Unified MILP Formulation for UAV Path Planning

Variable and Parameter Descriptions

Sets and Indices

- s: Base station (or sink) location
- $G_{\bar{s}}$: Set of all grid points (excluding the sink).
- C_i : Set of grid points in communication range of point i. Therefore C_s Set of grid points within the communication range of the sink location
- S_i: Set of grid points in sensing range of point i.

Parameters

- b_{mov}: Energy consumed for moving one grid step.
- b_{steady}: Energy consumed while idle for one time step.
- b_{full}: Maximum battery capacity for a single UAV.
- e_{base}: Minimum required battery level for a single UAV.
- N: Number of UAVs (or mobile nodes)
- $K_{\max}$: Maximum number of time-steps
- M: A large constant for Big-M method (e.g., b_{full}).
- $B_{\max}$: Maximum allowable total energy consumption for the entire fleet.
- $C_{\min}$: Minimum number of grid points that must be covered.
- $C_{\max}$: Total number of grid points to be covered (Note that, $C_{\max}$ = the cardinality of $G_{\bar{s}}$)
- cr: Area coverage ratio: ratio of the number of grid points covered at least once at any time-step and $C_{\max}$ ($0 \le cr \le 1$)
- $K_{\max}$: Maximum number of time-steps

Decision Variables

- $z_{i,n}^k \in \{0,1\}$: Binary variable; $z_{i,n}^k = 1$ if the nth UAV is located at grid point ith at the kth time-step, otherwise $z_{i,n}^k = 0$.
- $c_i \in [0,1]$: Continuous variable; $c_i = 1$ if the grid point i is covered by any UAV at any time-step, otherwise $c_i = 0$.
- $c_{i,n}^k \in [0,1]$: Continuous variable; $c_{i,n}^k = 1$ if the grid point i is covered kth by the nth UAV at the kth time-step, otherwise $c_{i,n}^k = 0$.
- $m_{i,j,n}^k \in \{0,1\}$: 1 if UAV n moves from i to j between k and $k+1$.
- $x_n^k \in \{0,1\}$: 1 if UAV n is charging at time k.
- $b_n^k \in \mathbb{R}^+$: Battery level of UAV n at time k.

Multi-Objective Formulations

Below are two distinct models for optimizing the UAV path planning based on different mission priorities. The common constraints applicable to both models are listed in the subsequent section.

Model: Maximize Coverage with a Battery Consumption Threshold

This model aims to achieve the maximum possible area coverage without exceeding a total energy budget for the fleet.

Objective Function

$$\max \sum_{i \in G_{\bar{s}}} c_i \quad \text{Maximize coverage} \tag{1}$$

Additional Constraint

$$\sum_{n \in N} \sum_{k \in K} -\left(b_n^k - b_n^{k-1}\right) \leq B_{\max} \quad \text{Total energy budget} \tag{17}$$

Common Constraints

The following constraints are applicable to both models presented above.

$$\sum_{i \in G_{\bar{s}}} z_{i,n}^k = 1 \quad n = 1, 2, ..., N; k = 1, 2, ..., K_{\max} \qquad \text{Unique position} \quad (2)$$

$$\sum_{n=1}^{N} z_{i,n}^k \leq 1 \quad \forall i \in G_{\bar{s}}; k = 1, 2, ..., K_{\max} \qquad \text{Collision avoidance} \quad (3)$$

$$\sum_{n=1}^{N} \sum_{p \in C_s} z_{p,n}^k \geq 1 \quad k = 1, 2, ..., K_{\max} \qquad \text{Sink connectivity} \quad (4)$$

$$z_{i,n}^k \leq \sum_{p \in C_i} z_{p,(n-1)}^k \quad \forall i \in G_{\bar{s}}; n = 2, 3, ...N; k = 1, 2, ..., K_{\max} \qquad \text{Inter-UAV link} \quad (5)$$

$$z_{i,n}^{k+1} \leq \sum_{p \in C_i} z_{p,n}^k \quad \forall i \in G_{\bar{s}}; n = 1, 2, ..., N; k = 1, 2, ..., (K_{\max} - 1) \qquad \text{Mobility rule} \quad (6)$$

$$m_{i,j,n}^k \leq z_{i,n}^k \quad \forall i, j, n, k \qquad \text{Movement definition} \quad (7a)$$

$$m_{i,j,n}^k \leq z_{j,n}^{k+1} \quad \forall i, j, n, k \qquad \text{Movement definition} \quad (7b)$$

$$m_{i,j,n}^k \geq z_{i,n}^k + z_{j,n}^{k+1} - 1 \quad \forall i, j, n, k \qquad \text{Movement definition} \quad (7c)$$

$$x_n^k \leq z_{s,n}^k \quad \forall n, k \qquad \text{Charging location} \quad (8)$$

$$b_n^{k+1} \leq b_n^k - \sum_{i,j,i \neq j} m_{i,j,n}^k b_{mov} - \sum_{i} m_{i,i,n}^k b_{steady} + M \cdot x_n^k \qquad \text{Battery discharge} \quad (9a)$$

$$b_n^{k+1} \geq b_n^k - \sum_{i,j,i \neq j} m_{i,j,n}^k b_{mov} - \sum_{i} m_{i,i,n}^k b_{steady} - M \cdot x_n^k \qquad \text{Battery discharge} \quad (9b)$$

$$b_n^{k+1} \leq b_{full} + M \cdot (1 - x_n^k) \quad \forall n, k \qquad \text{Battery charge} \quad (10a)$$

$$b_n^{k+1} \geq b_{full} - M \cdot (1 - x_n^k) \quad \forall n, k \qquad \text{Battery charge} \quad (10b)$$

$$b_n^k \leq b_{full} \quad \forall n, k \qquad \text{Max battery} \quad (11)$$

$$b_n^k \geq e_{base} \quad \forall n, k \qquad \text{Min battery} \quad (12)$$

$$c_{i,n}^k = \sum_{p \in S_i} z_{p,n}^k \quad \forall i \in G_{\bar{s}}; n = 1, 2, ..., N; k = 1, 2, ..., K_{\max} \qquad \text{Local coverage} \quad (13)$$

$$c_i \geq c_{i,n}^k \quad \forall i \in G_{\bar{s}}; n = 1, 2, ..., N; k = 1, 2, ..., K_{\max} \qquad \text{Global mapping} \quad (14)$$

$$c_i \leq \sum_{k=1}^{K_{\max}} \sum_{n=1}^{N} c_{i,n}^k \quad \forall i \in G_{\bar{s}} \qquad \text{Global coverage} \quad (15)$$

$$0 \leq c_i, c_{i,n}^k \leq 1 \quad \forall i \in G_{\bar{s}}; n = 1, 2, ..., N; k = 1, 2, ..., K_{\max} \qquad \text{Coverage bounds} \quad (16)$$