# Assignment - 3

Ans a) $\langle f, g \rangle = \int_{-1}^{1} (t+2)(t^2-3t+4)\, dt$

$= \int_{-1}^{1} (t^3 - t^2 - 2t + 8)\, dt$

$= 2\int_{0}^{1} (-t^2+8)\, dt$

$= 2\left[-\frac{t^3}{3} + 8t\right]_0^1$

$= \cancel{-\frac{8}{3}+16} \quad \frac{46}{3}$

$\cancel{= \frac{46}{3}}$

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$\|f\| = \int_{-1}^{1} (t^2+4t+4)\, dt$

$= 2\int_{0}^{1} (t^2+4)\, dt$

$= 2\left[\frac{t^3}{3} + 4t\right]_0^1 = \frac{26}{3}$

$\|g\| = \int_{-1}^{1} (t^4 + 9t^2 + 16t - 6t^3 - 24t + 8t^2 + 16)\, dt$

$= 2\int_{0}^{1} (t^4 + 17t^2 + 16)\, dt$

$= 2\left[\frac{t^5}{5} + \frac{17t^3}{3} + 16t\right]_0^1$

$= 2\left[\frac{1}{5} + \frac{17}{3} + 16\right]$

$= 2 \times \frac{328}{15} = \frac{656}{15}$

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b) same as (a)

c) Caushy-Schwarz inequality $\rightarrow$ $\langle f,g\rangle \leq \|f\|\cdot\|g\|$

for a , $\frac{\cancel{46}^{23}}{\cancel{8}} \leq \frac{\cancel{26}^{13}}{\cancel{8}} \times \frac{6056}{15}$

Clearly, its true.

Ans2 ~~a) $\langle\alpha,\beta\rangle$~~ let $\gamma = (z_1, z_2)$, to verify I.P. must satisfy:

a) $\langle\alpha+\gamma,\beta\rangle = \langle\alpha,\beta\rangle + \langle\gamma,\beta\rangle$

b) $\langle K\alpha,\beta\rangle = K\langle\alpha,\beta\rangle$

c) If $\langle\alpha,\alpha\rangle = 0$, then $\alpha = 0$

(a) i) $\langle\alpha+\gamma,\beta\rangle = \langle(x_1+z_1, x_2+z_2), (y_1,y_2)\rangle$

$= (x_1+z_1)y_1 - (x_1+z_1)y_2 - (x_2+z_2)y_1 + 4(x_2+z_2)y_2$

$= x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 + z_1y_1 - z_1y_2 - z_2y_1 + 4z_2y_2$

$= \langle\alpha,\beta\rangle + \langle\gamma,\beta\rangle$

ii) $\langle K\alpha,\beta\rangle = Kx_1y_1 - Kx_1y_2 - Kx_2y_1 + 4Kx_2y_2$

$= K\langle\alpha,\beta\rangle$

iii) $\langle\alpha,\alpha\rangle = x_1^2 - 2x_1x_2 + 4x_2^2$

$= (x_1-x_2)^2 + 3x_2^2$

$\geq 0$

Also, if $\langle\alpha,\alpha\rangle = 0 \Rightarrow x_1 = x_2$ & $x_2 = 0$

$\therefore x_1 = x_2 = 0$

Ans3 $\langle\alpha,\alpha\rangle \geq 0 \Rightarrow \alpha = 0$

$x_1^2 - 6x_1x_2 + ax_2^2 = 0$

$x_1^2 - 6x_1x_2 + 9x_2^2 + (a-9)x^2 = 0$

$\Rightarrow (x_1-3x_2)^2 + (a-9)x_2^2 = 0$

~~$x_1 = 3x_2$~~ & $a = 9$

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Ans4 Let $v:(x_1, x_2)$ & $w:(y_1, y_2)$

a) $\|v\| = \sqrt{\langle v,v\rangle} = \sqrt{x_1^2 + x_2^2} \geq 0$

If $\|v\| = 0 \Rightarrow x_1^2 + x_2^2 = 0$

$\therefore x_1 = x_2 = 0$

$\therefore v = 0$

b) $\|\lambda v\| = \sqrt{\langle \lambda v, \lambda v\rangle} = \sqrt{\lambda^2 x_1^2 + \lambda^2 x_2^2}$

$= |\lambda| \sqrt{x_1^2 + x_2^2}$

$= |\lambda| \sqrt{\langle v, v\rangle}$

$= |\lambda| \, \|v\|$

c) $\|v+w\| = \sqrt{(x_1+y_1)^2 + (x_2+y_2)^2}$

$= \sqrt{x_1^2 + y_1^2 + 2x_1y_1 + x_2^2 + y_2^2 + 2x_2y_2}$

$\leq \sqrt{x_1^2 + x_2^2} + \sqrt{y_1^2 + y_2^2}$

$\leq \|v\| + \|w\|$

Ans5 $w_1 = (1,1,1,1)$

$w_2 = \alpha_2 - \alpha_1 \dfrac{\langle \alpha_2, w_1\rangle}{\langle w_1, w_1\rangle}$

$= (1,-1,2,2) - (1,1,1,1) \dfrac{4}{4}$

$= (0,-2,1,1)$

$w_3 = \alpha_3 - \alpha_2 \dfrac{\langle \alpha_3, w_2\rangle}{\langle w_2, w_2\rangle}$

$= (1,2,-3,-4) - (1,-1,2,2) \left[\dfrac{-11}{6}\right]$

$= \dfrac{(17,1,4,2)}{6}$

$\therefore u_1 = \dfrac{w_1}{\|w_1\|} = \dfrac{1}{2}(1,1,1,1)$

$u_2 = \dfrac{w_2}{\|w_2\|} = \dfrac{1}{\sqrt{6}}(0,-2,1,1)$

$u_3 = \dfrac{w_3}{\|w_3\|}$

$=$ //

Ans 6 a) $w_1 = (1,0,1)$

$$w_2 = (1,0,-1) - (1,0,1)\left[\frac{0}{2}\right] = (1,0,-1)$$

$$w_3 = (0,3,4) - (1,0,-1)\left[\frac{\cancel{-4}^{\,2}}{\cancel{2}}\right]$$

$$= (2,3,2)$$

$$u_1 = \frac{1}{\sqrt{2}}(1,0,1)\ ,\ u_2 = \frac{1}{\sqrt{2}}(1,0,-1)\ \&\ u_3 = \frac{1}{\sqrt{17}}(2,3,2)$$

b) $w_1 = (1,0,1)$

$$w_2 = (1,0,-1) - (1,0,1)\left[\frac{1+0-3}{1+0+3}\right] = \left[\frac{3}{2}, 0, \frac{-1}{2}\right]$$

$$w_3 = (0,3,4) - (1,0,-1)\left[\frac{0+0-6}{9/4 + 3/4}\right] = (2,3,2)$$

$$\therefore\ u_1 = \frac{1}{\sqrt{2}}(1,0,1)\ ,\ u_2 = \frac{\sqrt{2}}{\sqrt{5}}\left[\frac{3}{2}, 0, \frac{-1}{2}\right]\ ,\ u_3 = \frac{1}{\sqrt{17}}(2,3,2)$$

coll.d $t^0$ $t^1$ $t^2$

Ans 7 $\quad w_1 = 1$

$$w_2 = t - 1\,\frac{\int_0^1 t\,dt}{\int_0^1 dt} = t - \frac{1}{2}$$

$$w_3 = t^2 - 1\,\frac{\int_0^1 t^2 dt}{\int_0^1 dt} - \frac{\int_0^1 t^2 (t - \frac{1}{2})\,dt}{\int_0^1 (t-\frac{1}{2})^2 dt}\,(t - \tfrac{1}{2})$$

$$= t^2 - \left[\frac{\left(\frac{1}{4} - \frac{1}{6}\right)}{\frac{1}{3} \times \frac{1}{2}}\right](t-\tfrac{1}{2}) - \frac{1}{3}$$

$$= t^2 - \frac{1}{3} - t + \frac{1}{2}$$

$$= t^2 - t - \frac{1}{6}$$

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