MATH-221: Probability and Statistics

Tutorial # 4 (Continuous Random Variable, pdf, CDF)

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1. The diameter of an electric cable, say X, is given by the function $f(x) = 6x(1-x)$, $0 \leq x \leq 1$.

   (a) Is $f(x)$ a valid p.d.f.?

   (b) Find CDF of $f(x)$,

   (c) Determine a number b such that $P(X < b) = P(X > b)$.

2. The kms X in thousands of kms car owner get with a certain kind of tyre is a random variable having probability density function:

$$f(x) = \begin{cases} \frac{1}{20}\exp\big(-\frac{x}{20}\big) & \text{if} \quad x > 0 \\ 0 & \text{if} \quad elsewhere \end{cases}.$$

   Find the probabilities that one of this certain type tyre will last (i) at most 10,000 kms, (ii) anywhere from 16,000 to 24,000 kms.

3. Let X be a continuous r.v. with p.d.f.:

$$f(x) = \begin{cases} ax & \text{if} \quad 0 \leq x \leq 1 \\ a & \text{if} \quad 1 \leq x \leq 2 \\ -ax + 3a & \text{if} \quad 2 \leq x \leq 3 \\ 0 & \text{if} \quad elsewhere \end{cases}.$$

   (a) Determine the constant a,

   (b) Find CDF of $f(x)$,

   (c) Compute $P(X > 1.5)$.

4. The marks obtained by a member of students for a certain subject are assumed to be approximately normally distributed with parameter $\mu = 65$ and with parameter $\sigma = 5$. If 3 students are taken at random from this set, What is the probability that exactly 2 of them will have marks over 70?

5. In a distribution exactly normal, 10.03 % of the items are under 25 kilogram weight and 89.97 % of the items are under 70 kilograms weight. What are the parameter μ and parameter σ of the distribution?

6. Let the random variable X denote the decay time of some radioactive particle and follows the exponential distribution function. Suppose λ is such that $P(X \geq 0.01) = \frac{1}{2}$. Find a number t such that $P(X \geq t) = 0.9$.

7. Let X have a normal distribution with parameters μ and $\sigma^2 = 0.25$. Find a constant c such that $P(|X - \mu| \leq c) = 0.9$. (Hint: Use Table for standard normal distribution function).

8. Subway trains on a certain line run every half an hour between mid-night(00:00 in 24 hour format) and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes?

9. Let X be a random variable with the following cdf

$$F(x) = \begin{cases} 0 & \text{if} \quad x \leq 0 \\ x & \text{if} \quad 0 < x \leq 1 \\ 1 & \text{if} \quad x > 1 \end{cases} .$$

Determine wether X is a discrete random variable or an continuous random variable? Accordingly find its pmf or pdf.

10. Let X be a random variable with the following cdf

$$f(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x^2 & \text{if} \quad 0 \leq x < \frac{1}{2} \\ \frac{3}{4} & \text{if} \quad \frac{1}{2} \leq x < 1 \\ 1 & \text{if} \quad x \geq 1 \end{cases} .$$

What type of random variable X is? Accordingly find its pmf or pdf.