CSE 325: Design and Analysis of Algorithm

Assignment – 1

January 25, 2018

1. Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $\mathcal{O}(logn)$ by substitution method
2. Show that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $\Omega(nlogn)$. Conclude the solution is in fact $\Theta(nlogn)$. Do this by substitution method.
3. Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by substitution method by making change of variables
4. Determine a good asymptotic upper bound on the recurrence $T(n) = 3T(n/2) + n$ by iteration
5. Show that any polynomial over n with degree d is $\Theta(n^d)$
6. Draw the recursion tree for $T(n) = 4T(\lfloor n/2 \rfloor) + n$ and provide tight asymptotic bounds on its solution
7. Use a recursion tree to solve the recurrence $T(n) = T(\alpha n) + T((1-\alpha)n) + n$, where α is a constant in the range $0 < \alpha < 1$
8. Use the master method to give tight asymptotic bounds for the following recurrences:
   (a) $T(n) = 4T(n/2) + n$
   (b) $T(n) = 4T(n/2) + n^2$
   (c) $T(n) = 4T(n/2) + n^3$
9. Show that $log(n!)$ is $\Theta(nlogn)$
10. Show that $\lceil logn \rceil!$ is not polynomially bounded but $\lceil loglogn \rceil!$ is polynomially bounded
11. The running time of an algorithm A is described by the recurrence $T(n) = 7T(n/2) + n^2$. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?