Government of Karnataka

MATHEMATICS

6

SIXTH STANDARD

PART - II

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Karnataka Textbook Society (R.)
6th Cross, Malleshwaram, Bengaluru-560 003

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Chapter 7

Fractions

0650CH07

7.1 Introduction

Subhash had learnt about fractions in Classes IV and V, so whenever possible he would try to use fractions. One occasion was when he forgot his lunch at home. His friend Farida invited him to share her lunch. She had five pooris in her lunch box. So, Subhash and Farida took two pooris each. Then Farida made two equal halves of the fifth poori and gave one-half to Subhash and took the other half herself. Thus, both Subhash and Farida had 2 full pooris and one-half poori.

2 pooris + half-poori–Subhash
2 pooris + half-poori–Farida

Where do you come across situations with fractions in your life?

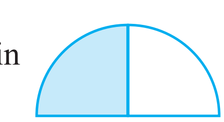


Fig 7.1

Subhash knew that one-half is written as $\frac{1}{2}$. While eating he further divided his half poori into two equal parts and asked Farida what fraction of the whole poori was that piece? (Fig 7.1)

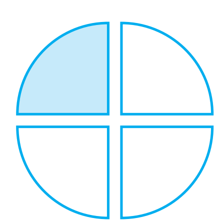


Fig 7.2

Without answering, Farida also divided her portion of the half puri into two equal parts and kept them beside Subhash's shares. She said that these four equal parts together make one whole (Fig 7.2). So, each equal part is one-fourth of one whole poori and 4 parts together will be $\frac{4}{4}$ or 1 whole poori.

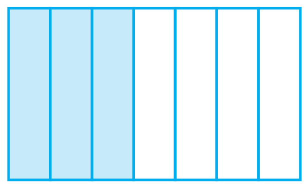


Fig 7.3

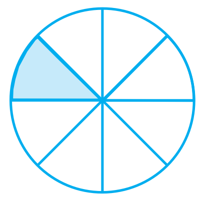


Fig 7.4

When they ate, they discussed what they had learnt earlier. Three parts out of 4 equal parts is $\frac{3}{4}$. Similarly, $\frac{3}{7}$ is obtained when we divide a whole into seven equal parts and take three parts (Fig 7.3). For $\frac{1}{8}$, we divide a whole into eight equal parts and take one part out of it (Fig 7.4).

Farida said that we have learnt that **a fraction is a number representing part of a whole. The whole may be a single object or a group of objects.** Subhash observed that **the parts have to be equal.**

7.2 A Fraction

Let us recapitulate the discussion.

A fraction means a part of a group or of a region.

$\frac{5}{12}$ is a fraction. We read it as "five-twelfths".

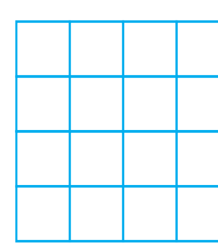
What does "12" stand for? It is the number of equal parts into which the whole has been divided.

What does "5" stand for? It is the number of equal parts which have been taken out.

Here 5 is called the numerator and 12 is called the denominator.

Name the numerator of $\frac{3}{7}$ and the denominator of $\frac{4}{15}$.

Play this Game



You can play this game with your friends.

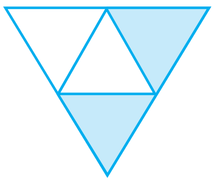
Take many copies of the grid as shown here.

Consider any fraction, say $\frac{1}{2}$.

Each one of you should shade $\frac{1}{2}$ of the grid.

EXERCISE 7.1

1. Write the fraction representing the shaded portion.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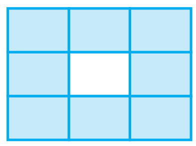
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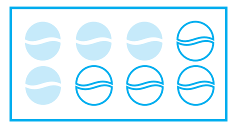
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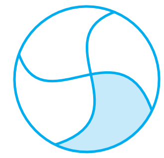
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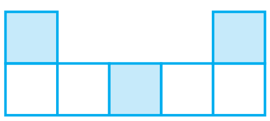
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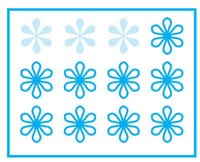
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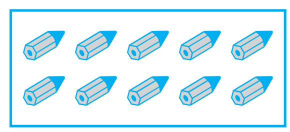
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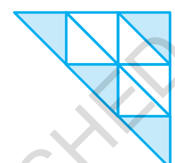
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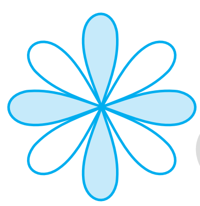
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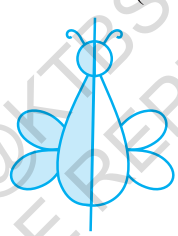
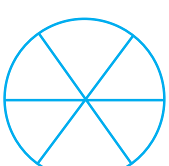
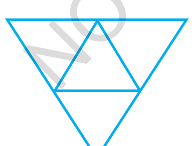
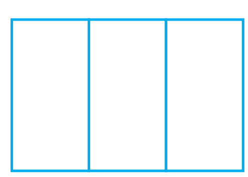

(x)

2. Colour the part according to the given fraction.


(i) $\frac{1}{6}$


(ii) $\frac{1}{4}$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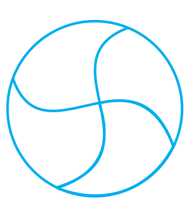
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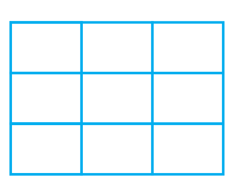
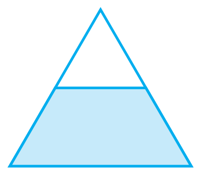
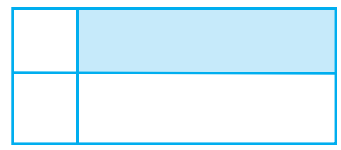

(v) $\frac{4}{9}$

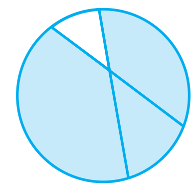
3. Identify the error, if any.



This is $\frac{1}{2}$



This is $\frac{1}{4}$

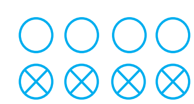


This is $\frac{3}{4}$

4. What fraction of a day is 8 hours?
5. What fraction of an hour is 40 minutes?
6. Arya, Abhimanyu, and Vivek shared lunch. Arya has brought two sandwiches, one made of vegetable and one of jam. The other two boys forgot to bring their lunch. Arya agreed to share his sandwiches so that each person will have an equal share of each sandwich.

   (a) How can Arya divide his sandwiches so that each person has an equal share?

   (b) What part of a sandwich will each boy receive?
7. Kanchan dyes dresses. She had to dye 30 dresses. She has so far finished 20 dresses. What fraction of dresses has she finished?
8. Write the natural numbers from 2 to 12. What fraction of them are prime numbers?
9. Write the natural numbers from 102 to 113. What fraction of them are prime numbers?
10. What fraction of these circles have X's in them?



11. Kristin received a CD player for her birthday. She bought 3 CDs and received 5 others as gifts. What fraction of her total CDs did she buy and what fraction did she receive as gifts?

7.3 Fraction on the Number Line

You have learnt to show whole numbers like 0,1,2... on a number line.

We can also show fractions on a number line. Let us draw a number line and try to mark $\frac{1}{2}$ on it.

We know that $\frac{1}{2}$ is greater than 0 and less than 1, so it should lie between 0 and 1.

Since we have to show $\frac{1}{2}$, we divide the gap between 0 and 1 into two equal parts and show 1 part as $\frac{1}{2}$ (as shown in the Fig 7.5).

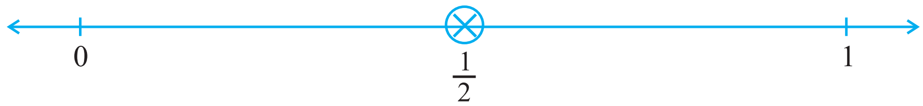


Fig 7.5

Suppose we want to show $\frac{1}{3}$ on a number line. Into how many equal parts should the length between 0 and 1 be divided? We divide the length between 0 and 1 into 3 equal parts and show one part as $\frac{1}{3}$ (as shown in the Fig 7.6)

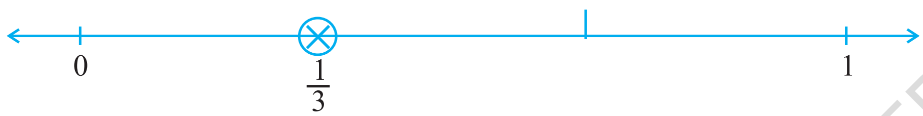


Fig 7.6

Can we show $\frac{2}{3}$ on this number line? $\frac{2}{3}$ means 2 parts out of 3 parts as shown (Fig 7.7).

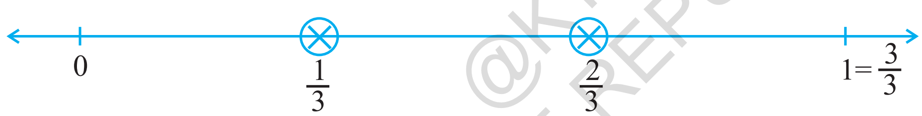


Fig 7.7

Similarly, how would you show $\frac{0}{3}$ and $\frac{3}{3}$ on this number line?

$\frac{0}{3}$ is the point zero whereas since $\frac{3}{3}$ is 1 whole, it can be shown by the point 1 (as shown in Fig 7.7)

So if we have to show $\frac{3}{7}$ on a number line, then, into how many equal parts should the length between 0 and 1 be divided? If P shows $\frac{3}{7}$ then how many equal divisions lie between 0 and P? Where do $\frac{0}{7}$ and $\frac{7}{7}$ lie?

Try These

1. Show $\frac{3}{5}$ on a number line.
2. Show $\frac{1}{10}, \frac{0}{10}, \frac{5}{10}$ and $\frac{10}{10}$ on a number line.
3. Can you show any other fraction between 0 and 1?
   Write five more fractions that you can show and depict them on the number line.
4. How many fractions lie between 0 and 1? Think, discuss and write your answer?

7.4 Proper Fractions

You have now learnt how to locate fractions on a number line. Locate the fractions $\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{0}{3}, \frac{5}{8}$ on separate number lines.

Does any one of the fractions lie beyond 1?

All these fractions lie to the left of 1as they are less than 1.

In fact, all the fractions we have learnt so far are less than 1. These are **proper fractions**. A proper fraction as Farida said (Sec. 7.1), is a number representing part of a whole. In a proper fraction the denominator shows the number of parts into which the whole is divided and the numerator shows the number of parts which have been considered. Therefore, in a proper fraction the numerator is always less than the denominator.

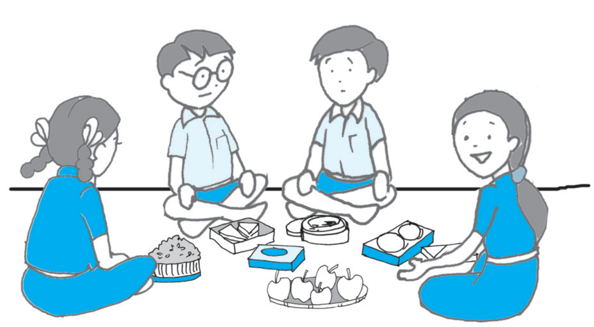
Try These

1. Give a proper fraction :
   (a) whose numerator is 5 and denominator is 7.
   (b) whose denominator is 9 and numerator is 5.
   (c) whose numerator and denominator add up to 10. How many fractions of this kind can you make?
   (d) whose denominator is 4 more than the numerator.
   (Give any five. How many more can you make?)
2. A fraction is given.
   How will you decide, by just looking at it, whether, the fraction is
   (a) less than 1?
   (b) equal to 1?
3. Fill up using one of these : '>', '<' or '='

   (a) $\frac{1}{2} \square 1$ (b) $\frac{3}{5} \square 1$ (c) $1 \square \frac{7}{8}$ (d) $\frac{4}{4} \square 1$ (e) $\frac{2005}{2005} \square 1$

7.5 Improper and Mixed Fractions

Anagha, Ravi, Reshma and John shared their tiffin. Along with their food, they had also, brought 5 apples. After eating the other food, the four friends wanted to eat apples.



How can they share five apples among four of them?

Anagha said, 'Let each of us have one full apple and a quarter of the fifth apple.'

Anagha Ravi Reshma John

Reshma said, 'That is fine, but we can also divide each of the five apples into 4 equal parts and take one-quarter from each apple.'

Anagha Ravi Reshma John

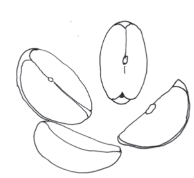
Ravi said, 'In both the ways of sharing each of us would get the same share, i.e., 5 quarters. Since 4 quarters make one whole, we can also say that each of us would get 1 whole and one quarter. The value of each share would be five divided by four. Is it written as $5 \div 4$?' John said, 'Yes the same as $\frac{5}{4}$'. Reshma added that in $\frac{5}{4}$, the numerator is bigger than the denominator. The fractions, where the numerator is bigger than the denominator are called **improper fractions.** Thus, fractions like $\frac{3}{2}, \frac{12}{7}, \frac{18}{5}$ are all improper fractions.

1. Write five improper fractions with denominator 7.
2. Write five improper fractions with numerator 11.

Ravi reminded John, 'What is the other way of writing the share? Does it follow from Anagha's way of dividing 5 apples?'

John nodded, 'Yes, It indeed follows from Anagha's way. In her way, each share is one whole and one quarter. It is $1+\frac{1}{4}$ and written in short as $1\frac{1}{4}$. Remember, $1\frac{1}{4}$ is the same as $\frac{5}{4}$'.

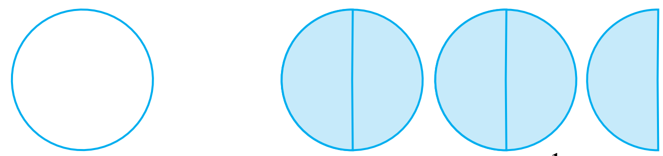
This is 1 (one)



Each of these is $\frac{1}{4}$ (one-fourth)

Fig 7.8

Recall the pooris eaten by Farida. She got $2\frac{1}{2}$ poories (Fig 7.9), i.e.



This is 1 This is $2\frac{1}{2}$

Fig 7.9

How many shaded halves are there in $2\frac{1}{2}$? There are 5 shaded halves.

So, the fraction can also be written as $\frac{5}{2}$. $2\frac{1}{2}$ is the same as $\frac{5}{2}$.

Fractions such as $1\frac{1}{4}$ and $2\frac{1}{2}$ are called **Mixed Fractions**. A mixed fraction has a combination of a whole and a part.

> **Do you know?**
> The grip-sizes of tennis racquets are often in mixed numbers. For example one size is '$3\frac{7}{8}$ inches' and '$4\frac{3}{8}$ inches' is another.

Where do you come across mixed fractions? Give some examples.

Example 1 : Express the following as mixed fractions :

(a) $\frac{17}{4}$ (b) $\frac{11}{3}$ (c) $\frac{27}{5}$ (d) $\frac{7}{3}$

Solution : (a) $\frac{17}{4}$

$$\begin{array}{r|l} & 4 \\ \hline 4 & 17 \\ & -16 \\ \hline & 1 \end{array}$$

i.e. 4 whole and $\frac{1}{4}$ more, or $4\frac{1}{4}$

(b) $\frac{11}{3}$

$$\begin{array}{r|l} & 3 \\ \hline 3 & 11 \\ & -9 \\ \hline & 2 \end{array}$$

i.e. 3 whole and $\frac{2}{3}$ more, or $3\frac{2}{3}$

$$\left[\textit{Alternatively}, \frac{11}{3} = \frac{9+2}{3} = \frac{9}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}\right]$$

Try (c) and (d) using both the methods for yourself.

Thus, we can express an improper fraction as a mixed fraction by dividing the numerator by denominator to obtain the quotient and the remainder. Then the mixed fraction will be written as $\text{Quotient}\frac{\text{Remainder}}{\text{Divisor}}$.

Example 2 : Express the following mixed fractions as improper fractions:

(a) $2\frac{3}{4}$ (b) $7\frac{1}{9}$ (c) $5\frac{3}{7}$

Solution : (a) $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2\times4}{4} + \frac{3}{4} = \frac{11}{4}$

(b) $7\frac{1}{9} = \frac{(7\times9)+1}{9} = \frac{64}{9}$

(c) $5\frac{3}{7} = \frac{(5\times7)+3}{7} = \frac{38}{7}$

Thus, we can express a mixed fraction as an improper fraction as

$\frac{(\text{Whole} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$.

EXERCISE 7.2

1. Draw number lines and locate the points on them :

(a) $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{4}{4}$ (b) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{7}{8}$ (c) $\frac{2}{5}, \frac{3}{5}, \frac{8}{5}, \frac{4}{5}$

2. Express the following as mixed fractions :

(a) $\frac{20}{3}$ (b) $\frac{11}{5}$ (c) $\frac{17}{7}$

(d) $\frac{28}{5}$ (e) $\frac{19}{6}$ (f) $\frac{35}{9}$

3. Express the following as improper fractions :

(a) $7\frac{3}{4}$ (b) $5\frac{6}{7}$ (c) $2\frac{5}{6}$ (d) $10\frac{3}{5}$ (e) $9\frac{3}{7}$ (f) $8\frac{4}{9}$

7.6 Equivalent Fractions

Look at all these representations of fraction (Fig 7.10).

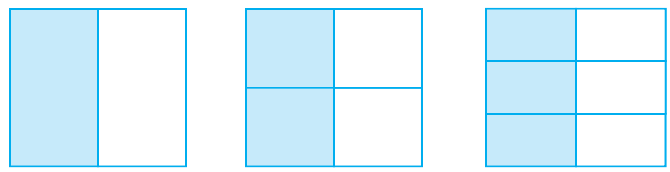
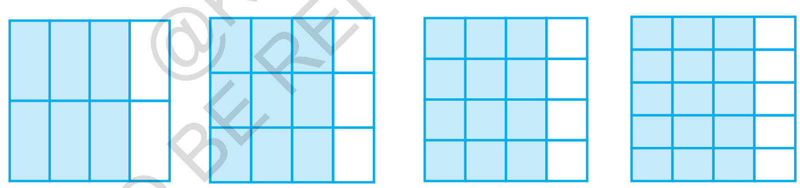


Fig 7.10

These fractions are $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, representing the parts taken from the total number of parts. If we place the pictorial representation of one over the other they are found to be equal. Do you agree?

Try These

1. Are $\frac{1}{3}$ and $\frac{2}{7}$; $\frac{2}{5}$ and $\frac{2}{7}$; $\frac{2}{9}$ and $\frac{6}{27}$ equivalent? Give reason.
2. Give example of four equivalent fractions.
3. Identify the fractions in each. Are these fractions equivalent?



These fractions are called **equivalent fractions**. Think of three more fractions that are equivalent to the above fractions.

Understanding equivalent fractions

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, ..., \frac{36}{72}, ...,$ are all equivalent fractions. They represent the same part of a whole.

Think, discuss and write

Why do the equivalent fractions represent the same part of a whole? How can we obtain one from the other?

We note $\frac{1}{2} = \frac{2}{4} = \frac{1 \times 2}{2 \times 2}$. Similarly, $\frac{1}{2} = \frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ and $\frac{1}{2} = \frac{4}{8} = \frac{1 \times 4}{2 \times 4}$

To find an equivalent fraction of a given fraction, you may multiply both the numerator and the denominator of the given fraction by the same number.

Rajni says that equivalent fractions of $\frac{1}{3}$ are :

$\frac{1\times 2}{3\times 2}=\frac{2}{6}$, $\frac{1\times 3}{3\times 3}=\frac{3}{9}$, $\frac{1\times 4}{3\times 4}=\frac{4}{12}$ and many more.

Do you agree with her? Explain.

Try These

1. Find five equivalent fractions of each of the following:

(i) $\frac{2}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{3}{5}$ (iv) $\frac{5}{9}$

Another way

Is there any other way to obtain equivalent fractions? Look at Fig 7.11.

$\frac{4}{6}$ is shaded here. $\frac{2}{3}$ is shaded here.

Fig 7.11

These include equal number of shaded things i.e. $\frac{4}{6}=\frac{2}{3}=\frac{4\div 2}{6\div 2}$

To find an equivalent fraction, we may divide both the numerator and the denominator by the same number.

One equivalent fraction of $\frac{12}{15}$ is $\frac{12\div 3}{15\div 3}=\frac{4}{5}$

Can you find an equivalent fraction of $\frac{9}{15}$ having denominator 5 ?

Example 3 : Find the equivalent fraction of $\frac{2}{5}$ with numerator 6.

Solution : We know 2 × 3 = 6. This means we need to multiply both the numerator and the denominator by 3 to get the equivalent fraction.

Hence, $\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$; $\frac{6}{15}$ is the required equivalent fraction.

Can you show this pictorially?

Example 4 : Find the equivalent fraction of $\frac{15}{35}$ with denominator 7.

Solution : We have $\frac{15}{35} = \frac{\square}{7}$

We observe the denominator and find 35 ÷ 5 = 7. We, therefore, divide both the numerator and the denominator of $\frac{15}{35}$ by 5.

Thus, $\frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}$.

An interesting fact

Let us now note an interesting fact about equivalent fractions. For this, complete the given table. The first two rows have already been completed for you.

Equivalent fractions	Product of the numerator of the 1st and the denominator of the 2nd	Product of the numerator of the 2nd and the denominator of the 1st	Are the products equal?
$\frac{1}{3} = \frac{3}{9}$	1 × 9 = 9	3 × 3 = 9	Yes
$\frac{4}{5} = \frac{28}{35}$	4 × 35 = 140	5 × 28 = 140	Yes
$\frac{1}{4} = \frac{4}{16}$			
$\frac{2}{3} = \frac{10}{15}$			
$\frac{3}{7} = \frac{24}{56}$			

What do we infer? The product of the numerator of the first and the denominator of the second is equal to the product of denominator of the first and the numerator of the second in all these cases. These two products are called cross products. Work out the cross products for other pairs of equivalent fractions. Do you find any pair of fractions for which cross products are not equal? This rule is helpful in finding equivalent fractions.

Example 5 : Find the equivalent fraction of $\frac{2}{9}$ with denominator 63.

Solution : We have $\frac{2}{9} = \frac{\square}{63}$

For this, we should have, $9 \times \square = 2 \times 63$.

But $63 = 7 \times 9$, so $9 \times \square = 2 \times 7 \times 9 = 14 \times 9 = 9 \times 14$

or $9 \times \square = 9 \times 14$

By comparison, $\square = 14$. Therefore, $\frac{2}{9} = \frac{14}{63}$.

7.7 Simplest Form of a Fraction

Given the fraction $\frac{36}{54}$, let us try to get an equivalent fraction in which the numerator and the denominator have no common factor except 1.

How do we do it? We see that both 36 and 54 are divisible by 2.

$$\frac{36}{54} = \frac{36 \div 2}{54 \div 2} = \frac{18}{27}$$

But 18 and 27 also have common factors other than one.

The common factors are 1, 3, 9; the highest is 9.

Therefore, $\frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3}$

Now 2 and 3 have no common factor except 1; we say that the fraction $\frac{2}{3}$ is in the simplest form.

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

The shortest way

The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF.

A Game

The equivalent fractions given here are quite interesting. Each one of them uses all the digits from 1 to 9 once!

$$\frac{2}{6} = \frac{3}{9} = \frac{58}{174}$$

$$\frac{2}{4} = \frac{3}{6} = \frac{79}{158}$$

Try to find two more such equivalent fractions.

Consider $\frac{36}{24}$.

The HCF of 36 and 24 is 12.

Therefore, $\frac{36}{24} = \frac{36 \div 12}{24 \div 12} = \frac{3}{2}$. The fraction $\frac{3}{2}$ is in the lowest form.

Thus, HCF helps us to reduce a fraction to its lowest form.

Try These

1. Write the simplest form of :

   (i) $\frac{15}{75}$ (ii) $\frac{16}{72}$

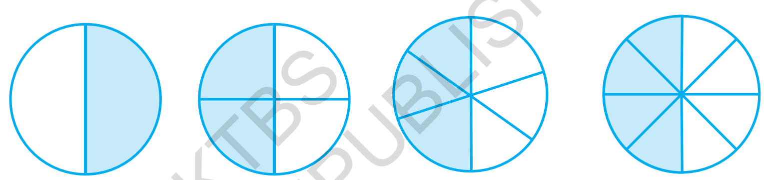
   (iii) $\frac{17}{51}$ (iv) $\frac{42}{28}$ (v) $\frac{80}{24}$

2. Is $\frac{49}{64}$ in its simplest form?

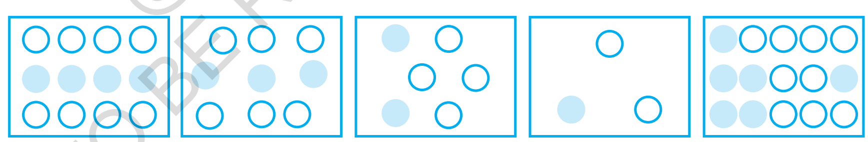
EXERCISE 7.3

1. Write the fractions. Are all these fractions equivalent?

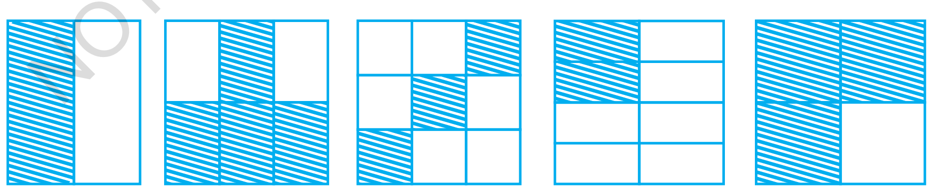
(a)



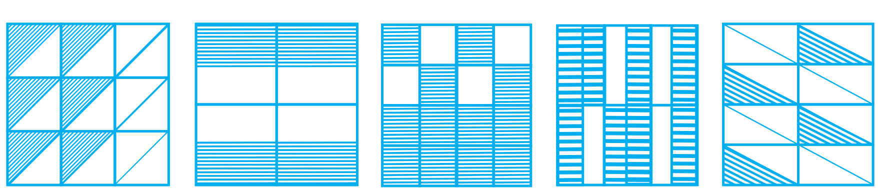
(b)



2. Write the fractions and pair up the equivalent fractions from each row.



(a) (b) (c) (d) (e)



(i) (ii) (iii) (iv) (v)

3. Replace $\square$ in each of the following by the correct number :

(a) $\frac{2}{7} = \frac{8}{\square}$ (b) $\frac{5}{8} = \frac{10}{\square}$ (c) $\frac{3}{5} = \frac{\square}{20}$ (d) $\frac{45}{60} = \frac{15}{\square}$ (e) $\frac{18}{24} = \frac{\square}{4}$

4. Find the equivalent fraction of $\frac{3}{5}$ having

(a) denominator 20 (b) numerator 9
(c) denominator 30 (d) numerator 27

5. Find the equivalent fraction of $\frac{36}{48}$ with

(a) numerator 9 (b) denominator 4

6. Check whether the given fractions are equivalent :

(a) $\frac{5}{9}, \frac{30}{54}$ (b) $\frac{3}{10}, \frac{12}{50}$ (c) $\frac{7}{13}, \frac{5}{11}$

7. Reduce the following fractions to simplest form :

(a) $\frac{48}{60}$ (b) $\frac{150}{60}$ (c) $\frac{84}{98}$ (d) $\frac{12}{52}$ (e) $\frac{7}{28}$

8. Ramesh had 20 pencils, Sheelu had 50 pencils and Jamaal had 80 pencils. After 4 months, Ramesh used up 10 pencils, Sheelu used up 25 pencils and Jamaal used up 40 pencils. What fraction did each use up? Check if each has used up an equal fraction of her/his pencils?

9. Match the equivalent fractions and write two more for each.

(i) $\frac{250}{400}$	(a) $\frac{2}{3}$	(iv) $\frac{180}{360}$	(d) $\frac{5}{8}$
(ii) $\frac{180}{200}$	(b) $\frac{2}{5}$	(v) $\frac{220}{550}$	(e) $\frac{9}{10}$
(iii) $\frac{660}{990}$	(c) $\frac{1}{2}$		

7.8 Like Fractions

Fractions with same denominators are called **like fractions**.

Thus, $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{8}{15}$ are all like fractions. Are $\frac{7}{27}$ and $\frac{7}{28}$ like fractions?

Their denominators are different. Therefore, they are not like fractions. They are called **unlike fractions**.

Write five pairs of like fractions and five pairs of unlike fractions.

7.9 Comparing Fractions

Sohni has $3\frac{1}{2}$ rotis in her plate and Rita has $2\frac{3}{4}$ rotis in her plate. Who has more rotis in her plate? Clearly, Sohni has 3 full rotis and more and Rita has less than 3 rotis. So, Sohni has more rotis.

Consider $\frac{1}{2}$ and $\frac{1}{3}$ as shown in Fig. 7.12. The portion of the whole corresponding to $\frac{1}{2}$ is clearly larger than the portion of the same whole corresponding to $\frac{1}{3}$.

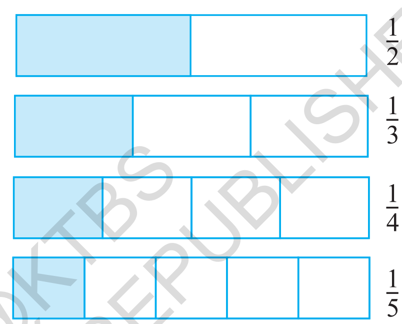


Fig 7.12

So $\frac{1}{2}$ is greater than $\frac{1}{3}$.

Try These

1. You get one-fifth of a bottle of juice and your sister gets one-third of the same size of a bottle of juice. Who gets more?

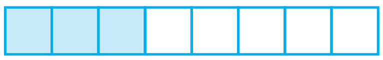
But often it is not easy to say which one out of a pair of fractions is larger. For example, which is greater, $\frac{1}{4}$ or $\frac{3}{10}$? For this, we may wish to show the fractions using figures (as in fig. 7.12), but drawing figures may not be easy especially with denominators like 13. We should therefore like to have a systematic procedure to compare fractions. It is particularly easy to compare like fractions. We do this first.

7.9.1 Comparing like fractions

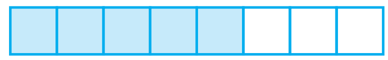
Like fractions are fractions with the same denominator. Which of these are like fractions?

$\frac{2}{5}, \frac{3}{4}, \frac{1}{5}, \frac{7}{2}, \frac{3}{5}, \frac{4}{5}, \frac{4}{7}$

Let us compare two like fractions: $\frac{3}{8}$ and $\frac{5}{8}$.



$\frac{3}{8}$



$\frac{5}{8}$

In both the fractions the whole is divided into 8 equal parts. For $\frac{3}{8}$ and $\frac{5}{8}$, we take 3 and 5 parts respectively out of the 8 equal parts. Clearly, out of 8 equal parts, the portion corresponding to 5 parts is larger than the portion corresponding to 3 parts. Hence, $\frac{5}{8} > \frac{3}{8}$. Note the number of the parts taken is given by the numerator. It is, therefore, clear that for two fractions with the same denominator, the fraction with the greater numerator is greater. Between $\frac{4}{5}$ and $\frac{3}{5}$, $\frac{4}{5}$ is greater. Between $\frac{11}{20}$ and $\frac{13}{20}$, $\frac{13}{20}$ is greater and so on.

Try These

1. Which is the larger fraction?

   (i) $\frac{7}{10}$ or $\frac{8}{10}$ (ii) $\frac{11}{24}$ or $\frac{13}{24}$ (iii) $\frac{17}{102}$ or $\frac{12}{102}$

   Why are these comparisons easy to make?

2. Write these in ascending and also in descending order.

   (a) $\frac{1}{8}, \frac{5}{8}, \frac{3}{8}$ (b) $\frac{1}{5}, \frac{11}{5}, \frac{4}{5}, \frac{3}{5}, \frac{7}{5}$ (c) $\frac{1}{7}, \frac{3}{7}, \frac{13}{7}, \frac{11}{7}, \frac{7}{7}$

7.9.2 Comparing unlike fractions

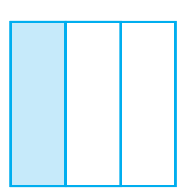
Two fractions are unlike if they have different denominators. For example, $\frac{1}{3}$ and $\frac{1}{5}$ are unlike fractions. So are $\frac{2}{3}$ and $\frac{3}{5}$.

Unlike fractions with the same numerator :

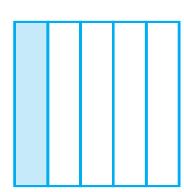
Consider a pair of unlike fractions $\frac{1}{3}$ and $\frac{1}{5}$, in which the numerator is the same.

Which is greater $\frac{1}{3}$ or $\frac{1}{5}$?

$\frac{1}{3}$



$\frac{1}{5}$



In $\frac{1}{3}$, we divide the whole into 3 equal parts and take one. In $\frac{1}{5}$, we divide the whole into 5 equal parts and take one. Note that in $\frac{1}{3}$, the whole is divided into a smaller number of parts than in $\frac{1}{5}$. The equal part that we get in $\frac{1}{3}$ is, therefore, larger than the equal part we get in $\frac{1}{5}$. Since in both cases we take the same number of parts (i.e. one), the portion of the whole showing $\frac{1}{3}$ is larger than the portion showing $\frac{1}{5}$, and therefore $\frac{1}{3} > \frac{1}{5}$.

In the same way we can say $\frac{2}{3} > \frac{2}{5}$. In this case, the situation is the same as in the case above, except that the common numerator is 2, not 1. The whole is divided into a large number of equal parts for $\frac{2}{5}$ than for $\frac{2}{3}$. Therefore, each equal part of the whole in case of $\frac{2}{3}$ is larger than that in case of $\frac{2}{5}$. Therefore, the portion of the whole showing $\frac{2}{3}$ is larger than the portion showing $\frac{2}{5}$ and hence, $\frac{2}{3} > \frac{2}{5}$.

We can see from the above example that **if the numerator is the same in two fractions, the fraction with the smaller denominator is greater of the two**.

Thus, $\frac{1}{8} > \frac{1}{10}$, $\frac{3}{5} > \frac{3}{7}$, $\frac{4}{9} > \frac{4}{11}$ and so on.

Let us arrange $\frac{2}{1}, \frac{2}{13}, \frac{2}{9}, \frac{2}{5}, \frac{2}{7}$ in increasing order. All these fractions are unlike, but their numerator is the same. Hence, in such case, the larger the denominator, the smaller is the fraction. The smallest is $\frac{2}{13}$, as it has the largest denominator. The next three fractions in order are $\frac{2}{9}, \frac{2}{7}, \frac{2}{5}$. The greatest fraction is $\frac{2}{1}$ (It is with the smallest denominator). The arrangement in increasing order, therefore, is $\frac{2}{13}, \frac{2}{9}, \frac{2}{7}, \frac{2}{5}, \frac{2}{1}$.

Try These

1. Arrange the following in ascending and descending order :

   (a) $\frac{1}{12}, \frac{1}{23}, \frac{1}{5}, \frac{1}{7}, \frac{1}{50}, \frac{1}{9}, \frac{1}{17}$

   (b) $\frac{3}{7}, \frac{3}{11}, \frac{3}{5}, \frac{3}{2}, \frac{3}{13}, \frac{3}{4}, \frac{3}{17}$

   (c) Write 3 more similar examples and arrange them in ascending and descending order.

Suppose we want to compare $\frac{2}{3}$ and $\frac{3}{4}$. Their numerators are different and so are their denominators. We know how to compare like fractions, i.e. fractions with the same denominator. We should, therefore, try to change the denominators of the given fractions, so that they become equal. For this purpose, we can use the method of equivalent fractions which we already know. Using this method we can change the denominator of a fraction without changing its value.

Let us find equivalent fractions of both $\frac{2}{3}$ and $\frac{3}{4}$.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = ...$ Similarly, $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = ...$

The equivalent fractions of $\frac{2}{3}$ and $\frac{3}{4}$ with the same denominator 12 are $\frac{8}{12}$ and $\frac{9}{12}$ repectively.

i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$. Since, $\frac{9}{12} > \frac{8}{12}$ we have, $\frac{3}{4} > \frac{2}{3}$.

Example 6 : Compare $\frac{4}{5}$ and $\frac{5}{6}$.

Solution : The fractions are unlike fractions. Their numerators are different too. Let us write their equivalent fractions.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = ...$$

and $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = ...$

The equivalent fractions with the same denominator are :

$$\frac{4}{5}=\frac{24}{30} \text{ and } \frac{5}{6}=\frac{25}{30}$$

Since, $\frac{25}{30}>\frac{24}{30}$ so, $\frac{5}{6}>\frac{4}{5}$

Note that the common denominator of the equivalent fractions is 30 which is 5×6. It is a common multiple of both 5 and 6.

So, when we compare two unlike fractions, we first get their equivalent fractions with a denominator which is a common multiple of the denominators of both the fractions.

Example 7 : Compare $\frac{5}{6}$ and $\frac{13}{15}$.

Solution : The fractions are unlike. We should first get their equivalent fractions with a denominator which is a common multiple of 6 and 15.

Now, $\frac{5\times 5}{6\times 5}=\frac{25}{30}, \frac{13\times 2}{15\times 2}=\frac{26}{30}$

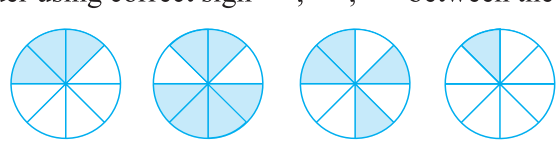
Since $\frac{26}{30}>\frac{25}{30}$ we have $\frac{13}{15}>\frac{5}{6}$.

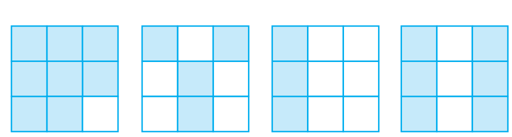
Why LCM?

The product of 6 and 15 is 90; obviously 90 is also a common multiple of 6 and 15. We may use 90 instead of 30; it will not be wrong. But we know that it is easier and more convenient to work with smaller numbers. So the common multiple that we take is as small as possible. This is why the LCM of the denominators of the fractions is preferred as the common denominator.

EXERCISE 7.4

1. Write shaded portion as fraction. Arrange them in ascending and descending order using correct sign '<', '=', '>' between the fractions:

(a) 

(b) 

(c) Show $\frac{2}{6}$, $\frac{4}{6}$, $\frac{8}{6}$ and $\frac{6}{6}$ on the number line. Put appropriate signs between the fractions given.

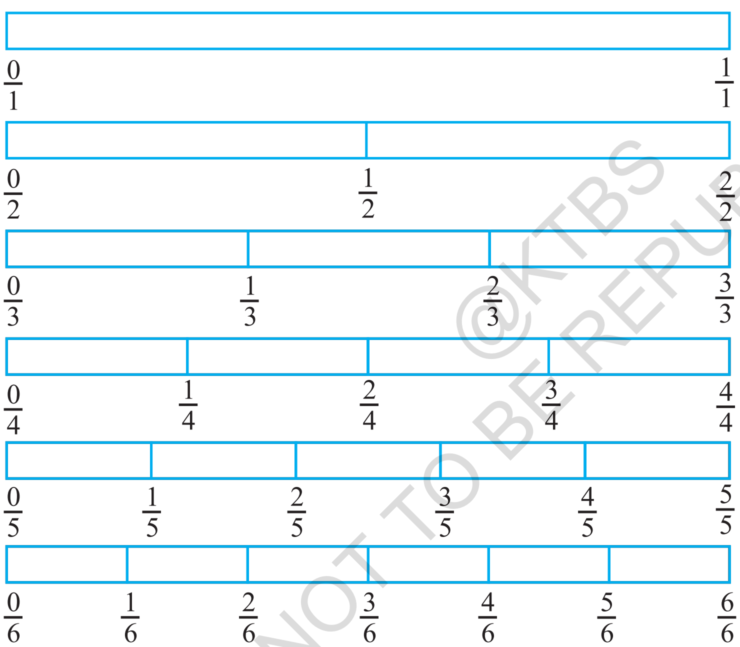
$\frac{5}{6} \square \frac{2}{6}$, $\quad \frac{3}{6} \square 0$, $\quad \frac{1}{6} \square \frac{6}{6}$, $\quad \frac{8}{6} \square \frac{5}{6}$

2. Compare the fractions and put an appropriate sign.

(a) $\frac{3}{6} \square \frac{5}{6}$ (b) $\frac{1}{7} \square \frac{1}{4}$ (c) $\frac{4}{5} \square \frac{5}{5}$ (d) $\frac{3}{5} \square \frac{3}{7}$

3. Make five more such pairs and put appropriate signs.

4. Look at the figures and write '<' or '>', '=' between the given pairs of fractions.



(a) $\frac{1}{6} \square \frac{1}{3}$ (b) $\frac{3}{4} \square \frac{2}{6}$ (c) $\frac{2}{3} \square \frac{2}{4}$ (d) $\frac{6}{6} \square \frac{3}{3}$ (e) $\frac{5}{6} \square \frac{5}{5}$

Make five more such problems and solve them with your friends.

5. How quickly can you do this? Fill appropriate sign. ('<', '=', '>')

(a) $\frac{1}{2} \square \frac{1}{5}$ (b) $\frac{2}{4} \square \frac{3}{6}$ (c) $\frac{3}{5} \square \frac{2}{3}$

(d) $\frac{3}{4} \square \frac{2}{8}$ (e) $\frac{3}{5} \square \frac{6}{5}$ (f) $\frac{7}{9} \square \frac{3}{9}$

(g) $\frac{1}{4} \square \frac{2}{8}$ (h) $\frac{7}{9} \square \frac{3}{9}$ (i) $\frac{3}{4} \square \frac{2}{8}$

(j) $\frac{6}{10} \square \frac{3}{5}$ (k) $\frac{5}{7} \square \frac{15}{21}$

6. The following fractions represent just three different numbers. Separate them into three groups of equivalent fractions, by changing each one to its simplest form.

(a) $\frac{2}{12}$ (b) $\frac{3}{15}$ (c) $\frac{8}{50}$ (d) $\frac{16}{100}$ (e) $\frac{10}{60}$ (f) $\frac{15}{75}$

(g) $\frac{12}{60}$ (h) $\frac{16}{96}$ (i) $\frac{12}{75}$ (j) $\frac{12}{72}$ (k) $\frac{3}{18}$ (l) $\frac{4}{25}$

7. Find answers to the following. Write and indicate how you solved them.

(a) Is $\frac{5}{9}$ equal to $\frac{4}{5}$? (b) Is $\frac{9}{16}$ equal to $\frac{5}{9}$?

(c) Is $\frac{4}{5}$ equal to $\frac{16}{20}$? (d) Is $\frac{1}{15}$ equal to $\frac{4}{30}$?

8. Ila read 25 pages of a book containing 100 pages. Lalita read $\frac{2}{5}$ of the same book. Who read less?

9. Rafiq exercised for $\frac{3}{6}$ of an hour, while Rohit exercised for $\frac{3}{4}$ of an hour. Who exercised for a longer time?

10. In a class A of 25 students, 20 passed with 60% or more marks; in another class B of 30 students, 24 passed with 60% or more marks. In which class was a greater fraction of students getting with 60% or more marks?

7.10 Addition and Subtraction of Fractions

So far in our study we have learnt about natural numbers, whole numbers and then integers. In the present chapter, we are learning about fractions, a different type of numbers.

Whenever we come across new type of numbers, we want to know how to operate with them. Can we combine and add them? If so, how? Can we take away some number from another? i.e., can we subtract one from the other? and so on. Which of the properties learnt earlier about the numbers hold now? Which are the new properties? We also see how these help us deal with our daily life situations.

Try These

1. My mother divided an apple into 4 equal parts. She gave me two parts and my brother one part. How much apple did she give to both of us together?
2. Mother asked Neelu and her brother to pick stones from the wheat. Neelu picked one fourth of the total stones in it and her brother also picked up one fourth of the stones. What fraction of the stones did both pick up together?
3. Sohan was putting covers on his note books. He put one fourth of the covers on Monday. He put another one fourth on Tuesday and the remaining on Wednesday. What fraction of the covers did he put on Wednesday?

Look at the following examples: A tea stall owner consumes in her shop $2\frac{1}{2}$ litres of milk in the morning and $1\frac{1}{2}$ litres of milk in the evening in preparing tea. What is the total amount of milk she uses in the stall?

Or Shekhar ate 2 chapatis for lunch and $1\frac{1}{2}$ chapatis for dinner. What is the total number of chapatis he ate?

Clearly, both the situations require the fractions to be added. Some of these additions can be done orally and the sum can be found quite easily.

Do This

Make five such problems with your friends and solve them.

7.10.1 Adding or subtracting like fractions

All fractions cannot be added orally. We need to know how they can be added in different situations and learn the procedure for it. We begin by looking at addition of like fractions.

Take a 7 × 4 grid sheet (Fig 7.13). The sheet has seven boxes in each row and four boxes in each column.

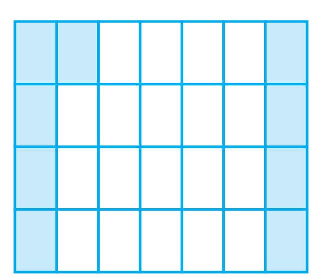


Fig 7.13

How many boxes are there in total?

Colour five of its boxes in green.

What fraction of the whole is the green region?

Now colour another four of its boxes in yellow.

What fraction of the whole is this yellow region?

What fraction of the whole is coloured altogether?

Does this explain that $\frac{5}{28}+\frac{4}{28}=\frac{9}{28}$?

Look at more examples

In Fig 7.14 (i) we have 2 quarter parts of the figure shaded. This means we have 2 parts out of 4 shaded or $\frac{1}{2}$ of the figure shaded.

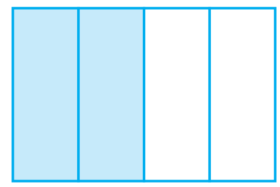


Fig. 7.14 (i)

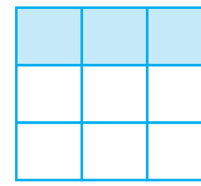


Fig. 7.14 (ii)

That is, $\frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{2}{4}=\frac{1}{2}$.

Look at Fig 7.14 (ii)

Fig 7.14 (ii) demonstrates $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1+1+1}{9}=\frac{3}{9}=\frac{1}{3}$.

What do we learn from the above examples? The sum of two or more like fractions can be obtained as follows :

Step 1 Add the numerators.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\dfrac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Try These

1. Add with the help of a diagram.
   (i) $\frac{1}{8}+\frac{1}{8}$ (ii) $\frac{2}{5}+\frac{3}{5}$ (iii) $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$
2. Add $\frac{1}{12}+\frac{1}{12}$. How will we show this pictorially? Using paper folding?
3. Make 5 more examples of problems given in 1 and 2 above.
   Solve them with your friends.

Let us, thus, add $\frac{3}{5}$ and $\frac{1}{5}$.

We have $\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}$

So, what will be the sum of $\frac{7}{12}$ and $\frac{3}{12}$?

Finding the balance

Sharmila had $\frac{5}{6}$ of a cake. She gave $\frac{2}{6}$ out of that to her younger brother. How much cake is left with her?

A diagram can explain the situation (Fig 7.15). (Note that, here the given fractions are like fractions).

We find that $\frac{5}{6}-\frac{2}{6}=\frac{5-2}{6}=\frac{3}{6}$ or $\frac{1}{2}$

(Is this not similar to the method of adding like fractions?)

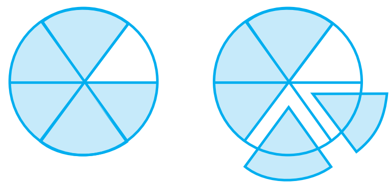


Fig 7.15

Thus, we can say that the difference of two like fractions can be obtained as follows:

Step 1 Subtract the smaller numerator from the bigger numerator.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\dfrac{\text{Result of Step 1}}{\text{Result of Step 2}}$

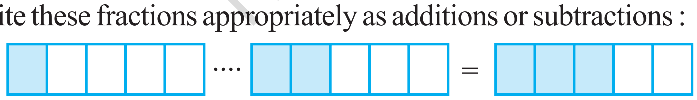
Can we now subtract $\dfrac{3}{10}$ from $\dfrac{8}{10}$?

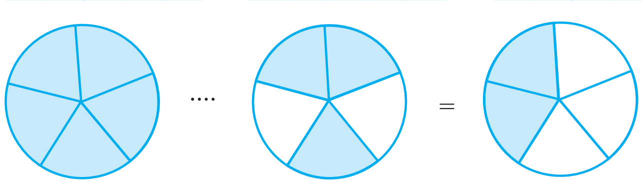
Try These

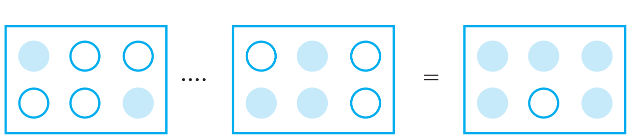
1. Find the difference between $\dfrac{7}{8}$ and $\dfrac{3}{8}$.
2. Mother made a gud patti in a round shape. She divided it into 5 parts. Seema ate one piece from it. If I eat another piece then how much would be left?
3. My elder sister divided the watermelon into 16 parts. I ate 7 out them. My friend ate 4. How much did we eat between us? How much more of the watermelon did I eat than my friend? What portion of the watermelon remained?
4. Make five problems of this type and solve them with your friends.

EXERCISE 7.5

1. Write these fractions appropriately as additions or subtractions :

(a) 

(b) 

(c) 

2. Solve :

(a) $\frac{1}{18}+\frac{1}{18}$ (b) $\frac{8}{15}+\frac{3}{15}$ (c) $\frac{7}{7}-\frac{5}{7}$ (d) $\frac{1}{22}+\frac{21}{22}$ (e) $\frac{12}{15}-\frac{7}{15}$

(f) $\frac{5}{8}+\frac{3}{8}$ (g) $1-\frac{2}{3}\left(1=\frac{3}{3}\right)$ (h) $\frac{1}{4}+\frac{0}{4}$ (i) $3-\frac{12}{5}$

3. Shubham painted $\frac{2}{3}$ of the wall space in his room. His sister Madhavi helped and painted $\frac{1}{3}$ of the wall space. How much did they paint together?

4. Fill in the missing fractions.

(a) $\frac{7}{10}-\square=\frac{3}{10}$ (b) $\square-\frac{3}{21}=\frac{5}{21}$ (c) $\square-\frac{3}{6}=\frac{3}{6}$ (d) $\square+\frac{5}{27}=\frac{12}{27}$

5. Javed was given $\frac{5}{7}$ of a basket of oranges. What fraction of oranges was left in the basket?

7.10.2 Adding and subtracting fractions

We have learnt to add and subtract like fractions. It is also not very difficult to add fractions that do not have the same denominator. When we have to add or subtract fractions we first find equivalent fractions with the same denominator and then proceed.

What added to $\frac{1}{5}$ gives $\frac{1}{2}$? This means subtract $\frac{1}{5}$ from $\frac{1}{2}$ to get the required number.

Since $\frac{1}{5}$ and $\frac{1}{2}$ are unlike fractions, in order to subtract them, we first find their equivalent fractions with the same denominator. These are $\frac{2}{10}$ and $\frac{5}{10}$ respectively.

This is because $\frac{1}{2}=\frac{1\times 5}{2\times 5}=\frac{5}{10}$ and $\frac{1}{5}=\frac{1\times 2}{5\times 2}=\frac{2}{10}$

Therefore, $\frac{1}{2}-\frac{1}{5}=\frac{5}{10}-\frac{2}{10}=\frac{5-2}{10}=\frac{3}{10}$

Note that 10 is the least common multiple (LCM) of 2 and 5.

Example 8 : Subtract $\frac{3}{4}$ from $\frac{5}{6}$.

Solution : We need to find equivalent fractions of $\frac{3}{4}$ and $\frac{5}{6}$, which have the same denominator. This denominator is given by the LCM of 4 and 6. The required LCM is 12.

Therefore, $\frac{5}{6}-\frac{3}{4}=\frac{5\times2}{6\times2}-\frac{3\times3}{4\times3}=\frac{10}{12}-\frac{9}{12}=\frac{1}{12}$

Example 9 : Add $\frac{2}{5}$ to $\frac{1}{3}$.

Solution : The LCM of 5 and 3 is 15.

Therefore, $\frac{2}{5}+\frac{1}{3}=\frac{2\times3}{5\times3}+\frac{1\times5}{3\times5}=\frac{6}{15}+\frac{5}{15}=\frac{11}{15}$

Example 10 : Simplify $\frac{3}{5}-\frac{7}{20}$

Solution : The LCM of 5 and 20 is 20.

Therefore, $\frac{3}{5}-\frac{7}{20}=\frac{3\times4}{5\times4}-\frac{7}{20}=\frac{12}{20}-\frac{7}{20}$

$=\frac{12-7}{20}=\frac{5}{20}=\frac{1}{4}$

Try These

1. Add $\frac{2}{5}$ and $\frac{3}{7}$.
2. Subtract $\frac{2}{5}$ from $\frac{5}{7}$.

How do we add or subtract mixed fractions?

Mixed fractions can be written either as a whole part plus a proper fraction or entirely as an improper fraction. One way to add (or subtract) mixed fractions is to do the operation seperately for the whole parts and the other way is to write the mixed fractions as improper fractions and then directly add (or subtract) them.

Example 11 : Add $2\frac{4}{5}$ and $3\frac{5}{6}$

Solution : $2\frac{4}{5}+3\frac{5}{6}=2+\frac{4}{5}+3+\frac{5}{6}=5+\frac{4}{5}+\frac{5}{6}$

Now $\frac{4}{5}+\frac{5}{6}=\frac{4\times6}{5\times6}+\frac{5\times5}{6\times5}$ (Since LCM of 5 and 6 = 30)

$=\frac{24}{30}+\frac{25}{30}=\frac{49}{30}=\frac{30+19}{30}=1+\frac{19}{30}$

Thus, $5+\frac{4}{5}+\frac{5}{6}=5+1+\frac{19}{30}=6+\frac{19}{30}=6\frac{19}{30}$

And, therefore, $2\frac{4}{5}+3\frac{5}{6}=6\frac{19}{30}$

Think, discuss and write

Can you find the other way of doing this sum?

Example 12 : Find $4\frac{2}{5} - 2\frac{1}{5}$

Solution : The whole numbers 4 and 2 and the fractional numbers $\frac{2}{5}$ and $\frac{1}{5}$ can be subtracted separately. (Note that $4 > 2$ and $\frac{2}{5} > \frac{1}{5}$)

$$\text{So, } 4\frac{2}{5} - 2\frac{1}{5} = (4-2) + \left(\frac{2}{5} - \frac{1}{5}\right) = 2 + \frac{1}{5} = 2\frac{1}{5}$$

Example 13 : Simplify: $8\frac{1}{4} - 2\frac{5}{6}$

Solution : Here $8 > 2$ but $\frac{1}{4} < \frac{5}{6}$. We proceed as follows:

$$8\frac{1}{4} = \frac{(8\times4)+1}{4} = \frac{33}{4} \text{ and } 2\frac{5}{6} = \frac{2\times6+5}{6} = \frac{17}{6}$$

$$\text{Now, } \frac{33}{4} - \frac{17}{6} = \frac{33\times3}{12} - \frac{17\times2}{12} \quad \text{(Since LCM of 4 and 6 = 12)}$$

$$= \frac{99-34}{12} = \frac{65}{12} = 5\frac{5}{12}$$

EXERCISE 7.6

1. Solve

(a) $\frac{2}{3}+\frac{1}{7}$ (b) $\frac{3}{10}+\frac{7}{15}$ (c) $\frac{4}{9}+\frac{2}{7}$ (d) $\frac{5}{7}+\frac{1}{3}$ (e) $\frac{2}{5}+\frac{1}{6}$

(f) $\frac{4}{5}+\frac{2}{3}$ (g) $\frac{3}{4}-\frac{1}{3}$ (h) $\frac{5}{6}-\frac{1}{3}$ (i) $\frac{2}{3}+\frac{3}{4}+\frac{1}{2}$ (j) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$

(k) $1\frac{1}{3}+3\frac{2}{3}$ (l) $4\frac{2}{3}+3\frac{1}{4}$ (m) $\frac{16}{5}-\frac{7}{5}$ (n) $\frac{4}{3}-\frac{1}{2}$

2. Sarita bought $\frac{2}{5}$ metre of ribbon and Lalita $\frac{3}{4}$ metre of ribbon. What is the total length of the ribbon they bought?

3. Naina was given $1\frac{1}{2}$ piece of cake and Najma was given $1\frac{1}{3}$ piece of cake. Find the total amount of cake was given to both of them.

4. Fill in the boxes : (a) $\square - \frac{5}{8} = \frac{1}{4}$ (b) $\square - \frac{1}{5} = \frac{1}{2}$ (c) $\frac{1}{2} - \square = \frac{1}{6}$

5. Complete the addition-subtraction box.

(a)

$\frac{2}{3}$	$\frac{4}{3}$	
$\frac{1}{3}$	$\frac{2}{3}$	

(b)

$\frac{1}{2}$	$\frac{1}{3}$	
$\frac{1}{3}$	$\frac{1}{4}$	

6. A piece of wire $\frac{7}{8}$ metre long broke into two pieces. One piece was $\frac{1}{4}$ metre long. How long is the other piece?

7. Nandini's house is $\frac{9}{10}$ km from her school. She walked some distance and then took a bus for $\frac{1}{2}$ km to reach the school. How far did she walk?

8. Asha and Samuel have bookshelves of the same size partly filled with books. Asha's shelf is $\frac{5}{6}$th full and Samuel's shelf is $\frac{2}{5}$th full. Whose bookshelf is more full? By what fraction?

9. Jaidev takes $2\frac{1}{5}$ minutes to walk across the school ground. Rahul takes $\frac{7}{4}$ minutes to do the same. Who takes less time and by what fraction?

What have we discussed?

1. (a) A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects.

   (b) When expressing a situation of counting parts to write a fraction, it must be ensured that all parts are equal.

2. In $\frac{5}{7}$, 5 is called the numerator and 7 is called the denominator.

3. Fractions can be shown on a number line. Every fraction has a point associated with it on the number line.

4. In a proper fraction, the numerator is less than the denominator. The fractions, where the numerator is greater than the denominator are called improper fractions. An improper fraction can be written as a combination of a whole and a part, and such fraction then called mixed fractions.

5. Each proper or improper fraction has many equivalent fractions. To find an equivalent fraction of a given fraction, we may multiply or divide both the numerator and the denominator of the given fraction by the same number.

6. A fraction is said to be in the simplest (or lowest) form if its numerator and the denominator have no common factor except 1.

Decimals

0650CH08

Chapter 8

8.1 Introduction

Savita and Shama were going to market to buy some stationary items. Savita said, “I have 5 rupees and 75 paise”. Shama said, “I have 7 rupees and 50 paise”.

They knew how to write rupees and paise using decimals.

So Savita said, I have ₹ 5.75 and Shama said, “I have ₹ 7.50”.

Have they written correctly?

We know that the dot represents a decimal point.

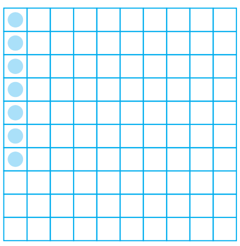
In this chapter, we will learn more about working with decimals.

8.2 Comparing Decimals

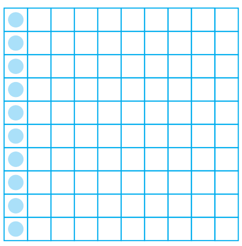
Can you tell which is greater, 0.07 or 0.1?

Take two pieces of square papers of the same size. Divide them into 100 equal parts. For 0.07 we have to shade 7 parts out of 100.

Now, $0.1 = \frac{1}{10} = \frac{10}{100}$, so, for 0.1, shade 10 parts out 100.



$0.07 = \frac{7}{100}$



$0.1 = \frac{1}{10} = \frac{10}{100}$

This means 0.1>0.07

Let us now compare the numbers 32.55 and 32.5. In this case , we first compare the whole part. We see that the whole part for both the nunbers is 32 and, hence, equal.

We, however, know that the two numbers are not equal. So, we now compare the tenth part. We find that for 32.55 and 32.5, the tenth part is also equal, then we compare the hundredth part.

We find,

$32.55 = 32 + \frac{5}{10} + \frac{5}{100}$ and $32.5 = 32 + \frac{5}{10} + \frac{0}{100}$, therefore, 32.55>32.5 as the hundredth part of 32.55 is more.

Example 1 : Which is greater?

(a) 1 or 0.99 (b) 1.09 or 1.093

Solution : (a) $1 = 1 + \frac{0}{10} + \frac{0}{100}$; $0.99 = 0 + \frac{9}{10} + \frac{9}{100}$

The whole part of 1 is greater than that of 0.99.

Therefore, 1 > 0.99

(b) $1.09 = 1 + \frac{0}{10} + \frac{9}{100} + \frac{0}{1000}$; $1.093 = 1 + \frac{0}{10} + \frac{9}{100} + \frac{3}{1000}$

In this case, the two numbers have same parts upto hundredth.

But the thousandths part of 1.093 is greater than that of 1.09.

Therefore, 1.093 > 1.09.

EXERCISE 8.1

1. Which is greater?

(a) 0.3 or 0.4 (b) 0.07 or 0.02 (c) 3 or 0.8 (d) 0.5 or 0.05

(e) 1.23 or 1.2 (f) 0.099 or 0.19 (g) 1.5 or 1.50 (h) 1.431 or 1.490

(i) 3.3 or 3.300 (j) 5.64 or 5.603

2. Make five more examples and find the greater number from them.

Try These

(i) Write 2 rupees 5 paise and 2 rupees 50 paise in decimals.

(ii) Write 20 rupees 7 paise and 21 rupees 75 paise in decimals?

8.3 Using Decimals

8.3.1 Money

We know that 100 paise = ₹ 1

Therefore, 1 paise = ₹ $\frac{1}{100}$ = ₹ 0.01

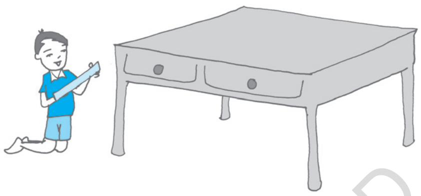
So, 65 paise $= ₹\ \dfrac{65}{100} = ₹\ 0.65$

and 5 paise $= ₹\ \dfrac{5}{100} = ₹\ 0.05$

What is 105 paise? It is ₹ 1 and 5 paise = ₹ 1.05

8.3.2 Length

Mahesh wanted to measure the length of his table top in metres. He had a 50 cm scale. He found that the length of the table top was 156 cm. What will be its length in metres?



Mahesh knew that

$1 \text{ cm} = \dfrac{1}{100} \text{ m} \quad \text{or} \quad 0.01 \text{ m}$

Therefore, $56 \text{ cm} = \dfrac{56}{100} \text{ m} = 0.56 \text{ m}$

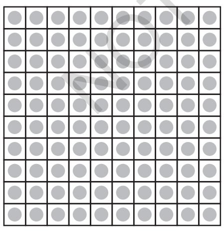
Thus, the length of the table top is 156 cm = 100 cm + 56 cm

$= 1 \text{ m} + \dfrac{56}{100} \text{ m} = 1.56 \text{ m}.$

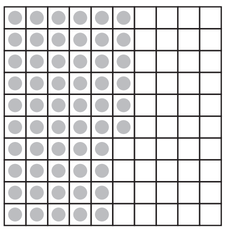
Try These

1. Can you write 4 mm in 'cm' using decimals?
2. How will you write 7cm 5 mm in 'cm' using decimals?
3. Can you now write 52 m as 'km' using decimals? How will you write 340 m as 'km' using decimals? How will you write 2008 m in 'km'?

Mahesh also wants to represent this length pictorially. He took squared papers of equal size and divided them into 100 equal parts. He considered each small square as one cm.



100 cm



56 cm

8.3.3 Weight

Nandu bought 500g potatoes, 250g capsicum, 700g onions, 500g tomatoes, 100g ginger and 300g radish. What is the total weight of the vegetables in the bag? Let us add the weight of all the vegetables in the bag.

500 g + 250 g + 700 g + 500 g + 100 g + 300 g

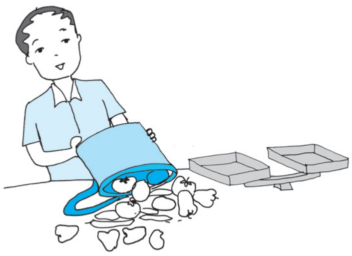
= 2350 g

Try These

1. Can you now write 456g as 'kg' using decimals?
2. How will you write 2kg 9g in 'kg' using decimals?

We know that 1000 g = 1 kg

Therefore, 1 g $= \frac{1}{1000}$ kg $= 0.001$ kg



Thus, 2350 g $= 2000 \text{ g} + 350 \text{ g}$

$$= \frac{2000}{1000} \text{ kg} + \frac{350}{1000} \text{ kg}$$

$$= 2 \text{ kg} + 0.350 \text{ kg} = 2.350 \text{ kg}$$

i.e. 2350 g $= 2 \text{ kg } 350 \text{ g} = 2.350 \text{ kg}$

Thus, the weight of vegetables in Nandu's bag is 2.350 kg.

EXERCISE 8.2

1. Express as rupees using decimals.
   (a) 5 paise (b) 75 paise (c) 20 paise
   (d) 50 rupees 90 paise (e) 725 paise
2. Express as metres using decimals.
   (a) 15 cm (b) 6 cm (c) 2 m 45 cm
   (d) 9 m 7 cm (e) 419 cm
3. Express as cm using decimals.
   (a) 5 mm (b) 60 mm (c) 164 mm
   (d) 9 cm 8 mm (e) 93 mm
4. Express as km using decimals.
   (a) 8 m (b) 88 m (c) 8888 m
   (d) 70 km 5 m
5. Express as kg using decimals.
   (a) 2 g (b) 100 g (c) 3750 g
   (d) 5 kg 8 g (e) 26 kg 50 g

8.4 Addition of Numbers with Decimals

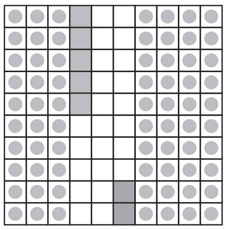
Do This

Add 0.35 and 0.42.

Take a square and divide it into 100 equal parts.

Mark 0.35 in this square by shading 3 tenths and colouring 5 hundredths.

Mark 0.42 in this square by shading 4 tenths and colouring 2 hundredths.



Now count the total number of tenths in the square and the total number of hundredths in the square.

	Ones	Tenths	Hundredths
	0	3	5
+	0	4	2
	0	7	7

Therefore, 0.35 + 0.42 = 0.77

Thus, we can add decimals in the same way as whole numbers.

Can you now add 0.68 and 0.54?

Try These

Find

(i) 0.29 + 0.36 (ii) 0.7 + 0.08

(iii) 1.54 + 1.80 (iv) 2.66 + 1.85

	Ones	Tenths	Hundredths
	0	6	8
+	0	5	4
	1	2	2

Thus, 0.68 + 0.54 = 1.22

Example 2 : Lata spent ₹ 9.50 for buying a pen and ₹ 2.50 for one pencil. How much money did she spend?

Solution : Money spent for pen = ₹ 9.50

Money spent for pencil = ₹ 2.50

Total money spent = ₹ 9.50 + ₹ 2.50

Total money spent = ₹ 12.00

Example 3 : Samson travelled 5 km 52 m by bus, 2 km 265 m by car and the rest 1km 30 m he walked. How much distance did he travel in all?

Solution: Distance travelled by bus = 5 km 52 m = 5.052 km

Distance travelled by car = 2 km 265 m = 2.265 km

Distance travelled on foot = 1 km 30 m = 1.030 km

Therefore, total distance travelled is

$$\begin{array}{r} 5.052 \text{ km} \\ 2.265 \text{ km} \\ + \quad 1.030 \text{ km} \\ \hline 8.347 \text{ km} \\ \hline \end{array}$$

Therefore, total distance travelled = 8.347 km

Example 4 : Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. Find the total weight of all the fruits he bought.

Solution : Weight of apples = 4 kg 90 g = 4.090 kg

Weight of grapes = 2 kg 60 g = 2.060 kg

Weight of mangoes = 5 kg 300 g = 5.300 kg

Therefore, the total weight of the fruits bought is

$$\begin{array}{r} 4.090 \text{ kg} \\ 2.060 \text{ kg} \\ + \quad 5.300 \text{ kg} \\ \hline 11.450 \text{ kg} \\ \hline \end{array}$$

Total weight of the fruits bought = 11.450 kg.

EXERCISE 8.3

1. Find the sum in each of the following :
   (a) 0.007 + 8.5 + 30.08
   (b) 15 + 0.632 + 13.8
   (c) 27.076 + 0.55 + 0.004
   (d) 25.65 + 9.005 + 3.7
   (e) 0.75 + 10.425 + 2
   (f) 280.69 + 25.2 + 38
2. Rashid spent ₹ 35.75 for Maths book and ₹ 32.60 for Science book. Find the total amount spent by Rashid.
3. Radhika's mother gave her ₹ 10.50 and her father gave her ₹ 15.80, find the total amount given to Radhika by the parents.
4. Nasreen bought 3 m 20 cm cloth for her shirt and 2 m 5 cm cloth for her trouser. Find the total length of cloth bought by her.
5. Naresh walked 2 km 35 m in the morning and 1 km 7 m in the evening. How much distance did he walk in all?

6. Sunita travelled 15 km 268 m by bus, 7 km 7 m by car and 500 m on foot in order to reach her school. How far is her school from her residence?
7. Ravi purchased 5 kg 400 g rice, 2 kg 20 g sugar and 10 kg 850g flour. Find the total weight of his purchases.

8.5 Subtraction of Decimals

Do This

Subtract 1.32 from 2.58

This can be shown by the table.

	Ones	Tenths	Hundredths
	2	5	8
–	1	3	2
	1	2	6

Thus, 2.58 – 1.32 = 1.26

Therefore, we can say that, subtraction of decimals can be done by subtracting hundredths from hundredths, tenths from tenths, ones from ones and so on, just as we did in addition.

Sometimes while subtracting decimals, we may need to regroup like we did in addition.

Let us subtract 1.74 from 3.5.

	Ones	Tenths	Hundredths
	3	5	0
–	1	7	4
	1	7	6

Subtract in the hundredth place.

Can't subtract !

so regroup

$$\begin{array}{r} {}^{2}\cancel{3}\;.\;\overset{14}{\cancel{5}}\;\overset{10}{0} \\ -\;1\;.\;7\;4 \\ \hline 1\;.\;7\;6 \\ \hline \end{array}$$

Thus, 3.5 – 1.74 = 1.76

Try These

1. Subtract 1.85 from 5.46 ;
2. Subtract 5.25 from 8.28 ;
3. Subtract 0.95 from 2.29 ;
4. Subtract 2.25 from 5.68.

Example 5 : Abhishek had ₹ 7.45. He bought toffees for ₹ 5.30. Find the balance amount left with Abhishek.

Solution : Total amount of money = ₹ 7.45

Amount spent on toffees = ₹ 5.30

Balance amount of money = ₹ 7.45 – ₹ 5.30 = ₹ 2.15

Example 6 : Urmila's school is at a distance of 5 km 350 m from her house. She travels 1 km 70 m on foot and the rest by bus. How much distance does she travel by bus?

Solution : Total distance of school from the house = 5.350 km

Distance travelled on foot = 1.070 km

Therefore, distance travelled by bus = 5.350 km – 1.070 km

= 4.280 km

Thus, distance travelled by bus = 4.280 km or 4 km 280 m

Example 7 : Kanchan bought a watermelon weighing 5 kg 200 g. Out of this she gave 2 kg 750 g to her neighbour. What is the weight of the watermelon left with Kanchan?

Solution : Total weight of the watermelon = 5.200 kg

Watermelon given to the neighbour = 2.750 kg

Therefore, weight of the remaining watermelon

= 5.200 kg – 2.750 kg = 2.450 kg

EXERCISE 8.4

1. Subtract :
   (a) ₹ 18.25 from ₹ 20.75
   (b) 202.54 m from 250 m
   (c) ₹ 5.36 from ₹ 8.40
   (d) 2.051 km from 5.206 km
   (e) 0.314 kg from 2.107 kg

2. Find the value of :
   (a) 9.756 – 6.28
   (b) 21.05 – 15.27
   (c) 18.5 – 6.79
   (d) 11.6 – 9.847

3. Raju bought a book for ₹ 35.65. He gave ₹ 50 to the shopkeeper. How much money did he get back from the shopkeeper?
4. Rani had ₹ 18.50. She bought one ice-cream for ₹ 11.75. How much money does she have now?

5. Tina had 20 m 5 cm long cloth. She cuts 4 m 50 cm length of cloth from this for making a curtain. How much cloth is left with her?

6. Namita travels 20 km 50 m every day. Out of this she travels 10 km 200 m by bus and the rest by auto. How much distance does she travel by auto?
7. Aakash bought vegetables weighing 10 kg. Out of this, 3 kg 500 g is onions, 2 kg 75 g is tomatoes and the rest is potatoes. What is the weight of the potatoes?

What have we discussed?

1. Every decimal can be written as a fraction.
2. Any two decimal numbers can be compared among themselves. The comparison can start with the whole part. If the whole parts are equal then the tenth parts can be compared and so on.
3. Decimals are used in many ways in our lives. For example, in representing units of money, length and weight.

Data Handling

Chapter 9

0650CH09

9.1 Introduction

You must have observed your teacher recording the attendance of students in your class everyday, or recording marks obtained by you after every test or examination. Similarly, you must have also seen a cricket score board. Two score boards have been illustrated here :

Name of the bowlers	Overs	Maiden overs	Runs given	Wickets taken
A	10	2	40	3
B	10	1	30	2
C	10	2	20	1
D	10	1	50	4

Name of the batsmen	Runs	Balls faced	Time (in min.)
E	45	62	75
F	55	70	81
G	37	53	67
H	22	41	55

You know that in a game of cricket the information recorded is not simply about who won and who lost. In the score board, you will also find some equally important information about the game. For instance, you may find out the time taken and number of balls faced by the highest run-scorer.

Similarly, in your day to day life, you must have seen several kinds of tables consisting of numbers, figures, names etc.

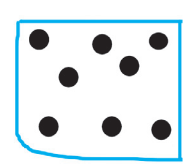
These tables provide 'Data'. *A data is a collection of numbers gathered to give some information.*

9.2 Recording Data

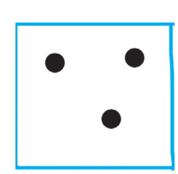
Let us take an example of a class which is preparing to go for a picnic. The teacher asked the students to give their choice of fruits out of banana, apple, orange or guava. Uma is asked to prepare the list. She prepared a list of all the children and wrote the choice of fruit against each name. This list would help the teacher to distribute fruits according to the choice.

Raghav	—	Banana	Bhawana	—	Apple
Preeti	—	Apple	Manoj	—	Banana
Amar	—	Guava	Donald	—	Apple
Fatima	—	Orange	Maria	—	Banana
Amita	—	Apple	Uma	—	Orange
Raman	—	Banana	Akhtar	—	Guava
Radha	—	Orange	Ritu	—	Apple
Farida	—	Guava	Salma	—	Banana
Anuradha	—	Banana	Kavita	—	Guava
Rati	—	Banana	Javed	—	Banana

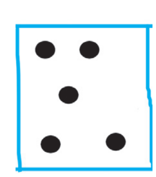
If the teacher wants to know the number of bananas required for the class, she has to read the names in the list one by one and count the total number of bananas required. To know the number of apples, guavas and oranges separately she has to repeat the same process for each of these fruits. How tedious and time consuming it is! It might become more tedious if the list has, say, 50 students.


Banana

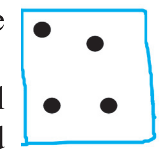
So, Uma writes only the names of these fruits one by one like, banana, apple, guava, orange, apple, banana, orange, guava, banana, banana, apple, banana, apple, banana, orange, guava, apple, banana, guava, banana.


Orange

Do you think this makes the teacher's work easier? She still has to count the fruits in the list one by one as she did earlier.


Apple

Salma has another idea. She makes four squares on the floor. Every square is kept for fruit of one kind only. She asks the students to put one pebble in the square which matches their choices. i.e. a student opting for banana will put a pebble in the square marked for banana and so on.



Guava

By counting the pebbles in each square, Salma can quickly tell the number of each kind of fruit required. She can get the required information quickly by systematically placing the pebbles in different squares.

Try to perform this activity for 40 students and with names of any four fruits. Instead of pebbles you can also use bottle caps or some other tokens.

9.3 Organisation of Data

To get the same information which Salma got, Ronald needs only a pen and a paper. He does not need pebbles. He also does not ask students to come and place the pebbles. He prepares the following table.

Banana	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓	8
Orange	✓ ✓ ✓	3
Apple	✓ ✓ ✓ ✓ ✓	5
Guava	✓ ✓ ✓ ✓	4

Do you understand Ronald's table?

What does one (✓) mark indicate?

Four students preferred guava. How many (✓) marks are there against guava?

How many students were there in the class? Find all this information.

Discuss about these methods. Which is the best? Why? Which method is more useful when information from a much larger data is required?

Example 1 : A teacher wants to know the choice of food of each student as part of the mid-day meal programme. The teacher assigns the task of collecting this information to Maria. Maria does so using a paper and a pencil. After arranging the choices in a column, she puts against a choice of food one (|) mark for every student making that choice.

<table>
<tr><th>Choice</th><th>Number of students</th></tr>
<tr><td>Rice only</td><td>| | | | | | | | | | | | | | | | |</td></tr>
<tr><td>Chapati only</td><td>| | | | | | | | | | | | |</td></tr>
<tr><td>Both rice and chapati</td><td>| | | | | | | | | | | | | | | | | | | |</td></tr>
</table>

Umesh, after seeing the table suggested a better method to count the students. He asked Maria to organise the marks (|) in a group of ten as shown below :

Choice	Tally marks	Number of students
Rice only	ǀǀǀǀǀǀǀǀǀǀ ǀǀǀǀǀǀǀ	17
Chapati only	ǀǀǀǀǀǀǀǀǀǀ ǀǀǀ	13
Both rice and chapati	ǀǀǀǀǀǀǀǀǀǀ ǀǀǀǀǀǀǀǀǀǀ	20

Rajan made it simpler by asking her to make groups of five instead of ten, as shown below :

Choice	Tally marks	Number of students
Rice only	ǀǀǀǀǀ ǀǀǀǀǀ ǀǀǀǀǀ ǀǀ	17
Chapati only	ǀǀǀǀǀ ǀǀǀǀǀ ǀǀǀ	13
Both rice and chapati	ǀǀǀǀǀ ǀǀǀǀǀ ǀǀǀǀǀ ǀǀǀǀǀ	20

Teacher suggested that the fifth mark in a group of five marks should be used as a cross, as shown by '~~ǀǀǀǀ~~ '. These are **tally marks**. Thus, ~~ǀǀǀǀ~~ ǀǀ shows the count to be five plus two (i.e. seven) and ~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ shows five plus five (i.e. ten).

With this, the table looks like :

Choice	Tally marks	Number of students
Rice only	~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ ǀǀ	17
Chapati only	~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ ǀǀǀ	13
Both rice and chapati	~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~ ~~ǀǀǀǀ~~	20

Example 2 : Ekta is asked to collect data for size of shoes of students in her Class VI. Her finding are recorded in the manner shown below :

5	4	7	5	6	7	6	5	6	6	5
4	5	6	8	7	4	6	5	6	4	6
5	7	6	7	5	7	6	4	8	7	

Javed wanted to know (i) the size of shoes worn by the maximum number of students. (ii) the size of shoes worn by the minimum number of students. Can you find this information?

Ekta prepared a table using tally marks.

Shoe size	Tally marks	Number of students
4	$\cancel{||||}$	5
5	$\cancel{||||}$ $|||$	8
6	$\cancel{||||}$ $\cancel{||||}$	10
7	$\cancel{||||}$ $||$	7
8	$||$	2

Now the questions asked earlier could be answered easily.

You may also do some such activity in your class using tally marks.

Do This

1. Collect information regarding the number of family members of your classmates and represent it in the form of a table. Find to which category most students belong.

Number of family members	Tally marks	Number of students with that many family members

Make a table and enter the data using tally marks. Find the number that appeared

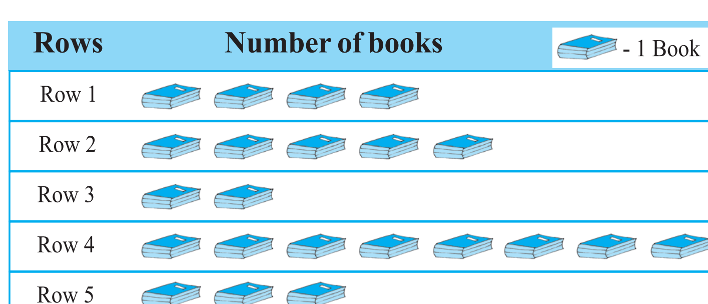
(a) the minimum number of times? (b) the maximum number of times?

(c) same number of times?

9.4 Pictograph

A cupboard has five compartments. In each compartment a row of books is arranged.

The details are indicated in the adjoining table :

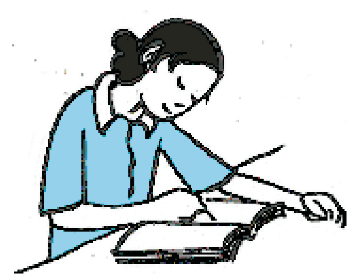


Which row has the greatest number of books? Which row has the least number of books? Is there any row which does not have books?

You can answer these questions by just studying the diagram. The picture visually helps you to understand the data. It is a **pictograph**.

A pictograph represents data through pictures of objects. It helps answer the questions on the data at a glance.

Do This



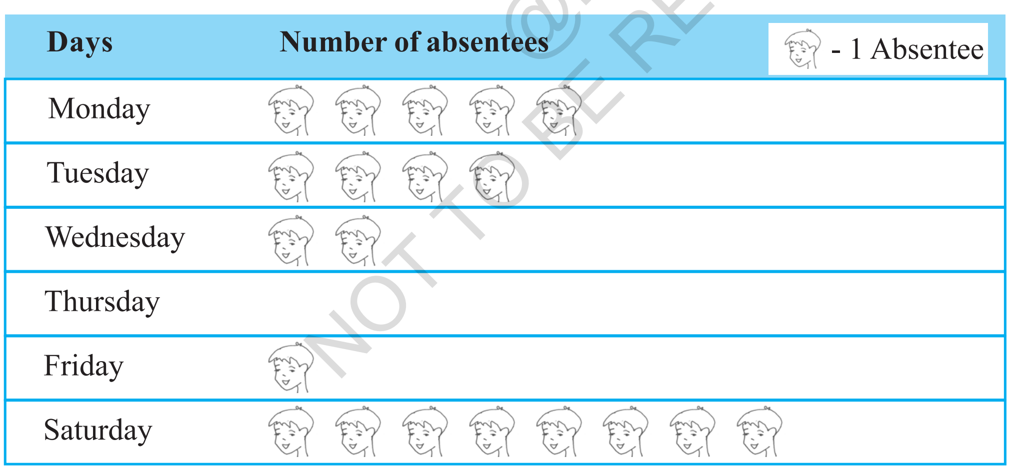
Pictographs are often used by dailies and magazines to attract readers attention.

Collect one or two such published pictographs and display them in your class. Try to understand what they say.

It requires some practice to understand the information given by a pictograph.

9.5 Interpretation of a Pictograph

Example 3 : The following pictograph shows the number of absentees in a class of 30 students during the previous week :



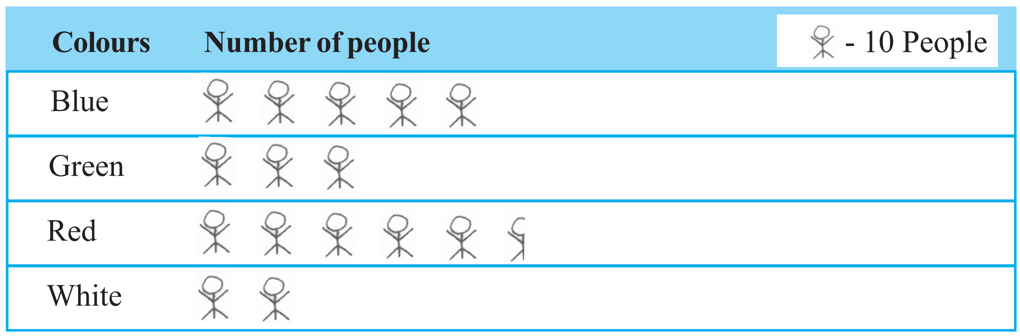
(a) On which day were the maximum number of students absent?
(b) Which day had full attendance?
(c) What was the total number of absentees in that week?

Solution : (a) Maximum absentees were on saturday. (There are 8 pictures in the row for saturday; on all other days, the number of pictures are less).

(b) Against thursday, there is no picture, i.e. no one is absent. Thus, on that day the class had full attendance.

(c) There are 20 pictures in all. So, the total number of absentees in that week was 20.

Example 4 : The colours of fridges preferred by people living in a locality are shown by the following pictograph :



(a) Find the number of people preferring blue colour.

(b) How many people liked red colour?

Solution : (a) Blue colour is preferred by 50 people.

[ = 10, so 5 pictures indicate 5×10 people].

(b) Deciding the number of people liking red colour needs more care.

For 5 complete pictures, we get $5 \times 10 = 50$ people.

For the last incomplete picture, we may roughly take it as 5.

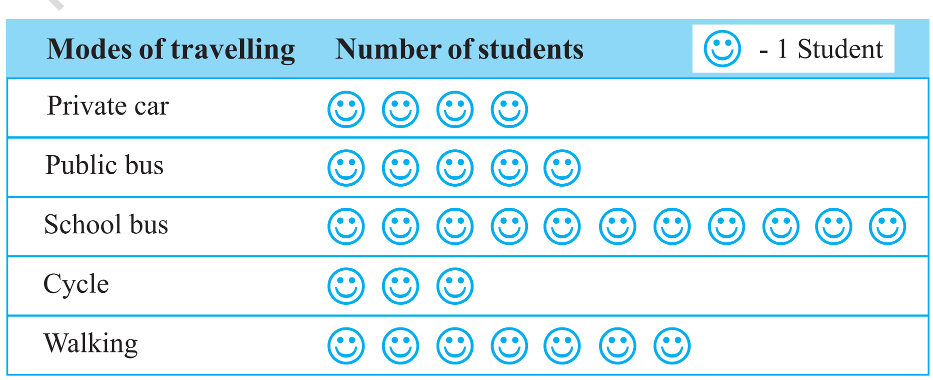
So, number of people preferring red colour is nearly 55.

Think, discuss and write

In the above example, the number of people who like red colour was taken as 50 + 5. If your friend wishes to take it as 50 + 8, is it acceptable?

Example 5 : A survey was carried out on 30 students of class VI in a school. Data about the different modes of transport used by them to travel to school was displayed as pictograph.

What can you conclude from the pictograph?



Solution : From the pictograph we find that:

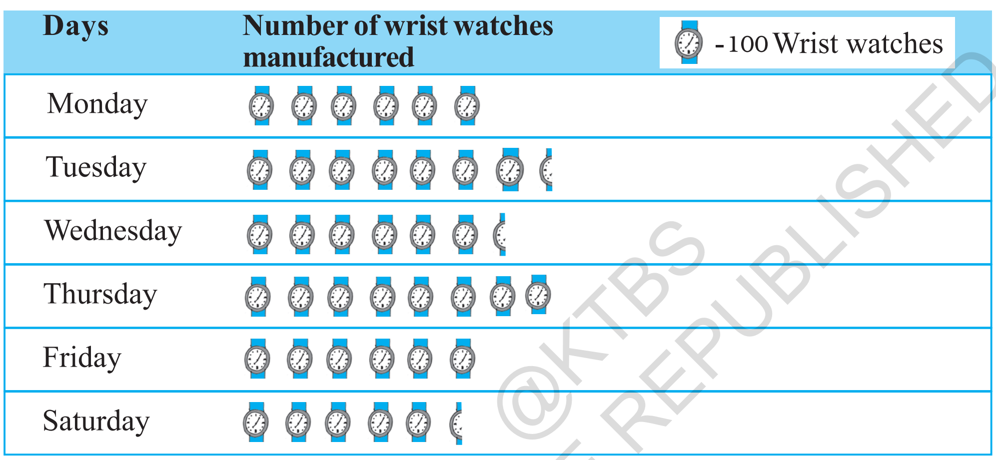
(a) The number of students coming by private car is 4.

(b) Maximum number of students use the school bus. This is the most popular way.

(c) Cycle is used by only three students.

(d) The number of students using the other modes can be similarly found.

Example 6 : Following is the pictograph of the number of wrist watches manufactured by a factory in a particular week.



(a) On which day were the least number of wrist watches manufactured?

(b) On which day were the maximum number of wrist watches manufactured?

(c) Find out the approximate number of wrist watches manufactured in the particular week?

Solution : We can complete the following table and find the answers.

Days	**Number of wrist watches manufactured**
Monday	600
Tuesday	More than 700 and less than 800
Wednesday	....
Thursday	....
Friday	....
Saturday	....

EXERCISE 9.1

1. In a Mathematics test, the following marks were obtained by 40 students. Arrange these marks in a table using tally marks.

8	1	3	7	6	5	5	4	4	2
4	9	5	3	7	1	6	5	2	7
7	3	8	4	2	8	9	5	8	6
7	4	5	6	9	6	4	4	6	6

(a) Find how many students obtained marks equal to or more than 7.

(b) How many students obtained marks below 4?

2. Following is the choice of sweets of 30 students of Class VI.

Ladoo, Barfi, Ladoo, Jalebi, Ladoo, Rasgulla, Jalebi, Ladoo, Barfi, Rasgulla, Ladoo, Jalebi, Jalebi, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo, Rasgulla, Ladoo, Ladoo, Barfi, Rasgulla, Rasgulla, Jalebi, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo.

(a) Arrange the names of sweets in a table using tally marks.

(b) Which sweet is preferred by most of the students?

3. Catherine threw a dice 40 times and noted the number appearing each time as shown below :

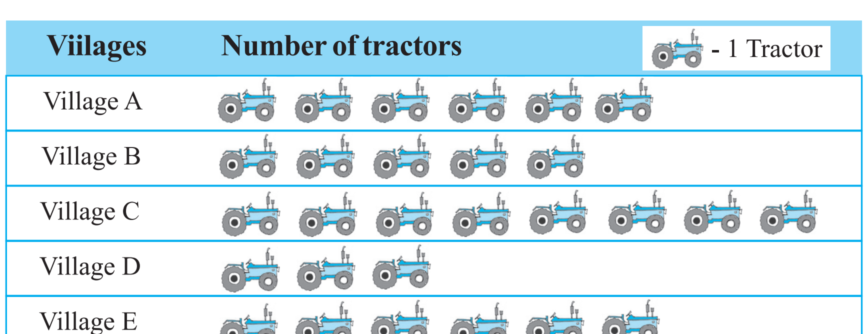
1	3	5	6	6	3	5	4	1	6
2	5	3	4	6	1	5	5	6	1
1	2	2	3	5	2	4	5	5	6
5	1	6	2	3	5	2	4	1	5

Make a table and enter the data using tally marks. Find the number that appeared.

(a) The minimum number of times (b) The maximum number of times

(c) Find those numbers that appear an equal number of times.

4. Following pictograph shows the number of tractors in five villages.



Observe the pictograph and answer the following questions.

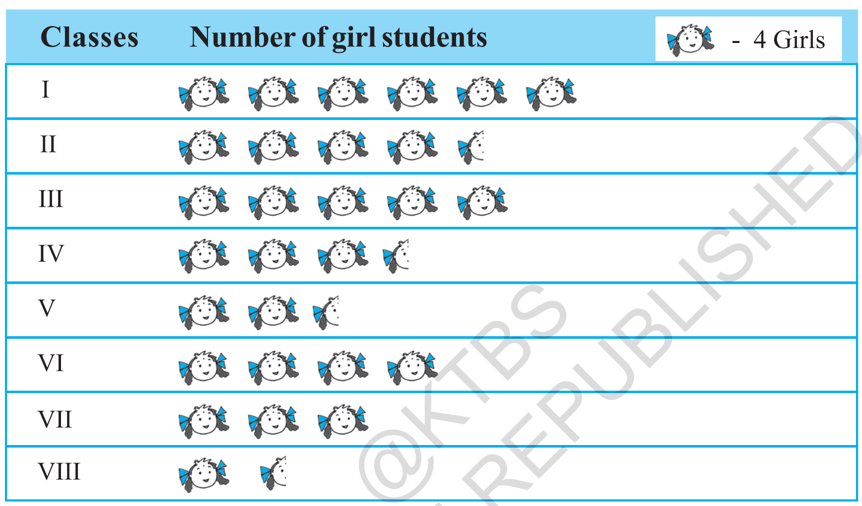
(i) Which village has the minimum number of tractors?

(ii) Which village has the maximum number of tractors?

(iii) How many more tractors village C has as compared to village B.

(iv) What is the total number of tractors in all the five villages?

5. The number of girl students in each class of a co-educational middle school is depicted by the pictograph :



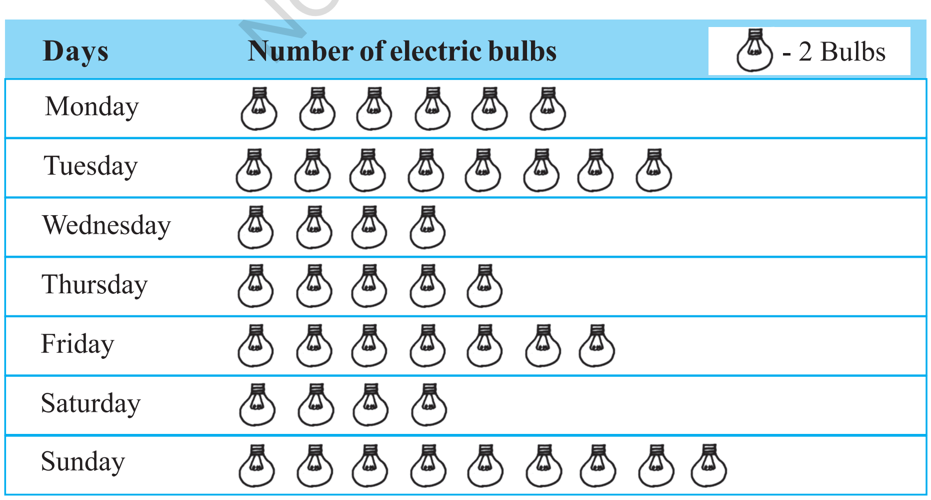
Observe this pictograph and answer the following questions :

(a) Which class has the minimum number of girl students?

(b) Is the number of girls in Class VI less than the number of girls in Class V?

(c) How many girls are there in Class VII?

6. The sale of electric bulbs on different days of a week is shown below :



Observe the pictograph and answer the following questions :

(a) How many bulbs were sold on Friday?

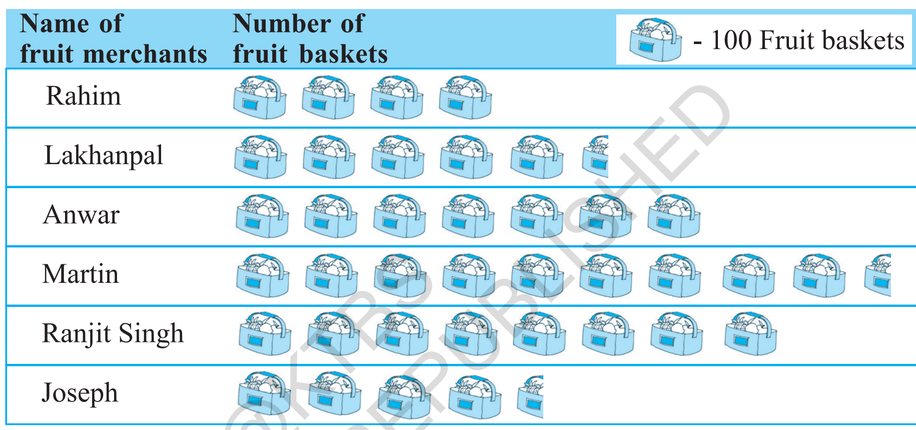
(b) On which day were the maximum number of bulbs sold?

(c) On which of the days same number of bulbs were sold?

(d) On which of the days minimum number of bulbs were sold?

(e) If one big carton can hold 9 bulbs. How many cartons were needed in the given week?

7. In a village six fruit merchants sold the following number of fruit baskets in a particular season :



Observe this pictograph and answer the following questions :

(a) Which merchant sold the maximum number of baskets?

(b) How many fruit baskets were sold by Anwar?

(c) The merchants who have sold 600 or more number of baskets are planning to buy a godown for the next season. Can you name them?

What have we discussed?

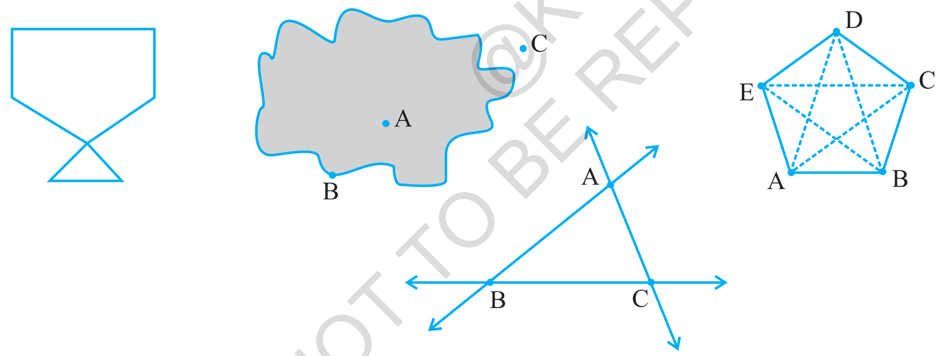
1. We have seen that data is a collection of numbers gathered to give some information.
2. To get a particular information from the given data quickly, the data can be arranged in a tabular form using tally marks.

Mensuration

Chapter 10

10.1 Introduction

When we talk about some plane figures as shown below we think of their regions and their boundaries. We need some measures to compare them. We look into these now.



10.2 Perimeter

Look at the following figures (Fig. 10.1). You can make them with a wire or a string.

If you start from the point S in each case and move along the line segments then you again reach the point S. You have made a complete round of the

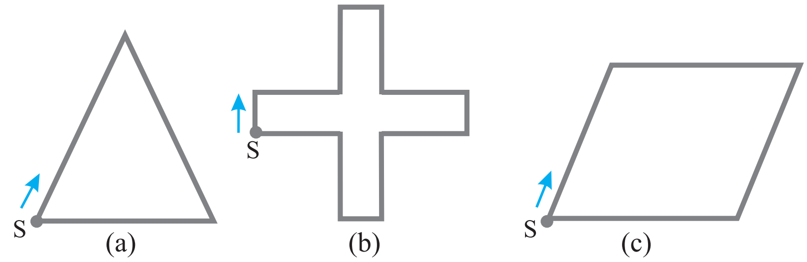


Fig 10.1

shape in each case (a), (b) & (c). The distance covered is equal to the length of wire used to draw the figure.

This distance is known as the **perimeter** of the closed figure. It is the length of the wire needed to form the figures.

The idea of perimeter is widely used in our daily life.

- A farmer who wants to fence his field.
- An engineer who plans to build a compound wall on all sides of a house.
- A person preparing a track to conduct sports.

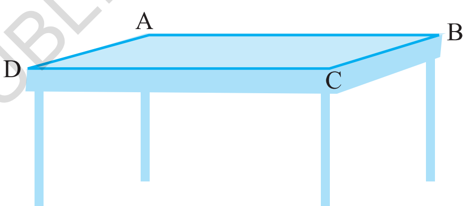
All these people use the idea of 'perimeter'.

Give five examples of situations where you need to know the perimeter.

Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.

Try These

1. Measure and write the length of the four sides of the top of your study table.



AB = ____ cm

BC = ____ cm

CD = ____ cm

DA = ____ cm

Now, the sum of the lengths of the four sides

= AB + BC + CD + DA

= ___ cm + ___ cm + ___ cm + ___ cm

= _____ cm

What is the perimeter?

2. Measure and write the lengths of the four sides of a page of your notebook. The sum of the lengths of the four sides

= AB + BC + CD + DA

= ___ cm + ___ cm + ___ cm + ___ cm

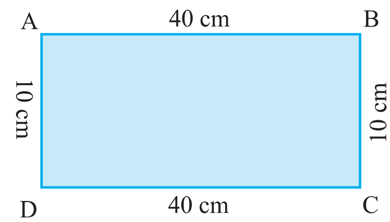
= _____ cm

What is the perimeter of the page?

3. Meera went to a park 150 m long and 80 m wide. She took one complete round on its boundary. What is the distance covered by her?

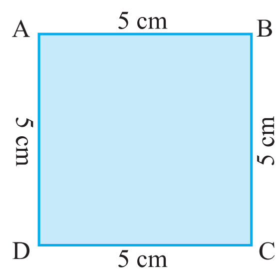
4. Find the perimeter of the following figures:

(a)



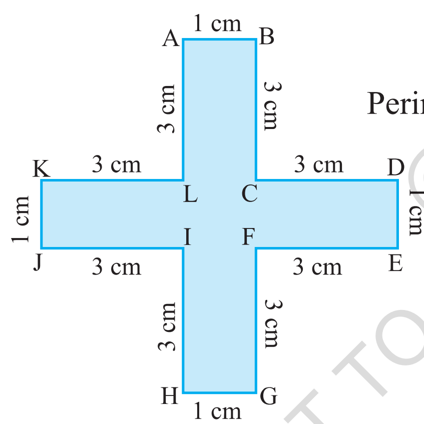
Perimeter = AB + BC + CD + DA
= __ + __ + __ + __
= ______

(b)



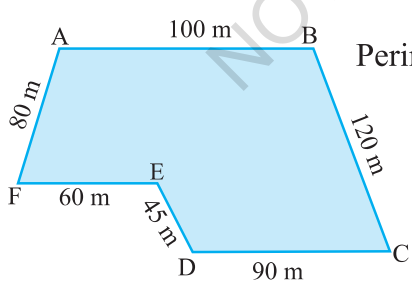
Perimeter = AB + BC + CD + DA
= __ + __ + __ + __
= ______

(c)

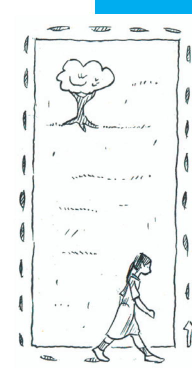


Perimeter = AB + BC + CD + DE
+ EF + FG + GH +HI
+ IJ + JK + KL + LA
= __ + __ + __ + __ + __ +
__ + __ + __ + __ + __
+ __ + __
= ______

(d)



Perimeter = AB + BC + CD + DE + EF
+ FA
= __ + __ + __ + __ + __ + __
= ______



So, how will you find the perimeter of any closed figure made up entirely of line segments? Simply find the sum of the lengths of all the sides (which are line segments).

10.2.1 Perimeter of a rectangle

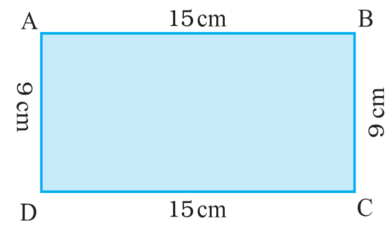
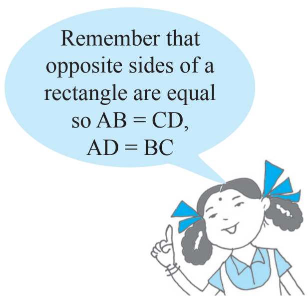


Fig 10.2

Let us consider a rectangle ABCD (Fig 10.2) whose length and breadth are 15 cm and 9 cm respectively. What will be its perimeter?

Perimeter of the rectangle = Sum of the lengths of its four sides.



= AB + BC + CD + DA

= AB + BC + AB + BC

= 2 × AB + 2 × BC

= 2 × (AB + BC)

= 2 × (15cm + 9cm)

= 2 × (24cm)

= 48 cm

Try These

Find the perimeter of the following rectangles:

Length of rectangle	Breadth of rectangle	Perimeter by adding all the sides	Perimeter by 2 × (Length + Breadth)
25 cm	12 cm	= 25 cm + 12 cm + 25 cm + 12 cm = 74 cm	= 2 ×(25 cm + 12 cm) = 2 × (37 cm) = 74 cm
0.5 m	0.25 m		
18 cm	15 cm		
10.5 cm	8.5 cm		

Hence, from the said example, we notice that

Perimeter of a rectangle = length + breadth + length + breadth

i.e. **Perimeter of a rectangle = 2 × (length + breadth)**

Let us now see practical applications of this idea :

Example 1 : Shabana wants to put a lace border all around a rectangular table cover (Fig 10.3), 3 m long and 2 m wide. Find the length of the lace required by Shabana.

Solution : Length of the rectangular table cover = 3 m

Breadth of the rectangular table cover = 2 m

Shabana wants to put a lace border all around the table cover. Therefore, the length of the lace required will be equal to the perimeter of the rectangular table cover.

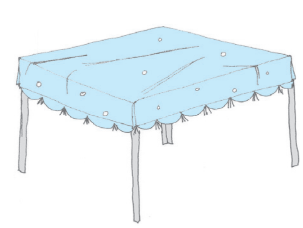


Fig 10.3

Now, perimeter of the rectangular table cover

= 2 × (length + breadth) = 2 × (3 m + 2 m) = 2 × 5 m = 10 m

So, length of the lace required is 10 m.

Example 2 : An athlete takes 10 rounds of a rectangular park, 50 m long and 25 m wide. Find the total distance covered by him.

Solution : Length of the rectangular park = 50 m

Breadth of the rectangular park = 25 m

Total distance covered by the athlete in one round will be the perimeter of the park.

Now, perimeter of the rectangular park

= 2 × (length + breadth)= 2 × (50 m + 25 m)

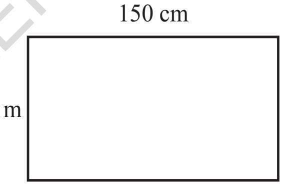
= 2 × 75 m = 150 m

So, the distance covered by the athlete in one round is 150 m.

Therefore, distance covered in 10 rounds = 10 × 150 m = 1500m

The total distance covered by the athlete is 1500 m.

Example 3 : Find the perimeter of a rectangle whose length and breadth are 150 cm and 1 m respectively.



Solution : Length = 150 cm

Breadth = 1m = 100 cm

Perimeter of the rectangle

= 2 × (length + breadth)

= 2 × (150 cm + 100 cm)

= 2 × (250 cm) = 500 cm = 5 m

Example 4 : A farmer has a rectangular field of length and breadth 240 m and 180 m respectively. He wants to fence it with 3 rounds of rope as shown in figure 10.4. What is the total length of rope he must use?

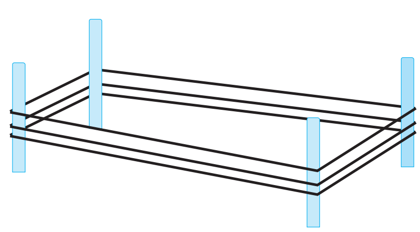


Fig 10.4

Solution : The farmer has to cover three times the perimeter of that field. Therefore, total length of rope required is thrice its perimeter.

Perimeter of the field = 2 × (length + breadth)

= 2 × (240 m + 180 m)

= 2 × 420 m = 840 m

Total length of rope required = 3 × 840 m = 2520 m

Example 5 : Find the cost of fencing a rectangular park of length 250 m and breadth 175 m at the rate of ₹ 12 per metre.

Solution : Length of the rectangular park = 250 m

Breadth of the rectangular park = 175 m

To calculate the cost of fencing we require perimeter.

Perimeter of the rectangle = 2 × (length + breadth)

= 2 × (250 m + 175 m)

= 2 × (425 m) = 850 m

Cost of fencing 1m of park = ₹ 12

Therefore, the total cost of fencing the park

= ₹ 12 × 850 = ₹ 10200

10.2.2 Perimeter of regular shapes

Consider this example.

Biswamitra wants to put coloured tape all around a square picture (Fig 10.5) of side 1m as shown. What will be the length of the coloured tape he requires?

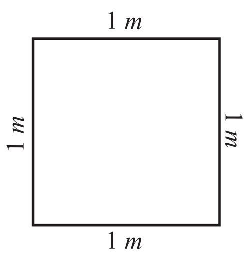


Fig 10.5

Since Biswamitra wants to put the coloured tape all around the square picture, he needs to find the perimeter of the picture frame.

Thus, the length of the tape required

= Perimeter of square = 1m + 1 m + 1 m + 1 m = 4 m

Now, we know that all the four sides of a square are equal, therefore, in place of adding it four times, we can multiply the length of one side by 4. Thus, the length of the tape required = 4 × 1 m = 4 m

From this example, we see that

Perimeter of a square = 4 × length of a side

Draw more such squares and find the perimeters.

Now, look at equilateral triangle (Fig 10.6) with each side equal to 4 cm. Can we find its perimeter?

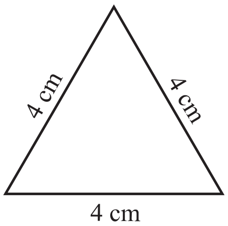


Fig 10.6

Perimeter of this equilateral triangle = 4 + 4 + 4 cm

= 3 × 4 cm = 12 cm

So, we find that

Perimeter of an equilateral triangle = 3 × length of a side

What is similar between a square and an equilateral triangle? They are figures having all the sides of equal length and all the angles of equal measure. Such

Try These

Find various objects from your surroundings which have regular shapes and find their perimeters.

figures are known as *regular closed figures*. Thus, a square and an equilateral triangle are regular closed figures.

You found that,

Perimeter of a square = 4 × length of one side

Perimeter of an equilateral triangle = 3 × length of one side

So, what will be the perimeter of a regular pentagon?

A regular pentagon has five equal sides.

Therefore, perimeter of a regular pentagon = 5 × length of one side and the perimeter of a regular hexagon will be _______ and of an octagon will be ______.

Example 6 : Find the distance travelled by Shaina if she takes three rounds of a square park of side 70 m.

Solution : Perimeter of the square park = 4 × length of a side = 4 × 70 m = 280 m

Distance covered in one round = 280 m

Therefore, distance travelled in three rounds = 3 × 280m = 840m

Example 7 : Pinky runs around a square field of side 75 m, Bob runs around a rectangular field with length 160 m and breadth 105 m. Who covers more distance and by how much?

Solution : Distance covered by Pinky in one round = Perimeter of the square
= 4 × length of a side
= 4 × 75 m = 300 m

Distance covered by Bob in one round = Perimeter of the rectangle
= 2 × (length + breadth)
= 2 × (160 m + 105 m)
= 2 × 265 m = 530 m

Difference in the distance covered = 530 m – 300 m = 230 m.

Therefore, Bob covers more distance by 230 m.

Example 8 : Find the perimeter of a regular pentagon with each side measuring 3 cm.

Solution : This regular closed figure has 5 sides, each with a length of 3 cm. Thus, we get

Perimeter of the regular pentagon = 5 × 3 cm = 15 cm

Example 9 : The perimeter of a regular hexagon is 18 cm. How long is its one side?

Solution : Perimeter = 18 cm

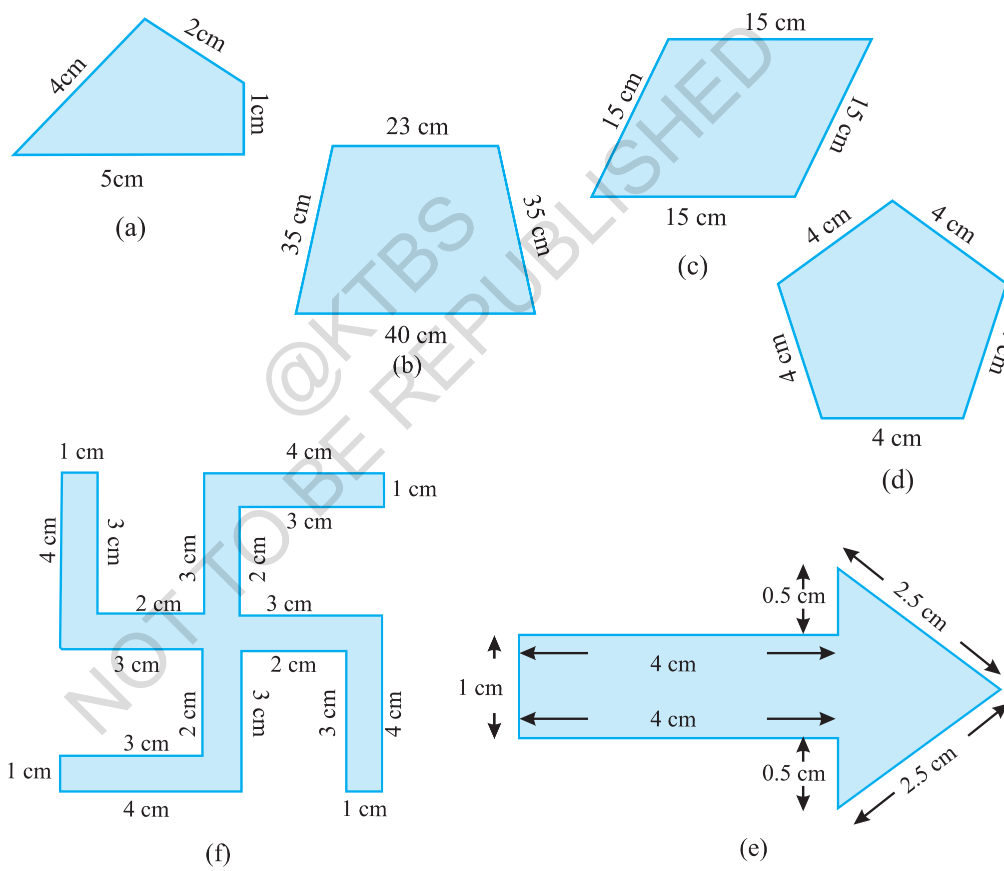
A regular hexagon has 6 sides, so we can divide the perimeter by 6 to get the length of one side.

One side of the hexagon = 18 cm ÷ 6 = 3 cm

Therefore, length of each side of the regular hexagon is 3 cm.

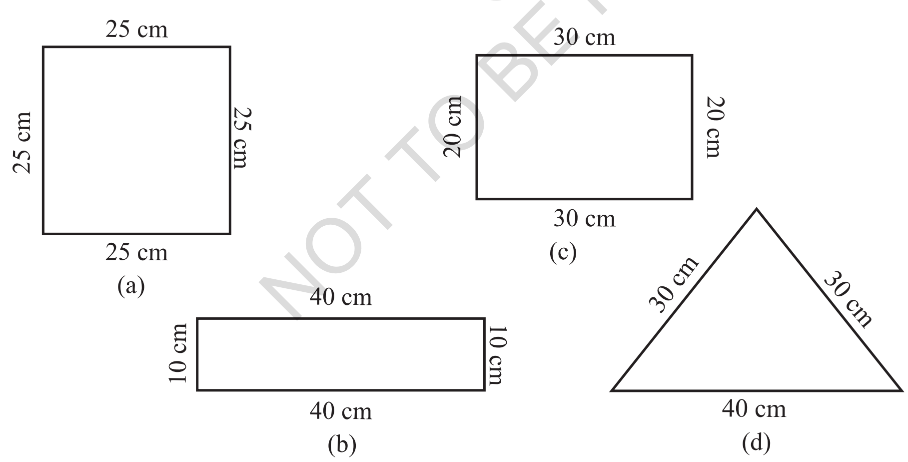
EXERCISE 10.1

1. Find the perimeter of each of the following figures :



2. The lid of a rectangular box of sides 40 cm by 10 cm is sealed all round with tape. What is the length of the tape required?
3. A table-top measures 2 m 25 cm by 1 m 50 cm. What is the perimeter of the table-top?
4. What is the length of the wooden strip required to frame a photograph of length and breadth 32 cm and 21 cm respectively?
5. A rectangular piece of land measures 0.7 km by 0.5 km. Each side is to be fenced with 4 rows of wires. What is the length of the wire needed?

6. Find the perimeter of each of the following shapes :
   (a) A triangle of sides 3 cm, 4 cm and 5 cm.
   (b) An equilateral triangle of side 9 cm.
   (c) An isosceles triangle with equal sides 8 cm each and third side 6 cm.
7. Find the perimeter of a triangle with sides measuring 10 cm, 14 cm and 15 cm.
8. Find the perimeter of a regular hexagon with each side measuring 8 m.
9. Find the side of the square whose perimeter is 20 m.
10. The perimeter of a regular pentagon is 100 cm. How long is its each side?
11. A piece of string is 30 cm long. What will be the length of each side if the string is used to form :
    (a) a square? (b) an equilateral triangle? (c) a regular hexagon?
12. Two sides of a triangle are 12 cm and 14 cm. The perimeter of the triangle is 36 cm. What is its third side?
13. Find the cost of fencing a square park of side 250 m at the rate of ₹ 20 per metre.
14. Find the cost of fencing a rectangular park of length 175 m and breadth 125 m at the rate of ₹ 12 per metre.
15. Sweety runs around a square park of side 75 m. Bulbul runs around a rectangular park with length 60 m and breadth 45 m. Who covers less distance?
16. What is the perimeter of each of the following figures? What do you infer from the answers?



17. Avneet buys 9 square paving slabs, each with a side of $\frac{1}{2}$ m. He lays them in the form of a square.
    (a) What is the perimeter of his arrangement [Fig 10.7(i)]?

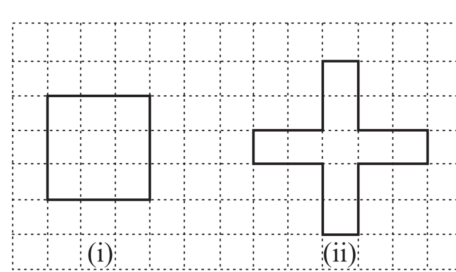


Fig 10.7

(b) Shari does not like his arrangement. She gets him to lay them out like a cross. What is the perimeter of her arrangement [(Fig 10.7 (ii)]?

(c) Which has greater perimeter?

(d) Avneet wonders if there is a way of getting an even greater perimeter. Can you find a way of doing this? (The paving slabs must meet along complete edges i.e. they cannot be broken.)

10.3 Area

Look at the closed figures (Fig 10.8) given below. All of them occupy some region of a flat surface. Can you tell which one occupies more region?

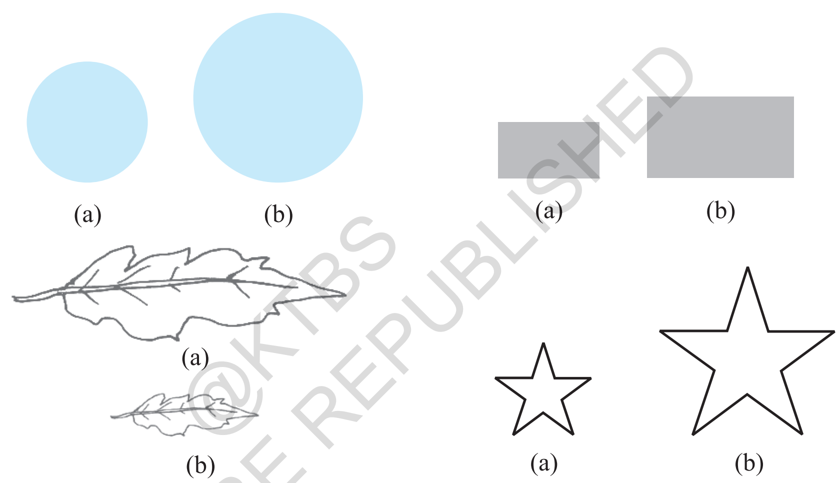


Fig 10.8

The amount of surface enclosed by a closed figure is called its **area.**

So, can you tell, which of the above figures has more area?

Now, look at the adjoining figures of Fig 10.9 :

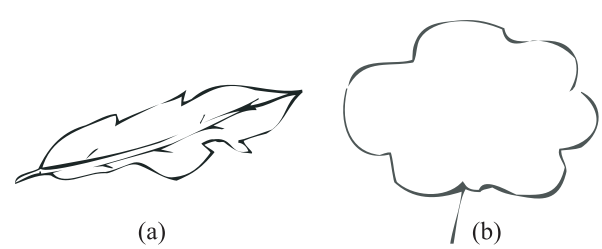


Fig 10.9

Which one of these has larger area? It is difficult to tell just by looking at these figures. So, what do you do?

Place them on a squared paper or graph paper where every square measures 1 cm × 1 cm.

Make an outline of the figure.

Look at the squares enclosed by the figure. Some of them are completely enclosed, some half, some less than half and some more than half.

The area is the number of centimetre squares that are needed to cover it.

But there is a small problem : the squares do not always fit exactly into the area you measure. We get over this difficulty by adopting a convention :

- The area of one full square is taken as 1 sq unit. If it is a centimetre square sheet, then area of one full square will be 1 sq cm.
- Ignore portions of the area that are less than half a square.
- If more than half of a square is in a region, just count it as one square.
- If exactly half the square is counted, take its area as $\frac{1}{2}$ sq unit.

Such a convention gives a fair estimate of the desired area.

Example 10 : Find the area of the shape shown in the figure 10.10.

Solution : This figure is made up of line-segments. Moreover, it is covered by full squares and half squares only. This makes our job simple.

(i) Fully-filled squares = 3

(ii) Half-filled squares = 3

Area covered by full squares

= 3 × 1 sq units = 3 sq units

Total area = $4\frac{1}{2}$ sq units.

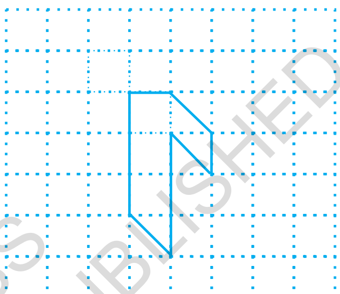


Fig 10.10

Example 11 : By counting squares, estimate the area of the figure 10.9 b.

Soultion : Make an outline of the figure on a graph sheet. (Fig 10.11)

Covered area	Number	Area estimate (sq units)
(i) Fully-filled squares	11	11
(ii) Half-filled squares	3	$3 \times \frac{1}{2}$
(iii) More than half-filled squares	7	7
(iv) Less than half-filled squares	5	0

Fig 10.11

Total area = $11 + 3 \times \frac{1}{2} + 7 = 19\frac{1}{2}$ sq units.

How do the squares cover it?

Example 12 : By counting squares, estimate the area of the figure 10.9 a.

Soultion : Make an outline of the figure on a graph sheet. This is how the squares cover the figure (Fig 10.12).

Try These

1. Draw any circle on a graph sheet. Count the squares and use them to estimate the area of the circular region.
2. Trace shapes of leaves, flower petals and other such objects on the graph paper and find their areas.

Covered area	Number	Area estimate (sq units)
(i) Fully-filled squares	1	1
(ii) Half-filled squares	–	–
(iii) More than half-filled squares	7	7
(iv) Less than half-filled squares	9	0

Total area = 1 + 7 = 8 sq units.

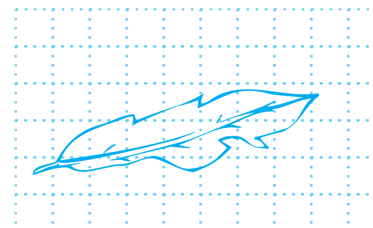
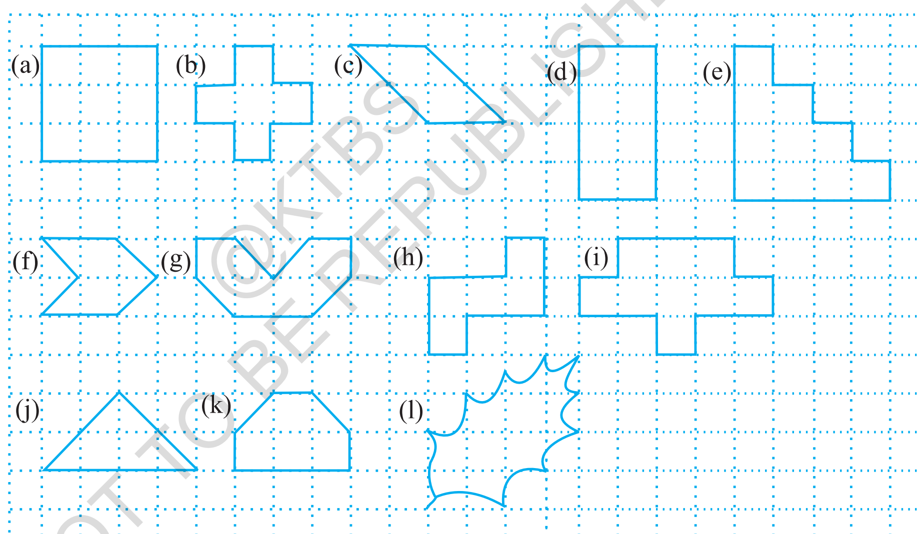
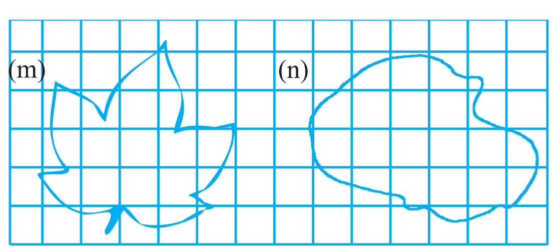


Fig 10.12

EXERCISE 10.2

1. Find the areas of the following figures by counting square:





10.3.1 Area of a rectangle

With the help of the squared paper, can we tell, what will be the area of a rectangle whose length is 5 cm and breadth is 3 cm?

Draw the rectangle on a graph paper having 1 cm × 1 cm squares (Fig 10.13). The rectangle covers 15 squares completely.

The area of the rectangle = 15 sq cm which can be written as 5 × 3 sq cm i.e. (length × breadth).

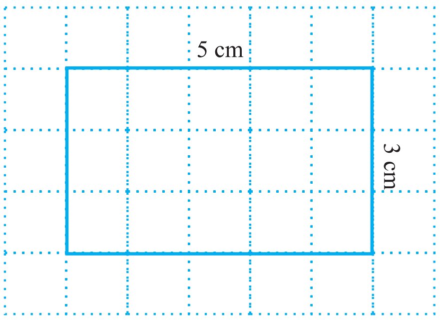


Fig 10.13

The measures of the sides of some of the rectangles are given. Find their areas by placing them on a graph paper and counting the number of square.

Length	Breadth	Area
3 cm	4 cm	----------
7 cm	5 cm	----------
5 cm	3 cm	----------

What do we infer from this?

We find,

Area of a rectangle = (length × breadth)

Without using the graph paper, can we find the area of a rectangle whose length is 6 cm and breadth is 4cm?

Yes, it is possible.

What do we infer from this?

We find that,

Area of the rectangle = length × breadth = 6 cm × 4 cm = 24 sq cm.

Try These

1. Find the area of the floor of your classroom.
2. Find the area of any one door in your house.

10.3.2 Area of a square

Let us now consider a square of side 4 cm (Fig 10.14).

What will be its area?

If we place it on a centimetre graph paper, then what do we observe?

It covers 16 squares i.e. the area of the square = 16 sq cm = 4 × 4 sq cm

Calculate areas of few squares by assuring length of one side of squares by yourself.

Find their areas using graph papers.

What do we infer from this?

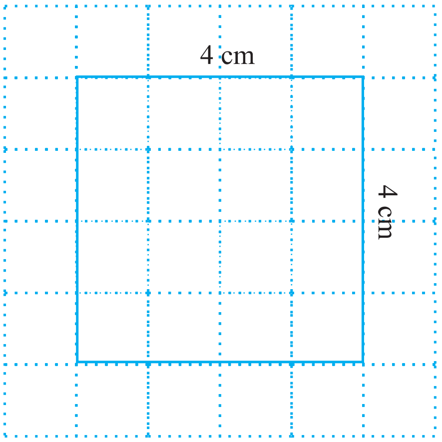


Fig 10.14

We find that in each case,

Area of the square = side × side

You may use this as a formula in doing problems.

Example 13 : Find the area of a rectangle whose length and breadth are 12 cm and 4 cm respectively.

Solution : Length of the rectangle = 12 cm

Breadth of the rectangle = 4 cm

Area of the rectangle = length × breadth

= 12 cm × 4 cm = 48 sq cm.

Example 14 : Find the area of a square plot of side 8 m.

Solution : Side of the square = 8 m

Area of the square = side × side

= 8 m × 8 m = 64 sq m.

Example 15 : The area of a rectangular piece of cardboard is 36 sq cm and its length is 9 cm. What is the width of the cardboard?

Solution : Area of the rectangle = 36 sq cm

Length = 9 cm

Width = ?

Area of a rectangle = length × width

So, width = $\frac{\text{Area}}{\text{Length}} = \frac{36}{9} = 4$ cm

Thus, the width of the rectangular cardboard is 4 cm.

Example 16 : Bob wants to cover the floor of a room 3 m wide and 4 m long by squared tiles. If each square tile is of side 0.5 m, then find the number of tiles required to cover the floor of the room.

Solution : Total area of tiles must be equal to the area of the floor of the room.

Length of the room = 4 m

Breadth of the room = 3 m

Area of the floor = length × breadth

= 4 m × 3 m = 12 sq m

Area of one square tile = side × side

= 0.5 m × 0.5 m

= 0.25 sq m

Number of tiles required = $\dfrac{\text{Area of the floor}}{\text{Area of one tile}} = \dfrac{12}{0.25} = \dfrac{1200}{25} = 48$ tiles.

Example 17 : Find the area in square metre of a piece of cloth 1m 25 cm wide and 2 m long.

Solution : Length of the cloth = 2 m

Breadth of the cloth = 1 m 25 cm = 1 m + 0. 25 m = 1.25 m

(since 25 cm = 0.25m)

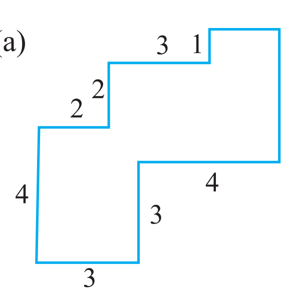
Area of the cloth = length of the cloth × breadth of the cloth

= 2 m × 1.25 m = 2.50 sq m

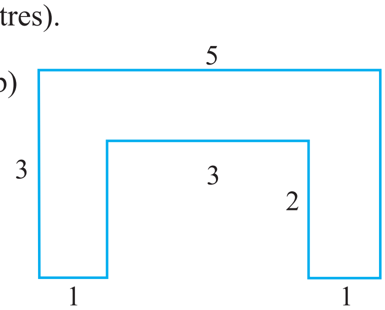
EXERCISE 10.3

1. Find the areas of the rectangles whose sides are :
   (a) 3 cm and 4 cm (b) 12 m and 21 m (c) 2 km and 3 km (d) 2 m and 70 cm
2. Find the areas of the squares whose sides are :
   (a) 10 cm (b) 14 cm (c) 5 m
3. The length and breadth of three rectangles are as given below :
   (a) 9 m and 6 m (b) 17 m and 3 m (c) 4 m and 14 m
   Which one has the largest area and which one has the smallest?
4. The area of a rectangular garden 50 m long is 300 sq m. Find the width of the garden.
5. What is the cost of tiling a rectangular plot of land 500 m long and 200 m wide at the rate of ₹ 8 per hundred sq m.?
6. A table-top measures 2 m by 1 m 50 cm. What is its area in square metres?
7. A room is 4 m long and 3 m 50 cm wide. How many square metres of carpet is needed to cover the floor of the room?
8. A floor is 5 m long and 4 m wide. A square carpet of sides 3 m is laid on the floor. Find the area of the floor that is not carpeted.
9. Five square flower beds each of sides 1 m are dug on a piece of land 5 m long and 4 m wide. What is the area of the remaining part of the land?
10. By splitting the following figures into rectangles, find their areas (The measures are given in centimetres).

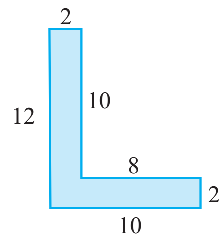
(a)



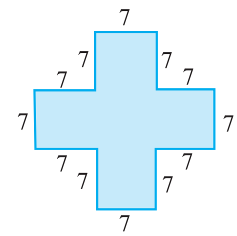
(b)



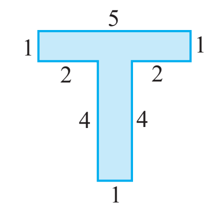
11. Split the following shapes into rectangles and find their areas. (The measures are given in centimetres)



(a)



(b)



(c)

12. How many tiles whose length and breadth are 12 cm and 5 cm respectively will be needed to fit in a rectangular region whose length and breadth are respectively:
(a) 100 cm and 144 cm (b) 70 cm and 36 cm.

A challenge!

On a centimetre squared paper, make as many rectangles as you can, such that the area of the rectangle is 16 sq cm (consider only natural number lengths).

(a) Which rectangle has the greatest perimeter?

(b) Which rectangle has the least perimeter?

If you take a rectangle of area 24 sq cm, what will be your answers?

Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter? With the least perimeter? Give example and reason.

What have we discussed?

1. Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.
2. (a) Perimeter of a rectangle = 2 × (length + breadth)
(b) Perimeter of a square = 4 × length of its side
(c) Perimeter of an equilateral triangle = 3 × length of a side
3. Figures in which all sides and angles are equal are called regular closed figures.
4. The amount of surface enclosed by a closed figure is called its area.
5. To calculate the area of a figure using a squared paper, the following conventions are adopted :
(a) Ignore portions of the area that are less than half a square.
(b) If more than half a square is in a region. Count it as one square.
(c) If exactly half the square is counted, take its area as $\frac{1}{2}$ sq units.
6. (a) Area of a rectangle = length × breadth
(b) Area of a square = side × side

Algebra

Chapter 11

0650CH11

11.1 Introduction

Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

11.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 11.1 (a).

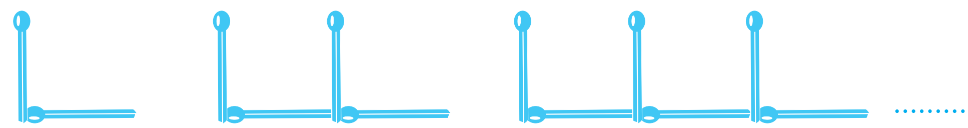


Fig 11.1

Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 11.1 (b)].

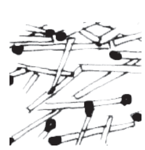
Then Ameena adds one more L and this goes on as shown by the dots in Fig 11.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.



While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = $2 \times$ number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write, Number of matchsticks required = $2 \times n$.

Instead of writing $2 \times n$, we write $2n$. Note that $2n$ is same as $2 \times n$.

Ameena tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required = $2 \times 1 = 2$

For $n = 2$, the number of matchsticks required = $2 \times 2 = 4$

For $n = 3$, the number of matchsticks required = $2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".
Do you agree with Sarita?

11.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was :

Number of matchsticks required = 2*n*

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4,.... Let us look at Table 1 once again. In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

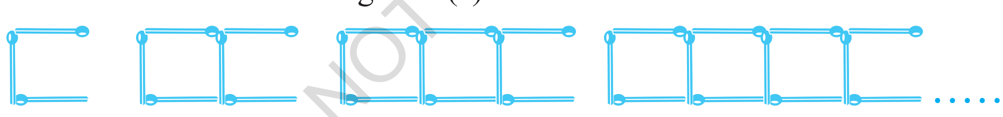
***n* is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... We wrote the rule for the number of matchsticks required using the variable *n*.**

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

11.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 11.2(a).



(a) (b) (c) (d)

Fig 11.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....	....

Can you complete the entries left blank in the table?

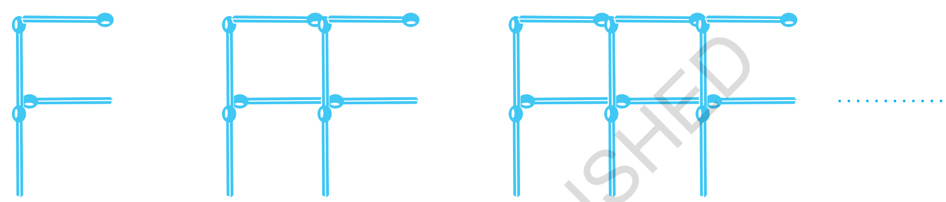
Sarita comes up with the rule :
Number of matchsticks required = 3*n*

She has used the letter n for the number of Cs; n is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita ?

Remember $3n$ is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 11.3(a).



(a) (b) (c)

Fig 11.3

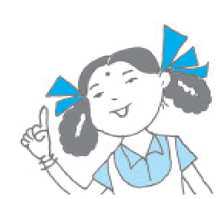
Can you now write the rule for making patterns of F?

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U (⊔), V (\/), triangle (△), square (□) etc. Choose any five and write the rules for making matchstick patterns with them.

11.5 More Examples of Variables

We have used the letter n to show a variable. Raju asks, "Why not *m*"? There is nothing special about n, any letter can be used.

One may use any letter as *m, l, p, x, y, z* etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But *n* in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4, ...



Let us now consider variables in a more familiar situation.

Students went to buy notebooks from the school bookstore. Price of one notebook is ₹ 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	.....	m	.....
Total cost in rupees	5	10	15	20	25	.....	$5m$	.....

The letter m stands for the number of notebooks a student wants to buy; m is a variable, which can take any value 1, 2, 3, 4, ... The total cost of m notebooks is given by the rule :

The total cost in rupees = 5 × number of note books required
= $5m$

If Munnu wants to buy 5 notebooks, then taking $m = 5$, we say that Munnu should carry ₹ 5 × 5 or ₹ 25 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig 11.4). How many children can there be in the drill?

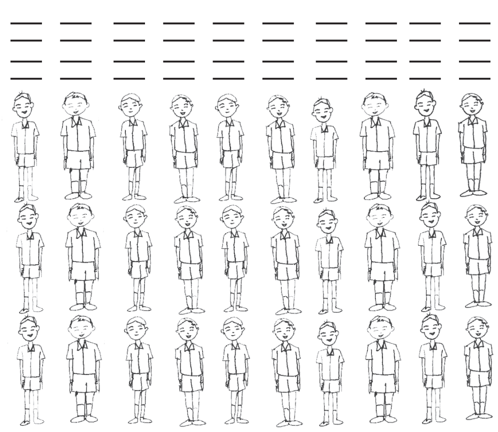


Fig 11.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2 × 10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here, r is a variable which stands for the number of rows and so takes on values 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles.

But we know that, Sarita's marbles = Ameena's marbles + 10.

We shall denote Ameena's marbles by the letter x. Here, x is a variable, which can take any value 1, 2, 3, 4,... ,10,... ,20,... ,30,... . Using x, we write Sarita's marbles = $x + 10$. The expression $(x + 10)$ is read as 'x plus ten'. It means 10 added to x. If x is 20, $(x + 10)$ is 30. If x is 30, $(x + 10)$ is 40 and so on.

> The expression $(x + 10)$ cannot be simplified further.
> Do not confuse $x + 10$ with $10x$, they are different.
> In $10x$, x is multiplied by 10. In $(x + 10)$, 10 is added to x.
> We may check this for some values of x.
> For example,
> If $x = 2$, $10x = 10 \times 2 = 20$ and $x + 10 = 2 + 10 = 12$.
> If $x = 10$, $10x = 10 \times 10 = 100$ and $x + 10 = 10 + 10 = 20$.

Raju and Balu are brothers. Balu is younger than Raju by 3 years. When Raju is 12 years old, Balu is 9 years old. When Raju is 15 years old, Balu is 12 years old. We do not know Raju's age exactly. It may have any value. Let x denote Raju's age in years, x is a variable. If Raju's age in years is x, then Balu's age in years is $(x – 3)$. The expression $(x – 3)$ is read as x minus three. As you would expect, when x is 12, $(x – 3)$ is 9 and when x is 15, $(x – 3)$ is 12.

EXERCISE 11.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable to write the rule.
   (a) A pattern of letter T as 
   (b) A pattern of letter Z as 

(c) A pattern of letter U as 

(d) A pattern of letter V as 

(e) A pattern of letter E as 

(f) A pattern of letter S as 

(g) A pattern of letter A as 

2. We already know the rule for the pattern of letters L, C and F. Some of the letters from Q.1 (given above) give us the same rule as that given by L. Which are these? Why does this happen?

3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)

4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)

5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)

6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)

7. Radha is drawing a dot Rangoli (a beautiful pattern of lines joining dots) with chalk powder. She has 9 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 8 rows? If there are 10 rows?

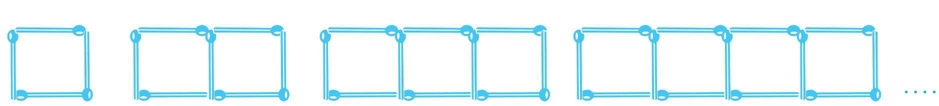
8. Leela is Radha's younger sister. Leela is 4 years younger than Radha. Can you write Leela's age in terms of Radha's age? Take Radha's age to be x years.

Fig 11.5

9. Mother has made laddus. She gives some laddus to guests and family members; still 5 laddus remain. If the number of laddus mother gave away is l, how many laddus did she make?

10. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?

11. (a) Look at the following matchstick pattern of squares (Fig 11.6). The squares are not separate. Two neighbouring squares have a common matchstick. Observe the patterns and find the rule that gives the number of matchsticks



(a) (b) (c) (d)

Fig 11.6

in terms of the number of squares. (Hint : If you remove the vertical stick at the end, you will get a pattern of Cs.)

(b) Fig 11.7 gives a matchstick pattern of triangles. As in Exercise 11 (a) above, find the general rule that gives the number of matchsticks in terms of the number of triangles.

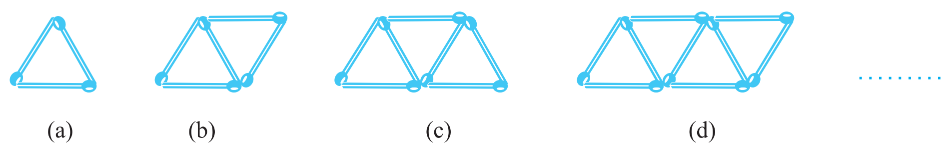


Fig 11.7

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,... . It is a variable, denoted by some letter like n.
2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.
3. We may use any letter $n, l, m, p, x, y, z,$ etc. to show a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3$, $x + 3$, $2n$, $5m$, $\frac{p}{3}$, $2y + 3$, $3l - 5$, etc.

Ratio and Proportion

Chapter 12

12.1 Introduction

In our daily life, many a times we compare two quantities of the same type. For example, Avnee and Shari collected flowers for scrap notebook. Avnee collected 30 flowers and Shari collected 45 flowers. So, we may say that Shari collected $45 - 30 = 15$ flowers more than Avnee.

Also, if height of Rahim is 150 cm and that of Avnee is 140 cm then, we may say that the height of Rahim is 150 cm – 140 cm = 10 cm more than Avnee. This is one way of comparison by taking difference.

If we wish to compare the lengths of an ant and a grasshopper, taking the difference does not express the comparison. The grasshopper's length, typically 4 cm to 5 cm is too long as compared to the ant's length which is a few mm. Comparison will be better if we try to find that how many ants can be placed one behind the other to match the length of grasshopper. So, we can say that 20 to 30 ants have the same length as a grasshopper.

Consider another example.

Cost of a car is ₹ 2,50,000 and that of a motorbike is ₹ 50,000. If we calculate the difference between the costs, it is ₹ 2,00,000 and if we compare by division; i.e. $\dfrac{2,50,000}{50,000} = \dfrac{5}{1}$

We can say that the cost of the car is five times the cost of the motorbike. Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference. The comparison by division is the Ratio. In the next section, we shall learn more about 'Ratios'.

12.2 Ratio

Consider the following:

Isha's weight is 25 kg and her father's weight is 75 kg. How many times Father's weight is of Isha's weight? It is three times.

Cost of a pen is ₹ 10 and cost of a pencil is ₹ 2. How many times the cost of a pen that of a pencil? Obviously it is five times.

In the above examples, we compared the two quantities in terms of 'how many times'. This comparison is known as the Ratio. We denote ratio using symbol ':'

Consider the earlier examples again. We can say,

The ratio of father's weight to Isha's weight = $\frac{75}{25} = \frac{3}{1} = 3:1$

The ratio of the cost of a pen to the cost of a pencil = $\frac{10}{2} = \frac{5}{1} = 5:1$

Let us look at this problem.

In a class, there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.

(b) Number of boys to the total number of students.

First we need to find the total number of students, which is,

Number of girls + Number of boys = 20 + 40 = 60.

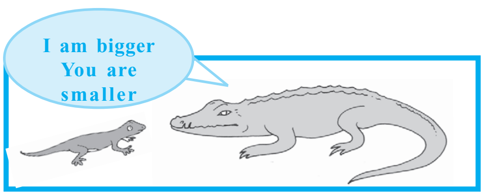
Then, the ratio of number of girls to the total number of students is $\frac{40}{60} = \frac{2}{3} = 2:3$

Find the answer of part (b) in the similar manner.

Now consider the following example.

Length of a house lizard is 20 cm and the length of a crocodile is 4 m.

"I am 5 times bigger than you", says the lizard. As we can see this



Try These

1. In a class, there are 20 boys and 40 girls. What is the ratio of the number of boys to the number of girls?
2. Ravi walks 6 km in an hour while Roshan walks 4 km in an hour. What is the ratio of the distance covered by Ravi to the distance covered by Roshan?

is really absurd. A lizard's length cannot be 5 times of the length of a crocodile. So, what is wrong? Observe that the length of the lizard is in centimetres and length of the crocodile is in metres. So, we have to convert their lengths into the same unit.

Length of the crocodile = 4 m = 4 × 100 = 400 cm.

Therefore, ratio of the length of the crocodile to the length of the lizard

$$= \frac{400}{20} = \frac{20}{1} = 20:1.$$

Two quantities can be compared only if they are in the same unit.

Now what is the ratio of the length of the lizard to the length of the crocodile?

It is $\frac{20}{400} = \frac{1}{20} = 1:20$.

Observe that the two ratios 1 : 20 and 20 : 1 are different from each other. The ratio 1 : 20 is the ratio of the length of the lizard to the length of the crocodile whereas, 20 : 1 is the ratio of the length of the crocodile to the length of the lizard.

Now consider another example.

Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length? Since the length and the diameter of the pencil are given in different units, we first need to convert them into same unit.

Thus, length of the pencil = 18 cm = 18 × 10 mm = 180 mm.

The ratio of the diameter of the pencil to that of the length of the pencil

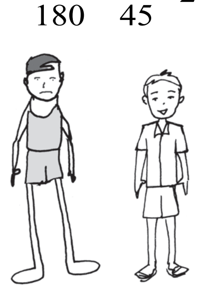
$$= \frac{8}{180} = \frac{2}{45} = 2:45.$$

Try These

1. Saurabh takes 15 minutes to reach school from his house and Sachin takes one hour to reach school from his house. Find the ratio of the time taken by Saurabh to the time taken by Sachin.
2. Cost of a toffee is 50 paise and cost of a chocolate is ₹ 10. Find the ratio of the cost of a toffee to the cost of a chocolate.
3. In a school, there were 73 holidays in one year. What is the ratio of the number of holidays to the number of days in one year?

Think of some more situations where you compare two quantities of same type in different units.

We use the concept of ratio in many situations of our daily life without realising that we do so.

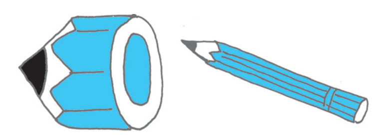


A B

Compare the drawings A and B. B looks more natural than A. Why?

The legs in the picture A are too long in comparison to the other body parts. This is because we normally expect a certain ratio of the length of legs to the length of whole body.

Compare the two pictures of a pencil. Is the first one looking like a full pencil? No.



Why not? The reason is that the thickness and the length of the pencil are not in the correct ratio.

Same ratio in different situations :

Consider the following :

- Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room = $\frac{30}{20}=\frac{3}{2}=3:2$
- There are 24 girls and 16 boys going for a picnic. Ratio of the number of girls to the number of boys = $\frac{24}{16}=\frac{3}{2}=3:2$

  The ratio in both the examples is 3 : 2.
- Note the ratios 30 : 20 and 24 : 16 in lowest form are same as 3 : 2. These are equivalent ratios.
- Can you think of some more examples having the ratio 3 : 2?

  It is fun to write situations that give rise to a certain ratio. For example, write situations that give the ratio 2 : 3.
- Ratio of the breadth of a table to the length of the table is 2 : 3.
- Sheena has 2 marbles and her friend Shabnam has 3 marbles.

Then, the ratio of marbles that Sheena and Shabnam have is 2 : 3.

Can you write some more situations for this ratio? Give any ratio to your friends and ask them to frame situations.

Ravi and Rani started a business and invested money in the ratio 2 : 3. After one year the total profit was ₹ 4,00,000.

Ravi said "we would divide it equally", Rani said "I should get more as I have invested more".

It was then decided that profit will be divided in the ratio of their investment.

Here, the two terms of the ratio 2 : 3 are 2 and 3.

Sum of these terms = 2 + 3 = 5

What does this mean?

This means if the profit is ₹ 5 then Ravi should get ₹ 2 and Rani should get ₹ 3. Or, we can say that Ravi gets 2 parts and Rani gets 3 parts out of the 5 parts.

i.e., Ravi should get $\frac{2}{5}$ of the total profit and Rani should get $\frac{3}{5}$ of the total profit.

If the total profit were ₹ 500

Ravi would get ₹ $\frac{2}{5} \times 500 =$ ₹ 200

and Rani would get $\frac{3}{5} \times 500 =$ ₹ 300

Now, if the profit were ₹ 4,00,000 could you find the share of each?

Ravi's share = ₹ $\frac{2}{5} \times 4{,}00{,}000$ = ₹ 1,60,000

And Rani's share = ₹ $\frac{3}{5} \times 4{,}00{,}000$ = ₹ 2,40,000

Can you think of some more examples where you have to divide a number of things in some ratio? Frame three such examples and ask your friends to solve them.

Let us look at the kind of problems we have solved so far.

Try These

1. Find the ratio of number of notebooks to the number of books in your bag.
2. Find the ratio of number of desks and chairs in your classroom.
3. Find the number of students above twelve years of age in your class. Then, find the ratio of number of students with age above twelve years and the remaining students.
4. Find the ratio of number of doors and the number of windows in your classroom.
5. Draw any rectangle and find the ratio of its length to its breadth.

Example 1 : Length and breadth of a rectangular field are 50 m and 15 m respectively. Find the ratio of the length to the breadth of the field.

Solution : Length of the rectangular field = 50 m

Breadth of the rectangular field = 15 m

The ratio of the length to the breadth is 50 : 15

The ratio can be written as $\frac{50}{15} = \frac{50 \div 5}{15 \div 5} = \frac{10}{3} = 10 : 3$

Thus, the required ratio is 10 : 3.

Example 2 : Find the ratio of 90 cm to 1.5 m.

Solution : The two quantities are not in the same units. Therefore, we have to convert them into same units.

1.5 m = 1.5 × 100 cm = 150 cm.

Therefore, the required ratio is 90 : 150.

$$= \frac{90}{150} = \frac{90 \times 30}{150 \times 30} = \frac{3}{5}$$

Required ratio is 3 : 5.

Example 3 : There are 45 persons working in an office. If the number of females is 25 and the remaining are males, find the ratio of:

(a) The number of females to number of males.

(b) The number of males to number of females.

Solution : Number of females = 25

Total number of workers = 45

Number of males = 45 – 25 = 20

Therefore, the ratio of number of females to the number of males

= 25 : 20 = 5 : 4

And the ratio of number of males to the number of females

= 20 : 25 = 4 : 5.

(Notice that there is a difference between the two ratios 5 : 4 and 4 : 5).

Example 4 : Give two equivalent ratios of 6 : 4.

Solution : Ratio 6 : 4 = $\frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{12}{8}$.

Therefore, 12 : 8 is an equivalent ratio of 6 : 4

Similarly, the ratio 6 : 4 = $\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$

So, 3:2 is another equivalent ratio of 6 : 4.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Write two more equivalent ratios of 6 : 4.

Example 5 : Fill in the missing numbers :

$$\frac{14}{21} = \frac{\square}{3} = \frac{6}{\square}$$

Solution : In order to get the first missing number, we consider the fact that $21 = 3 \times 7$. i.e. when we divide 21 by 7 we get 3. This indicates that to get the missing number of second ratio, 14 must also be divided by 7.

When we divide, we have, 14 ÷ 7 = 2

Hence, the second ratio is $\frac{2}{3}$.

Similarly, to get third ratio we multiply both terms of second ratio by 3. (Why?)

Hence, the third ratio is $\frac{6}{9}$

Therefore, $\frac{14}{21} = \frac{\boxed{2}}{3} = \frac{6}{\boxed{9}}$ [These are all equivalent ratios.]

Example 6 : Ratio of distance of the school from Mary's home to the distance of the school from John's home is 2 : 1.

(a) Who lives nearer to the school?

(b) Complete the following table which shows some possible distances that Mary and John could live from the school.

Distance from Mary's home to school (in km.)	10	$\boxed{}$	4	$\boxed{}$	$\boxed{}$
Distance from John's home to school (in km.)	5	4	$\boxed{}$	3	1

(c) If the ratio of distance of Mary's home to the distance of Kalam's home from school is 1 : 2, then who lives nearer to the school?

Solution : (a) John lives nearer to the school (As the ratio is 2 : 1).

(b)

Distance from Mary's home to school (in km.)	10	$\boxed{8}$	4	$\boxed{6}$	$\boxed{2}$
Distance from John's home to school (in km.)	5	4	$\boxed{2}$	3	1

(c) Since the ratio is 1 : 2, so Mary lives nearer to the school.

Example 7 : Divide ₹ 60 in the ratio 1 : 2 between Kriti and Kiran.

Solution : The two parts are 1 and 2.

Therefore, sum of the parts = 1 + 2 = 3.

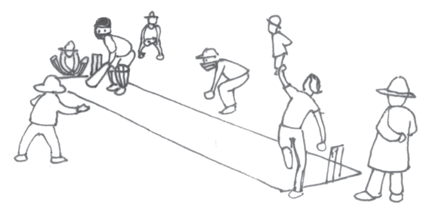
This means if there are ₹ 3, Kriti will get ₹ 1 and Kiran will get ₹ 2. Or, we can say that Kriti gets 1 part and Kiran gets 2 parts out of every 3 parts.

Therefore, Kriti's share $= \frac{1}{3} \times 60 =$ ₹ 20

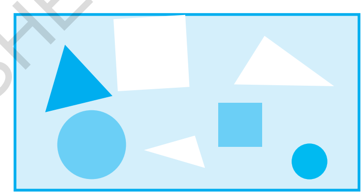
And Kiran's share $= \frac{2}{3} \times 60 =$ ₹ 40.

EXERCISE 12.1

1. There are 20 girls and 15 boys in a class.
   (a) What is the ratio of number of girls to the number of boys?
   (b) What is the ratio of number of girls to the total number of students in the class?
2. Out of 30 students in a class, 6 like football, 12 like cricket and remaining like tennis. Find the ratio of
   (a) Number of students liking football to number of students liking tennis.
   (b) Number of students liking cricket to total number of students.



3. See the figure and find the ratio of
   (a) Number of triangles to the number of circles inside the rectangle.
   (b) Number of squares to all the figures inside the rectangle.
   (c) Number of circles to all the figures inside the rectangle.



4. Distances travelled by Hamid and Akhtar in an hour are 9 km and 12 km. Find the ratio of speed of Hamid to the speed of Akhtar.
5. Fill in the following blanks :

$$\frac{15}{18} = \frac{\square}{6} = \frac{10}{\square} = \frac{\square}{30}$$ [Are these equivalent ratios?]

6. Find the ratio of the following :
   (a) 81 to 108 (b) 98 to 63
   (c) 33 km to 121 km (d) 30 minutes to 45 minutes
7. Find the ratio of the following :
   (a) 30 minutes to 1.5 hours (b) 40 cm to 1.5 m
   (c) 55 paise to ₹ 1 (d) 500 mL to 2 litres
8. In a year, Seema earns ₹ 1,50,000 and saves ₹ 50,000. Find the ratio of
   (a) Money that Seema earns to the money she saves.
   (b) Money that she saves to the money she spends.
9. There are 102 teachers in a school of 3300 students. Find the ratio of the number of teachers to the number of students.
10. In a college, out of 4320 students, 2300 are girls. Find the ratio of
    (a) Number of girls to the total number of students.
    (b) Number of boys to the number of girls.

(c) Number of boys to the total number of students.

11. Out of 1800 students in a school, 750 opted basketball, 800 opted cricket and remaining opted table tennis. If a student can opt only one game, find the ratio of
    (a) Number of students who opted basketball to the number of students who opted table tennis.
    (b) Number of students who opted cricket to the number of students opting basketball.
    (c) Number of students who opted basketball to the total number of students.
12. Cost of a dozen pens is ₹ 180 and cost of 8 ball pens is ₹ 56. Find the ratio of the cost of a pen to the cost of a ball pen.
13. Consider the statement: Ratio of breadth and length of a hall is 2 : 5. Complete the following table that shows some possible breadths and lengths of the hall.
14. Divide 20 pens between Sheela and Sangeeta in the ratio of 3 : 2.

Breadth of the hall (in metres)	10	$\boxed{}$	40
Length of the hall (in metres)	25	50	$\boxed{}$

15. Mother wants to divide ₹ 36 between her daughters Shreya and Bhoomika in the ratio of their ages. If age of Shreya is 15 years and age of Bhoomika is 12 years, find how much Shreya and Bhoomika will get?

16. Present age of father is 42 years and that of his son is 14 years. Find the ratio of
    (a) Present age of father to the present age of son.
    (b) Age of the father to the age of son, when son was 12 years old.
    (c) Age of father after 10 years to the age of son after 10 years.
    (d) Age of father to the age of son when father was 30 years old.

12.3 Proportion

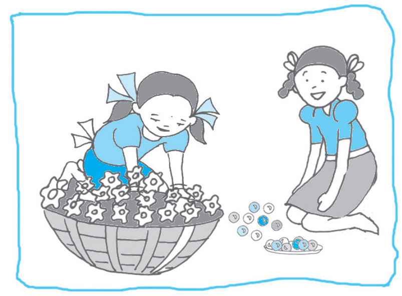
Consider this situation :

Raju went to the market to purchase tomatoes. One shopkeeper tells him that the cost of tomatoes is ₹ 40 for 5 kg. Another shopkeeper gives the cost as 6 kg for ₹ 42. Now, what should Raju do? Should he purchase tomatoes from the first shopkeeper or from the second? Will the comparison by taking the difference help him decide? No. Why not?

Think of some way to help him. Discuss with your friends.

Consider another example.

Bhavika has 28 marbles and Vini has 180 flowers. They want to share these among themselves. Bhavika gave 14 marbles to Vini and Vini gave 90



flowers to Bhavika. But Vini was not satisfied. She felt that she had given more flowers to Bhavika than the marbles given by Bhavika to her.

What do you think? Is Vini correct?

To solve this problem both went to Vini's mother Pooja.

Pooja explained that out of 28 marbles, Bhavika gave 14 marbles to Vini.

Therefore, ratio is 14 : 28 = 1 : 2.

And out of 180 flowers, Vini had given 90 flowers to Bhavika.

Therefore, ratio is 90 : 180 = 1 : 2.

Since both the ratios are the same, so the distribution is fair.

Two friends Ashma and Pankhuri went to market to purchase hair clips. They purchased 20 hair clips for ₹ 30. Ashma gave ₹ 12 and Pankhuri gave ₹ 18. After they came back home, Ashma asked Pankhuri to give 10 hair clips to her. But Pankhuri said, "since I have given more money so I should get more clips. You should get 8 hair clips and I should get 12".

Can you tell who is correct, Ashma or Pankhuri? Why?

Ratio of money given by Ashma to the money given by Pankhuri

= ₹ 12 : ₹ 18 = 2 : 3

According to Ashma's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 10 : 10 = 1 : 1

According to Pankhuri's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 8 : 12 = 2 : 3

Now, notice that according to Ashma's distribution, ratio of hair clips and the ratio of money given by them is not the same. But according to the Pankhuri's distribution the two ratios are the same.

Hence, we can say that Pankhuri's distribution is correct.

Sharing a ratio means something!

Consider the following examples :

- Raj purchased 3 pens for ₹ 15 and Anu purchased 10 pens for ₹ 50. Whose pens are more expensive?

  Ratio of number of pens purchased by Raj to the number of pens purchased by Anu = 3 : 10.

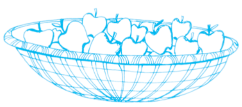
  Ratio of their costs = 15 : 50 = 3 : 10

  Both the ratios 3 : 10 and 15 : 50 are equal. Therefore, the pens were purchased for the same price by both.

- Rahim sells 2 kg of apples for ₹ 180 and Roshan sells 4 kg of apples for ₹ 360. Whose apples are more expensive?

  Ratio of the weight of apples = 2 kg : 4 kg = 1 : 2

  Ratio of their cost = ₹ 180 : ₹ 360 = 6 : 12 = 1 : 2



So, the ratio of weight of apples = ratio of their cost.

Since both the ratios are equal, hence, we say that they are in proportion. They are selling apples at the same rate.

If two ratios are equal, we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

For the first example, we can say 3, 10, 15 and 50 are in proportion which is written as 3 : 10 :: 15 : 50 and is read as 3 is to 10 as 15 is to 50 or it is written as 3 : 10 = 15 : 50.

For the second example, we can say 2, 4, 180 and 360 are in proportion which is written as 2 : 4 :: 180 : 360 and is read as 2 is to 4 as 180 is to 360.

Let us consider another example.

A man travels 35 km in 2 hours. With the same speed would he be able to travel 70 km in 4 hours?

Now, ratio of the two distances travelled by the man is 35 to 70 = 1 : 2 and the ratio of the time taken to cover these distances is 2 to 4 = 1 : 2 .

Hence, the two ratios are equal i.e. 35 : 70 = 2 : 4.

Therefore, we can say that the four numbers 35, 70, 2 and 4 are in proportion.

Hence, we can write it as 35 : 70 :: 2 : 4 and read it as 35 is to 70 as 2 is to 4. Hence, he can travel 70 km in 4 hours with that speed.

Now, consider this example.

Cost of 2 kg of apples is ₹ 180 and a 5 kg watermelon costs ₹ 45.

Now, ratio of the weight of apples to the weight of watermelon is 2 : 5.

And ratio of the cost of apples to the cost of the watermelon is 180 : 45 = 4 : 1.

Here, the two ratios 2 : 5 and 180 : 45 are not equal, i.e. 2 : 5 $\neq$ 180 : 45

Therefore, the four quantities 2, 5, 180 and 45 are not in proportion.

Try These

Check whether the given ratios are equal, i.e. they are in proportion.

If yes, then write them in the proper form.

1. 1 : 5 and 3 : 15
2. 2 : 9 and 18 : 81
3. 15 : 45 and 5 : 25
4. 4 : 12 and 9 : 27
5. ₹ 10 to ₹ 15 and 4 to 6

If two ratios are not equal, then we say that they are not in proportion. In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as extreme terms. Second and third terms are known as middle terms.

For example, in 35 : 70 : : 2 : 4;

35, 70, 2, 4 are the four terms. 35 and 4 are the extreme terms. 70 and 2 are the middle terms.

Example 8 : Are the ratios 25g : 30g and 40 kg : 48 kg in proportion?

Solution : 25 g : 30 g = $\frac{25}{30}$ = 5 : 6

40 kg : 48 kg = $\frac{40}{48}$ = 5 : 6 So, 25 : 30 = 40 : 48.

Therefore, the ratios 25 g : 30 g and 40 kg : 48 kg are in proportion, i.e. 25 : 30 :: 40 : 48

The middle terms in this are 30, 40 and the extreme terms are 25, 48.

Example 9 : Are 30, 40, 45 and 60 in proportion?

Solution : Ratio of 30 to 40 = $\frac{30}{40}$ = 3 : 4.

Ratio of 45 to 60 = $\frac{45}{60}$ = 3 : 4.

Since, 30 : 40 = 45 : 60.

Therefore, 30, 40, 45, 60 are in proportion.

Example 10 : Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution : Ratio of 15 cm to 2 m = 15 : 2 × 100 (1 m = 100 cm)
= 3 : 40

Ratio of 10 sec to 3 min = 10 : 3 × 60 (1 min = 60 sec)
= 1 : 18

Since, 3 : 40 ≠ 1 : 18, therefore, the given ratios do not form a proportion.

EXERCISE 12.2

1. Determine if the following are in proportion.

(a) 15, 45, 40, 120 (b) 33, 121, 9,96 (c) 24, 28, 36, 48

(d) 32, 48, 70, 210 (e) 4, 6, 8, 12 (f) 33, 44, 75, 100

2. Write True (T) or False (F) against each of the following statements :

(a) 16 : 24 :: 20 : 30 (b) 21: 6 :: 35 : 10 (c) 12 : 18 :: 28 : 12

(d) 8 : 9 :: 24 : 27 (e) 5.2 : 3.9 :: 3 : 4 (f) 0.9 : 0.36 :: 10 : 4

3. Are the following statements true?
   (a) 40 persons : 200 persons = ₹ 15 : ₹ 75
   (b) 7.5 litres : 15 litres = 5 kg : 10 kg
   (c) 99 kg : 45 kg = ₹ 44 : ₹ 20
   (d) 32 m : 64 m = 6 sec : 12 sec
   (e) 45 km : 60 km = 12 hours : 15 hours
4. Determine if the following ratios form a proportion. Also, write the middle terms and extreme terms where the ratios form a proportion.
   (a) 25 cm : 1 m and ₹ 40 : ₹ 160 (b) 39 litres : 65 litres and 6 bottles : 10 bottles
   (c) 2 kg : 80 kg and 25 g : 625 g (d) 200 mL : 2.5 litre and ₹ 4 : ₹ 50

12.4 Unitary Method

Consider the following situations:

- Two friends Reshma and Seema went to market to purchase notebooks. Reshma purchased 2 notebooks for ₹ 24. What is the price of one notebook?
- A scooter requires 2 litres of petrol to cover 80 km. How many litres of petrol is required to cover 1 km?

These are examples of the kind of situations that we face in our daily life. How would you solve these?

Reconsider the first example: Cost of 2 notebooks is ₹ 24.

Therefore, cost of 1 notebook = ₹ 24 ÷ 2 = ₹ 12.

Now, if you were asked to find cost of 5 such notebooks. It would be = ₹ 12 × 5 = ₹ 60

Reconsider the second example: We want to know how many litres are needed to travel 1 km.

For 80 km, petrol needed = 2 litres.

Therefore, to travel 1 km, petrol needed = $\frac{2}{80} = \frac{1}{40}$ litres.

Now, if you are asked to find how many litres of petrol are required to cover 120 km?

Then petrol needed = $\frac{1}{40} \times 120$ litres = 3 litres.

The method in which first we find the value of one unit and then the value of required number of units is known as Unitary Method.

Try These

1. Prepare five similar problems and ask your friends to solve them.
2. Read the table and fill in the boxes.

Time	Distance travelled by Karan	Distance travelled by Kriti
2 hours	8 km	6 km
1 hour	4 km	$\boxed{}$
4 hours	$\boxed{}$	$\boxed{}$

We see that,
Distance travelled by Karan in 2 hours = 8 km

Distance travelled by Karan in 1 hour $= \frac{8}{2}$ km = 4 km

Therefore, distance travelled by Karan in 4 hours = 4 × 4 = 16 km

Similarly, to find the distance travelled by Kriti in 4 hours, first find the distance travelled by her in 1 hour.

Example 11 : If the cost of 6 cans of juice is ₹ 210, then what will be the cost of 4 cans of juice?

Solution : Cost of 6 cans of juice = ₹ 210

Therefore, cost of one can of juice $= \frac{210}{6}$ = ₹ 35

Therefore, cost of 4 cans of juice = ₹ 35 × 4 = ₹ 140.

Thus, cost of 4 cans of juice is ₹ 140.

Example 12 : A motorbike travels 220 km in 5 litres of petrol. How much distance will it cover in 1.5 litres of petrol?

Solution : In 5 litres of petrol, motorbike can travel 220 km.

Therefore, in 1 litre of petrol, motor bike travels $= \frac{220}{5}$ km

Therefore, in 1.5 litres, motorbike travels $= \frac{220}{5} \times 1.5$ km

$$= \frac{220}{5} \times \frac{15}{10} \text{ km} = 66 \text{ km}.$$

Thus, the motorbike can travel 66 km in 1.5 litres of petrol.

Example 13 : If the cost of a dozen soaps is ₹ 153.60, what will be the cost of 15 such soaps?

Solution : We know that 1 dozen = 12

Since, cost of 12 soaps = ₹ 153.60

Therefore, cost of 1 soap $= \frac{153.60}{12} = ₹\ 12.80$

Therefore, cost of 15 soaps = ₹ 12.80 × 15 = ₹ 192

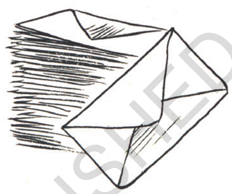
Thus, cost of 15 soaps is ₹ 192.

Example 14 : Cost of 105 envelopes is ₹ 350. How many envelopes can be purchased for ₹ 100?

Solution : In ₹ 350, the number of envelopes that can be purchased = 105

Therefore, in ₹ 1, number of envelopes that can be purchased $= \frac{105}{350}$

Therefore, in ₹ 100, the number of envelopes that can be purchased $= \frac{105}{350} \times 100 = 30$



Thus, 30 envelopes can be purchased for ₹ 100.

Example 15 : A car travels 90 km in $2\frac{1}{2}$ hours.

(a) How much time is required to cover 30 km with the same speed?

(b) Find the distance covered in 2 hours with the same speed.

Solution : (a) In this case, time is unknown and distance is known. Therefore, we proceed as follows :

$2\frac{1}{2}$ hours $= \frac{5}{2}$ hours $= \frac{5}{2} \times 60$ minutes = 150 minutes.

90 km is covered in 150 minutes

Therefore, 1 km can be covered in $\frac{150}{90}$ minutes

Therefore, 30 km can be covered in $\frac{150}{90} \times 30$ minutes i.e., 50 minutes

Thus, 30 km can be covered in 50 minutes.

(b) In this case, distance is unknown and time is known. Therefore, we proceed as follows :

Distance covered in $2\frac{1}{2}$ hours (i.e., $\frac{5}{2}$ hours) = 90 km

Therefore, distance covered in 1 hour $= 90 \div \frac{5}{2}$ km $= 90 \times \frac{2}{5} = 36$ km

Therefore, distance covered in 2 hours = 36 × 2 = 72 km.

Thus, in 2 hours, distance covered is 72 km.

EXERCISE 12.3

1. If the cost of 7 m of cloth is ₹ 1470, find the cost of 5 m of cloth.
2. Ekta earns ₹ 3000 in 10 days. How much will she earn in 30 days?
3. If it has rained 276 mm in the last 3 days, how many cm of rain will fall in one full week (7 days)? Assume that the rain continues to fall at the same rate.
4. Cost of 5 kg of wheat is ₹ 91.50.
   (a) What will be the cost of 8 kg of wheat?
   (b) What quantity of wheat can be purchased in ₹ 183?
5. The temperature dropped 15 degree celsius in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?
6. Shaina pays ₹ 15000 as rent for 3 months. How much does she has to pay for a whole year, if the rent per month remains same?
7. Cost of 4 dozen bananas is ₹ 180. How many bananas can be purchased for ₹ 90?
8. The weight of 72 books is 9 kg. What is the weight of 40 such books?
9. A truck requires 108 litres of diesel for covering a distance of 594 km. How much diesel will be required by the truck to cover a distance of 1650 km?
10. Raju purchases 10 pens for ₹ 150 and Manish buys 7 pens for ₹ 84. Can you say who got the pens cheaper?
11. Anish made 42 runs in 6 overs and Anup made 63 runs in 7 overs. Who made more runs per over?

What have we discussed?

1. For comparing quantities of the same type, we commonly use the method of taking difference between the quantities.
2. In many situations, a more meaningful comparison between quantities is made by using division, i.e. by seeing how many times one quantity is to the other quantity. This method is known as comparison by ratio.

   For example, Isha's weight is 25 kg and her father's weight is 75 kg. We say that Isha's father's weight and Isha's weight are in the ratio 3 : 1.
3. For comparison by ratio, the two quantities must be in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.
4. The same ratio may occur in different situations.
5. Note that the ratio 3 : 2 is different from 2 : 3. Thus, the order in which quantities are taken to express their ratio is important.

6. A ratio may be treated as a fraction, thus the ratio 10 : 3 may be treated as $\frac{10}{3}$.

7. Two ratios are equivalent, if the fractions corresponding to them are equivalent. Thus, 3 : 2 is equivalent to 6 : 4 or 12 : 8.

8. A ratio can be expressed in its lowest form. For example, ratio 50 : 15 is treated as $\frac{50}{15}$; in its lowest form $\frac{50}{15} = \frac{10}{3}$. Hence, the lowest form of the ratio 50 : 15 is 10 : 3.

9. Four quantities are said to be in proportion, if the ratio of the first and the second quantities is equal to the ratio of the third and the fourth quantities. Thus, 3, 10, 15, 50 are in proportion, since $\frac{3}{10} = \frac{15}{50}$. We indicate the proportion by 3:10 :: 15:50, it is read as 3 is to 10 as 15 is to 50. In the above proportion, 3 and 50 are the extreme terms and 10 and 15 are the middle terms.

10. The order of terms in the proportion is important. 3, 10, 15 and 50 are in proportion, but 3, 10, 50 and 15 are not, since $\frac{3}{10}$ is not equal to $\frac{50}{15}$.

11. The method in which we first find the value of one unit and then the value of the required number of units is known as the unitary method. Suppose the cost of 6 cans is ₹ 210. To find the cost of 4 cans, using the unitary method, we first find the cost of 1 can. It is ₹ $\frac{210}{6}$ or ₹ 35. From this, we find the price of 4 cans as ₹ 35 × 4 or ₹ 140.

ANSWERS

EXERCISE 7.1

1. (i) $\frac{2}{4}$ (ii) $\frac{8}{9}$ (iii) $\frac{4}{8}$ (iv) $\frac{1}{4}$ (v) $\frac{3}{7}$ (vi) $\frac{3}{12}$

(vii) $\frac{10}{10}$ (viii) $\frac{4}{9}$ (ix) $\frac{4}{8}$ (x) $\frac{1}{2}$

3. Shaded portions do not represent the given fractions.

4. $\frac{8}{24}$ **5.** $\frac{40}{60}$

6. (a) Arya will divide each sandwich into three equal parts, and give one part of each sandwich to each one of them.

(b) $\frac{1}{3}$ **7.** $\frac{2}{3}$

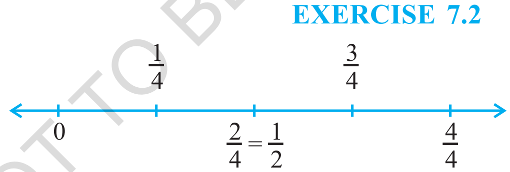
8. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; $\frac{5}{11}$

9. 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113; $\frac{4}{12}$

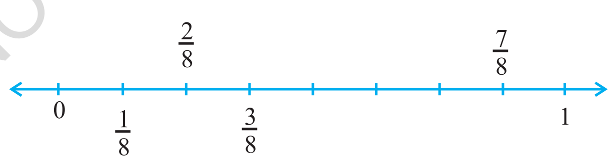
10. $\frac{4}{8}$ **11.** $\frac{3}{8}, \frac{5}{8}$

EXERCISE 7.2

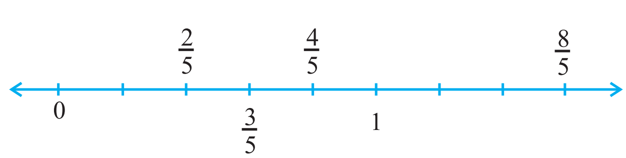
1. (a)



(b)



(c)



2. (a) $6\frac{2}{3}$ (b) $2\frac{1}{5}$ (c) $2\frac{3}{7}$ (d) $5\frac{3}{5}$ (e) $3\frac{1}{6}$ (f) $3\frac{8}{9}$

3. (a) $\frac{31}{4}$ (b) $\frac{41}{7}$ (c) $\frac{17}{6}$ (d) $\frac{53}{5}$ (e) $\frac{66}{7}$ (f) $\frac{76}{9}$

EXERCISE 7.3

1. (a) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; Yes (b) $\frac{4}{12}, \frac{3}{9}, \frac{2}{6}, \frac{1}{3}, \frac{6}{15}$; No

2. (a) $\frac{1}{2}$ (b) $\frac{4}{6}$ (c) $\frac{3}{9}$ (d) $\frac{2}{8}$ (e) $\frac{3}{4}$ (i) $\frac{6}{18}$

(ii) $\frac{4}{8}$ (iii) $\frac{12}{16}$ (iv) $\frac{8}{12}$ (v) $\frac{4}{16}$

(a), (ii); (b), (iv); (c), (i); (d), (v); (e), (iii)

3. (a) 28 (b) 16 (c) 12 (d) 20 (e) 3

4. (a) $\frac{12}{20}$ (b) $\frac{9}{15}$ (c) $\frac{18}{30}$ (d) $\frac{27}{45}$

5. (a) $\frac{9}{12}$ (b) $\frac{3}{4}$

6. (a) equivalent (b) not equivalent (c) not equivalent

7. (a) $\frac{4}{5}$ (b) $\frac{5}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{13}$ (e) $\frac{1}{4}$

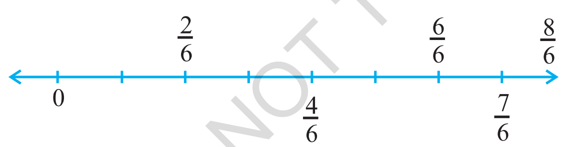
8. Ramesh $\rightarrow \frac{10}{20} = \frac{1}{2}$, Sheelu $\rightarrow \frac{25}{50} = \frac{1}{2}$, Jamaal $\rightarrow \frac{40}{80} = \frac{1}{2}$. Yes

9. (i) $\rightarrow$ (d) (ii) $\rightarrow$ (e) (iii) $\rightarrow$ (a) (iv) $\rightarrow$ (c) (v) $\rightarrow$ (b)

EXERCISE 7.4

1. (a) $\frac{1}{8} < \frac{3}{8} < \frac{4}{8} < \frac{6}{8}$ (b) $\frac{3}{9} < \frac{4}{9} < \frac{6}{9} < \frac{8}{9}$

(c)



$\frac{5}{6} > \frac{2}{6}, \frac{3}{6} > \frac{0}{6}, \frac{1}{6} < \frac{6}{6}, \frac{8}{6} > \frac{5}{6}$

2. (a) $\frac{3}{6} < \frac{5}{6}$ (b) $\frac{1}{7} < \frac{1}{4}$ (c) $\frac{4}{5} < \frac{5}{5}$ (d) $\frac{3}{5} > \frac{3}{7}$

4. (a) $\frac{1}{6} < \frac{1}{3}$ (b) $\frac{3}{4} > \frac{2}{6}$ (c) $\frac{2}{3} > \frac{2}{4}$ (d) $\frac{6}{6} = \frac{3}{3}$

(e) $\frac{5}{6} < \frac{5}{5}$

5. (a) $\frac{1}{2} > \frac{1}{5}$ (b) $\frac{2}{4} = \frac{3}{6}$ (c) $\frac{3}{5} < \frac{2}{3}$ (d) $\frac{3}{4} > \frac{2}{8}$

(e) $\frac{3}{5} < \frac{6}{5}$ (f) $\frac{7}{9} > \frac{3}{9}$ (g) $\frac{1}{4} = \frac{2}{8}$ (h) $\frac{6}{10} < \frac{4}{5}$

(i) $\frac{3}{4} < \frac{7}{8}$ (j) $\frac{6}{10} = \frac{3}{5}$ (k) $\frac{5}{7} = \frac{15}{21}$

6. (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{4}{25}$ (d) $\frac{4}{25}$ (e) $\frac{1}{6}$ (f) $\frac{1}{5}$

(g) $\frac{1}{5}$ (h) $\frac{1}{6}$ (i) $\frac{4}{25}$ (j) $\frac{1}{6}$ (k) $\frac{1}{6}$ (l) $\frac{4}{25}$

(a), (e), (h), (j), (k) ; (b), (f), (g) ; (c), (d), (i), (l)

7. (a) No ; $\frac{5}{9} = \frac{25}{45}, \frac{4}{5} = \frac{36}{45}$ and $\frac{25}{45} \neq \frac{36}{45}$

(b) No ; $\frac{9}{16} = \frac{81}{144}, \frac{5}{9} = \frac{80}{144}$ and $\frac{81}{144} \neq \frac{80}{144}$ (c) Yes ; $\frac{4}{5} = \frac{16}{20}$

(d) No ; $\frac{1}{15} = \frac{2}{30}$ and $\frac{2}{30} \neq \frac{4}{30}$

8. Ila has read less **9.** Rohit

10. Same fraction $\left(\frac{4}{5}\right)$ of students got first class in both the classes.

EXERCISE 7.5

1. (a) + (b) – (c) +

2. (a) $\frac{1}{9}$ (b) $\frac{11}{15}$ (c) $\frac{2}{7}$ (d) 1 (e) $\frac{1}{3}$

(f) 1 (g) $\frac{1}{3}$ (h) $\frac{1}{4}$ (i) $\frac{3}{5}$

3. The complete wall.

4. (a) $\frac{4}{10}\left(=\frac{2}{5}\right)$ (b) $\frac{8}{21}$ (c) $\frac{6}{6}$ (=1) (d) $\frac{7}{27}$

5. $\frac{2}{7}$

EXERCISE 7.6

1. (a) $\frac{17}{21}$ (b) $\frac{23}{30}$ (c) $\frac{46}{63}$ (d) $\frac{22}{21}$ (e) $\frac{17}{30}$

(f) $\frac{22}{15}$ (g) $\frac{5}{12}$ (h) $\frac{3}{6}\left(=\frac{1}{2}\right)$ (i) $\frac{23}{12}$ (j) $\frac{6}{6}$ (= 1) (k) 5

(l) $\frac{95}{12}$ (m) $\frac{9}{5}$ (n) $\frac{5}{6}$

2. $\frac{23}{20}$ metre **3.** $2\frac{5}{6}$

4. (a) $\frac{7}{8}$ (b) $\frac{7}{10}$ (c) $\frac{1}{3}$

5. (a)

$\frac{2}{3}$	$\frac{4}{3}$	2
$\frac{1}{3}$	$\frac{2}{3}$	1
$\frac{1}{3}$	$\frac{2}{3}$	1

(b)

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{7}{12}$
$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$

6. Length of the other piece = $\frac{5}{8}$ metre

7. The distance walked by Nandini = $\frac{4}{10}\left(=\frac{2}{5}\right)$ km

8. Asha's bookshelf is more full; by $\frac{13}{30}$

9. Rahul takes less time; by $\frac{9}{20}$ minutes

EXERCISE 8.1

1. (a) 0.4 (b) 0.07 (c) 3 (d) 0.5 (e) 1.23
(f) 0.19 (g) both are same (h) 1.490 (i) both are same (j) 5.64

EXERCISE 8.2

1. (a) ₹ 0.05 (b) ₹ 0.75 (c) ₹ 0.20 (d) ₹ 50.90 (e) ₹ 7.25
2. (a) 0.15 m (b) 0.06 m (c) 2.45 m (d) 9.07 m (e) 4.19 m
3. (a) 0.5 cm (b) 6.0 cm (c) 16.4 cm (d) 9.8 cm (e) 9.3 cm
4. (a) 0.008 km (b) 0.088 km (c) 8.888 km (d) 70.005 km
5. (a) 0.002 kg (b) 0.1 kg (c) 3.750 kg (d) 5.008 kg (e) 26.05 kg

EXERCISE 8.3

1. (a) 38.587 (b) 29.432 (c) 27.63 (d) 38.355 (e) 13.175 (f) 343.89
2. ₹ 68.35 **3.** ₹ 26.30 **4.** 5.25 m
5. 3.042 km **6.** 22.775 km **7.** 18.270 kg

EXERCISE 8.4

1. (a) ₹ 2.50 (b) 47.46 m (c) ₹ 3.04 (d) 3.155 km (e) 1.793 kg
2. (a) 3.476 (b) 5.78 (c) 11.71 (d) 1.753
3. ₹ 14.35 **4.** ₹ 6.75 **5.** 15.55 m
6. 9.850 km **7.** 4.425 kg

EXERCISE 9.1

1.

Marks	Tally marks	Number of students
1	||	2
2	|||	3
3	|||	3
4	卌 ||	7
5	卌 |	6
6	卌 ||	7
7	卌	5
8	||||	4
9	|||	3

(a) 12 (b) 8

2.

Sweets	Tally marks	Number of students
Ladoo	卌 卌 |	11
Barfi	|||	3
Jalebi	卌 ||	7
Rasgulla	卌 ||||	9
		30

(b) Ladoo

3.

Numbers	Tally marks	How many times?
1	卌 ||	7
2	卌 |	6
3	卌	5
4	||||	4
5	卌 卌 |	11
6	卌 ||	7

(a) 4 (b) 5 (c) 1 and 6

4. (i) Village D (ii) Village C (iii) 3 (iv) 28
5. (a) VIII (b) No (c) 12
6. (a) Number of bulbs sold on Friday are 14. Similarly, number of bulbs sold on other days can be found.

(b) Maximum number of bulbs were sold on Sunday.
(c) Same number of bulbs were sold on Wednesday and Saturday.
(d) Minimum number of bulbs were sold on Wednesday and Saturday.
(e) 10 Cartons

7. (a) Martin (b) 700 (c) Anwar, Martin, Ranjit Singh

EXERCISE 10.1

1. (a) 12 cm (b) 133 cm (c) 60 cm (d) 20 cm (e) 15 cm
(f) 52 cm **2.** 100 cm or 1 m **3.** 7.5 m **4.** 106 cm
5. 9.6 km **6.** (a) 12 cm (b) 27 cm (c) 22 cm
7. 39 cm **8.** 48 m **9.** 5 m **10.** 20 cm
11. (a) 7.5 cm (b) 10 cm (c) 5 cm **12.** 10 cm
13. ₹ 20,000 **14.** ₹ 7200 **15.** Bulbul
16. (a) 100 cm (b) 100 cm (c) 100 cm (d) 100 cm
All the figures have same perimeter.
17. (a) 6 m (b) 10 m (c) Cross has greater perimeter

EXERCISE 10.2

1. (a) 9 sq units (b) 5 sq units (c) 4 sq units (d) 8 sq units (e) 10 sq units
(f) 4 sq units (g) 6 sq units (h) 5 sq units (i) 9 sq units (j) 4 sq units
(k) 5 sq units (l) 8 sq units (m) 14 sq units (n) 18 sq units

EXERCISE 10.3

1. (a) 12 sq cm (b) 252 sq cm (c) 6 sq km (d) 1.40 sq m
2. (a) 100 sq cm (b) 196 sq cm (c) 25 sq m
3. (c) largest area (b) smallest area
4. 6 m **5.** ₹ 8000 **6.** 3 sq m **7.** 14 sq m
8. 11 sq m **9.** 15 sq m
10. (a) 28 sq cm (b) 9 sq cm
11. (a) 40 sq cm (b) 245 sq cm (c) 9 sq cm
12. (a) 240 tiles (b) 42 tiles

EXERCISE 11.1

1. (a) $2n$ (b) $3n$ (c) $3n$ (d) $2n$ (e) $5n$
(f) $5n$ (g) $6n$
2. (a) and (d); The number of matchsticks required in each of them is 2
3. $5n$ **4.** $50b$ **5.** $5s$
6. t km **7.** $8r$, 64, 80 **8.** $(x - 4)$ years **9.** $l + 5$
10. $2x + 10$
11. (a) $3x + 1$, x = number of squares
(b) $2x + 1$, x = number of triangles

EXERCISE 12.1

1.	(a) 4 : 3	(b) 4 : 7		
2.	(a) 1 : 2	(b) 2 : 5		
3.	(a) 3 : 2	(b) 2 : 7	(c) 2 : 7	
4.	3 : 4	5. 5, 12, 25, Yes		
6.	(a) 3 : 4	(b) 14 : 9	(c) 3 : 11	(d) 2 : 3
7.	(a) 1 : 3	(b) 4 : 15	(c) 11 : 20	(d) 1 : 4
8.	(a) 3 : 1	(b) 1 : 2		
9.	17 : 550			
10.	(a) 115 : 216	(b) 101 : 115	(c) 101 : 216	
11.	(a) 3 : 1	(b) 16 : 15	(c) 5 : 12	
12.	15 : 7	13. 20 ; 100	14. 12 and 8	15. ₹ 20 and ₹ 16
16.	(a) 3 : 1	(b) 10 : 3	(c) 13 : 6	(d) 15 : 1

EXERCISE 12.2

1.	(a) Yes	(b) No	(c) No	(d) No	
	(e) Yes	(f) Yes			
2.	(a) T	(b) T	(c) F	(d) T	
	(e) F	(f) T			
3.	(a) T	(b) T	(c) T	(d) T	(e) F

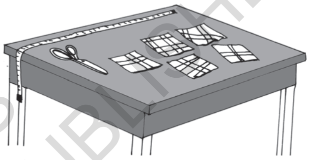
4. (a) Yes, Middle Terms – 1 m, ₹ 40; Extreme Terms – 25 cm, ₹ 160
   (b) Yes, Middle Terms – 65 litres, 6 bottles; Extreme Terms – 39 litres, 10 bottles
   (c) No.
   (d) Yes, Middle Terms – 2.5 litres, ₹ 4 ; Extreme Terms – 200 ml, ₹ 50

EXERCISE 12.3

1. ₹ 1,050	2. ₹ 9,000	3. 64.4 cm	
4. (a) ₹ 146.40	(b) 10 kg		
5. 5 degrees	6. ₹ 60, 000	7. 24 bananas	8. 5 kg
9. 300 litres	10. Manish	11. Anup	

BRAIN-TEASERS

1. From a basket of mangoes when counted in twos there was one extra, counted in threes there were two extra, counted in fours there were three extra, counted in fives there were four extra, counted in sixes there were five extra. But counted in sevens there were no extra. Atleast how many mangoes were there in the basket?
2. A boy was asked to find the LCM of 3, 5, 12 and another number. But while calculating, he wrote 21 instead of 12 and yet came with the correct answer. What could be the fourth number?
3. There were five pieces of cloth of lengths 15 m, 21 m, 36 m, 42 m, 48 m. But all of them could be measured in whole units of a measuring rod. What could be the largest length of the rod?



4. There are three cans. One of them holds exactly 10 litres of milk and is full. The other two cans can hold 7 litres and 3 litres respectively. There is no graduation mark on the cans. A customer asks for 5 litres of milk. How would you give him the amount he ask? He would not be satisfied by eye estimates.
5. Which two digit numbers when added to 27 get reversed?
6. Cement mortar was being prepared by mixing cement to sand in the ratio of 1:6 by volume. In a cement mortar of 42 units of volume, how much more cement needs to be added to enrich the mortar to the ratio 2:9?
7. In a solution of common salt in water, the ratio of salt to water was 30:70 as per weight. If we evaporate 100 grams of water from one kilogram of this solution, what will be the ratio of the salt to water by weight?
8. Half a swarm of bees went to collect honey from a mustard field. Three fourth of the rest went to a rose garden. The rest ten were still undecided. How many bees were there in all?

9. Fifteen children are sitting in a circle. They are asked to pass a handkerchief to the child next to the child immediately after them.

   The game stops once the handkerchief returns to the child it started from. This can be written as follows : 1→3→5→7→9→11→13→15→2→4→6→8 → 10→12 →14→1. Here, we see that every child gets the handkerchief.

(i) What would happen if the handkerchief were passed to the left leaving two children in between? Would every child get the handkerchief?

(ii) What if we leave three children in between? What do you see?

In which cases every child gets the handkerchief and in which cases not?

Try the same game with 16, 17, 18, 19, 20 children. What do you see?

10. Take two numbers 9 and 16. Divide 9 by 16 to get the remainder. What is the remainder when 2 × 9 is divided by 16, 3 × 9 divided by 16, 4 × 9 divided by 16, 5 × 9 divided by 16... 15 × 9 divided by 16. List the remainders. Take the numbers 12 and 14. List the remainders of 12, 12 × 2, 12 × 3, 12 × 4, 12 × 5, 12 × 6, 12 × 7, 12 × 8, 12 × 9, 12 × 10, 12 × 11, 12 × 12, 12 × 13 when divided by 14. Do you see any difference between above two cases?

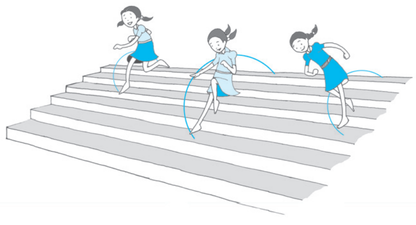
11. You have been given two cans with capacities 9 and 5 litres respectively. There is no graduation marks on the cans nor is eye estimation possible. How can you collect 3 litres of water from a tap? (You are allowed to pour out water from the can). If the cans had capacities 8 and 6 litres respectively, could you collect 5 litres?

12. The area of the east wall of an auditorium is 108 sq m, the area of the north wall is 135 sq m and the area of the floor is 180 sq m. Find the height of the auditorium.

13. If we subtract 4 from the digit at the units place of a two digit number and add 4 to the digit at the tens place then the resulting number is doubled. Find the number.

14. Two boatmen start simultaneously from the opposite shores of a river and they cross each other after 45 minutes of their starting from the respective shores. They rowed till they reached the opposite shore and returned immediately after reaching the shores. When will they cross each other again?

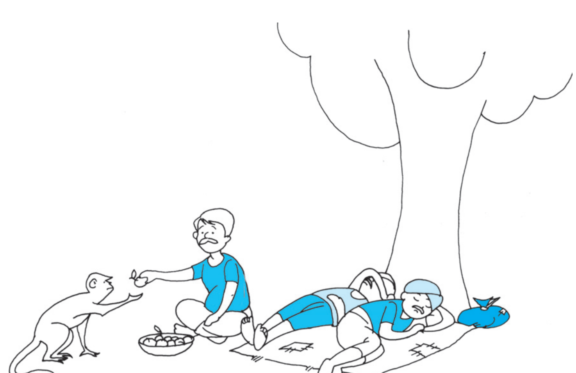
15. Three girls are climbing down a staircase. One girl climbs down two steps at one go. The second girl three steps at one go and the third climbs down four steps. They started together from the beginning of the staircase



leaving their foot marks. They all came down in complete steps and had their foot marks together at the bottom of the staircase. In how many steps would there be only one pair of foot mark?

Are there any steps on which there would be no foot marks.

16. A group of soldiers was asked to fall in line making rows of three. It was found that there was one soldier extra. Then they were asked to stand in rows of five. It was found there were left 2 soldiers. They were asked to stand in rows of seven. Then there were three soldiers who could not be adjusted. At least how many soldiers were there in the group?
17. Get 100 using four 9's and some of the symbols like +, –, ×, , etc.
18. How many digits would be in the product $2 \times 2 \times 2 \ldots\ldots \times 2$ (30 times)?
19. A man would be 5 minutes late to reach his destination if he rides his bike at 30 km. per hour. But he would be 10 minutes early if he rides at the speed of 40 km per hour. What is the distance of his destination from where he starts?
20. The ratio of speeds of two vehicles is 2:3. If the first vehicle covers 50 km in 3 hours, what distance would the second vehicle covers in 2 hours?
21. The ratio of income to expenditure of Mr. Natarajan is 7:5. If he saves ₹ 2000 a month, what could be his income?
22. The ratio of the length to breadth of a lawn is 3:5. It costs ₹ 3200 to fence it at the rate of ₹ 2 a metre. What would be the cost of developing the lawn at the rate of ₹10 per square metre.
23. If one counts one for the thumb, two for the index finger, three for the middle finger, four for the ring finger, five for the little finger and continues counting backwards, six for the ring finger, seven for the middle finger, eight for the index finger, 9 for the thumb, ten for the index finger, eleven for the middle finger, twelve for the ring finger, thirteen for the little finger, fourteen for the ring finger and so on. Which finger will be counted as one thousand?
24. Three friends plucked some mangoes from a mango grove and collected them together in a pile and took nap after that. After some time, one of the friends woke up and divided the mangoes into three equal numbers. There was one



mango extra. He gave it to a monkey nearby, took one part for himself and slept again. Next the second friend got up unaware of what has happened, divided the rest of the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and slept again. Next the third friend got up not knowing what happened and divided the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and went to sleep again. After some time, all of them got up together to find 30 mangoes. How many mangoes did the friends pluck initially?

25. **The peculiar number**

    There is a number which is very peculiar. This number is three times the sum of its digits. Can you find the number?

26. Ten saplings are to be planted in straight lines in such way that each line has exactly four of them.

27. What will be the next number in the sequence?
    (a) 1, 5, 9, 13, 17, 21, ...
    (b) 2, 7, 12, 17, 22, ...
    (c) 2, 6, 12, 20, 30, ...
    (d) 1, 2, 3, 5, 8, 13, ...
    (e) 1, 3, 6, 10, 15, ...

28. Observe the pattern in the following statement:

    $31 \times 39 = 13 \times 93$

    The two numbers on each side are co-prime and are obtained by **reversing the digits** of respective numbers. Try to write some more pairs of such numbers.

ANSWERS

1. 119
2. 28
3. 3 m
4. The man takes an empty vessel other than these.

   With the help of 3 litres can he takes out 9 litres of milk from the 10 litres can and pours it in the extra can. So, 1 litre milk remains in the 10 litres can. With the help of 7 litres can he takes out 7 litres of milk from the extra can and pours it in the 10 litres can. The 10 litres can now has $1 + 7 = 8$ litres of milk.

With the help of 3 litres can he takes out 3 litres milk from the 10 litres can. The 10 litres can now has 8 – 3 = 5 litres of milk, which he gives to the customer.

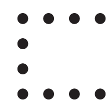
5. 14, 25, 36, 47, 58, 69
6. 2 units
7. 1 : 2
8. 80
9. (i) No, all children would not get it.

   (ii) All would get it.
10. 9, 2, 11, 4, 13, 6, 15, 8, 1, 10, 3, 12, 5, 14, 7.

    12, 10, 8, 6, 4, 2, 0, 12, 10, 8, 6, 4.
11. Fill the 9 litres can. Remove 5 litres from it using the 5 litres can. Empty the 5 litres can. Pour 4 litres remaining in the 9 litres can to the 5 litres can.

    Fill the 9 litres can again. Fill the remaining 5 litres can from the water in it. This leaves 8 litres in the 9 litres can. Empty the 5 litres can. Fill it from the 9 litres can. You now have 3 litres left in the 9 litres can.
12. Height = 9m
13. 36
14. 90 minutes
15. Steps with one pair of foot marks – 2, 3, 9, 10

    Steps with no foot marks – 1, 5, 7, 11
16. 52
17. $99 + \frac{9}{9}$
18. 10
19. 30 km
20. 50 km
21. ₹ 7000 per month

22. ₹ 15,00,000
23. Index finger
24. 106 mangoes
25. 27
26. One arrangement could be 
27. (a) 25 (b) 27 (c) 42 (d) 21 (e) 21
28. One such pair is $13 \times 62 = 31 \times 26$.