The LNM Institute of Information Technology

Department of Computer Science & Engineering

CS325 Design and Analysis of Algorithm

Sample Question Paper for Midterm

1. Students have given ratings for their teachers in an institution. Teachers have also been rated by an external body through some historical data. Assume that every student and the external body have rated all the faculties in the decreasing order of their liking and there are no ties in that. Now the institution wants to see who have given similar ratings to the rating given by the external body.

   (a) Which algorithm is the best one to use for this problem from the set of algorithms that we have learned till now in the class and why? (3)

   (b) Present an algorithm for this, explain how it works and analyze the running time complexity of this algorithm. (5)

2. Given are n points in the xy-plane. Following algorithm finds the closest pair of points with respect to Euclidean distance.

   CLOSEST-PAIR $(p_1, p_2, \ldots, p_n)$

   Compute separation line L such that half the points are on each side of the line.

   $\delta_1 \leftarrow$ CLOSEST-PAIR (points in left half).

   $\delta_2 \leftarrow$ CLOSEST-PAIR (points in right half).

   $\delta \leftarrow \min\{\delta_1, \delta_2\}$.

   Delete all points further than δ from line L.

   Sort remaining points by y-coordinate.

   Scan points in y-order and compare distance between each point and next 11 neighbors. If any of these distances is less than δ, update δ.

   RETURN δ.

   Discuss the complexity of this algorithm. Check the possibility of improving the efficiency of this algorithm with respect to running time complexity. If it is possible to make it more efficient, how you will make the algorithm better? If not, argue that you will not be able to find a more optimum way of solving this problem. (8)

3. Describe an efficient Greedy algorithm that, given a set of points $\{x_1, x_2, \cdots, x_n\}$ on the real line, determines the smallest set of unit-length closed intervals that contains all of the given

points. Analyze the complexity of your algorithm and argue that your algorithm is correct. (6)

Example: Suppose $\{0.7, 1.0, 2.0, 1.5, 2.6, 3.1, 3.6, 3.9, 4.2, 2.3, 5.2, 5.5, 4.7, 4.2\}$ is the given set of points then the output will be $\{[0.7, 1.0, 1.5], [2.0, 2.3, 2.6], [3.1, 3.6, 3.9], [4.2, 4.7, 5.2], [5.5]\}$.

4. Professors Ram, Ryan and Rahim have proposed the following sorting algorithm:

Algorithm 0.1: STOOGE-SORT(A, i, j)

if $A[i] > A[j]$
  then *Exchange* $A[i]$ *and* $A[j]$
if $i + 1 >= j$
  then *return*
$k \leftarrow \lfloor (j - i + 1)/3 \rfloor$
STOOGE-SORT$(A, i, j - k)$
STOOGE-SORT$(A, i + k, j)$
STOOGE-SORT$(A, i, j - k)$

(a) Give a recurrence equation for the worst-case running time of STOOGE-SORT and solve it to get a tight asymptotic bound on the worst-case running time. (5)

(b) Compare the worst-case running time of STOOGE-SORT with that insertion sort, merge sort, heapsort and quicksort. Do the professors deserve a tenure in LNMIIT!? (2)

5. Solve the following recurrence equations: (3+3)

(a) $T(n) = 27T(n/3) + n$

(b) $T(n) = 27T(n/3) + n^3$