfloor (12305-1)/13 \rfloor + 1$

$= 947.63$

Que 8 $\sum_{i=m}^{n} i = \sum_{i=m}^{n} (n+m-i)$

RHS $\sum_{i=m}^{n} i = m + (m+1) + (m+2) \cdots (n-1) + n$

RHS $\sum_{i=m}^{n} (n+m-i) = (n+\cancel{m}-\cancel{m}) + (n+m-(m+1)) +$
$(n+m-(m+2)) + \cdots (n+m-(n-1))$
$+ (\cancel{n}+m-\cancel{n})$

$= n + (n-1) + (n-2) + \cdots + (m+1) + m$

$= m + (m+1) + (m+2) + \cdots (n-2) + (n-1) + n$

LHS = RHS. (True)

Que 9 ⓐ $S = \sum_{i=m+1}^{n} (a_i - a_{i-1})$

$= (a_{m+1} - a_{m+1-1}) + (a_{m+2} - a_{m+2-1}) +$
$(a_{m+3} - a_{m+3-1}) + \cdots + (a_n - a_{n-1})$

$= (\cancel{a_{m+1}} - a_m) + (\cancel{a_{m+2}} - \cancel{a_{m+1}}) + (a_{m+3} - \cancel{a_{m+2}})$
$+ \cdots + (a_n - \cancel{a_{n-1}})$

$= -a_m + a_n$

$= a_n - a_m$

identity $\frac{1}{i(i+1)} = \frac{1}{i} + \frac{1}{i(i+1)}$

ⓑ $\sum_{i=1}^{n} \frac{1}{i(i+1)}$

$= \frac{1}{1(1+1)} + \frac{1}{2(2+1)} + \frac{1}{3(3+1)} + \cdots + \frac{1}{(n-1)(m-1+1)} + \frac{1}{n(n+1)}$

$= \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{(n-1)n} + \frac{1}{n(n+1)}$

by identity $= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{n} - \frac{1}{(n+1)}$

$= 1 - \frac{1}{(n+1)}$

$= \frac{(n+1) - 1}{(n+1)} = \frac{n}{n+1}$

Que 16 (a)

$x \leftarrow 0$
for $i=1$ to n do
$x = x + (2i-1)$

at first step $i=1$ $\quad x = 0 + (2\times1-1) = 1 = 1^2$

$i=2 \quad x = 1 + (2\times2-1) = 4 = 2^2$

$i=3 \quad x = 4 + (2\times3-1) = 9 = 3^2$

$i=4 \quad x = 9 + (2\times4-1) = 16 = 4^2$

$\vdots$

$i=n \quad \boxed{x = \sum_{i=1}^{n} (2i-1) = n^2}$

(b) $x \leftarrow 0$
for $i=1$ to n do
for $j=1$ to i do
$x \leftarrow x+1$.

$i=1 \quad j=1 \quad x = 0+1 = 1$

$i=2 \quad j=1 \quad x = 1+1 = 2$

$j=2 \quad x = 2+1 = 3$

$i=3 \quad j=1 \quad x = 3+1 = 4$

$j=2 \quad x = 4+1 = 5$

$j=3 \quad x = 5+1 = 6$

$i=n$ upto $j=n \quad x = 1+2+3+\cdots n$

value of $x = 1+2+3+\cdots n$

$= \frac{n(n+1)}{2}$

Que 11 $\quad A = m\times n \quad B = p\times q \quad C = r\times s$

a) $A+B \quad \underset{A}{(m\times n)} + \underset{B}{(p\times q)}$ ~~only~~ possible if $m=p \quad n=q$

b) $B-C \quad (p\times q) - (r\times s)$ if $p=r$ and $q=s$

c) $BC \quad (p\times q) \times (r\times s)$ if $q=r$

d) $A^2 \quad (m\times n) \times (m\times n)$ if $m=n$

Que 12

No of days	7	14	21	28
Insuline	A	B	C	D
Semi	25	40	35	0
Lente	20	0	15	15
Ultra	20	0	30	40

daily dose

A stays in hotel for 7 days
B for 14 days
C for 21 days
D – 28 days.

ⓐ Calculation for total usage of insuline

insuline	A	B	C	D	Total
S	175	560	735	0	1470
L	140	0	315	420	875
U	140	0	630	1120	1890

Cost of

Semi	10 ~~per~~ cent
Lente	11
Ultra	12

ⓑ total cost $= (1470 \times 10) + (875 \times 11) + (1890 \times 12)$
$= 47005$

ⓒ if guest stays additional 3, 5, 8, and 13 days respectively,
A stays for 10, B = 19, C = 29, D = 41

	10	19	29	41	
Insuline	A	B	C	D	total.
S	250	760	1015	0	2025
L	200	0	435	615	1250
U	200	0	870	1640	2750

ⓓ if guest stays 3 times their orignal time.
A = 7×3 = 21 days
B = 14×3 = 42 days
C = 21×3 = 63 days
D = 28×3 = 84 days

	21	42	63	84	
Insulin	A	B	C	D	total
S	525	1680	2205	0	4410
L	420	0	945	1260	2625
U	420	0	1890	3360	5670

Assignment 2

Ques 1

- Proof (by induction)

Let x be any element in Σ^*. Let $P(y)$ denote the predicate that $\|xy\| = \|x\| + \|y\|$, where $y \in \Sigma^*$. Since $y \in \Sigma^*$, y can be the null word λ or a nonempty word.

<u>Basis step</u>: to show $P(\lambda)$ is true; that is $\|x\lambda\| = \|x\| + \|\lambda\|$

Since $x\lambda = x$, ~~$\|x\lambda\| = \|x\| = \|x\| + \|\lambda\|$~~

$$\|x\lambda\| = \|x\|$$
$$= \|x\| + 0$$
$$= \|x\| + \|\lambda\|$$

So $P(\lambda)$ is true)

<u>Induction step</u> Assume $P(y)$ is true, that is $\|xy\| = \|x\| + \|y\|$ (induction hypothesis)

→ we must show that $P(ys)$ is true that is

$$\|xys\| = \|x\| + \|ys\|$$

Notice that $xys = (xy)s$ association property of concatenation.

Then

$\|xys\|$	$= \|(xy)s\|$	length is a function.
	$= \|xy\| + 1$	recursive def of length
	$= (\|x\| + \|y\|) + 1$	induction hypothesis
	$= \|x\| + (\|y\| + 1)$	association rule
	$= \|x\| + \|ys\|$	recursive def of length

Therefore $P(ys)$ is true. Thus $P(y)$ implies $P(ys)$

Therefore, by induction, $P(y)$ is true for every $y \in \Sigma^*$ that is $\|xy\| = \|x\| + \|y\|$ for every $x, y \in \Sigma^*$

- if $w \in \Sigma^*$ and $s \in \Sigma^*$ then $ws \in \Sigma^*$

furthermore, the length $\|w\|$ of a word w over Σ can be defined recursively as follows:

- $\|\lambda\| = 0$
- if $w \in \Sigma^*$ and $s \in \Sigma$, then $\|ws\| = \|w\| + 1$