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Quiz-1, Section - B (Solution)

1. Find the infimum of the set $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. **[5]**

Ans. Clearly $S_2 \neq \Phi$ and 0 is a lower bound for S_2.

So in order to show that 0 is the infimum, it remains to show that 0 is the greatest among all the lower bounds of the set. For this it is enough to show that a number $\alpha > 0$ cannot be a lower bound of the given set.

Let $\alpha > 0$ be arbitrary then by the Archimedean property $\exists n \in \mathbb{N}$ such that $\frac{1}{n} < \alpha$. $\frac{1}{n} \in S$ and so α can not be a lower bound.

2. Discuss the convergence of the following recursive sequence: **[5]**

$$S_1 = 1 \text{ and } S_{n+1} = \sqrt{3S_n} \text{ for } n \in \mathbb{N}.$$

Ans. By Mathematical Induction, Clearly $S_2 > S_1$. Suppose $S_n > S_{n-1}$ then $\sqrt{3S_n} > \sqrt{3S_{n-1}} \Rightarrow S_{n+1} > S_n$. So S_n is monotonic increasing.

Again by Mathematical Induction, $S_1 < 3$. Suppose $S_n < 3$ then $\sqrt{3S_n} < \sqrt{9} \Rightarrow S_{n+1} < 3$ for all $n \in \mathbb{N}$. So S_n is Bounded above. Hence, the sequence S_n, being monotonically increasing and bounded is convergent.

Suppose it converges to l. then $l = 3$.