Ans 6 :- Let

X : # of defective stamps in the sheet.

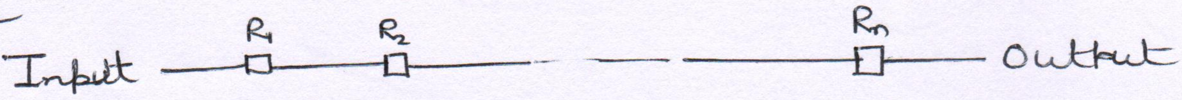
Then X is a Binomial r.v. with prob dist^n

$$P(X=x) = {}^5C_x (0.03)^x (0.97)^{5-x}, \quad x = 0,1,2,3,4,5$$

Now

$$P(\text{sheet goes into circulation}) = P(X=0)$$
$$= (0.97)^5 = \boxed{0.859}$$

Ans 7 :-



Let X : r.v. which counts the # flips in the n relays

R_X : $0, 1, 2, \ldots, n$.

$$\Longrightarrow \quad X \sim B(n, p)$$

$$P(X = j) = {}^nC_j \, p^j (1-p)^{n-j}$$

Input = Output $\Longleftrightarrow$ X is even

$$\therefore P(\text{Input} = \text{Output}) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} P(X = 2k)$$

where $\lfloor x \rfloor$ = floor x = largest integer not greater than x.

$$\Rightarrow P(\qquad\qquad) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} {}^nC_{2k} \, p^{2k} (1-p)^{n-2k}$$

$$= (1-p)^n \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} {}^nC_{2k} \left(\frac{p}{1-p}\right)^{2k} = \frac{1 + (1-2p)^n}{2}$$

$$\left.\begin{aligned} (1+x)^n &= \sum_{k=0}^{n} {}^nC_k x^k \\ (1-x)^n &= \sum_{k=0}^{n} {}^nC_k (-x)^k \end{aligned}\right] \Longrightarrow \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} {}^nC_{2k} x^{2k} = \frac{1}{2}\left[(1+x)^n + (1-x)^n\right]$$

Ans 8:- (a) $\lambda = np = 12 \times 0.05 = 0.6$

$$P(X \geq 2) = 1 - P(X<2) = 1 - P(X=0) - P(X=1)$$
$$= 1 - e^{-0.6} - e^{-0.6}(0.6)$$

(b) $\displaystyle\sum_{x=4}^{8} \frac{e^{-0.6}(0.6)^x}{\lfloor x}$

Ans 9:- $\lambda = np = 100 \times (0.0024) = 0.24$

P(at most 2 errors before 100 pages)

$= P(X=0) + P(X=1) + P(X=2)$

$= e^{-0.24} + \dfrac{e^{-0.24}(0.24)}{\lfloor 1} + \dfrac{e^{-0.24}(0.24)^2}{\lfloor 2}$

$= \boxed{0.9979}$

Ans 10:- (a) $\lambda = 0.03$.

X : Count the breakdown in a week

R_X : 0, 1, 2, — —.

P(even no. of breakdown) $= \displaystyle\sum_{k=0}^{\infty} P(X=2k)$

$$= \sum_{k=0}^{\infty} \frac{e^{-0.03}(0.03)^{2k}}{\lfloor 2k}$$

(b)(i) P(at least two weeks have no breakdown)

Let Y : Count the no. of week having no breakdown

R_Y : 0, 1, 2, ——, 10

P(at least two weeks have no breakdown)

$= P(2 \leq Y \leq 10) = 1 - P(Y=0) - P(Y=1)$ —— ①

Clearly Y is Binomial r.v. with parameters n = 10, $p = e^{-10\lambda}$.

where $p = e^{-10\lambda} = P(\text{there is no breakdown in 10 weeks})$

$P(\text{there is no breakdown in a week}) = e^{-\lambda} = e^{-0.03}$

We are repeating this experiment independently 10 times $\Longrightarrow P(\text{there is no breakdown in 10 weeks})$

$$= \underbrace{e^{-\lambda} \cdot e^{-\lambda} \text{———} e^{-\lambda}}_{10 \text{ times}} = e^{-10\lambda} = e^{-0.3}$$

Thus from (1),

$$P(2 \leq Y \leq 10) = 1 - (1-p)^{10} - {}^{10}C_1\, p(1-p)^9$$

$$= 1 - (1-e^{-0.3})^{10} - 10\, e^{-0.3}(1-e^{-0.3})^9$$

(ii) $P(10\text{th is the first week to have a breakdown})$

$= P(\text{there is no breakdown in first 9 week and a breakdown in 10 th week})$

$$= e^{-9\lambda}(1-e^{-\lambda}) = e^{-0.27}(1-e^{-0.03}).$$

Ans 11: $P(X=1 / X \leq 1) = \dfrac{P(X=1 \cap X \leq 1)}{P(X \leq 1)}$

$$= \frac{P(X=1)}{P(X=0)+P(X=1)} = 0.8$$

i.e. $\dfrac{e^{-\lambda} \cdot \lambda}{e^{-\lambda} + e^{-\lambda} \cdot \lambda} = 0.8 \Longrightarrow \dfrac{\lambda}{1+\lambda} = 0.8$

$\Longrightarrow \lambda = 0.8 + 0.8\lambda \Longrightarrow 0.2\lambda = 0.8$

$\Longrightarrow \boxed{\lambda = 0.4}$