Assignment - 2

Ans 1 a) R(Q) let $x, y \in R$, $K \in Q$

$x+y \in R$

$Kx \in R$

b) C(Q) let $x, y \in C$, $K \in Q$

$x+y \in C$

$Kx \in C$

c) R(C) let $x, y \in R$, $K \in C$

$x+y \in R$

but Kx may or may not $\in R$

d) R(R) let $x, y \in R$, $K \in R$

$x+y \in R$ $Kx \in R$

Ans 3 a) let $X : (x_1, y_1, z_1)$ $x_1 \geq 0$

$Y : (x_2, y_2, z_2)$ $x_2 \geq 0$

$\therefore X+Y = (x_1+x_2, y_1+y_2 + z_1+z_2)$ $x_1+x_2 \geq 0$

For $\alpha \in R$

$\alpha X = (\alpha x_1, \alpha y_1, \alpha z_1)$ But αx_1 may or may not ≥ 0

Hence not a subspace.

b) let $X : (x_1, y_1, z_1) \mid x_1 + y_1 = z_1$

$Y : (x_2, y_2, z_2) \mid x_2 + y_2 = z_2$

$\therefore X + Y : (x_1 + x_2, y_1 + y_2, z_1 + z_2) \mid x_1 + x_2 + y_1 + y_2 = z_1 + z_2$

$\alpha X : (\alpha x_1, \alpha y_1, \alpha z_1) \mid \alpha x_1 + \alpha y_1 = \alpha z_1$

$\alpha \in R$ Hence subspace

c) let $X : (x_1, y_1) \mid x_1 = y_1^2$

$Y : (x_2, y_2) \mid x_2 = y_2^2$

$\therefore X+Y : (x_1+x_2, y_1+y_2) \mid x_1+x_2 = y_1^2 + y_2^2 \neq (y_1+y_2)^2$

Hence $X+Y \notin R$ $\therefore$ not a subspace

d) $X : (x_1, y_1) \mid x_1 y_1 = 0$

$Y : (x_2, y_2) \mid x_2 y_2 = 0$

$X+Y : (x_1+x_2, y_1+y_2) \mid (x_1+x_2)(y_1+y_2) = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$

$= x_1y_2 + x_2y_1 \neq 0$

Hence, $X+Y \notin R$ $\therefore$ not a subspace.

Ans4 $V = \alpha p_1 + \beta_2 p_2 + \gamma p_3$

$\Rightarrow x^2 + 4x - 3 = (\alpha + 2\beta) x^2 + (-2\alpha - 3\beta + \gamma) x + (5\alpha - \gamma)$

By comparing, $\alpha + 2\beta = 1$

$-2\alpha - 3\beta + \gamma = 4$

$5\alpha - \gamma = -3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ -2 & -3 & 1 & 4 \\ 5 & 0 & -1 & 3 \end{array}\right] \xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 + 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & -10 & -1 & -2 \end{array}\right]$$

$R_3 \to R_3 + 10R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 9 & 8 \end{array}\right]$$

$\therefore \gamma = 8/9$

$\beta = 6 - 8/9 = \frac{46}{9}$, $\alpha = 1 - \frac{92}{9} = -\frac{83}{9}$

Ans5 $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 3 & 2 & 1 \\ 4 & 1 & 2 & 2 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & -6 & -2 \end{bmatrix}$

$\approx \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -6 & -2 \end{bmatrix}$

Hence, linear independent.

b) $A = \begin{bmatrix} 1 & 2 & 6 \\ -1 & 3 & 4 \\ -1 & -4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & 2 \\ 0 & -2 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

So, linear independent

c) $a(u+v) + b(v+w) + c(u+w) = (a+c)u + (a+b)v + (b+c)w = 0$

$\left.\begin{aligned} a+c &= 0 \\ a+b &= 0 \\ b+c &= 0 \end{aligned}\right\}$ $a=b=c=0$ So L.I.

since $\{u, v, w\}$ are independent.

d) $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & -7 & 1 \end{bmatrix} \xrightarrow{R_3 - R_2}$

So, linear dependent

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Ans 6 $a\sin x + b e^x + c x^2 = 0$ is possible iff $a = b = c = 0$

$\therefore$ linear independent.

Ans 7 If $\{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ are L.D., so we can say, $a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n = 0$ where $a_i \neq 0$.

$\Rightarrow \alpha_n = \left[\frac{-a_1}{a_n}\right]\alpha_1 + \left[\frac{-a_2}{a_n}\right]\alpha_2 + \cdots + \left[\frac{-a_{n-1}}{a_n}\right]\alpha_3$

where, $n \in I$

Clearly, at least one of them can be written in terms of others.

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Ans8
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since, there is no zero rows in row-echelon form. Also they span R^4. Hence, they form basis.

Ans9 Let $P_2(t)$ be defined as $a_2 x^2 + a_1 x^1 + a_0 x^0$.
where $a_i \neq 0$.

(a) $\{1, x, x^2\}$ $\rightarrow$
$$\begin{array}{l} \text{coeff of } x^0 \\ \text{coeff of } x^1 \\ \text{coeff of } x^2 \end{array}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) $\{1+x+x^2\}$ $\rightarrow$
$$\begin{array}{l} \text{c. of } x^0 \\ \text{c. of } x^1 \\ \text{c. of } x^2 \end{array}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
2 zero rows $\therefore$ not basis.

(c) $\{1, x, x^2, 1+x, x^2\}$ $\rightarrow$
$$\begin{array}{l} \text{c. of } x^0 \\ \text{c. of } x^1 \\ \text{c. of } x^2 \end{array}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
$\therefore$ not basis

zero column

(d) $\{1, 1+x, 1+x+x^2\}$ $\rightarrow$
$$\begin{array}{ccc} \text{c. of } x^0 & x^1 & x^2 \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$
$$\Downarrow$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \therefore$$

zero vector → zero dimension

Ans 10 Dimension —

a) $2 \times 3 = 6$

b) $\frac{3 \times 4}{2} = 6$ $\left[\because \frac{n(n+1)}{2}\right]$

c) $\frac{3 \times 2}{2} = 3$ $\left[\because \frac{n(n+1)}{2}\right]$

d) $\frac{2 \times 1}{2} = 1$ $\left[\because \frac{n(n-1)}{2}\right]$

Ans 11 $M_{2,3} =$ $\left[\begin{array}{ccc:ccc} 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 4 & 3 & 7 & 5 & 6 \\ 1 & 2 & 3 & 5 & 7 & 6 \end{array}\right]$

$\sim \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Clearly, one row is zero, so two rows will form the basis of W. Hence, dimension = 2.

Ans 12 $A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 0 & 0 & K-3 \end{bmatrix}$ $\therefore K-3=0$

$R_3 = R_3 - 2R_2 - R_1$ $\Rightarrow K = 3$

$B = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -6 & -1 \\ 0 & 2 & -6 & K-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 0 & 0 & K-3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\therefore K-3=0$

$\therefore K=3$

Ans 2 a) To prove ~~that~~ vector space —

$f(x) + g(x) = (f+g)(x)$ — given

$(rf)(x) = r f(x)$ — given

i) $(f+g)(x) + h(x) = f(x) + (g+h)(x)$

ii) $\alpha(f+g)(x) = \alpha(f(x) + g(x)) = (\alpha f)(x) + (\alpha g)(x)$

iii) Let $0(x)$ be a vector s.t. $0(x) = 0$.

$\therefore \quad f(x) + 0(x) = (f+0)(x) = f(x)$

iv) $f(x) + (-f)(x) = (f+(-f))(x) = 0(x) = 0$

v) $f(x) + g(x) = (f+g)(x) = (g+f)(x) = g(x) + f(x)$

vi) $1 \cdot f(x) = (1 \cdot f)(x) = f(x)$

vii) $(\alpha+\beta) f(x) = (\alpha+\beta) f(x) = (\alpha f)(x) + (\beta f)(x)$

viii) $((\alpha\beta) f)(x) = \alpha\beta f(x) = \alpha \cdot (\beta f)(x)$

b) Let $f(x)$ & $g(x)$ be two func^n(s) on $[a,b]$.

$\therefore$ i) $\int f(x)\,dx + \int g(x)\,dx = \int (f(x) + g(x))\,dx$

ii) $\alpha \int f(x)\,dx = \int \alpha f(x)\,dx = \int (\alpha f)(x)\,dx$