Assignment (4) Simple Linear Regression-1

Delivery_time -> Predict delivery time using sorting time

Build a simple linear regression model by performing EDA and do necessary transformations and select the best model using R or Python.

Importing libraries

In [2]:

```
import pandas as pd
import numpy as np
import scipy.stats as stats
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as smf
import statsmodels.formula.api as sm
import warnings
warnings.filterwarnings('ignore')
```

Step 1

Importing data

In [3]:
```
df = pd.read_csv('delivery_time.csv')
df
```

Out[3]:

	Delivery Time	Sorting Time
0	21.00	10
1	13.50	4
2	19.75	6
3	24.00	9
4	29.00	10
5	15.35	6
6	19.00	7
7	9.50	3
8	17.90	10
9	18.75	9
10	19.83	8
11	10.75	4
12	16.68	7
13	11.50	3
14	12.03	3
15	14.88	4
16	13.75	6
17	18.11	7
18	8.00	2
19	17.83	7
20	21.50	5

Step 2

Performing EDA On Data

Renaming columns

In [4]:

```
df1 = df.rename({'Delivery Time':'Delivery_Time','Sorting Time':'Sorting_Time
df1
```

Out[4]:

	Delivery_Time	Sorting_Time
0	21.00	10
1	13.50	4
2	19.75	6
3	24.00	9
4	29.00	10
5	15.35	6
6	19.00	7
7	9.50	3
8	17.90	10
9	18.75	9
10	19.83	8
11	10.75	4
12	16.68	7
13	11.50	3
14	12.03	3
15	14.88	4
16	13.75	6
17	18.11	7
18	8.00	2
19	17.83	7
20	21.50	5

Checking Datatype

In [5]:

```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 21 entries, 0 to 20
Data columns (total 2 columns):
 #   Column         Non-Null Count  Dtype  
---  ------         --------------  -----  
 0   Delivery Time  21 non-null     float64
 1   Sorting Time   21 non-null     int64  
dtypes: float64(1), int64(1)
memory usage: 464.0 bytes
```

In [6]:
```
df.describe()
```

Out[6]:

	Delivery Time	Sorting Time
count	21.000000	21.000000
mean	16.790952	6.190476
std	5.074901	2.542028
min	8.000000	2.000000
25%	13.500000	4.000000
50%	17.830000	6.000000
75%	19.750000	8.000000
max	29.000000	10.000000

Checking for Null Values

In [7]:
```
df.isnull().sum()
```

Out[7]:
```
Delivery Time    0
Sorting Time     0
dtype: int64
```

Checking for Duplicate Values

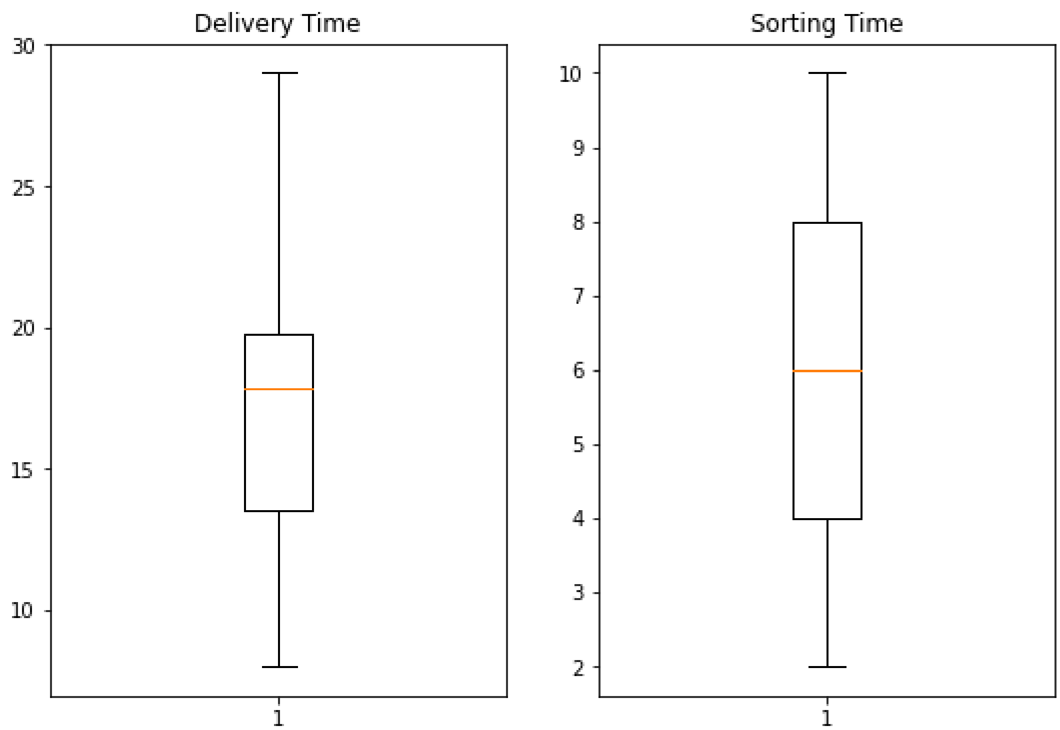
In [8]:
```
df[df.duplicated()].shape
```

Out[8]:
```
(0, 2)
```

Step 3

Plotting the data to check for outliers

In [9]:
```
plt.subplots(figsize = (9,6))
plt.subplot(121)
plt.boxplot(df['Delivery Time'])
plt.title('Delivery Time')
plt.subplot(122)
plt.boxplot(df['Sorting Time'])
plt.title('Sorting Time')
plt.show()
```



As you can see there are no Outliers in the data

Step 4

Checking the Correlation between variables

In [10]:
```
df.corr()
```

Out[10]:

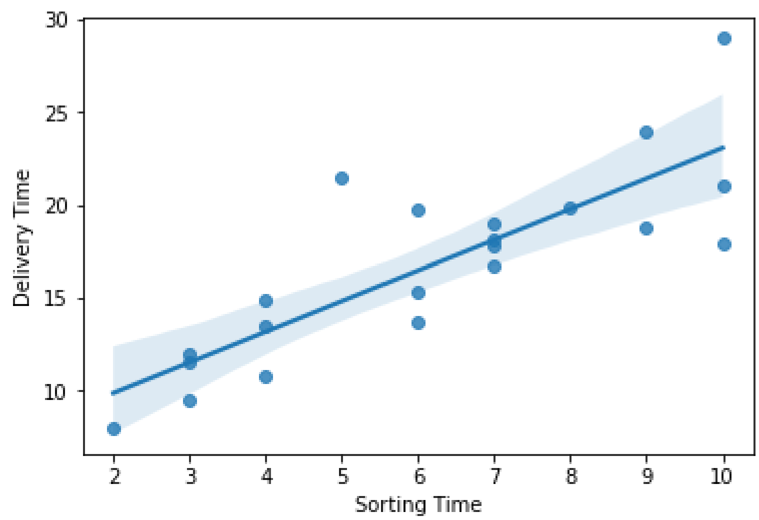
	Delivery Time	Sorting Time
Delivery Time	1.000000	0.825997
Sorting Time	0.825997	1.000000

Visualization of Correlation beteen x and y

regplot = regression plot

In [11]:
```
sns.regplot(x=df['Sorting Time'],y=df['Delivery Time'])
```

Out[11]: <AxesSubplot:xlabel='Sorting Time', ylabel='Delivery Time'>



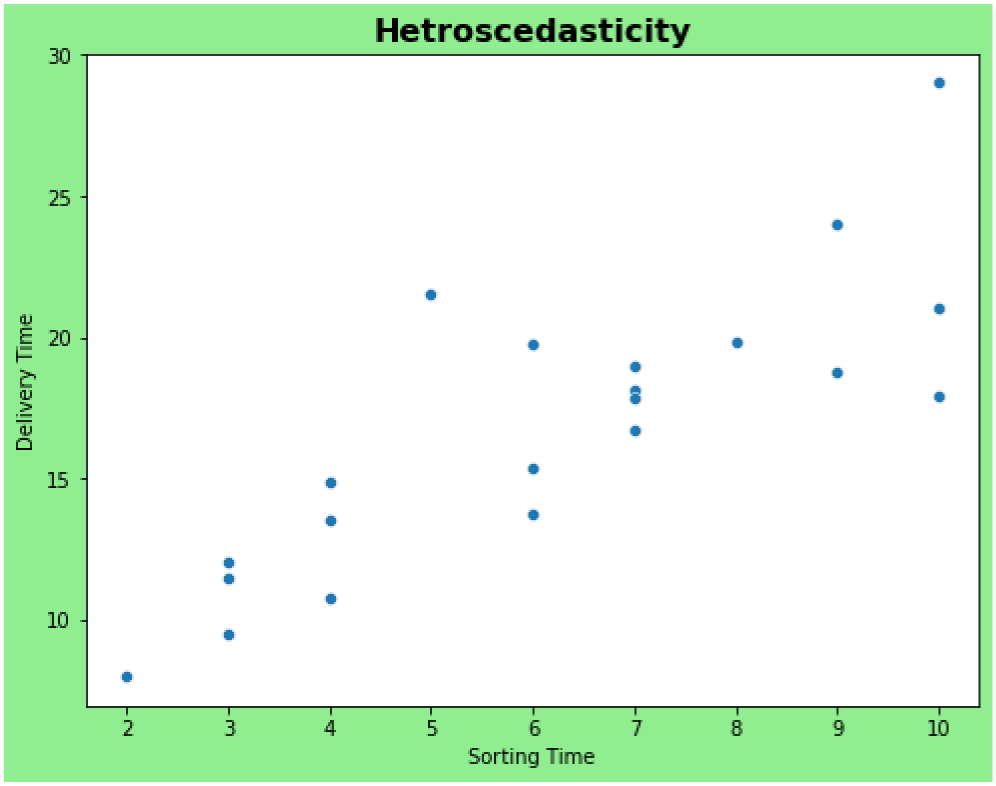
As you can see above

```
* There is good correlation between the two variabl
* The score is more than 0.8 which is a good sign
```

Step 5

Checking for Homoscedasticity or Hetroscedasticity

In [12]:
```
plt.figure(figsize=(8,6),facecolor='lightgreen')
sns.scatterplot(x=df['Sorting Time'],y=df['Delivery Time'])
plt.title('Hetroscedasticity',fontweight = 'bold', fontsize = 16)
plt.show()
```



In [13]:
```
df.var()
```

Out[13]:
```
Delivery Time    25.754619
Sorting Time      6.461905
dtype: float64
```

As you can see in above graph

```
 * It shows as the Sorting Time Increases Delivery Time also increases w
ith much variation along the way
 * The data doesn't have any specific pattern in the variation, but we c
an't say the variation is homoscedasticity
```

Step 6

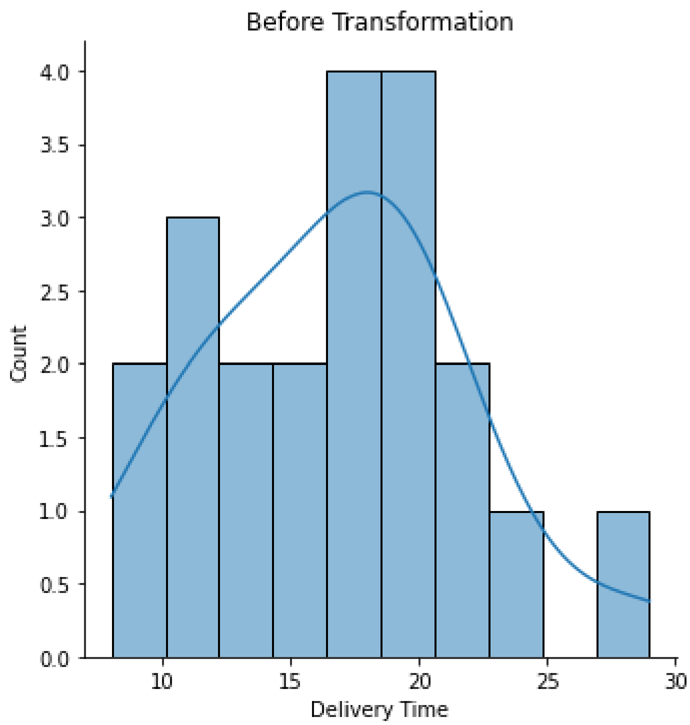
Feature Engineering

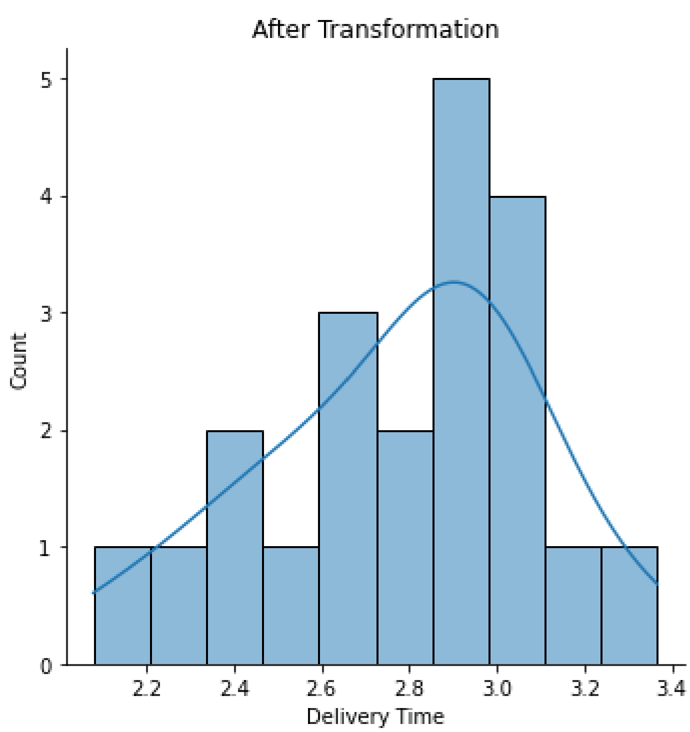
Feature Engineering

Trying different transformation of data to estimate normal distribution and to remove any skewness

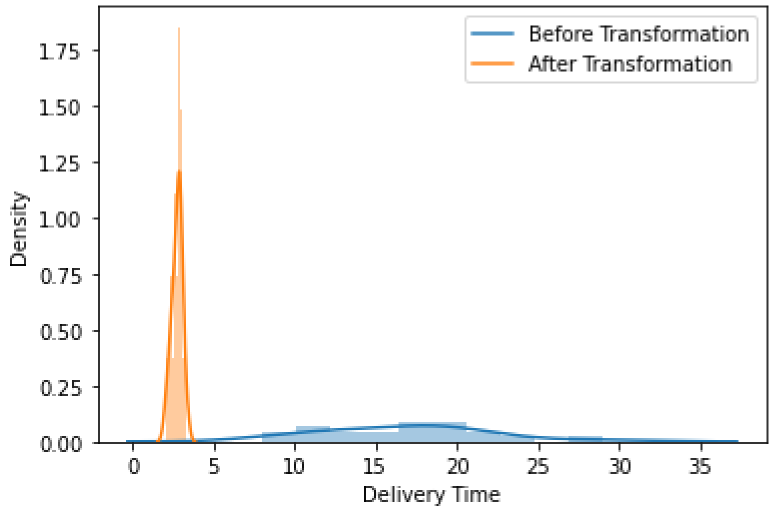
In [14]:

```
sns.displot(df['Delivery Time'],bins = 10,kde= True)
plt.title('Before Transformation')
sns.displot(np.log(df['Delivery Time']),bins = 10,kde= True)
plt.title('After Transformation')
plt.show()
```





In [15]:
```
labels = ['Before Transformation','After Transformation']
sns.distplot(df['Delivery Time'], bins = 10, kde = True)
sns.distplot(np.log(df['Delivery Time']), bins = 10, kde = True)
plt.legend(labels)
plt.show()
```

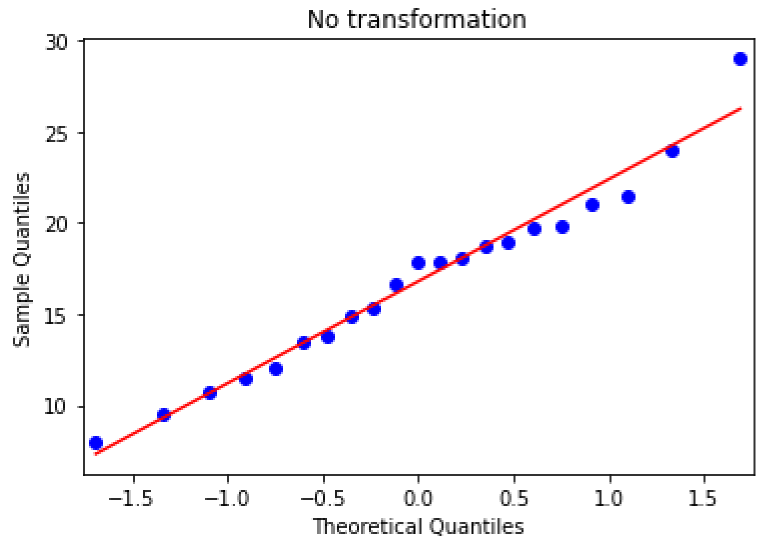


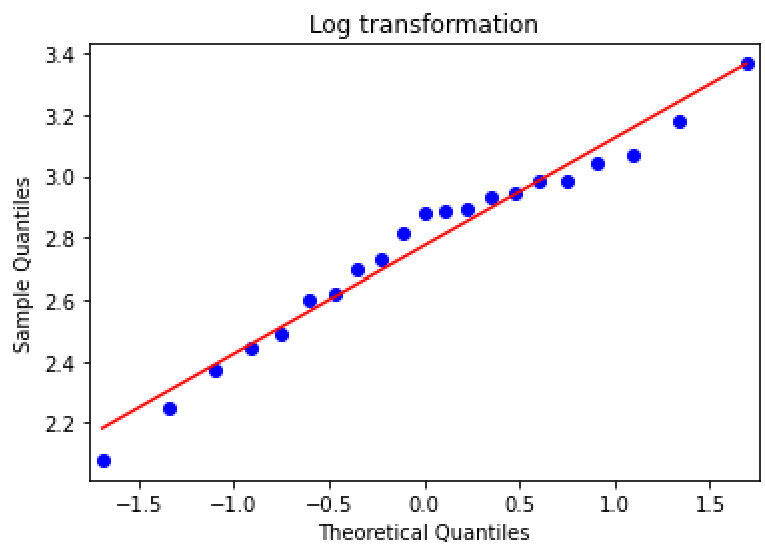
As you can see

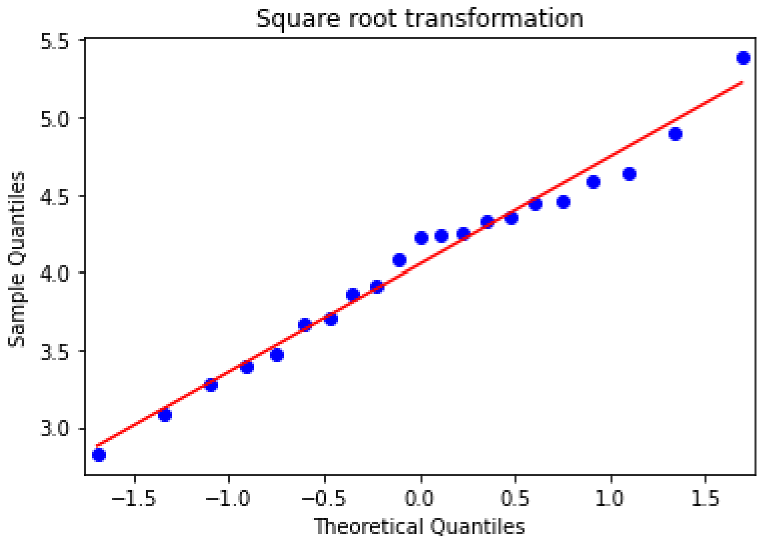
How log transformation affects the data and it scales the values down. Before prediction it is necessary to reverse scaled the values, even for calculating RMSE for the models.(Errors)

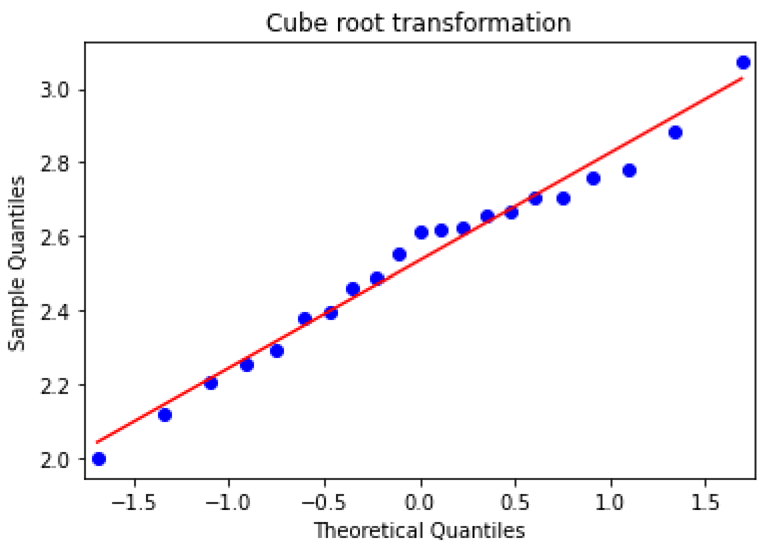
In [16]:

```
smf.qqplot(df['Delivery Time'], line = 'r')
plt.title('No transformation')
smf.qqplot(np.log(df['Delivery Time']), line = 'r')
plt.title('Log transformation')
smf.qqplot(np.sqrt(df['Delivery Time']), line = 'r')
plt.title('Square root transformation')
smf.qqplot(np.cbrt(df['Delivery Time']), line = 'r')
plt.title('Cube root transformation')
plt.show()
```



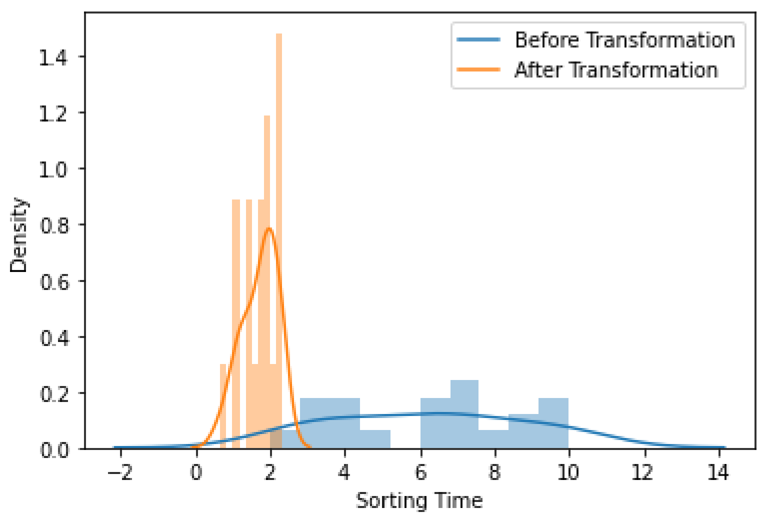




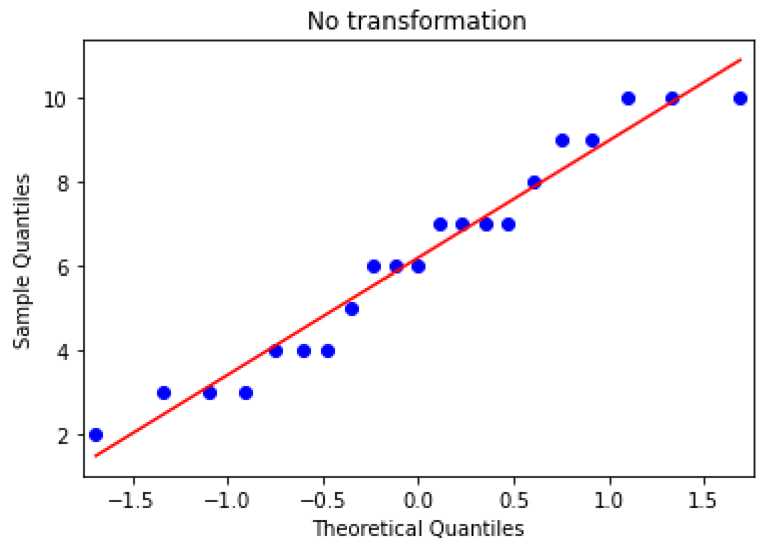


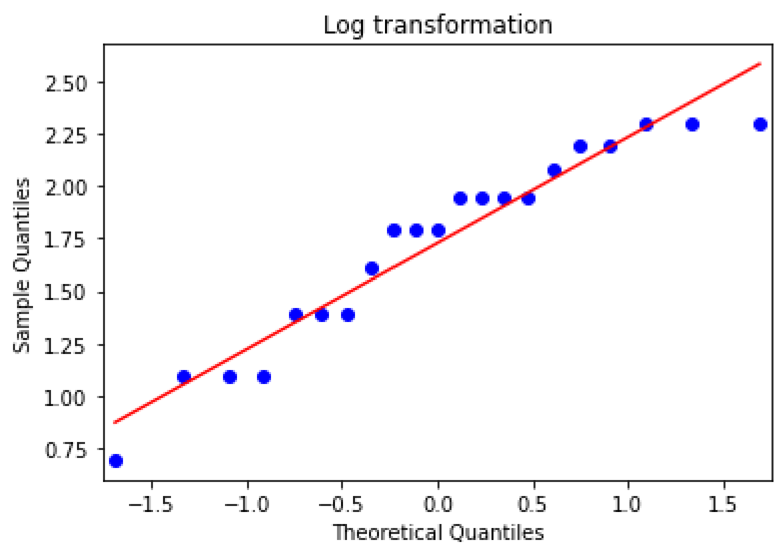
In [17]:

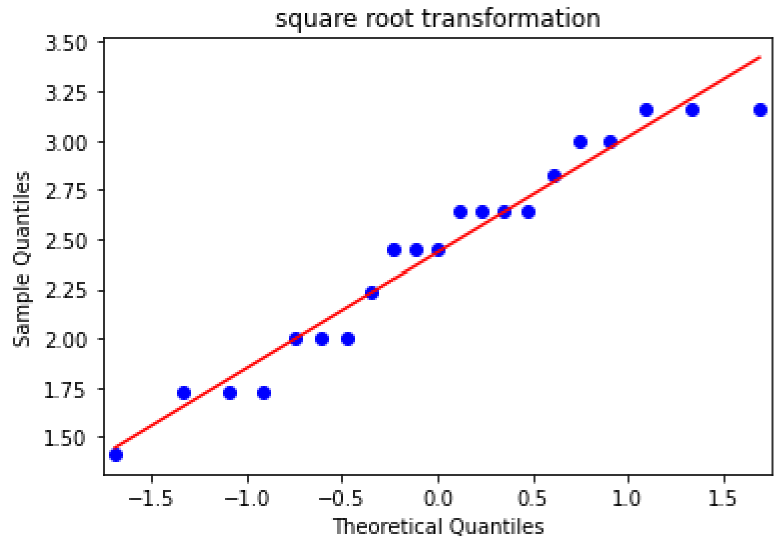
```
labels = ['Before Transformation','After Transformation']
sns.distplot(df['Sorting Time'], bins = 10, kde = True)
sns.distplot(np.log(df['Sorting Time']), bins = 10, kde = True)
plt.legend(labels)
plt.show()
```

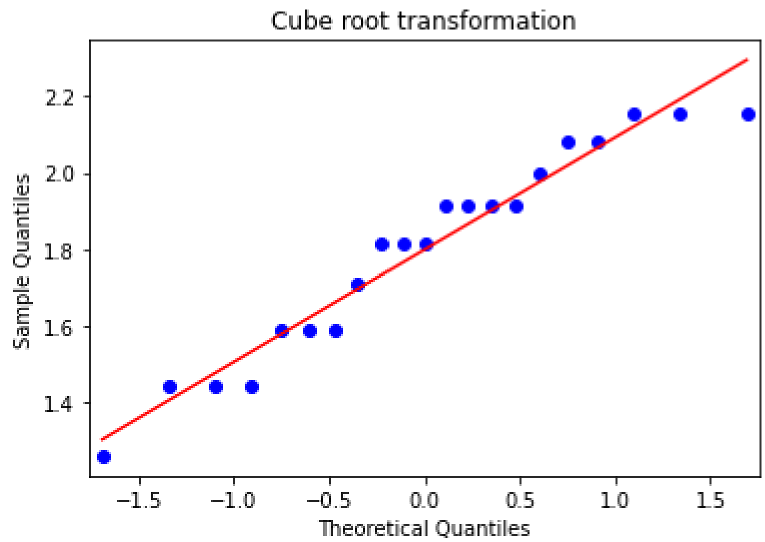


In [18]:
```
smf.qqplot(df['Sorting Time'], line = 'r')
plt.title('No transformation')
smf.qqplot(np.log(df['Sorting Time']), line = 'r')
plt.title('Log transformation')
smf.qqplot(np.sqrt(df['Sorting Time']), line = 'r')
plt.title('square root transformation')
smf.qqplot(np.cbrt(df['Sorting Time']), line = 'r')
plt.title('Cube root transformation')
plt.show()
```









Important Note:

```
We only Perform any data transformation when the data is skewed or not n
ormal
```

Step 7

Fitting a Linear Regression Model

Using Ordinary least squares (OLS) regression

It is a statistical method of analysis that estimates the relationship between one or more independent variables and a dependent variable; the method estimates the relationship by minimizing the sum of the squares in the difference between the observed and predicted values of the dependent variable configured as a straight line

In [19]:
```
model= sm.ols('Delivery_Time~Sorting_Time',data=df1).fit()
```

In [20]:
```
model.summary()
```

Out[20]:

OLS Regression Results

Dep. Variable:	Delivery_Time	**R-squared:**	0.682
Model:	OLS	**Adj. R-squared:**	0.666
Method:	Least Squares	**F-statistic:**	40.80
Date:	Wed, 08 Jun 2022	**Prob (F-statistic):**	3.98e-06
Time:	12:07:41	**Log-Likelihood:**	-51.357
No. Observations:	21	**AIC:**	106.7
Df Residuals:	19	**BIC:**	108.8
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	6.5827	1.722	3.823	0.001	2.979	10.186
Sorting_Time	1.6490	0.258	6.387	0.000	1.109	2.189

Omnibus:	3.649	**Durbin-Watson:**	1.248
Prob(Omnibus):	0.161	**Jarque-Bera (JB):**	2.086
Skew:	0.750	**Prob(JB):**	0.352
Kurtosis:	3.367	**Cond. No.**	18.3

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

As you can notice in the above model

```
The R-squared and Adjusted R-squared scores are still below 0.85.
(It is a thumb rule to consider Adjusted R-squared to be greater than 0.
8 for a good model for prediction)
Lets Try some data transformation to check whether these scores can get
 any higher than this.
```

Square Root transformation on data

In [21]:

```
model1 = sm.ols('np.sqrt(Delivery_Time)~np.sqrt(Sorting_Time)', data = df1).f
model1.summary()
```

Out[21]:

OLS Regression Results

Dep. Variable:	np.sqrt(Delivery_Time)	**R-squared:**	0.729
Model:	OLS	**Adj. R-squared:**	0.715
Method:	Least Squares	**F-statistic:**	51.16
Date:	Wed, 08 Jun 2022	**Prob (F-statistic):**	8.48e-07
Time:	12:07:42	**Log-Likelihood:**	-5.7320
No. Observations:	21	**AIC:**	15.46
Df Residuals:	19	**BIC:**	17.55
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.6135	0.349	4.628	0.000	0.884	2.343
np.sqrt(Sorting_Time)	1.0022	0.140	7.153	0.000	0.709	1.295

Omnibus:	2.869	**Durbin-Watson:**	1.279
Prob(Omnibus):	0.238	**Jarque-Bera (JB):**	1.685
Skew:	0.690	**Prob(JB):**	0.431
Kurtosis:	3.150	**Cond. No.**	13.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

As you can notice in the above model

```
After Square Root transformation on the Data, R-squared and Adjusted R-s
quared scores have increased but they are still          below 0.85 which
is a thumb rule we consider for a good model for prediction.
Lets Try other data transformation to check whether these scores can get
any higher than this.
```

Cube Root transformation on Data

In [22]:
```
model2 = sm.ols('np.cbrt(Delivery_Time)~np.cbrt(Sorting_Time)', data = df1).f
model2.summary()
```

Out[22]:

OLS Regression Results

Dep. Variable:	np.cbrt(Delivery_Time)	**R-squared:**	0.744
Model:	OLS	**Adj. R-squared:**	0.731
Method:	Least Squares	**F-statistic:**	55.25
Date:	Wed, 08 Jun 2022	**Prob (F-statistic):**	4.90e-07
Time:	12:07:42	**Log-Likelihood:**	13.035
No. Observations:	21	**AIC:**	-22.07
Df Residuals:	19	**BIC:**	-19.98
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.0136	0.207	4.900	0.000	0.581	1.447
np.cbrt(Sorting_Time)	0.8456	0.114	7.433	0.000	0.607	1.084

Omnibus:	2.570	**Durbin-Watson:**	1.292
Prob(Omnibus):	0.277	**Jarque-Bera (JB):**	1.532
Skew:	0.661	**Prob(JB):**	0.465
Kurtosis:	3.075	**Cond. No.**	16.4

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

As you can notice in the above model

```
After Cueb root transformation on the Data, R-squared and Adjusted R-squ
ared scores have increased but they are still below     0.85 which is a
 thumb rule we consider for a good model for prediction.
Lets Try other data transformation to check whether these scores can get
any higher than this.
```

Log transformation on Data

In [23]:
```
model3 = sm.ols('np.log(Delivery_Time)~np.log(Sorting_Time)', data = df1).fit
model3.summary()
```

Out[23]:

OLS Regression Results

Dep. Variable:	np.log(Delivery_Time)	**R-squared:**	0.772
Model:	OLS	**Adj. R-squared:**	0.760
Method:	Least Squares	**F-statistic:**	64.39
Date:	Wed, 08 Jun 2022	**Prob (F-statistic):**	1.60e-07
Time:	12:07:43	**Log-Likelihood:**	10.291
No. Observations:	21	**AIC:**	-16.58
Df Residuals:	19	**BIC:**	-14.49
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.7420	0.133	13.086	0.000	1.463	2.021
np.log(Sorting_Time)	0.5975	0.074	8.024	0.000	0.442	0.753

Omnibus:	1.871	**Durbin-Watson:**	1.322
Prob(Omnibus):	0.392	**Jarque-Bera (JB):**	1.170
Skew:	0.577	**Prob(JB):**	0.557
Kurtosis:	2.916	**Cond. No.**	9.08

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

As you can notice in the above model

```
* After log transformation on the Data, This Model has scored the highes
t R-squared and Adjusted R-squared scores
  than the previous model
* Yet both Adjusted R-squared and R-squared scores are still below 0.85
 which is a thumb rule we consider for a
  good model for prediction.
* Though it is now close to 0.8 which for a single feature/predictor var
iable or single independent variable is
  expected to be low. Hence , we can stop here.
```

Model Testing

As Y = Beta0 + Beta1*(X)

Finding Coefficient Parameters (Beta0 and Beta1 values)

In [24]:
```
model.params
```

Out[24]:
```
Intercept       6.582734
Sorting_Time    1.649020
dtype: float64
```

Here, (Intercept) Beta0 value = 6.58 & (Sorting Time) Beta1 value = 1.64

Hypothesis testing of X variable by finding test_statistics and P_values for Beta1 i.e if (P_value < α=0.05 ; Reject Null)

Null Hypothesis as Beta1=0 (No Slope) and Alternate Hypthesis as Beta1≠0 (Some or significant Slope)

In [25]:
```
print(model.tvalues,'\n',model.pvalues)
```

```
Intercept       3.823349
Sorting_Time    6.387447
dtype: float64
 Intercept       0.001147
Sorting_Time    0.000004
dtype: float64
```

(Intercept) Beta0: tvalue=3.82 , pvalue=0.001147

(daily) Beta1: tvalue=6.38, pvalue=0.000004

As (pvalue=0)<(α=0.05); Reject Null hyp. Thus, X(Sorting Time) variable has good slope and variance w.r.t Y(Delivery Time) variable.

R-squared measures the strength of the relationship between your model and the dependent variable on a 0 – 100% scale.

Measure goodness-of-fit by finding rsquared values (percentage of variance)

In [26]:
```
model.rsquared,model.rsquared_adj
```

Out[26]:
```
(0.6822714748417231, 0.6655489208860244)
```

Determination Coefficient = rsquared value = 0.68 ; very good fit >= 85%

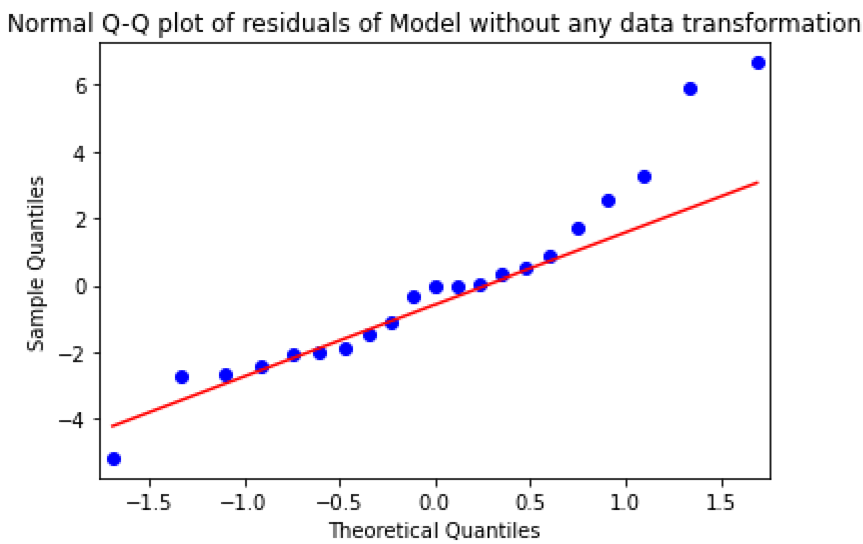
Step 8

Residual Analysis

Test for Normality of Residuals (Q-Q Plot)

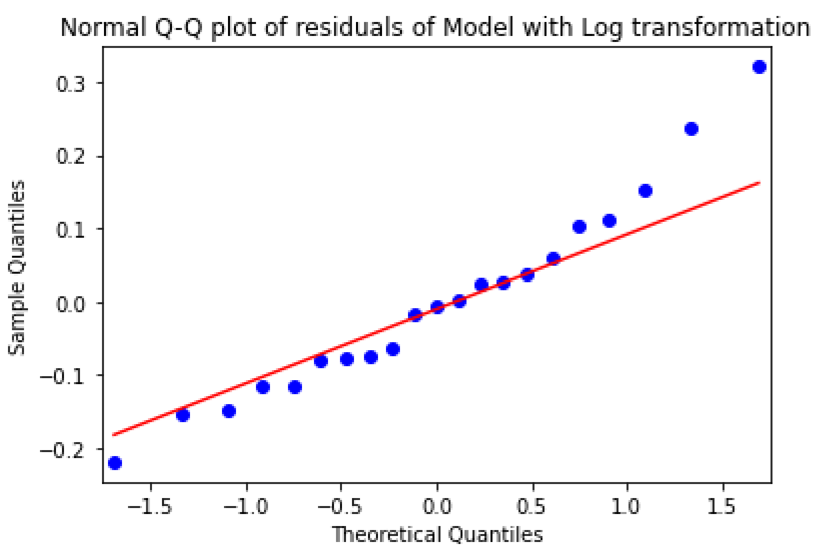
In [27]:

```
import statsmodels.api as sm
sm.qqplot(model.resid,line='q')
plt.title('Normal Q-Q plot of residuals of Model without any data transformat
plt.show()
```



In [28]:

```
sm.qqplot(model2.resid, line = 'q')
plt.title('Normal Q-Q plot of residuals of Model with Log transformation')
plt.show()
```



As you can notice in the above plots

As you can notice in the above plots

```
Both The Model have slightly different plots
The first model is right skewed and doesn't follow normal distribution
The second model after log-transformation follows normal distributon wit
h less skewness than first model
```

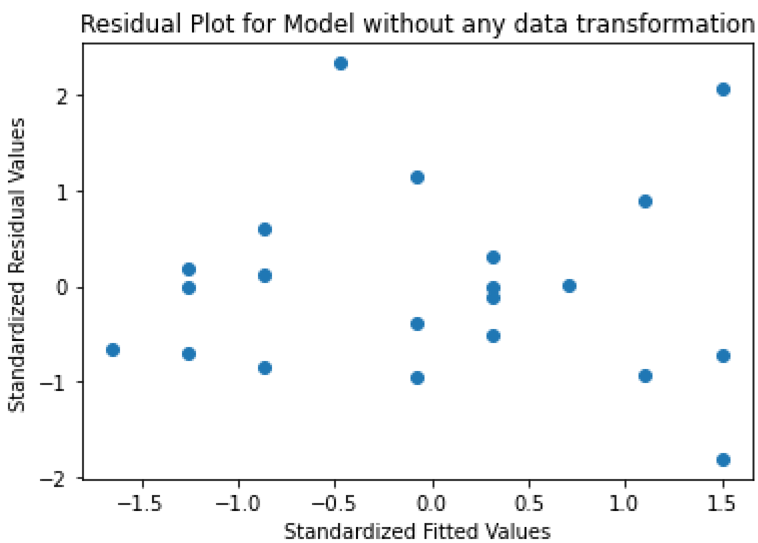
Residual Plot to check Homoscedasticity or Hetroscedasticity

In [29]:

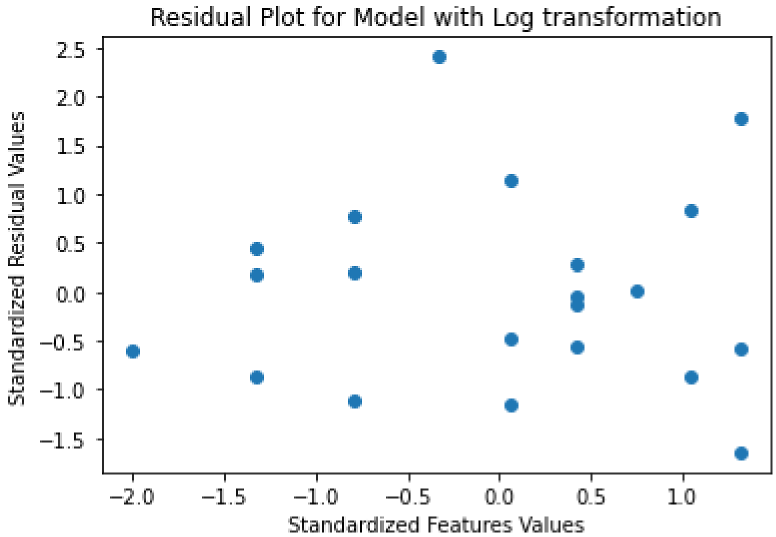
```python
def  get_standardized_values( vals ):
    return (vals-vals.mean())/vals.std()
```

In [30]:

```python
plt.scatter(get_standardized_values(model.fittedvalues), get_standardized_val
plt.title('Residual Plot for Model without any data transformation')
plt.xlabel('Standardized Fitted Values')
plt.ylabel('Standardized Residual Values')
plt.show()
```



In [31]:
```
plt.scatter(get_standardized_values(model2.fittedvalues), get_standardized_va
plt.title('Residual Plot for Model with Log transformation')
plt.xlabel('Standardized Features Values')
plt.ylabel('Standardized Residual Values')
plt.show()
```



As you can notice in the above plots

```
Both The Model have Homoscedasciticity.
The Residual(i.e Residual = Actual Value - Predicted Value) and the Fitt
ed values do not share any Pattern.
Hence, there is no relation between the Residual and the Fitted Value. I
t is Randomly distributed
```

Step 9

Model Validation

Comparing different models with respect to their Root Mean Squared Errors

We will analyze Mean Squared Error (MSE) or Root Mean Squared Error (RMSE) — AKA the average distance (squared to get rid of negative numbers) between the model's predicted target value and the actual target value.

In [32]:
```
from sklearn.metrics import mean_squared_error
```

In [34]:
```
model1_pred_y = np.square(model.predict(df1['Sorting_Time']))
model2_pred_y = pow(model2.predict(df1['Sorting_Time']),3)
model3_pred_y =np.exp(model3.predict(df1['Sorting_Time']))
```

In [36]:
```
model1_rmse =np.sqrt(mean_squared_error(df1['Delivery_Time'], model1_pred_y))
model2_rmse =np.sqrt(mean_squared_error(df1['Delivery_Time'], model2_pred_y))
model3_rmse =np.sqrt(mean_squared_error(df1['Delivery_Time'], model3_pred_y))
print('model=', np.sqrt(model.mse_resid),'\n' 'model1=', model1_rmse,'\n' 'mo
```

```
model= 2.9349037688901394
model1= 312.52867343522814
model2= 2.755584309893575
model3= 2.7458288976145497
```

In [37]:
```
data = {'model': np.sqrt(model.mse_resid), 'model1': model1_rmse, 'model2': m
min(data, key=data.get)
```

Out[37]: 'model2'

As model2 has the minimum RMSE and highest Adjusted R-squared score. Hence, we are going to use model2 to predict our values

Model2 is the model where we did log transformation on both dependent variable as well as on independent variable

Step 10

Predicting values from Model with Log Transformation on the Data

In [39]:

```
predicted = pd.DataFrame()
predicted['Sorting_Time'] =  df1.Sorting_Time
predicted['Delivery_Time'] = df1.Delivery_Time
predicted['Predicted_Delivery_Time'] = pd.DataFrame(np.exp(model2.predict(pre
predicted
```

Out[39]:

	Sorting_Time	Delivery_Time	Predicted_Delivery_Time
0	10	21.00	17.035997
1	4	13.50	10.547128
2	6	19.75	12.808396
3	9	24.00	15.997918
4	10	29.00	17.035997
5	6	15.35	12.808396
6	7	19.00	13.889274
7	3	9.50	9.328887
8	10	17.90	17.035997
9	9	18.75	15.997918
10	8	19.83	14.950443
11	4	10.75	10.547128
12	7	16.68	13.889274
13	3	11.50	9.328887
14	3	12.03	9.328887
15	4	14.88	10.547128
16	6	13.75	12.808396
17	7	18.11	13.889274
18	2	8.00	7.996000
19	7	17.83	13.889274
20	5	21.50	11.698973

Predicitng from Original Model without any data transformation

In [40]:
```
predicted1 = pd.DataFrame()
predicted1['Sorting_Time'] = df1.Sorting_Time
predicted1['Delivery_Time'] = df1.Delivery_Time
predicted1['Predicted_Delivery_Time'] = pd.DataFrame(model.predict(predicted1
predicted1
```

Out[40]:

	Sorting_Time	Delivery_Time	Predicted_Delivery_Time
0	10	21.00	23.072933
1	4	13.50	13.178814
2	6	19.75	16.476853
3	9	24.00	21.423913
4	10	29.00	23.072933
5	6	15.35	16.476853
6	7	19.00	18.125873
7	3	9.50	11.529794
8	10	17.90	23.072933
9	9	18.75	21.423913
10	8	19.83	19.774893
11	4	10.75	13.178814
12	7	16.68	18.125873
13	3	11.50	11.529794
14	3	12.03	11.529794
15	4	14.88	13.178814
16	6	13.75	16.476853
17	7	18.11	18.125873
18	2	8.00	9.880774
19	7	17.83	18.125873
20	5	21.50	14.827833

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]: