Cylindrical coordinate system

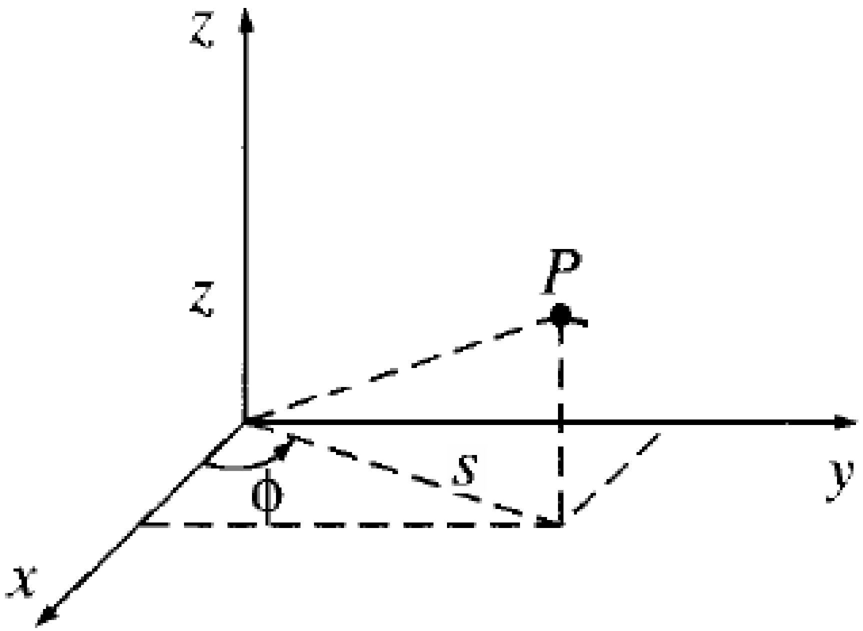
s

Cylindrical Coordinates

$$0 \le s \le \infty$$

$$0 \le \varphi \le 2\pi$$

$$-\infty \le z \le \infty$$



s

φ

z

$$x = s\cos\phi, \qquad y = s\sin\phi, \qquad z = z$$

$$s = \sqrt{x^2 + y^2} \qquad \varphi = \tan^{-1}\left(\frac{y}{x}\right) \qquad z = z$$

Unit Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{s} = \frac{\frac{\partial \vec{r}}{\partial s}}{\left|\frac{\partial \vec{r}}{\partial s}\right|} \qquad \hat{\varphi} = \frac{\frac{\partial \vec{r}}{\partial \varphi}}{\left|\frac{\partial \vec{r}}{\partial \varphi}\right|} \qquad \hat{z} = \frac{\frac{\partial \vec{r}}{\partial z}}{\left|\frac{\partial \vec{r}}{\partial z}\right|}$$

$$\vec{r} = s\cos\varphi\,\hat{i} + s\sin\varphi\,\hat{j} + z\hat{k}$$

$$\hat{s} = \cos\varphi\,\hat{i} + \sin\varphi\,\hat{j}$$

$$\hat{\varphi} = -\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j}$$

$$\hat{z} = \hat{k}$$

$$\vec{A} = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

$$\vec{A} = a_\rho \hat{\rho} + a_\phi \hat{\phi} + a_z \hat{z}$$

$$a_x = a_\rho \cos\phi - a_\phi \sin\phi$$
$$a_y = a_\rho \sin\phi + a_\phi \cos\phi$$
$$a_z = a_z$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\varphi \\ a_z \end{bmatrix}$$

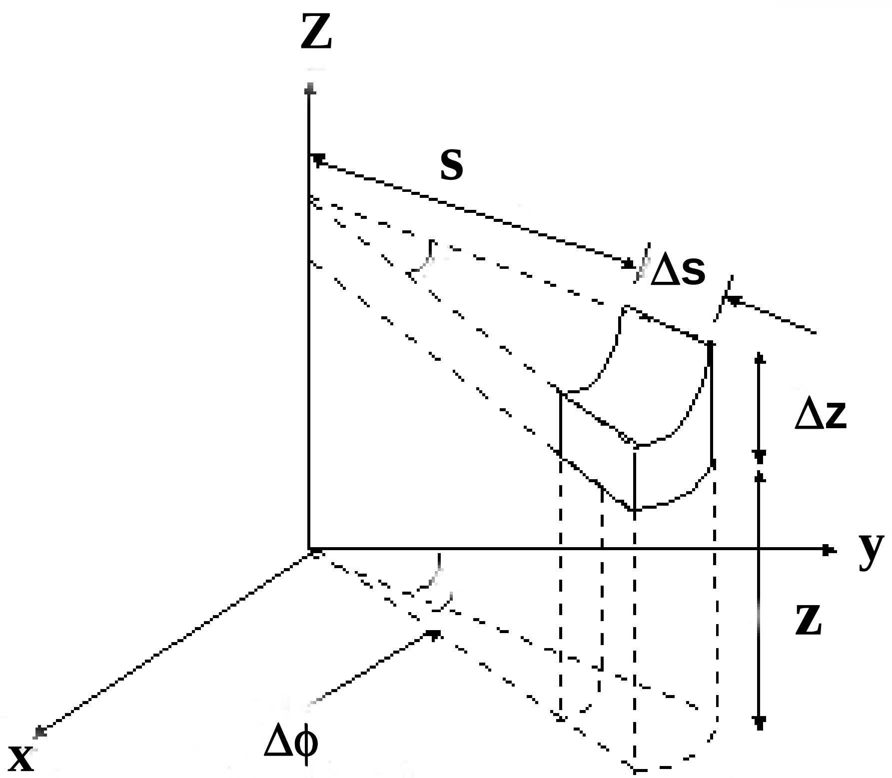
$$a_\rho = a_x \cos\phi + a_y \sin\phi$$
$$a_\phi = -a_x \sin\phi + a_y \cos\phi$$
$$a_z = a_z$$

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

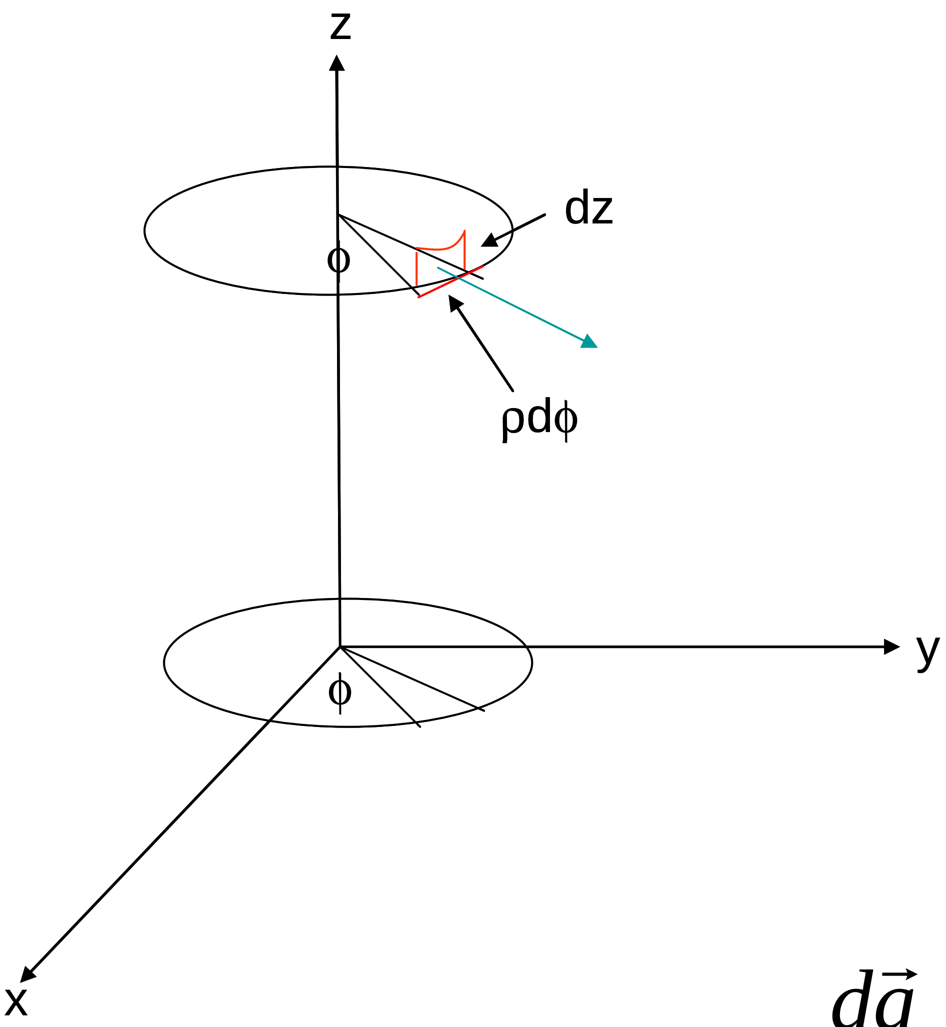
Infinitesimal vector

$$dl_s = ds, \qquad dl_\phi = s\,d\phi, \qquad dl_z = dz$$

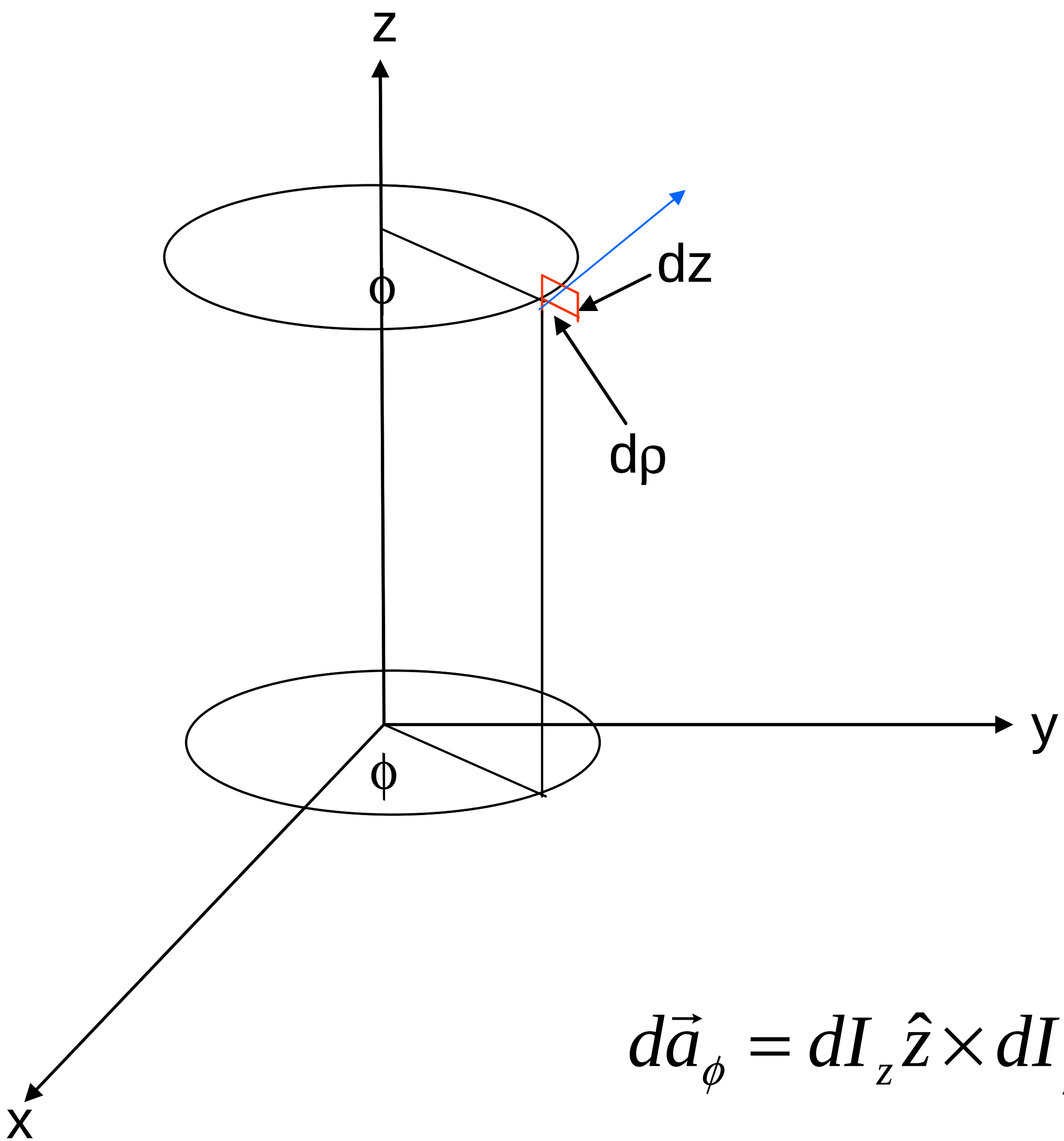
$$d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$



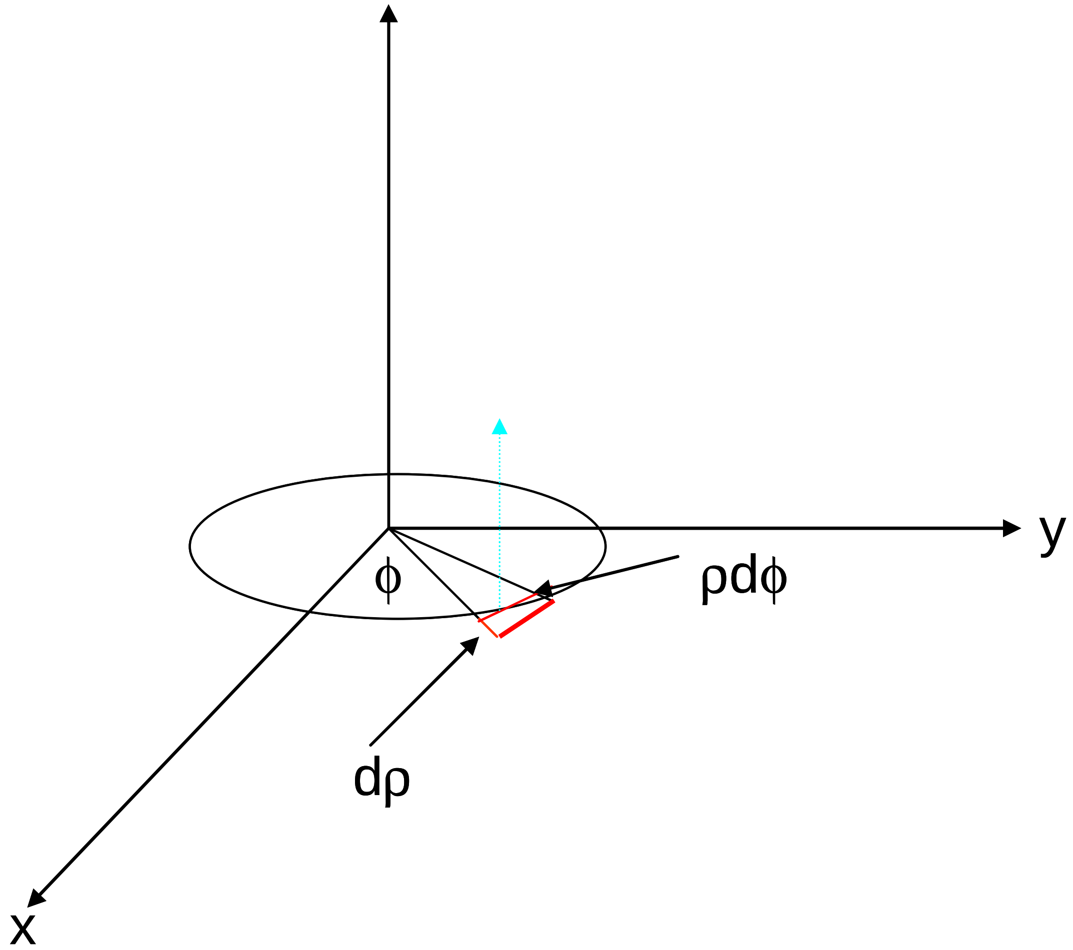
Differential Surface Element



$$d\vec{a}_s = dI_z \hat{z} \times dI_\phi \hat{\phi} = s\,d\phi\,dz\,\hat{s}$$

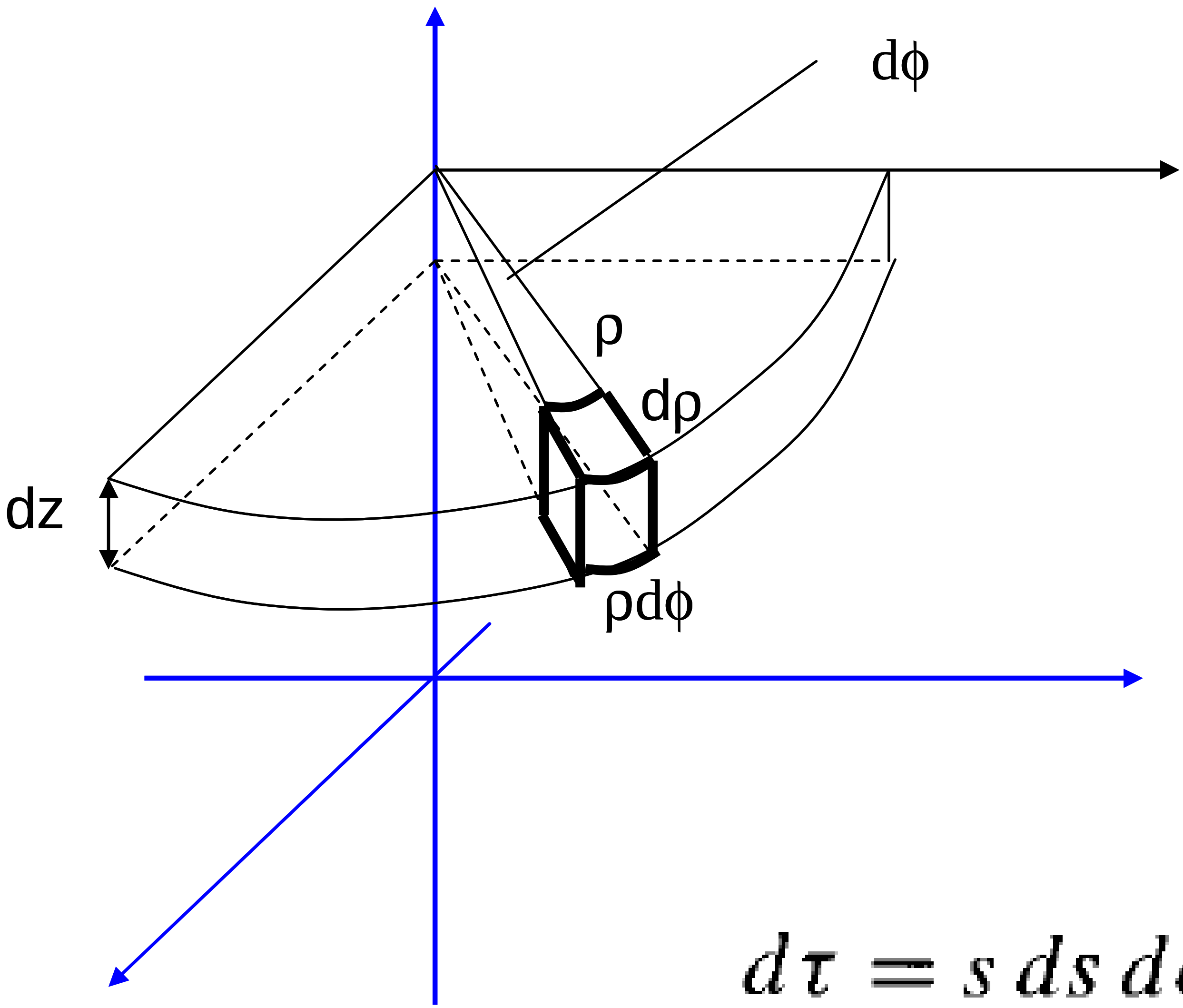


$$d\vec{a}_{\phi} = dI_z\hat{z} \times dI_{\rho}\hat{\rho} = dzd\rho\hat{\phi}$$



$$d\vec{a}_z = dI_\rho \hat{\rho} \times dI_\phi \hat{\phi} = \rho d\phi d\rho \hat{z}$$

Volume element in cylindrical coordinate system



$$d\tau = s\, ds\, d\phi\, dz$$

Infinitesimal vector

$$\vec{V} = \hat{\rho} + \sin\phi\hat{\phi} + z\hat{z}$$

$$\vec{V} = \rho\hat{\rho} + \hat{\phi} + z\hat{z}$$

$$\nabla T = \frac{\partial T}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\,\hat{\mathbf{z}}.$$

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$