Unified MILP Formulation for UAV Path Planning

Variable and Parameter Descriptions

Sets and Indices

- $i, j \in G$: Set of all grid points.
- $n \in N$: Set of all UAVs.
- $k \in K$: Set of all time steps.
- $s \in G$: Location of the base station (sink).
- $C_i \subset G$: Set of grid points in communication range of point i.
- $S_i \subset G$: Set of grid points in sensing range of point i.

Parameters

- b_{mov}: Energy consumed for moving one grid step.
- b_{steady}: Energy consumed while idle for one time step.
- b_{full}: Maximum battery capacity for a single UAV.
- e_{base}: Minimum required battery level for a single UAV.
- M: A large constant for Big-M method (e.g., b_{full}).
- $B_{\max}$: Maximum allowable total energy consumption for the entire fleet.
- $C_{\min}$: Minimum number of grid points that must be covered.

Decision Variables

- $z_{i,n}^k \in \{0, 1\}$: 1 if UAV n is at grid point i at time k.
- $c_i \in [0, 1]$: 1 if grid point i is covered at any time.
- $c_{i,n}^k \in [0, 1]$: 1 if grid point i is covered by UAV n at time k.
- $m_{i,j,n}^k \in \{0, 1\}$: 1 if UAV n moves from i to j between k and $k + 1$.
- $x_n^k \in \{0, 1\}$: 1 if UAV n is charging at time k.
- $b_n^k \in \mathbb{R}^+$: Battery level of UAV n at time k.

Multi-Objective Formulations

Below are two distinct models for optimizing the UAV path planning based on different mission priorities. The common constraints applicable to both models are listed in the subsequent section.

Model 1: Maximize Coverage with a Battery Consumption Threshold

This model aims to achieve the maximum possible area coverage without exceeding a total energy budget for the fleet.

Objective Function

$$\max \sum_{i \in G} c_i \quad \text{Maximize coverage} \tag{1}$$

Additional Constraint

$$\sum_{n \in N} \sum_{k \in K} \left(b_{\text{mov}} \sum_{i,j \in G, i \neq j} m_{i,j,n}^k + b_{\text{steady}} \sum_{i \in G} m_{i,i,n}^k \right) \leq B_{\max} \quad \text{Total energy budget} \tag{16}$$

Model 2: Minimize Battery Consumption with a Coverage Threshold

This model aims to find the most energy-efficient flight paths while ensuring a minimum required area is surveyed.

Objective Function

$$\min \sum_{n \in N} \sum_{k \in K} \left(b_{\text{mov}} \sum_{i,j \in G, i \neq j} m_{i,j,n}^k + b_{\text{steady}} \sum_{i \in G} m_{i,i,n}^k \right) \quad \text{Minimize total energy} \tag{2}$$

Additional Constraint

$$\sum_{i \in G} c_i \geq C_{\min} \quad \text{Minimum coverage} \tag{17}$$

Common Constraints

The following constraints are applicable to both models presented above.

$$\sum_{i \in G} z_{i,n}^k = 1 \quad \forall n, k \qquad \text{Unique position} \tag{2}$$

$$\sum_{n \in N} z_{i,n}^k \leq 1 \quad \forall i \neq s, k \qquad \text{Collision avoidance} \tag{3}$$

$$\sum_{n \in N} \sum_{p \in C_s} z_{p,n}^k \geq 1 \quad \forall k \qquad \text{Sink connectivity} \tag{4}$$

$$z_{i,n}^k \leq \sum_{p \in C_i} z_{p,n-1}^k \quad \forall i, n > 1, k \qquad \text{Inter-UAV link} \tag{5}$$

$$z_{i,n}^{k+1} \leq \sum_{p \in C_i} z_{p,n}^k \quad \forall i, n, k < K_{max} \qquad \text{Mobility rule} \tag{6}$$

$$m_{i,j,n}^k \leq z_{i,n}^k \quad \forall i, j, n, k \qquad \text{Movement definition} \tag{7a}$$

$$m_{i,j,n}^k \leq z_{j,n}^{k+1} \quad \forall i, j, n, k \qquad \text{Movement definition} \tag{7b}$$

$$m_{i,j,n}^k \geq z_{i,n}^k + z_{j,n}^{k+1} - 1 \quad \forall i, j, n, k \qquad \text{Movement definition} \tag{7c}$$

$$x_n^k \leq z_{s,n}^k \quad \forall n, k \qquad \text{Charging location} \tag{8}$$

$$b_n^{k+1} \leq b_n^k - \sum_{i,j,i \neq j} m_{i,j,n}^k b_{mov} - \sum_i m_{i,i,n}^k b_{steady} + M \cdot x_n^k \qquad \text{Battery discharge} \tag{9a}$$

$$b_n^{k+1} \geq b_n^k - \sum_{i,j,i \neq j} m_{i,j,n}^k b_{mov} - \sum_i m_{i,i,n}^k b_{steady} + M \cdot x_n^k \qquad \text{Battery discharge} \tag{9b}$$

$$b_n^{k+1} \leq b_{full} + M \cdot (1 - x_n^k) \quad \forall n, k \qquad \text{Battery charge} \tag{10a}$$

$$b_n^{k+1} \geq b_{full} - M \cdot (1 - x_n^k) \quad \forall n, k \qquad \text{Battery charge} \tag{10b}$$

$$b_n^k \leq b_{full} \quad \forall n, k \qquad \text{Max battery} \tag{11}$$

$$b_n^k \geq e_{base} \quad \forall n, k \qquad \text{Min battery} \tag{12}$$

$$c_{i,n}^k = \sum_{p \in S_i} z_{p,n}^k \quad \forall i, n, k \qquad \text{Local coverage} \tag{13}$$

$$c_{i,n}^k \leq c_i \quad \forall i, n, k \qquad \text{Global mapping} \tag{14}$$

$$c_i \leq \sum_{n \in N} \sum_{k \in K} c_{i,n}^k \quad \forall i \qquad \text{Global coverage} \tag{15}$$