The LNM Institute of Information Technology Jaipur, Rajasthan

MATH-II # Assignment 5

1. Find the order and degree of the following differential equations, where $y' = \frac{dy}{dx}$

(i)
$$y'' + x^2y' = \sin y$$

 $\sqrt{y}\cos x$

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$$y'' + x^2y' = \sin y$$
 (ii) $(y')^{\frac{3}{2}} = 2x(ye^{-x^2} - y - 3x)$ (iii) $y''' + 2xy' + x^2 = 2x(ye^{-x^2} - y - 3x)$

(iii)
$$y''' + 2xy' + x^2 =$$

2. Which of the following differential equations is/are linear

(i)
$$y' + 2y = x^{\frac{2}{3}}$$

(ii)
$$yy'' + 3y' = e^{3x^2}$$

(iii)
$$y'' + 4y' = x \cos y$$

(i)
$$y' + 2y = x^{\frac{2}{3}}$$
 (ii) $yy'' + 3y' = e^{3x^2}$ (iii) $y'' + 4y' = x \cos y$ (iv) $x^2y'' + 4\sqrt{x} \ y = \cos x$ (v) $y'' + 4y^2 = 3x$ (vi) $y'' + 2x^2y' = \sqrt[3]{x}$

(v)
$$y'' + 4y^2 = 3x$$

(vi)
$$y'' + 2x^2y' = \sqrt[3]{x}$$

3. Solve (i)
$$\log(\frac{dy}{dx}) = 2x + 3y$$
 (ii) $\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$

(ii)
$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$$

4. Use the suitable transformation on the following differential equations to get separable equation form:

(i)
$$\frac{dy}{dx} = (4x + y + 1)^2$$
 (ii) $(x+y)^2 \frac{dy}{dx} = a^2$ (iii) $\frac{dy}{dx} = \sec(x+y)$

(ii)
$$(x+y)^2 \frac{dy}{dx} = a^2$$

(iii)
$$\frac{dy}{dx} = \sec(x+y)$$

5. Solve the following homogeneous equations:

(i)
$$x \cos \frac{y}{x}(y \ dx + x \ dy) = y \sin \frac{y}{x}(x \ dy - y \ dx)$$
 (ii) $x \frac{dy}{dx} = y(\log y - \log x + 1)$

(ii)
$$x \frac{dy}{dx} = y(\log y - \log x + 1)$$

6. Reducible to homogenous form (i) $\frac{dy}{dx} = \frac{(x+2y-3)}{2x+y-3}$ (ii) $\frac{dy}{dx} = \frac{(-x+y+2)}{x-2y-3}$

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$$\frac{dy}{dx} = \frac{(x+2y-3)}{2x+y-3}$$

(ii)
$$\frac{dy}{dx} = \frac{(-x+y+2)}{x-2y-3}$$

7. Show that the following equations are exact and hence find their general solution:

(i)
$$\{y(1+\frac{1}{x})+\cos y\}dx + (x+\log x - x\sin y)dy = 0$$
 (ii) $xdx + ydy = \frac{ydx - xdy}{x^2 + y^2}$.

(ii)
$$xdx + ydy = \frac{ydx - xdy}{x^2 + y^2}$$

8. Find the value of constant λ such that $\frac{dy}{dx} = -\frac{(3x^2 + \lambda e^y)}{2xe^y + 3y^2}$ is exact. Further, for the value of λ , solve the equation.

9. Find the integrating factor for the following differential equations

(i)
$$y(x + y) dx + (x + 2y - 1) dy = 0$$

(i)
$$y(x + y) dx + (x + 2y - 1) dy = 0$$
 (ii) $y(x + y + 1) dx + x(x + 3y + 2) dy = 0$

Supplementary problems from "Advanced Engg. Maths. by E. Kreyszig (8^{th} Edn.)

(i) Page 18, Q. 9,11,15,22,25