Assignment#7

M2

1. (i) $xy'' + y' = y'^2$

$\Rightarrow xp' + p = p^2$ $\quad y' = p$

$\Rightarrow y'' = p'$

$$\frac{p'}{p^2 - p} = \frac{1}{x}$$ → Reduced

$$\int \frac{(1)\,dp}{p(p-1)} = \frac{1\,dx}{x}$$

$$\Rightarrow \log \frac{p}{p-1} = \log x + \log c$$

$$p = xc(p-1)$$

$$\frac{dy}{dx} = xc\left(\frac{dy}{dx} - 1\right)$$

$$\frac{dy}{dx} = \frac{xc - 1 + 1}{xc - 1}$$

$$y = \int dx + \frac{dx}{xc-1} + \cancel{c}$$

$$y = x + \frac{1}{c}\log|xc - 1| + K$$

(ii) $yy'' - y'^2 = 0 \rightarrow y' = p$

$$y\cancel{p}\frac{dp}{dy} = p^{\cancel{2}} \qquad y'' = p\frac{dp}{dy}$$

$$\int \frac{dp}{p} = \int \frac{dy}{y} \Rightarrow p = cy$$

$$\frac{dy}{dx} = cy \Rightarrow$$

(iii) $yy'' + y'^2 + 1 = 0$.

$y' = p$

$y'' = p\frac{dp}{dy}$

$yp\frac{dp}{dy} + p^2 + 1 = 0$

$y\frac{dp}{dy} = \frac{-p}{\cancel{y}} - \frac{1}{p} = \frac{-(p^2+1)}{p}$

$\int \frac{p\,dp}{(p^2+1)} = \int \frac{-dy}{y} \Rightarrow \frac{1}{2}\log(p^2+1) = \log\frac{c}{y}$

$p^2 + 1 = \frac{c^2}{y^2}$

$\left(\frac{dy}{dx}\right)^{\cancel{2}} = \sqrt{\frac{c^2}{y^2} - 1}$

$\frac{1}{(-2)}\int \frac{(-2y)\,dy}{\sqrt{c^2 - y^2}} = \int dx$

(iv)

$y'' - 2y'\coth x = 0$.

$y' = p$

$y'' = p'$

$p' - 2p\coth x = 0$

$\int \frac{dp}{2p} = \int \coth x\, dx$

$\boxed{\ln p = 2\ln|\sinh x| + c}$

(i) $y'' + \frac{2(1-2x)}{x(1-x)} y' - \frac{2y}{x(1-x)} = 0$

$\Rightarrow y_2 = \int \frac{x^2 e^{-\int \frac{2(-2x+1)}{x(1-x)} dx}}{\phi} dx$ ← ott $y_1 = \frac{1}{x}$

t

(ii)

$y'' - \frac{2x}{1-x^2} y' + \frac{2y}{1-x^2} = 0$

$y_2 = \int \frac{e^{+\int \frac{2x}{1-x^2} dx}}{x^2} dx$

3.

$x^2 y'' + xy' + (x^2 - \frac{1}{4}) y = 0$

~~$y_1 \Leftarrow$ is a soln. So it will Satisfy~~

$= \frac{\sin x}{\sqrt{x}}$

$y_1' = \frac{1}{\sqrt{x}} \cos x - \frac{1}{2}\frac{\sin x}{\sqrt{x^3}}$

$p(x) = \frac{x}{x^2} = 1/x$

$y_2 = \int \frac{e^{-\int p(x) dx}}{y_1^2} dx$

$= \int \frac{1/x \cdot x}{\sin^2 x} = \int \csc^2 x \, dx$

$y_2 = -\cot x$

4. i) $y'' - 4y' + 3y = 0$

$y = e^{\lambda x}$

~~$y' = p \quad y'' = p$~~

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda = 1, 3$

$\Rightarrow$

$y(x) = c_1 e^{x} + c_2 e^{3x}$

(ii) $y'' + 2y' + (\omega^2 + 1) y = 0.$

$\omega \to$ Re

$\lambda^2 + 2\lambda + (\omega^2 + 1) = 0.$

$(\lambda + 1)^2 + \omega^2 = 0.$

$\lambda = \pm \omega i - 1$

$\Rightarrow$

$y(x) = e^{-x} (c_1 \cos \omega x + c_2 \sin \omega x)$

5] i) C.E $\to$ Characteristic Equation

$\Downarrow$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda = -2$

$y(x) = c_1 e^{-2x} + c_2 x e^{-2x}$

$y(0) = 1, \quad y'(0) = -1$

$y(0) = c_1 + c_2(0) \Rightarrow \boxed{c_1 = 1}$

$y'(x) = c_1(-2) e^{-2x} + -2 c_2 e^{-2x} x + c_2 e^{-2x}$

$y'(0) = -2c_1 + c_2 = -1$

-2

$c_2 = 2 - 1 = 1$

$y(x) = (1+x)e^{-2x}$

(i) $y'' - 2y' - 3y = 0$

$y = e^{\lambda x}$

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda - 1)^2 - 4 = 0$

$(\lambda - 1 - 2)(\lambda - 1 + 2) = 0$

$(\lambda - 3)(\lambda + 1) = 0$

$\lambda = 3, -1$

$y = c_1 e^{3x} + c_2 e^{-x}$

$\boxed{IVP}$

(6). $x = e^t \Rightarrow dx = x\,dt$

$\frac{dx}{dy} = \frac{e^t\,dt}{dy} \Rightarrow \frac{dy}{dx} = e^{-t}\frac{dy}{dt}$

$\frac{d^2y}{dx^2} = -e^{-t}\frac{dy}{dt} + \frac{d^2y}{dt^2}(e^{-t})\frac{dt}{dx} = e^{-t}$

substituting

$e^{-t}e^{2t}\left(e^{-t}\left(-\frac{dy}{dt} + \frac{d^2y}{dt^2}\right)\right) + a\,e^t e^{-t}\frac{dy}{dt} + by = 0$

$\Rightarrow e^t\frac{d^2y}{dt^2} - \frac{dy}{dt} \times e^t + a\frac{dy}{dt} + by = 0$

Redmild

6 (i) $x^2 y'' + 2x y' - 12y = 0.$

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 12y = 0.$$

$$\frac{dy}{dt} = e^{\lambda t} \Rightarrow \lambda^2 + \lambda - 12 = 0$$

$$\lambda = -4, +3$$

$$y(t) = c_1 e^{-4t} + c_2 e^{+3t}$$

$$x = e^t$$

$$\Rightarrow y(x) = c_1 x^{-4} + c_2 x^3$$

(ii)

(iii) Similarly.

7. (i) $y'' + 4y = \underbrace{2\cos^2 x + 10e^x}_{r(x)}$

$y'' + 4y = 0$ ← Homogeneous

$y_1 \; y_2$

$\lambda^2 + 4 = 0$

$\Rightarrow \lambda = \pm 2i$

$y_1 = \cos 2x$

$y_2 = \sin 2x$

$$y_h(x) = c_1 \cos 2x + c_2 \sin 2x.$$

$$y_p(x) = y_1 \int \frac{w_1(x)}{w(x)} r(x)\,dx + y_2 \int \frac{w_2(x)}{w(x)} \ldots$$

$$= y_1 \int \frac{-y_2 r}{w} dx + y_2 \int \frac{y_1 r}{w} dx$$

$$= \cos 2x \int \frac{-\sin 2x\,(2\cos^2 x + 10e^x)}{2}\,dx$$

$$+ \sin 2x \int \frac{\cos 2x\,(2\cos^2 x + 10e^x)}{2}\,dx$$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2(1) = 2$$

Assignment (4)

(1) $y'' - \frac{4xy'}{x^2} + \frac{(x^2+6)}{x^2}y = 0$ —— (1)

$y_1 = 0 \Rightarrow y' = 0 \Rightarrow y'' = 0.$ satisfy (1).

$y_2 = x^2 \sin x \Rightarrow y' = 2x\sin x + x^2\cos x, \therefore \rightarrow$ satisfy (1).

verified $\Rightarrow$ These both are solution,

since, at initial value, $x = 0$, $p(x)$ & $q(x)$ are not continuous.

$\Rightarrow$ We can't apply uniqueness theorem.
Also wronskian $= 0 \Rightarrow$ Linearly dependent (y_1, y_2)

$\Rightarrow$ It contradict the uniqueness.

(2) $y'' = y' \Rightarrow y = c_1 + c_2 e^x$

$\hookrightarrow \lambda_1 = 0, \lambda_2 = 1$

$y' = 1$ at $(0,0)$. $\qquad y' = c_2 e^x$

$c_2 = 1$

$y = 0$ at $x = 0$.

$\Rightarrow 0 = c_1 + 1e^0 \Rightarrow c_1 = -1$

$\Rightarrow y = e^x - 1$