Unified MILP Formulation

Variable and Parameter Descriptions

Sets and Indices

- $i, j \in G$: Set of all grid points.
- $n \in N$: Set of all UAVs.
- $k \in K$: Set of all time steps.
- $s \in G$: Location of the base station (sink).
- $C_i \subset G$: Set of grid points in communication range of point i.
- $S_i \subset G$: Set of grid points in sensing range of point i.

Parameters

- b_{mov}: Energy consumed for moving one grid step.
- b_{steady}: Energy consumed while idle for one time step.
- b_{full}: Maximum battery capacity.
- e_{base}: Minimum required battery level.
- M: A large constant for Big-M method (e.g., b_{full}).

Decision Variables

- $z_{i,n}^k \in \{0, 1\}$: 1 if UAV n is at grid point i at time k.
- $c_i \in [0, 1]$: 1 if grid point i is covered at any time.
- $c_{i,n}^k \in [0, 1]$: 1 if grid point i is covered by UAV n at time k.
- $m_{i,j,n}^k \in \{0, 1\}$: 1 if UAV n moves from i to j between k and $k + 1$.
- $x_n^k \in \{0, 1\}$: 1 if UAV n is charging at time k.
- $b_n^k \in \mathbb{R}^+$: Battery level of UAV n at time k.

Objective Function

$$\max \sum_{i \in G} c_i \quad \text{Maximize coverage} \tag{1}$$

Constraints

$$\sum_{i \in G} z_{i,n}^k = 1 \quad \forall n, k \qquad \text{Unique position} \tag{2}$$

$$\sum_{n \in N} z_{i,n}^k \leq 1 \quad \forall i \neq s, k \qquad \text{Collision avoidance} \tag{3}$$

$$\sum_{n \in N} \sum_{p \in C_s} z_{p,n}^k \geq 1 \quad \forall k \qquad \text{Sink connectivity} \tag{4}$$

$$z_{i,n}^k \leq \sum_{p \in C_i} z_{p,n-1}^k \quad \forall i, n > 1, k \qquad \text{Inter-UAV link} \tag{5}$$

$$z_{i,n}^{k+1} \leq \sum_{p \in C_i} z_{p,n}^k \quad \forall i, n, k < K_{max} \qquad \text{Mobility rule} \tag{6}$$

$$m_{i,j,n}^k \leq z_{i,n}^k \quad \forall i, j, n, k \qquad \text{Movement definition} \tag{7a}$$

$$m_{i,j,n}^k \leq z_{j,n}^{k+1} \quad \forall i, j, n, k \qquad \text{Movement definition} \tag{7b}$$

$$m_{i,j,n}^k \geq z_{i,n}^k + z_{j,n}^{k+1} - 1 \quad \forall i, j, n, k \qquad \text{Movement definition} \tag{7c}$$

$$x_n^k \leq z_{s,n}^k \quad \forall n, k \qquad \text{Charging location} \tag{8}$$

$$b_n^{k+1} \leq b_n^k - \sum_{i,j,i \neq j} m_{i,j,n}^k b_{mov} - \sum_i m_{i,i,n}^k b_{steady} + M \cdot x_n^k \qquad \text{Battery discharge} \tag{9a}$$

$$b_n^{k+1} \geq b_n^k - \sum_{i,j,i \neq j} m_{i,j,n}^k b_{mov} - \sum_i m_{i,i,n}^k b_{steady} - M \cdot x_n^k \qquad \text{Battery discharge} \tag{9b}$$

$$b_n^{k+1} \leq b_{full} + M \cdot (1 - x_n^k) \quad \forall n, k \qquad \text{Battery charge} \tag{10a}$$

$$b_n^{k+1} \geq b_{full} - M \cdot (1 - x_n^k) \quad \forall n, k \qquad \text{Battery charge} \tag{10b}$$

$$b_n^k \leq b_{full} \quad \forall n, k \qquad \text{Max battery} \tag{11}$$

$$b_n^k \geq e_{base} \quad \forall n, k \qquad \text{Min battery} \tag{12}$$

$$c_{i,n}^k = \sum_{p \in S_i} z_{p,n}^k \quad \forall i, n, k \qquad \text{Local coverage} \tag{13}$$

$$c_{i,n}^k \leq c_i \quad \forall i, n, k \qquad \text{Global mapping} \tag{14}$$

$$c_i \leq \sum_{n \in N} \sum_{k \in K} c_{i,n}^k \quad \forall i \qquad \text{Global coverage} \tag{15}$$