The LNM Institute of Information Technology, Jaipur

Mathematics - II (MidTerm) **Part-A** **Solution**

1 Which **statement is true** for the matrix: $\begin{pmatrix} 1 & -1 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{pmatrix}$ [2 Marks]

(A) columns are linearly dependent (B) rows are linearly independent (C) matrix rank is four (D) rank of matrix is two (E) matrix is invertible.

Ans (A) columns are linearly dependent

Justification:

Clearly $C_2 = (-1)C_1$. It shows that columns are linearly dependent.

2 For which value of k, the following system of linear equations **has no solution** [2 Marks]

$$\begin{aligned} x + y &= 1 \\ 2x + ky &= 3 \end{aligned}$$

(A) 1 **(B) 2** (C) 3 (D) 4 (E) none.

Ans (B) 2

Justification: ....

The system can be written as

$\begin{pmatrix} 1 & 1 & 1 \\ 2 & k & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & k-2 & 1 \end{pmatrix}$

Where $A = \begin{pmatrix} 1 & 1 \\ 0 & k-2 \end{pmatrix}$ and $(A|b) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & k-2 & 1 \end{pmatrix}$

For K= 2

$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ and $(A|b) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

It is clear that k =2

rank of A $\neq$ rank of $(A|b)$. Thus, system is inconstant and have no solution.

3 Let $V(F)$ denotes the vector space over the field F, Q denotes the set of all rational numbers and R denotes the set of all real numbers, then which of the following vector space **is not a finite** dimensional: [2 Marks]

(A) $R(R)$ (B) $R^9(R)$ **(C)** $R(Q)$ (D) $Q(Q)$ (E) none.

Ans ...(C) $R(Q)$............

Justification: The basis of $R(Q)$ comprises all the irrational numbers. Thus, it is not a finite dimensional vector space over the field Q.

4 Classify the following differential equation: $\frac{dy}{dx} = 1 + 2y + 2xy + x$ [2 Marks]

(A) Both separable and linear (B) Separable and not linear
(C) Linear and not separable (D) Exact (E) none.

Ans .... (A) Both separable and linear

Justification: separable: $\frac{dy}{dx} = 1 + 2y + 2xy + x = (1+x)(1+2y)$

linear: $\frac{dy}{dx} + y(-2-2x) = 1 + x$

5 Consider the initial value problem $\frac{dy}{dx} = 1 + y^2, \quad y(0) = 0$ over the rectangle $R = \{(x, y) : |x| < 9, |y| < 2\}$. Then, **solution is guaranteed for the following interval** for x. [2 Marks]

(A) $|x| < 9$ (B) $|x| < 2$ (C) $|x| < \infty$ (D) $|x| < \frac{5}{2}$ (E) $|x| < \frac{2}{5}$.

Ans (E) $|x| < \frac{2}{5}$

Justification:

Using existence and uniqueness theorem, solution exist in $|x - x_0| < \alpha$, where $\alpha = min\{a, \frac{b}{k}\}$.

Given $f(x, y) = 1 + y^2$, $a = 9, \; b = 2$,

Then, $|f(x, y)| \leq 1 + |y|^2 \leq 1 + 4 = 5$, so $\alpha = min\{9, \frac{2}{5}\} = \frac{2}{5}$.

Thus, the solution is guaranteed for the interval $|x - 0| < \frac{2}{5}$.