The LNM Institute of Information Technology
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Optimization Techniques & Applications
Self Practice Problems

Linear Programming

Simplex Method

1. Maximize $Z = \ 3x_1 + x_2 + 3x_3$ subject to

$$\begin{gathered} 2x_1 + x_2 + x_3 \leq 2 \\ x_1 + 2x_2 + 3x_3 \leq 5 \\ 2x_1 + 2x_2 + x_3 \leq 6 \\ x_1, x_2 \text{ and } x_3 \geq 0 \end{gathered}$$

   (Ans: $x_1 = \ \frac{1}{5}, x_2 = 0, x_3 = \frac{8}{5}$, Max $Z = -\frac{27}{5}$.

2. Maximize $Z = \ 3x_1 + 2x_2 + 5x_3$
   subject to

$$\begin{gathered} x_1 + 2x_2 + x_3 \leq 430 \\ 3x_1 + 2x_3 \leq 460 \\ x_1 + 4x_3 \leq 420 \end{gathered}$$

   $x_1, x_2, x_3 \geq 0$.
   (Ans: $x_1 = 0, x_2 = 100, x_3 = 230$ and Max $Z = \ 1350$

3. Maximize $Z = \ x_1 - x_2 + 3x_3$ subject to

$$\begin{gathered} x_1 + x_2 + x_3 \leq 10 \\ 2x_1 - x_3 \leq 2 \\ 2x_1 - 2x_2 + 3x_3 \leq 0 \\ x_1, x_2, x_3 \geq 0 \end{gathered}$$

   (Ans: $x_1 = 0, x_2 = 6, x_3 = 4$ and Max $Z = 6$

4. Minimize $Z = \ 2x_1 + 4x_2 + x_3 + x_4$ subject to

$$\begin{gathered} x_1 + 3x_2 + x_4 \leq 4 \\ 2x_1 + x_2 \leq 3 \\ x_2 + 4x_3 + x_4 \leq 3 \\ x_i \geq 0, i = \ 1, 2, 3, 4 \end{gathered}$$

   (Ans : $x_1 = 1, x_2 = 1, x_3 = 1/2$, Max $Z = \ 13/2$)

Two-Phase Simplex Method

1. Maximize $Z = 3x_1 - x_2$
   subject to

$$\begin{gathered} 2x_1 + x_2 \geq 2 \\ x_1 + 3x_2 \leq 2 \\ x_2 \leq 4 \end{gathered}$$

   $x_1, x_2 \geq 0$ (Ans: $x_1 = 1, x_2 = 0$ and Max $Z = 6$

2. Minimize $Z = -3x_1 + x_2 + 3x_3 - x_4$ Subject to

$$\begin{gathered} x_1 + 2x_2 - x_3 + x_4 = 0 \\ 2x_1 - 2x_2 + 3x_3 - x_4 = 9 \\ x_1 - x_2 + 2x_3 - x_4 = 6 \\ x_i \geq 0,\ i = 1, 2, 3, 4 \end{gathered}$$

3. Minimize $Z = x_1 + 6x_2 - 7x_3 + x_4 + 5x_5$ subject to

$$\begin{gathered} 5x_1 - 4x_2 + 13x_3 - 2x_4 + x_5 = 20 \\ x_1 - x_2 + 5x_3 - x_4 + x_5 = 8 \\ x_i \geq 0, i = 1, 2, 3, 4, 5 \end{gathered}$$

4. Maximize $Z = 3x_1 + 2x_2 + 2x_3$
   Subject to

$$\begin{gathered} 5x_1 + 7x_2 + 4x_3 \leq 7 \\ -4x_1 + 7x_2 + 5x_3 \geq -2 \\ 3x_1 + 4x_2 - 6x_3 \geq \frac{97}{7} \end{gathered}$$

   $x_i \geq 0, i = 1, 2, 3$

Big-M Method

1. Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$
   subject to

$$\begin{gathered} x_1 + 2x_2 + 3x_3 = 15 \\ 2x_1 + x_2 + 5x_3 = 20 \\ x_1 + 2x_2 + x_3 + x_4 = 10 \\ x_i \geq 0, i = 1, 2, 3, 4 \end{gathered}$$

   (Ans: $x_1 = \frac{15}{6}, x_2 = \frac{15}{6}, x_3 = \frac{15}{6}$, Max $Z = 15$

2. Minimize $Z = 3x_1 - x_2$
   subject to

$$\begin{gathered} 2x_1 + x_2 \geq 2 \\ x_1 + 3x_2 \leq 3 \\ x_2 \geq 4 \end{gathered}$$

   $x_1, x_2 \geq 0$

3. Maximize $Z = x_1 + x_2 + x_4$
   subject to

$$x_1 + x_2 + x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 = 4$$

$x_1, x_2, x_3 \geq 0$ (Ans $x_1 = 0, x_2 = 0, x_3 = 0, x_5 = 0$, Max $Z = 4$)

4. Minimize $Z = x_1 - 3x_2 + 2x_3$
   subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

   $x_1, x_2, x_3 \geq 0$

(Ans $x_1 = 4, x_2 = 5$ and Min $Z = -11$.

Duality

1. Minimize $Z_x = 7x_1 + 3x_2 + 8x_3$
   subject to
   $8x_1 + 2x_2 + x_3 \geq 3$

$$8x_1 + 2x_2x_3 \geq 3$$
$$3x_1 + 6x_2 + 4x_3 \geq 4$$
$$4x_1 + x_2 + 5x_3 \geq 1$$
$$x_1 + 5x_2 + 2x_3 \geq 7$$

   $x_1, x_2, x_3 \geq 0$

2. $Z_x = x_1 + x_2 + x_3$
   subject to

$$x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_2 \leq 3$$
$$2x_2 - x_3 \geq 4$$

   $x_1, x_2 \geq 0$ and x_3 is unrestricted.

Dual Simplex Method

1. Maximize $Z = -2x_1 - x_3$
   subject to the constraints

$$x_1 + x_2 - x_3 \geq 5$$
$$x_1 - 2x_2 + 4x_3 \geq 8$$

   and $x_1, x_2, x_3 \geq 0$
   (Ans: $x_1 = 0, x_2 = 14, x_3 = 9$ and Max $Z = -9$

2. Min $Z = 3x_1 + 2x_2 + x_3 + 4x_4$
   Subject to

$$2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$$
$$3x_1 - x_2 + 7x_3 - 2x_4 \leq 2$$
$$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$$

$x_i \geq 0, i = 1, 2, 3, 4$
(Ans : $x_1 = \frac{65}{23}, x_2 = 1, x_3 = \frac{23}{23}$ and Min $Z = \frac{215}{23}$

3. Minimize $Z = 6x_1 + 7x_2 + 5x_4$ subject to

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$
$$x_2 + 5x_3 - 6x_4 \geq 10$$
$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$

$x_i \geq 0$, $i = 1, 2, 3, 4$
(Ans: $x_1 = \frac{16}{13}, x_2 = \frac{6}{13}, x_3 = \frac{8}{13}$ and Min $z = \frac{2280}{13}$)

End