The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-II ■ Assignment 4

1. Compute the dimension and a basis of the solution space W of each homogenous system:

$$\text{(a)}\quad \begin{aligned} x + y + 2z &= 0 \\ 2x + 3y + 3z &= 0 \\ x + 3y + 5z &= 0 \end{aligned} \qquad \text{(b)}\quad \begin{aligned} x + 2y + 2z - s + 3t &= 0 \\ x + 2y + 3z + s + t &= 0 \\ 3x + 6y + 8z + s + 5t &= 0 \end{aligned}$$

2. Find the basis of the row space, column space and null space of the matrices 'A'. Additionally, find rank and nullity. Show that the row space is a subspace of R^4,

where: $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$

3. For the given matrix $A = \begin{pmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{pmatrix}$

   (a) Compute the rank of M_k, for $k = 1, 2, 3, 4, 5, 6$, where M_k is the submatrix of A consisting of the first k column of A.

   (b) Find the columns of A that forms a basis for the column space of A.

   (c) Express column 4 as the linear combination of columns of part (b).

   (d) Find the rank of A.

4. If A is a 3×4 non-zero matrix, What is the largest possible dimension of the row space of A? What is the largest possible dimension of the null space of A? Justify each answer.

5. If the null space of a 4×6 matrix A is 4-dimension, then what is the dimension of the row space of A? Justify

6. If A be an $n \times n$ matrix, then A^t and A have the same eigenvalues. Do they have the same eigenvectors?

7. Find the eigenvalues and corresponding eigenvectors of matrices

   (a) $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$.

8. Let eigenvalue of A is 2, then the basis of the corresponding eigenspace, where $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$