

**Note:** Each question carry 3 marks for correct answer and carry a **negative marking of 1 mark for wrong answer**. Questions (1-4) may have more than one correct options. Encircle/Tick all the **correct** answer/s.

1. Which of the following is/are NOT correct ?

- (A) Every non-empty bounded subset of  $\mathbb{R}$  has a supremum in  $\mathbb{R}$ .
- (B) Every non-empty bounded subset of  $\mathbb{Q}$  has a supremum in  $\mathbb{Q}$ .
- (C) Every non-empty bounded subset of  $\mathbb{Q}$  has a supremum in  $\mathbb{R}$ .
- (C) Every non-empty bounded subset of  $\mathbb{Z}$  has a supremum in  $\mathbb{Z}$ .

Ans. (B)

2. Which of the following statements is/are correct ?

- A)  $\lim_{x \rightarrow 0} \sqrt{x} \sin \frac{1}{x}$  does not exist.
- (B)  $\lim_{x \rightarrow 0^+} \sqrt{x} \sin \frac{1}{x}$  exists.
- (C)  $\lim_{x \rightarrow 0^-} \sqrt{x} \sin \frac{1}{x}$  exists.

Ans. (B)

3. Let  $a, b \in \mathbb{R}, a < b$ . Suppose  $f : [a, b] \rightarrow \mathbb{R}$  is continuous, then

- (A)  $f$  is bounded.
- (B)  $f$  has a local maximum.
- (C)  $f$  has intermediate value property.
- (D)  $f$  does not have a fixed point.

Ans. (A), (C).

4. The interval/s on which function  $f(x) = \frac{x}{x+1}$ ,  $x \neq -1$  is convex

- (A)  $(-\infty, -1)$ .
- (B)  $(-\infty, \infty)$ .
- (C)  $(0, \infty)$ .
- (D)  $(-1, \infty)$ .

Ans. (A)

$$f'(x) = \frac{1}{x+1} - \frac{x}{(x+1)^2} = \frac{1}{(x+1)^2}, \quad f''(x) = \frac{-2}{(x+1)^3} > 0; \quad x < -1.$$

5. Any Newton sequence for  $f(x) = x^5 - x - 1$ , with its initial point  $x_0 \in [1, 2]$  will converge to the unique root of  $f(x) = 0$  in  $[1, 2]$ . TRUE/ FALSE \_\_\_\_\_.

Ans. TRUE.

Note that  $f(1) < 0, f(2) > 0$ . As  $f$  is a polynomial so its infinitely times differentiable everywhere. Now  $f'(x) = 5x^4 - 1$  is nonzero on  $[1, 2]$  and  $f''(x) = 20x^3 > 0$  on  $[1, 2]$ .