November 26, 2021 End Sem CS205M

No of Questions: 3 *Maximum Marks: 75* *Maximum Time: 2 Hrs.*

1. (a) [6Marks] Construct a DFA equivalent to the NFA $N = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_2\})$, (using the subset construction method) where the transition function, δ is given below.

	0	1
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	ϕ	$\{q_2\}$
q_2	$\{q_1\}$	ϕ

   (b) [6Marks] Either prove or disprove the following.
   The language $\{xwx^R | x, w \in \{0, 1\}^* \text{ and } |x|, |w| > 0\}$ is regular.

2. (a) [5Marks] Let $M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_1, Z_1, F_1)$ and $M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_2, Z_2, F_2)$ be two PDAs over the same alphabet Σ. Write a formal description of a PDA that recognizes the language $L(M_1) \cup L(M_2)$. No proof of correctness is required.

   (b) [5Marks] Let (N_1, Σ, P_1, S_1) be a CFG for a language $L_1 \subseteq \Sigma^*$ and let (N_2, Σ, P_2, S_2) be a CFG for $L_2 \subseteq \Sigma^*$. Assume that N_1 and N_2 are disjoint. Specify, formally, a CFG for the language $L_1^* \cup L_2$. No proof of correctness is required.

   (c) [6Marks] Consider the CFG: $S \to Sa|A|BC,\ A \to a,\ B \to ab,\ C \to bC$.
   Remove useless symbols and useless productions, if any, from the above grammar and give the equivalent simplified grammar. What language does the grammar generates?

   (d) [8Marks] Give a PDA to accept the language $\{w0x1 | w, x \in \{0, 1\}^*, |w| = |x|\}$.

   (e) [12Marks] For each of the following languages determine whether it is context-free or not. If it is context-free, then give a PDA for it. Otherwise, use the pumping lemma to prove that it is not context-free.
   a) $L = \{a^i b^i c^j | i \leq j \leq 2i\}$
   b) $L = \{a^i b^j c^k d^l | i + j = k + l\}$

3. (a) [5Marks] What is the language generated by the following unrestricted grammar? Give a brief justification.
   $S \to BS \,|\, bC$
   $Bb \to bbB$
   $BC \to C$
   $C \to \epsilon$

   (b) [4Marks] Consider the Turing machine: $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b\}, \delta, q_0, B, \{q_2\})$, where δ is given below.
   $\delta(q_0, a) = (q_1, a, R),\ \delta(q_0, b) = (q_0, b, R),\ \delta(q_1, a) = (q_0, a, R)$
   $\delta(q_1, b) = (q_1, b, R),\ \delta(q_0, B) = (q_2, B, R)$
   What is the language accepted by M?

   (c) [9Marks] Give the transition diagram of a Turing machine to accept the language $L = \{w \in \{0, 1\}^* | w \text{ has equal numbers of 0's and 1's}\}$.

   (d) [9Marks] Construct a standard Turing machine (give the transition diagram only) to compute the function $f(n) = 2n + 1$, $n \geq 0$. (Assume that the integer n is represented as 0^n and the initial configuration of the Turing machine is $q_0 0^n 1$, where q_0 is the intial state.)