MILP Formulation for UAV Path Planning

Variable and Parameter Descriptions

Sets and Indices

- $i, j \in G$: Set of all grid points[cite: 4].
- $n \in N$: Set of all UAVs[cite: 5].
- $k \in K$: Set of all time steps[cite: 6].
- $s \in G$: Location of the base station (sink)[cite: 7].
- $C_i \subset G$: Set of grid points in communication range of point i[cite: 8].
- $S_i \subset G$: Set of grid points in sensing range of point i[cite: 9].

Parameters

- b_{mov}: Energy consumed for moving one grid step[cite: 11].
- b_{steady}: Energy consumed while idle for one time step[cite: 11].
- b_{full}: Maximum battery capacity for a single UAV[cite: 12].
- e_{base}: Minimum required battery level for a single UAV[cite: 13].
- M: A large constant for Big-M method (e.g., b_{full})[cite: 14].
- $B_{\max}$: Maximum allowable total energy consumption for the entire fleet[cite: 15].
- $C_{\min}$: Minimum number of grid points that must be covered[cite: 16].

Decision Variables

- $z_{i,n}^k \in \{0, 1\}$: 1 if UAV n is at grid point i at time k[cite: 17].
- $c_i \in [0, 1]$: 1 if grid point i is covered at any time[cite: 18].
- $c_{i,n}^k \in [0, 1]$: 1 if grid point i is covered by UAV n at time k[cite: 19].
- $m_{i,j,n}^k \in \{0, 1\}$: 1 if UAV n moves from i to j between k and $k+1$[cite: 20].
- $x_n^k \in \{0, 1\}$: 1 if UAV n is charging at time k[cite: 21].
- $b_n^k \in \mathbb{R}^+$: Battery level of UAV n at time k[cite: 22].

Multi-Objective Formulations

Below are two distinct models for optimizing the UAV path planning based on different mission priorities. The common constraints applicable to both models are listed in the subsequent section.

Model 1: Maximize Coverage with a Battery Consumption Threshold

This model aims to achieve the maximum possible area coverage without exceeding a total energy budget for the fleet[cite: 27].

Objective Function

$$\max \sum_{i \in G} c_i \quad \text{Maximize coverage} \tag{1}$$

Additional Constraint

$$\sum_{n \in N} \sum_{k \in K} \left(b_{\text{mov}} \sum_{i,j \in G, i \neq j} m_{i,j,n}^k + b_{\text{steady}} \sum_{i \in G} m_{i,i,n}^k \right) \leq B_{\max} \quad \text{Total energy budget} \tag{16}$$

Model 2: Minimize Battery Consumption with a Coverage Threshold

This model aims to find the most energy-efficient flight paths while ensuring a minimum required area is surveyed[cite: 37].

Objective Function

$$\min \sum_{n\in N}\sum_{k\in K}\left(b_{\text{mov}}\sum_{i,j\in G,i\neq j} m^k_{i,j,n} + b_{\text{steady}}\sum_{i\in G} m^k_{i,i,n}\right) \quad \text{Minimize total energy} \tag{2}$$

Additional Constraint

$$\sum_{i\in G} c_i \geq C_{\min} \quad \text{Minimum coverage} \tag{17}$$

Common Constraints

The following constraints are applicable to both models presented above[cite: 45].

$$\sum_{i\in G} z^k_{i,n} = 1 \quad \forall n,k \qquad \text{Unique position} \tag{2}$$

$$\sum_{n\in N} z^k_{i,n} \leq 1 \quad \forall i\neq s,k \qquad \text{Collision avoidance} \tag{3}$$

$$\sum_{n\in N}\sum_{p\in C_s} z^k_{p,n} \geq 1 \quad \forall k \qquad \text{Sink connectivity} \tag{4}$$

$$z^k_{i,n} \leq \sum_{p\in C_i} z^k_{p,n-1} \quad \forall i,n>1,k \qquad \text{Inter-UAV link} \tag{5}$$

$$z^{k+1}_{i,n} \leq \sum_{p\in C_i} z^k_{p,n} \quad \forall i,n,k<K_{max} \qquad \text{Mobility rule} \tag{6}$$

$$m^k_{i,j,n} \leq z^k_{i,n} \quad \forall i,j,n,k \qquad \text{Movement definition} \tag{7a}$$

$$m^k_{i,j,n} \leq z^{k+1}_{j,n} \quad \forall i,j,n,k \qquad \text{Movement definition} \tag{7b}$$

$$m^k_{i,j,n} \geq z^k_{i,n} + z^{k+1}_{j,n} - 1 \quad \forall i,j,n,k \qquad \text{Movement definition} \tag{7c}$$

$$x^k_n \leq z^k_{s,n} \quad \forall n,k \qquad \text{Charging location} \tag{8}$$

$$b^{k+1}_n \leq b^k_n - \sum_{i,j,i\neq j} m^k_{i,j,n} b_{mov} - \sum_i m^k_{i,i,n} b_{steady} + M\cdot x^k_n \qquad \text{Battery discharge} \tag{9a}$$

$$b^{k+1}_n \geq b^k_n - \sum_{i,j,i\neq j} m^k_{i,j,n} b_{mov} - \sum_i m^k_{i,i,n} b_{steady} \qquad \text{Battery discharge} \tag{9b}$$

$$b^{k+1}_n \leq b_{full} + M\cdot(1-x^k_n) \quad \forall n,k \qquad \text{Battery charge} \tag{10a}$$

$$b^{k+1}_n \geq b_{full} - M\cdot(1-x^k_n) \quad \forall n,k \qquad \text{Battery charge} \tag{10b}$$

$$b^k_n \leq b_{full} \quad \forall n,k \qquad \text{Max battery} \tag{11}$$

$$b^k_n \geq e_{base} \quad \forall n,k \qquad \text{Min battery} \tag{12}$$

$$c^k_{i,n} = \sum_{p\in S_i} z^k_{p,n} \quad \forall i,n,k \qquad \text{Local coverage} \tag{13}$$

$$c^k_{i,n} \leq c_i \quad \forall i,n,k \qquad \text{Global mapping} \tag{14}$$

$$c_i \leq \sum_{n\in N}\sum_{k\in K} c^k_{i,n} \quad \forall i \qquad \text{Global coverage} \tag{15}$$

Hierarchical Decomposition for Scalability

To address the scalability issues of the unified MILP, we decompose the problem into a two-stage hierarchical structure: a high-level Master Problem for coarse-grained regional assignment and a low-level Subproblem for fine-grained path planning within each region.

Stage 1: The Master Problem MILP Formulation

This problem operates on a coarse grid of regions, assigning UAVs to sequences of these regions over large time intervals.

Sets and Indices

- $n \in N$: Set of all UAVs.
- $p, q \in R$: Set of disjoint regions that partition the main grid G.
- $t \in T$: Set of coarse time intervals.
- $R_s \in R$: The region containing the base station s.

Parameters

- C_p^{est}: Estimated maximum coverage value in region p.
- E_p^{fly}: Estimated energy for a UAV to operate in region p for one interval t.
- E_{pq}^{travel}: Estimated energy for a UAV to travel from region p to adjacent region q.
- $B_{\max}$: Maximum total energy consumption for the fleet.

Decision Variables

- $A_{n,p,t} \in \{0, 1\}$: 1 if UAV n is assigned to region p during interval t.
- $Y_{n,p,q,t} \in \{0, 1\}$: 1 if UAV n travels from region p to q at the end of interval t.

Objective Function

$$\max \sum_{t \in T} \sum_{p \in R} \sum_{n \in N} C_p^{est} \cdot A_{n,p,t} \quad \text{Maximize total potential coverage} \tag{1}$$

Constraints

$$\sum_{p \in R} A_{n,p,t} = 1 \quad \forall n \in N, t \in T \qquad \text{UAV Assignment} \tag{2}$$

$$A_{n,p,t} + A_{n,q,t+1} - 1 \leq Y_{n,p,q,t} \quad \forall n, t < |T|, \text{adj } p, q \qquad \text{Transition Logic} \tag{3}$$

$$\sum_{t,p,n} E_p^{fly} \cdot A_{n,p,t} + \sum_{t,n,p,q} E_{pq}^{travel} \cdot Y_{n,p,q,t} \leq B_{\max} \qquad \text{Total Energy Budget} \tag{4}$$

$$A_{n,R_s,1} = 1 \quad \forall n \in N \qquad \text{Initial Location} \tag{5}$$

Stage 2: The Subproblem MILP Formulation

For each assignment (n, p, t) from the Master Problem, a path-planning subproblem is solved within the specific region R_p for a specific UAV n over a set of fine-grained time steps K_p.

Inputs from Master Problem

- Region: $R_p \subset G$.
- UAV: $n \in N$.
- Time Horizon: $K_p \subset K$.
- Initial State: Entry location i_{entry} at time k_{start} (Note this is an arbitrary entry).

- Terminal State: Exit location i_{exit} at time k_{end} (Note this is the corresponding exit to the arbitrary entry).
- Allocated Budget: $B^{alloc}_{n,p,t}$.

Objective Function

$$\max \sum_{i \in R_p} c_i \quad \text{Maximize regional coverage [cf. eq. (1)]} \tag{6}$$

Constraints The constraints are adapted from the original formulation, scoped to the smaller domain of region R_p.

$$\sum_{k \in K_p} \left(b_{\text{mov}} \sum_{\substack{i,j \in R_p \\ i \neq j}} m^k_{i,j,n} + b_{\text{steady}} \sum_{i \in R_p} m^k_{i,i,n} \right) \leq B^{alloc}_{n,p,t} \quad \text{Regional Energy Budget} \tag{7}$$

$$z^{k_{start}}_{i_{entry},n} = 1 \quad \text{Entry Constraint} \tag{8}$$

$$z^{k_{end}}_{i_{exit},n} = 1 \quad \text{Exit Constraint} \tag{9}$$

$$b^{k_{start}}_n = b_{entry} \quad \text{Initial Battery} \tag{10}$$

$$\sum_{i \in R_p} z^k_{i,n} = 1 \quad \forall k \in K_p \quad \text{Adapted Unique position} \tag{11}$$

$$z^{k+1}_{i,n} \leq \sum_{j \in C_i \cap R_p} z^k_{j,n} \quad \forall i \in R_p, k < k_{end} \quad \text{Adapted Mobility rule} \tag{12}$$

All other common constraints (3-5, 7a-15) from the above formulation are also applied, but are restricted to the sets $i, j \in R_p$, the single UAV n, and time steps $k \in K_p$. The charging constraint (8) is only active if the base station s is within the current region ($s \in R_p$).