"Assignment-6"

Q1 →

(a) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

$yx(x - 2y)dx - x^2(x - 3y)dy = 0$

$x(x-3y)dy - y(x-2y)dx = 0$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{(xy - 2y^2)x - (x^2 - 3xy)y}$$

$$= \frac{1}{x^2y - 2xy^2 - x^2y + 3xy^2} = \boxed{\frac{1}{xy^2}}$$

$$\frac{xy - 2y^2}{xy^2} = \int\left(\frac{1}{y} - \frac{2}{x}\right)dx$$

$$= \left[\frac{x}{y} - 2\ln x\right] + C$$

(b) $\frac{1}{Mx + Ny}$:-

(c) $\frac{1}{Mx - Ny}$:-

Q2 →

$$(x + 2y^3)\frac{dy}{dx} = y$$

$$\frac{dx}{dy} = \frac{x}{y} + 2y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$e^{-\frac{1}{y}dy}$

$I = \frac{1}{y}$

$$\frac{x}{y} = \int 2y + C = \int \frac{1}{y} \cdot 2y^2 dy + C$$

(b) $(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$.

$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$

$(x-e^{-\tan^{-1}y})\frac{dy}{dx}=(-1-y^2)$

$\frac{dx}{dy}(-1-y^2)=x-e^{-\tan^{-1}y}$

$\frac{dx}{dy}=\frac{x}{(1+y^2)}$

$\frac{1}{1+x}dy$

$e^{\tan^{-1}y}$

Q3 → $x\frac{dy}{dx}+y\log y=xye^x$

$\frac{dy}{dx}+\frac{y}{x}\log y=ye^x$

$\frac{1}{y}\frac{dy}{dx}+\frac{\log y}{x}=e^x$ → v

$\frac{dv}{dx}$

$\frac{dy}{dx}-ye^x=-\frac{y}{x}\log y$.

$I=e^{\int e^x dx}$

$x\frac{dy}{dx}+y\log y=xye^x$

$\frac{x}{y}\frac{dy}{dx}+\log y=xe^x$

$=e^{e^x}$.

$x\frac{dy}{dx}-ye^x=-y\log y$

(d) $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x}e^x$

$y/x=v$

$y=vx$ $\quad v+x\frac{dv}{dx}+v\log(vx).=ve^x$

$\frac{dy}{dx}=v+x\frac{dv}{dx}$.

(b) $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^2\cos^2 y$

$\sec^2 y \frac{dy}{dx} + \frac{1}{x}\frac{2\sin y \cos y}{\cos y} = x^2$

$\sec^2 y \frac{dy}{dx} + \frac{2}{x}\tan y = x^2$

$\tan y = v$.

$\sec^2 y \frac{dv}{dx} = \frac{dv}{dx}$

$\frac{dv}{dx} + \frac{2}{x}v = x^2$.

(c) $(xy^2 + e^{-1/x^3})dx - x^2 y\,dy = 0$.

$(xy^2 + e^{-1/x^3})dx = x^2 y\,dy$

$\frac{dy}{dx} = \frac{xy^2}{x^2 y} + \frac{e^{-1/x^3}}{x^2 y}$

$\frac{dy}{dx} - \frac{y^2 x}{x^2} = \frac{e^{-1/x^3}}{x^2}$

$y\frac{dy}{dx} - \frac{x(y^2)}{x^2} = e^{-1/x^3}$

$y^2 = v$

$2y\frac{dy}{dx} = dv$

$e^{(-1/x^2)}$

e^{-1/x^3}

$1/x^3$

(4) →

(a) $y = ax^2$

$\frac{dy}{dx} = 2ax.$

$\frac{dy}{dx} = -\frac{1}{2ax}.$

$dy = -\int \frac{1}{x} dx$

$y = -\ln x + C$

(b) $x^2 + y^2 = 2ax$

$2x + 2y\frac{dy}{dx} = 2a.$

$2y\frac{dy}{dx} = 2a - 2x$

$\frac{dy}{dx} = \frac{2a-2x}{2y}$

$\frac{dy}{dx} = \frac{2y}{(2a-2x)}$

$\frac{dy}{2y} = \frac{dx}{2(a-x)}$

(5) → $\frac{ds}{dQ}$ Solve it

(6) → $\frac{ds}{dQ}$ Same as above